IOP Publishing

Nuclear Data
A collective motion view
David Jenkins and John L Wood

Appendix A

Derivation of commutator bracket relations for spin in a body-fixed frame

The commutator brackets for the components of angular momentum, e.g., for three Cartesian axes x, y, z in (3, R), the space in which we live—the 'laboratory' frame, take the familiar form

$$[L_x, L_y] = i\hbar L_z, \tag{A.1}$$

with cyclic permutations of x, y, z. The commutator brackets for the components of nuclear spin in the body-frame of the nucleus—the 'intrinsic' frame, e.g., for three body-fixed axes 1, 2, 3 take the form

$$[L_1, L_2] = -i\hbar L_3 \tag{A.2}$$

with cyclic permutations of 1, 2, 3. Note the minus sign. The reason for this is not obvious. The following provides a concise derivation of equation (A.2) from equation (A.1).

The key to connecting equations (A.1) and (A.2) is the recognition that there is a rotational transformation between the intrinsic frame of the nucleus and the laboratory frame. Rotational transformations about an axis 'n' take the generic form

$$R_n(\phi_n) = \exp\left\{-\frac{i(\boldsymbol{n}\cdot\boldsymbol{L})\phi_n}{\hbar}\right\}, \tag{A.3}$$

where $\boldsymbol{n}\cdot\boldsymbol{L}$ is the component of angular momentum along the n-axis and the rotation angle is ϕ_n. Thus, one can build a language of successive rotational transformations and effect a rotation between any two frames of reference in a given space. To ensure that this language correctly describes rotations in (3, R) it is important to recall that rotations about different axes do not commute, even classically. Thus, for rotations about the x- and y-axes, using $R_x(\phi_x) = \exp\{-iL_x\phi_x/\hbar\}$ and $R_y(\phi_y) = \exp\left\{-iL_y\phi_y/\hbar\right\}$, consider

doi:10.1088/978-0-7503-5643-5ch8

$$R_y(\phi_y)R_x(\phi_x) = \exp\left\{-iL_y\phi_y/\hbar\right\}\exp\left\{-iL_x\phi_x/\hbar\right\} \tag{A.4}$$

and

$$R_x(\phi_x)R_y(\phi_y) = \exp\left\{-iL_x\phi_x/\hbar\right\}\exp\left\{-iL_y\phi_y/\hbar\right\} \tag{A.5}$$

i.e., rotation about the x-axis through an angle ϕ_x followed by rotation about the y-axis through an angle ϕ_y, and these two operations in reversed order. Recall that one reads the actions of such operators on an operand on the right, not shown. The key to manipulating such operations is to consider infinitesimal steps: all continuous transformations are effected from a very large number of infinitesimal steps. We illustrate this in detail in the following.

For infinitesimal angles, $\phi_x = \boldsymbol{\varepsilon_x}$, $\phi_y = \boldsymbol{\varepsilon_y}$ (counterclockwise rotations, right-hand rule),

$$R_x(\boldsymbol{\varepsilon_x}) = \exp\{-iL_x\boldsymbol{\varepsilon_x}/\hbar\} \sim I - iL_x\boldsymbol{\varepsilon_x}/\hbar \tag{A.6}$$

and

$$R_y(\boldsymbol{\varepsilon_y}) = \exp\left\{-iL_y\boldsymbol{\varepsilon_y}/\hbar\right\} \sim I - iL_y\boldsymbol{\varepsilon_y}/\hbar, \tag{A.7}$$

whence

$$R_y(\boldsymbol{\varepsilon_y})R_x(\boldsymbol{\varepsilon_x}) = \{I - iL_y\boldsymbol{\varepsilon_y}/\hbar\}\{I - iL_x\boldsymbol{\varepsilon_x}/\hbar\}, \tag{A.8}$$

$$\therefore R_y(\boldsymbol{\varepsilon_y})R_x(\boldsymbol{\varepsilon_x}) = \{I - iL_y\boldsymbol{\varepsilon_y}/\hbar - iL_x\boldsymbol{\varepsilon_x}/\hbar - L_yL_x\boldsymbol{\varepsilon_y}\boldsymbol{\varepsilon_x}/\hbar^2\}. \tag{A.9}$$

Similarly,

$$R_x(\boldsymbol{\varepsilon_x})R_y(\boldsymbol{\varepsilon_y}) = \{I - iL_x\boldsymbol{\varepsilon_x}/\hbar - iL_y\boldsymbol{\varepsilon_y}/\hbar - L_xL_y\boldsymbol{\varepsilon_x}\boldsymbol{\varepsilon_y}/\hbar^2\}, \tag{A.10}$$

which leads to

$$\begin{aligned} R_x(\boldsymbol{\varepsilon_x})R_y(\boldsymbol{\varepsilon_y}) - R_y(\boldsymbol{\varepsilon_y})R_x(\boldsymbol{\varepsilon_x}) = {} & \cancel{I} - \cancel{iL_x\boldsymbol{\varepsilon_x}/\hbar} - \cancel{iL_y\boldsymbol{\varepsilon_y}/\hbar} - L_xL_y\boldsymbol{\varepsilon_x}\boldsymbol{\varepsilon_y}/\hbar^2 \\ & \cancel{I} + \cancel{iL_y\boldsymbol{\varepsilon_y}/\hbar} + \cancel{iL_x\boldsymbol{\varepsilon_x}/\hbar} + L_yL_x\boldsymbol{\varepsilon_y}\boldsymbol{\varepsilon_x}/\hbar^2, \end{aligned} \tag{A.11}$$

$$\begin{aligned} \therefore R_x(\boldsymbol{\varepsilon_x})R_y(\boldsymbol{\varepsilon_y}) - R_y(\boldsymbol{\varepsilon_y})R_x(\boldsymbol{\varepsilon_x}) &= (L_yL_x - L_xL_y)\boldsymbol{\varepsilon_x}\boldsymbol{\varepsilon_y}/\hbar^2 \\ &= -i\hbar L_z\boldsymbol{\varepsilon_x}\boldsymbol{\varepsilon_y}/\hbar^2 \\ &= R_z(\boldsymbol{\varepsilon_x}\boldsymbol{\varepsilon_y}) - I, \end{aligned} \tag{A.12}$$

i.e., the difference in the order of performance of the infinitesimal rotations is a rotation about the z-axis through the infinitesimal angle $\boldsymbol{\varepsilon_x}\boldsymbol{\varepsilon_y}$. Only the form of this relationship is needed in order to proceed to consideration of rotations in the intrinsic frame. The similar set of operations in the intrinsic frame follows.

Consider, $R_1(\boldsymbol{\varepsilon}_1)R_2(\boldsymbol{\varepsilon}_2) - R_2(\boldsymbol{\varepsilon}_2)R_1(\boldsymbol{\varepsilon}_1)$, and *for the instantaneous orientation* 1-axis with the x-axis, 2-axis with y-axis, 3-axis with z-axis, we can replace $R_2(\boldsymbol{\varepsilon}_2)$ with $R_y(\boldsymbol{\varepsilon}_2)$, for the R_1R_2 (first term), viz.

$$R_1(\boldsymbol{\varepsilon}_1)R_2(\boldsymbol{\varepsilon}_2) = R_1(\boldsymbol{\varepsilon}_1)R_y(\boldsymbol{\varepsilon}_2), \tag{A.13}$$

but then no further replacement is valid because following $R_y(\boldsymbol{\varepsilon}_2)$, the x-axis is no longer collinear with the 1-axis. We designate the newly oriented 1-axis the x'-axis and obtain for the R_1R_2 term

$$R_1(\boldsymbol{\varepsilon}_1)R_2(\boldsymbol{\varepsilon}_2) = R_{x'}(\boldsymbol{\varepsilon}_1)R_y(\boldsymbol{\varepsilon}_2). \tag{A.14}$$

The challenge here is to handle the $R_{x'}(\boldsymbol{\varepsilon}_1)$ term. To do this we use a similarity transformation $R_n^{-1}(\psi_n)R_{x'}(\boldsymbol{\varepsilon}_1)R_n(\psi_n)$. To understand this in practical terms (the concept is equally valid in the space in which we live and in state vector space, i.e. Hilbert space), consider the following task. A book on a tightly packed bookshelf has its title upside-down. Take the book out [translation, T], rotate the title right-side up [rotation, R], return the book to its original position on the shelf [inverse translation, T^{-1}], the result is the execution of $T^{-1}\ R\ T$, a similarity transformation. Above we have defined a similar task, but with a translation replaced by a rotation and its inverse. We need to rotate the x'-axis so that it becomes the x-axis. At the instant after the infinitesimal transformation $R_y(\boldsymbol{\varepsilon}_2)$ was made, the rotation that we seek is $R^{-1} = R_y(\boldsymbol{\varepsilon}_2)$, so $R = R_y(-\boldsymbol{\varepsilon}_2)$. Thus, we obtain

$$R_{x'}(\boldsymbol{\varepsilon}_1) = R_y(\boldsymbol{\varepsilon}_2)R_x(\boldsymbol{\varepsilon}_1)R_y(-\boldsymbol{\varepsilon}_2). \tag{A.15}$$

Whence, for the R_1R_2 term

$$\begin{aligned} R_1(\boldsymbol{\varepsilon}_1)R_2(\boldsymbol{\varepsilon}_2) &= R_{x'}(\boldsymbol{\varepsilon}_1)R_y(\boldsymbol{\varepsilon}_2) \\ &= R_y(\boldsymbol{\varepsilon}_2)R_x(\boldsymbol{\varepsilon}_1)\cancel{R_y(-\boldsymbol{\varepsilon}_2)R_y(\boldsymbol{\varepsilon}_2)} \\ &= R_y(\boldsymbol{\varepsilon}_2)R_x(\boldsymbol{\varepsilon}_1). \end{aligned} \tag{A.16}$$

Similarly, for the R_2R_1 term, noting that the 1-axis = x-axis rotation occurs first for this term

$$R_2(\boldsymbol{\varepsilon}_2)R_1(\boldsymbol{\varepsilon}_1) = R_x(\boldsymbol{\varepsilon}_1)R_y(\boldsymbol{\varepsilon}_2). \tag{A.17}$$

It then follows by inspection that

$$\begin{aligned} R_1(\boldsymbol{\varepsilon}_1)R_2(\boldsymbol{\varepsilon}_2) - R_2(\boldsymbol{\varepsilon}_2)R_1(\boldsymbol{\varepsilon}_1) &= R_y(\boldsymbol{\varepsilon}_2)R_x(\boldsymbol{\varepsilon}_1) - R_x(\boldsymbol{\varepsilon}_1)R_y(\boldsymbol{\varepsilon}_2) \\ &= -\{R_x(\boldsymbol{\varepsilon}_1)R_y(\boldsymbol{\varepsilon}_2) - R_y(\boldsymbol{\varepsilon}_2)R_x(\boldsymbol{\varepsilon}_1)\}, \end{aligned} \tag{A.18}$$

i.e. a minus sign is involved for the body-frame commutator brackets, cf equation (A.18) with equation (A.12) and (A.1); thus, equation (A.2) follows.

IOP Publishing

Nuclear Data

A collective motion view

David Jenkins and John L Wood

Appendix B

A generic two-band mixing formalism

Mixing of states with the same spin-parity in two or more bands occurs widely in nuclei. This has been addressed in a specific situation under the title of Mikhailov theory in section 2.4. Here, a general formalism is presented which is designed to keep track of magnitudes of the matrix elements of the electric quadrupole, $E2$ and electric monopole, $E0$ operators and how they contribute to electromagnetic transition strengths.

For the mixing of two configurations of spin J, from bands a and b,

$$|\, J_1\,\rangle = \alpha_J |\, J^a\,\rangle + \beta_J |\, J^b\,\rangle, \qquad |\, J_2\,\rangle = -\beta_J |\, J^a\,\rangle + \alpha_J |\, J^b\,\rangle, \tag{B.1}$$

can be written, where the mixing amplitudes obey $\alpha_J^2 + \beta_J^2 = 1$.

For $E2$ properties, intrinsic matrix elements,

$$M^a_{20} = \langle 2^a | T(E2) | 0^a \rangle, \tag{B.2}$$

$$M^b_{20} = \langle 2^b | T(E2) | 0^b \rangle, \tag{B.3}$$

are introduced, and then all other matrix elements are defined by the axially symmetric rotor model using Clebsch–Gordan coefficients, viz.

$$M^a_{20} = \sqrt{B(E2;\, 0^+_a \rightarrow 2^+_a)}\,, \tag{B.4}$$

$$M^a_{J,\,J-2} = \frac{\sqrt{3J(J-1)}}{\sqrt{2(2J-1)}} M^a_{20}, \tag{B.5}$$

$$M^a_{J,\,J} = -\frac{\sqrt{J(J+1)(2J+1)}}{\sqrt{(2J-1)(2J+3)}} M^a_{20}, \tag{B.6}$$

with similar expressions for band b. The $E2$ matrix elements for the resulting mixed bands are then obtained directly, viz.

doi:10.1088/978-0-7503-5643-5ch9

$$M_{2_1 0_1} = \alpha_0 \alpha_2 M^a_{20} + \beta_0 \beta_2 M^b_{20}, \tag{B.7}$$

$$M_{4_1 2_1} = (\alpha_2 \alpha_4 M^a_{20} + \beta_2 \beta_4 M^b_{20})(1.604), \tag{B.8}$$

$$M_{2_2 0_2} = \beta_0 \beta_2 M^a_{20} + \alpha_0 \alpha_2 M^b_{20}, \tag{B.9}$$

$$M_{2_2 0_1} = -\alpha_0 \beta_2 M^a_{20} + \alpha_2 \beta_0 M^b_{20}, \tag{B.10}$$

$$M_{2_2 2_1} = \alpha_2 \beta_2 (M^a_{20} - M^b_{20})(-1.195), \tag{B.11}$$

$$M_{0_2 2_1} = -\alpha_2 \beta_0 M^a_{20} + \alpha_0 \beta_2 M^b_{20}. \tag{B.12}$$

These matrix elements follow from the substitution of equations (B.1) and (B.2) into, e.g.

$$M_{2_1 0_1} = \langle 2_1 | T(E2) | 0_1 \rangle, \tag{B.13}$$

etc and adopting zero values for the inter-band matrix elements.

For $E0$ properties, the intrinsic matrix elements are introduced

$$\langle r^2 \rangle_a = \langle 0^a | r^2 | 0^a \rangle, \tag{B.14}$$

$$\langle r^2 \rangle_b = \langle 0^b | r^2 | 0^b \rangle, \tag{B.15}$$

and zero values for the inter-band matrix elements are adopted.

Note the following:

1. The minus signs in (B.10)–(B.12) result in cancellations (destructive interference);

2. table B.1 reveals a natural hierarchy of inter-band transition strengths which match the cancellations.

The $E0$ properties for the resulting mixed bands are then obtained directly, viz.

$$\rho_J(E0) = \alpha_J \beta_J (\langle r^2 \rangle_a - \langle r^2 \rangle_b) = \alpha_J \beta_J \Delta \langle r^2 \rangle, \tag{B.16}$$

$$\delta \langle r^2 \rangle_{2_1 0_1} = \left(\beta_2^2 - \beta_0^2 \right) \Delta r^2. \tag{B.17}$$

The strength of $E0$ transitions is expressed as

$$\rho_J^2 10^3 = \alpha_J^2 \beta_J^2 (\Delta \langle r^2 \rangle)^2 10^3 Z^2 / R^4, \tag{B.18}$$

where $R = 1.2A^{1/3}$ fm, and the factor 10^3 is by convention. The quantity $\delta \langle r^2 \rangle_{2_1 0_1}$ is sometimes called the isomer shift (between the ground and the first excited state).

Table B.1. Details of the mixing for the $K = 0_1$ (ground-state band) and $K = 0_2$ (first excited $K = 0$) band in ^{152}Sm. The parameters are $M^a_{20} = 1.650$ e.b, $M^b_{20} = 2.300$ e.b and the amplitudes $\alpha_0 = 0.8458$, etc given in the box.

Two-band mixing calculation for E2 properties of ^{152}Sm

	$Q(2^+_1)$	
expt.	-1.683^{18}	b
calc.	-1.690	b

Rotor matrix elements

$$M_{J,J-2} = \sqrt{\frac{3J(J-1)}{2(2J-1)}}M_{20}$$

$$M_{J,J} = -\sqrt{\frac{J(J+1)(2J+1)}{(2J-1)(2J+3)}}M_{20}$$

$$M_{20} = \sqrt{B(E2;\ 0^+_1 \to 2^+_1)}$$

$$M^a_{20} = 1.650 \quad e \cdot b$$

$$M^b_{20} = 2.300 \quad e \cdot b$$

e.g.,

$$M_{2_1 0_1} = \alpha_0\alpha_2 M^a_{20} + \beta_0\beta_2 M^b_{20} = 1.848\ e \cdot b$$

$$M_{4_1 2_1} = (\alpha_2\alpha_4 M^a_{20} + \beta_2\beta_4 M^b_{20})(1.604) = 3.024$$

$$M_{2_2 0_2} = \beta_0\beta_2 M^a_{20} + \alpha_0\alpha_2 M^b_{20} = 2.097$$

$$M_{2_2 0_1} = -\alpha_0\beta_2 M^a_{20} + \alpha_2\beta_0 M^b_{20} = 0.197$$

$$M_{2_2 2_1} = \alpha_2\beta_2(M^b_{20} - M^a_{20})(-1.195) = -0.366$$

$$M_{0_2 2_1} = -\alpha_2\beta_0 M^a_{20} + \alpha_0\beta_2 M^b_{20} = 0.404$$

$$B(E2) = \frac{M^2}{2J_i + 1} \times 207.5\ W.u.$$

	calc.	exp.
$2_1 \to 0_1$	141.7	145.0^{16}
$4_1 \to 2_1$	210.8	209.5^{22}
$6_1 \to 4_1$	245.7	240^{4}
$8_1 \to 6_1$	271.6	293^{4}
$2_2 \to 0_2$	182.5	170^{12}
$4_2 \to 2_2$	249.4	250^{40}
$0_2 \to 2_1$	33.9	33.3^{12}
$2_2 \to 4_1$	24.5	18.0^{12}
$4_2 \to 6_1$	23.1	17^{3}
$2_2 \to 2_1$	5.56	5.7^{4}
$4_2 \to 4_1$	5.52	5.0^{+10}_{-7}
$2_2 \to 0_1$	1.61	0.94^{6}
$4_2 \to 2_1$	1.32	0.74^{12}

J	0	2	4	6	8
α_J	0.8458	0.8167	0.7656	0.7071	0.6536
β_J	0.5334	0.5770	0.6433	0.7071	0.7569

B.1 Exercises

B-1. Derive equations for $M_{4_1 2_2}$, $M_{4_2 2_2}$, $M_{4_2 2_1}$, $M_{4_2 4_1}$, $M_{4_1 4_1}$, $M_{4_2 4_2}$.

B-2. Derive equations for $M_{6_1 4_1}$, $M_{6_1 6_1}$.

B-3. Derive equation (B.18).

B-3. Using data for ^{152}Sm in ENSDF, what is the Grodzins product? (Note that the product is for states which are mixed configurations with different deformations.)

B-4. Complete all of the computational steps for the calculated (calc.) values in table B.1, using the input values given for mixing amplitudes and model parameters.

B-5. What modifications would be needed to the above formalism to handle the mixing of two $K = 2$ bands?

B-6. What modifications would be needed to the above formalism to handle the mixing of a $K = 0$ band with a $K = 2$ band?

IOP Publishing

Nuclear Data

A collective motion view

David Jenkins and John L Wood

Appendix C

*E*2 matrix elements for selected even–even nuclei and selected transitions

See tables C.1, C.2, C.3, C.4, C.5 and C.6 for selected even–even nuclei and selected transitions.

Table C.1. Compilation of $E2$ matrix elements in units of e.b for selected transitions in selected even–even zinc ($Z = 30$) and germanium ($Z = 32$) nuclei. The values for $0_1 \rightarrow 2_1$, shown in blue, are derived from $B(E2; 0_1 \rightarrow 2_1)$ values in $e^2 \cdot b^2$ given in [1]. Other data are taken from: ^{66}Zn [2], ^{68}Zn [3], ^{70}Ge [4], ^{72}Ge [5], ^{74}Ge [6], and ^{76}Ge [7].

	^{66}Zn	^{68}Zn	^{70}Ge	^{72}Ge	^{74}Ge	^{76}Ge
$0_1 \rightarrow 2_1$	$+0.370^{4}$	$+0.346^{3}$	$+0.423^{4}$	$+0.457^{3}$	$+0.553^{14}$	$+0.523^{3}$
$0_1 \rightarrow 2_2$	$+0.0048^{7}_{5}$	$+0.069^{3}$	-0.0434^{13}	$+0.030^{1}$	$+0.058^{10}$	$+0.089^{3}$
$2_1 \rightarrow 2_2$	$+0.57^{10}$	-0.39^{4}	$+0.42^{7}$	$+0.65^{1}_{2}$	$+0.50^{4}$	$+0.535^{3}_{7}$
$2_1 \rightarrow 2_1$	$+0.32^{10}$	$+0.12^{4}$	$+0.05^{4}$	-0.16^{7}_{2}	-0.25^{3}	-0.24^{2}
$2_2 \rightarrow 2_2$		$+0.12^{8}$	-0.09^{5}	$+0.179^{3}_{6}$	$+0.34^{8}$	$+0.26^{2}_{5}$
$2_1 \rightarrow 4_1$	$+0.500^{10}$	$+0.441^{7}$	$+0.54^{10}$	$+0.90^{2}$	$+0.850^{25}$	$+0.795^{5}$
$4_1 \rightarrow 6_1$	$+1.39^{6}$			$+1.11^{4}_{5}$		$+1.11^{3}_{2}$
$4_1 \rightarrow 4_1$			$+0.29^{7}$	-0.14^{9}_{4}		-0.26^{1}_{7}
$6_1 \rightarrow 6_1$				-0.20^{8}_{25}		-0.23^{9}_{4}
$2_1 \rightarrow 4_2$				$+0.035^{6}$		-0.22^{5}_{3}
$2_2 \rightarrow 4_1$		$+0.31^{5}$	-0.52^{12}	-0.06^{3}_{4}	$+0.05^{25}$	$+0.09^{2}$
$2_2 \rightarrow 4_2$				$+0.58^{5}_{1}$		$+0.472^{6}$
$4_1 \rightarrow 4_2$				$+0.43^{10}$		$+0.61^{1}$

Table C.2. Compilation of $E2$ matrix elements in units of e.b for selected transitions in selected even–even selenium ($Z = 34$) nuclei. The values for $0_1 \rightarrow 2_1$, shown in blue, are derived from $B(E2; 0_1 \rightarrow 2_1)$ values in $e^2 \cdot b^2$ given in [1]. Other data are taken from: ^{76}Se [8], ^{78}Se [9], and ^{80,82} Se [10].

	^{76}Se	^{78}Se	^{80}Se	^{82}Se
$0_1 \rightarrow 2_1$	$+0.657^{11}_{5}$	$+0.586^{10}$	$+0.502^{8}$	$+0.428^{12}$
$0_1 \rightarrow 2_2$	$+0.110^{2}$	$+0.08^{1}$	$+0.106^{6}$	$+0.120^{6}$
$2_1 \rightarrow 2_2$	$+0.640^{11}$	$+0.45^{4}$	$+0.38^{2}$	$+0.19^{2}$
$2_1 \rightarrow 2_1$	-0.46^{5}	-0.27^{9}	-0.26^{4}_{3}	-0.30^{4}_{3}
$2_2 \rightarrow 2_2$	$+0.25^{6}$	$+0.23^{12}$	$+0.53^{3}$	$+0.45^{4}_{5}$
$2_1 \rightarrow 4_1$	$+1.108^{12}_{11}$	$+0.81^{6}$	$+0.82^{4}$	$+0.63^{3}$
$4_1 \rightarrow 6_1$	$+1.39^{6}$		$+1.14^{29}_{15}$	
$4_1 \rightarrow 4_1$	-0.39^{6}_{5}	-0.90^{20}	-0.85^{11}_{6}	-0.76^{7}_{8}
$6_1 \rightarrow 6_1$				
$2_1 \rightarrow 4_2$	$+0.039^{35}_{7}$		$\pm 0.01^{13}_{6}$	$+0.09^{1}$
$2_2 \rightarrow 4_1$	$+0.05^{4}_{3}$	$\pm 0.09^{4}_{5}$	$+0.08^{4}_{13}$	
$2_2 \rightarrow 4_2$	$+0.77^{4}$		$+0.67^{8}_{18}$	$+0.71^{3}_{9}$
$4_1 \rightarrow 4_2$	$+0.73^{5}_{4}$			$+0.28^{5}_{4}$

Table C.3. Compilation of $E2$ matrix elements in units of e.b for selected transitions in selected even–even Krypton ($Z = 36$) nuclei. The values for $0_1 \rightarrow 2_1$, shown in blue, are derived from $B(E2; 0_1 \rightarrow 2_1)$ values in $e^2 \cdot b^2$ given in [1]. Other data are taken from: ^{74,76}Kr [11], ^{78}Kr [12], ^{80}Kr [13], ^{82}Kr [14], and ^{84}Kr [15].

	^{74}Kr	^{76}Kr	^{78}Kr	^{80}Kr	^{82}Kr	^{84}Kr
$0_1 \rightarrow 2_1$	$+0.792^{20}$	$+0.871^{15}$	$+0.796^{10}$	$+0.617^{10}$	$+0.474^{7}$	$+0.356^{16}$
$0_1 \rightarrow 2_2$	-0.199^{18}_{11}	$+0.183\ ^{8}_{6}$	$+0.157^{3}_{4}$	$+0.078^{15}$	-0.035^{11}_{8}	0.17^{2}
$2_1 \rightarrow 2_2$	$+0.49^{4}$	-0.09^{4}	$+0.26^{6}_{5}$	$+0.73^{14}$	-0.28^{91}_{6}	0.35^{14}
$2_1 \rightarrow 2_1$	-0.7^{3}	-0.9^{3}	-0.80^{4}	-0.43^{7}_{3}		
$2_2 \rightarrow 2_2$	$+0.3^{3}_{2}$	-1.0^{5}	$+0.58^{4}_{8}$	$+0.4^{17}$		
$2_1 \rightarrow 4_1$	$+1.60^{3}$	$+1.49^{1}$	$+1.27^{3}_{3}$		$+0.78^{13}_{16}$	$+0.69^{9}$
$4_1 \rightarrow 6_1$	$+1.98^{10}_{9}$	$+1.90^{11}_{3}$	$+1.61^{6}_{8}$	$+1.68^{12}$	$+0.74^{17}_{23}$	
$4_1 \rightarrow 4_1$	-1.0^{6}_{2}	-2.3^{4}	-0.73^{15}_{14}	-0.77^{22}		
$6_1 \rightarrow 6_1$	-1.8^{7}_{5}	-2.9^{4}				
$2_1 \rightarrow 4_2$		$+0.09^{1}_{19}$	$+0.073^{2}_{5}$		$+0.15^{2}$	
$2_2 \rightarrow 4_1$	$+0.47^{3}_{2}$	-0.62^{4}_{5}	$+0.32^{5}_{4}$			
$2_2 \rightarrow 4_2$	$+0.55^{16}_{8}$		$+0.91^{6}_{4}$		$+0.41^{8}$	
$4_1 \rightarrow 4_2$		$+0.43^{3}$	-0.60^{2}_{3}		$+0.87^{11}$	

Table C.4. Compilation of $E2$ matrix elements in units of e.b for selected transitions in selected even–even ruthenium ($Z = 44$), palladium ($Z = 46$) and cadmium ($Z = 48$) nuclei. The values for $0_1 \rightarrow 2_1$, shown in blue, are derived from $B(E2; 0_1 \rightarrow 2_1)$ values in $e^2 \cdot b^2$ given in [1]. Other data are taken from: ^{104}Ru [16], ^{106,108}Pd [17], ^{110}Pd [18], and ^{114}Cd [19].

	^{104}Ru	^{106}Pd	^{108}Pd	^{110}Pd	^{114}Cd
$0_1 \rightarrow 2_1$	$+0.909^{9}$	$+0.812^{20}$	$+0.874^{11}$	$+0.930^{12}$	$+0.732^{17}$
$0_1 \rightarrow 2_2$	-0.156^{2}	-0.114^{6}	-0.98^{5}	-0.096^{2}_{3}	$+0.091^{3}$
$2_1 \rightarrow 2_2$	-0.75^{4}	-0.76^{4}	-0.88^{4}	-0.863^{11}_{16}	$+0.684^{21}$
$2_1 \rightarrow 2_1$	-0.71^{11}	-0.72^{6}_{7}	-0.81^{4}_{9}	-0.87^{17}_{15}	-0.36^{1}_{3}
$2_2 \rightarrow 2_2$		$+0.52^{6}_{5}$	$+0.73^{9}_{7}$	$+0.70^{9}_{32}$	$+0.92^{4}_{5}$
$2_1 \rightarrow 4_1$	$+1.43^{4}$	$+1.38^{7}$	$+1.42^{7}$	$+1.579^{4}_{37}$	$+1.35^{4}$
$4_1 \rightarrow 6_1$	$+2.04^{8}$	$+1.86^{10}_{14}$	$+2.06^{11}$	$+2.08^{8}_{3}$	$+2.3^{3}$
$4_1 \rightarrow 4_1$	-0.79^{15}	-1.02^{7}_{11}	-0.78^{11}_{10}	-1.6^{4}_{2}	-0.95^{4}_{11}
$6_1 \rightarrow 6_1$	-0.7^{3}_{2}	-1.41^{23}_{13}	-0.76^{18}	-1.4^{2}_{4}	-3.5^{9}
$2_1 \rightarrow 4_2$	-0.107^{8}	-0.014^{5}_{4}		-0.066^{15}_{12}	$+0.11^{1}$
$2_2 \rightarrow 4_1$		$+0.14^{30}_{3}$	$+0.18^{9}_{13}$	$+0.51^{11}_{32}$	-0.35^{7}_{2}
$2_2 \rightarrow 4_2$	$+1.12^{5}$	-0.30^{18}_{5}	$+1.23^{7}_{6}$	$+0.97^{4}_{3}$	$+0.97^{17}_{3}$
$4_1 \rightarrow 4_2$	-0.88^{5}	$+0.79^{4}$	-0.91^{7}_{8}	-0.94^{5}_{4}	$+0.61^{8}_{4}$

Table C.5. Compilation of $E2$ matrix elements in units of e.b for selected transitions in selected even–even tellurium ($Z = 52$), xenon ($Z = 54$) and neodymium ($Z = 60$) nuclei. The values for $0_1 \rightarrow 2_1$, shown in blue, are derived from $B(E2; 0_1 \rightarrow 2_1)$ values in $e^2 \cdot b^2$ given in [1]. Other data are taken from: ^{122}Te [18], ^{126,128}Xe [20], and ^{148}Nd [21].

	^{122}Te	^{126}Xe	^{128}Xe	^{148}Nd
$0_1 \rightarrow 2_1$	$+0.806^{19}$	$+0.91^{3}$	$+0.889^{21}$	$+1.157^{13}$
$0_1 \rightarrow 2_2$	$+0.110^{2}$	$+0.119^{9}$	0.105^{8}	$+0.123^{5}_{4}$
$2_1 \rightarrow 2_2$	$+0.640^{11}$	$+1.00^{4}$	$+0.92^{4}$	-0.65^{2}
$2_1 \rightarrow 2_1$	-0.46^{5}	-1.0^{2}	-0.58^{12}_{15}	-1.85^{4}_{5}
$2_2 \rightarrow 2_2$	$+0.25^{6}$	$+0.14^{9}$	$+0.01^{9}_{10}$	-1.15^{18}_{12}
$2_1 \rightarrow 4_1$	$+1.08^{12}_{11}$	$+1.48^{4}$	$+1.38^{4}$	$+2.00^{4}$
$4_1 \rightarrow 6_1$	$+1.39^{6}$	$+2.07^{9}$	$+1.95^{12}$	$+2.62^{7}$
$4_1 \rightarrow 4_1$	-0.39^{6}_{5}	-0.78^{16}	-1.38^{13}	-1.40^{17}
$6_1 \rightarrow 6_1$				-1.72^{19}_{20}
$2_1 \rightarrow 4_2$	$+0.39^{35}_{7}$			$+0.072^{5}$
$2_2 \rightarrow 4_1$	$+0.05^{4}_{3}$			$+1.12^{3}_{5}$
$2_2 \rightarrow 4_2$	$+0.77^{4}$	$+0.97^{6}$		$+2.06^{8}_{7}$
$4_1 \rightarrow 4_2$	$+0.73^{5}_{4}$			-0.338^{20}_{15}

Table C.6. Compilation of $E2$ matrix elements in units of e.b for selected transitions in selected even–even osmium ($Z = 76$) and platinum ($Z = 78$) nuclei. The values for $0_1 \rightarrow 2_1$, shown in blue, are derived from $B(E2; 0_1 \rightarrow 2_1)$ values in $e^2 \cdot b^2$ given in [1]. Other data are taken from: ^{186,188,190,192}Os [22], ^{194}Pt [22], and ^{196}Pt [23].

	^{186}Os	^{188}Os	^{190}Os	^{192}Os	^{194}Pt	^{196}Pt
$0_1 \rightarrow 2_1$	$+1.750^{21}$	$+1.581^{11}$	$+1.53^{3}$	$+1.43^{4}$	$+1.277^{27}$	$+1.184^{29}$
$0_1 \rightarrow 2_2$	$+0.545^{13}_{7}$	$+0.483^{2}_{9}$	$+0.444^{9}_{7}$	$+0.430^{8}_{4}$	$+0.0888^{12}$	0.000
$2_1 \rightarrow 2_2$	$+0.897^{67}_{14}$	$+0.865^{11}$	$+1.065^{20}_{37}$	$+1.230^{34}_{16}$	$+1.517^{11}_{18}$	$+1.36^{1}$
$2_1 \rightarrow 2_1$	-1.75^{22}_{13}	-1.73^{19}_{5}	-1.25^{22}_{13}	-1.21^{6}_{17}	$+0.54^{8}_{6}$	$+0.82^{10}$
$2_2 \rightarrow 2_2$	$+2.12^{6}_{22}$	$+2.10^{9}_{6}$	$+1.53^{6}_{31}$	$+0.99^{6}_{9}$	-0.40^{12}_{5}	-0.52^{20}
$2_1 \rightarrow 4_1$	$+2.76^{6}_{7}$	$+2.642^{25}_{20}$	$+2.37^{3}$	$+2.12^{3}$	$+1.935^{21}_{13}$	$+1.91^{3}$
$4_1 \rightarrow 6_1$	$+3.89^{8}_{5}$	$+3.31^{4}$	$+2.97^{6}_{4}$	$+2.93^{7}_{4}$	$+2.90^{10}_{4}$	$+2.42^{7}$
$4_1 \rightarrow 4_1$	-2.02^{39}_{18}	-2.00^{9}_{20}	-1.28^{27}_{19}	-0.73^{26}_{6}	$+1.00^{12}_{14}$	$+1.35^{16}$
$6_1 \rightarrow 6_1$	-1.67^{29}_{21}	-1.60^{18}_{33}	-0.91^{24}_{15}	-1.16^{11}_{26}	$+0.28^{12}_{27}$	-0.3^{4}
$2_1 \rightarrow 4_2$	$+0.419^{27}_{15}$	$+0.283^{8}_{7}$	$+0.203^{7}$	$+0.130^{5}_{8}$	$+0.220^{9}$	$+0.11^{7}$
$2_2 \rightarrow 4_1$		$+0.38^{5}_{6}$	$+0.19^{12}_{9}$	$+0.35^{16}_{4}$	$+0.25^{4}_{6}$	
$2_2 \rightarrow 4_2$	$+1.97^{9}_{7}$	$+1.78^{7}_{5}$	$+1.87^{4}$	$+1.637^{24}_{33}$	$+1.78^{5}_{3}$	$+1.28^{6}$
$4_1 \rightarrow 4_2$	$+1.22^{6}$	$+1.10^{3}$	$+1.44^{4}$	$+1.35^{8}_{4}$	$+1.51^{6}_{5}$	$+0.87^{7}$

C.1 Exercises

C-1. Test the triangular relationships depicted in figures 2.5(a) and (b) for deduced Q_0 values. Example: for ^{188}Os, $(0.865^2 + 1.73^2) \times 56\pi/25 = 5.13$ e.b cf $(1.581^2 + 0.483^2) \times 16\pi/5 = 5.24$ e.b. (Note that the signs on the matrix elements are ignored because they are being combined in quadrature.)

C-2. Check the agreement of the Q_0 values deduced from the two triangle relationships within the experimental uncertainties given, e.g. $0.865^{11} = 0.865 \pm 0.011$.

References

[1] Pritychenko B, Birch M, Singh B and Horoi M 2016 tables of $E2$ transition probabilities from the first 2^+ states in even-even nuclei *At. Data Nucl. Data Tables* **107** 1–139

[2] Rocchini M *et al* 2021 Onset of triaxial deformation in ^{66}Zn and properties of its first excited 0^+ state studied by means of Coulomb excitation *Phys. Rev.* C **103** 014311

[3] Koizumi M *et al* 2004 Multiple Coulomb excitation experiment of ^{68}Zn *Nucl. Phys.* A **730** 46

[4] Sugawara M 2003 Multiple Coulomb excitation of a ^{70}Ge beam and the interpretation of the 0_2^+ state as a deformed intruder *Eur. Phys. J.* A **16** 409–14

[5] Ayangeakaa A D *et al* 2016 Shape coexistence and the role of axial symmetry in ^{72}Ge *Phys. Lett.* B **754** 254–9
[6] Toh Y *et al* 2000 Coulomb excitation of ^{74}Ge beam *Eur. Phys. J.* A **9** 353–6
[7] Ayangeakaa A D *et al* 2023 Triaxiality and the nature of low-energy excitations in ^{76}Ge *Phys. Rev.* C **107** 044314
[8] Henderson J *et al* 2019 Triaxiality in selenium-76 *Phys. Rev.* C **99** 054313
[9] Hayakawa T *et al* 2003 Projectile Coulomb excitation of ^{78}Se *Phys. Rev.* C **67** 064310
[10] Kavka A E *et al* 1995 Coulomb excitation of ^{76,80,82}Se *Nucl. Phys.* A **593** 177–211
[11] Clément E *et al* 2007 Shape coexistence in neutron-deficient krypton isotopes *Phys. Rev.* C **75** 054313
[12] Becker F *et al* 2006 Coulomb excitation of ^{78}Kr *Nucl. Phys.* A **770** 107–25
[13] Gillespie S A *et al* 2021 Coulomb excitation of ^{80,82} Kr and a change in structure approaching $N = Z = 40$ *Phys. Rev.* C **104** 044313
[14] Brüssermann S, Lieb K P, Sona P, Emling H, Grosse E and Stachel J 1985 Multiple Coulomb excitation of the transitional nucleus ^{82}Kr *Phys. Rev.* C **32** 1521
[15] Osa A *et al* 2002 First measurement of the quadrupole moment in the 2_1^+ state of ^{84}Kr *Phys. Lett.* B **546** 48–54
[16] Srebrny J *et al* 2006 Experimental and theoretical investigations of quadrupole collective degrees of freedom in ^{104}Ru *Nucl. Phys.* A **766** 25–51
[17] Svensson L E *et al* 1995 Multiphonon vibrational states in ^{106,108}Pd *Nucl. Phys.* A **584** 547–72
[18] Svensson L E 1989 Coulomb excitation of vibrational nuclei *PhD thesis* University of Uppsala
[19] Fahlander C *et al* 1988 Quadrupole collective properties of ^{114}Cd *Nucl. Phys.* A **485** 327–59
[20] Kisyov S *et al* 2022 Structure of ^{126,128}Xe studied in coulomb excitation measurements *Phys. Rev.* C **106** 034311
[21] Ibbotson R W *et al* 1997 Quadrupole and octupole collectivity in ^{148}Nd *Nucl. Phys.* A **619** 213–40
[22] Wu C Y *et al* 1996 Quadrupole collectivity and shapes of Os-Pt nuclei *Nucl. Phys.* A **607** 178–234
[23] Lim C S, Spear R H, Fewell M P and Gyapong G J 1992 Measurements of static electric quadrupole moments of the 2_1^+, 2_2^+, 4_1^+ and 6_1^+ states of ^{196}Pt *Nucl. Phys.* A **548** 308