Part I

Basics of photonics and lasers

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An Introduction to Photonics and Laser Physics with Applications

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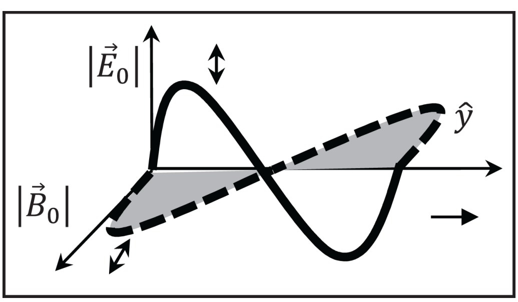
Chapter 1

The photon and photonics

The full form of the acronym **'laser'** *is light amplification by stimulated emission of radiation.* The laser was invented in 1960 as a so-called 'tool looking for an application'. Within six decades, lasers have found applications in all walks of life, including industries based on them. Micromachining, waveguide fabrication, welding, cutting, nondestructive testing, and pulsed-laser deposition of thin films are some of their applications in materials science. In the field of electrical engineering, fiber optics has revolutionized the field of optical communication. In aerospace engineering applications involving jet and scramjet engines, studies of the mixing of fuel sprays require laser-induced fluorescence techniques. The defense, medical, and cosmetic fields, as well as scientific research, are interdisciplinary areas that make extensive use of lasers. Like the flow of electrons that completes *electronic circuits* in *electronics*, photons are related to *photonic circuits* in the field of *photonics.* Maxwell's equations suggest that light is an electromagnetic (EM) wave. Therefore, this chapter connects optics with EM theory. The figure shows an electric field (E, in the y direction) and a magnetic field (B, in the x direction), which are mutually perpendicular to each other. The EM wave is propagating in the y direction; the details are given in this chapter.



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Learning objectives
After reading this chapter, the learner will be able to:
Identify the various branches of photonics;
Relate Maxwell's equation to optics;
Define radiation pressure and the angular momentum of light;
Explain radiation using accelerating charge;
Describe the refractive index and dispersion of a material;
Differentiate between electronic and photonic circuits.

1.1 The photon

Max Planck suggested the idea of energy packets known as 'quanta' in 1900. While Einstein called these packets 'energy particles' in 1905, it took until 1923 for this idea to be reinforced by the discovery of the Compton effect. These particles were named *photons* by Gilbert Lewis in 1926; this term denoted 'carriers of radiant energy'. Just as the electron is associated with electricity, light of wavelength λ or frequency ν consists of photons of energy $h\nu$. Here, h is Planck's constant.

The photon:

(i) has no charge,
(ii) is considered to have zero rest mass but
(iii) carries momentum ($p = h/\lambda$),
(iv) has a constant velocity (c)$\sim 10^8$ m s^{-1} in vacuum,
(v) carries a spin of 1 and thus follows Bose–Einstein statistics,
(vi) is immune to EM noise (as it has no charge), and
(vii) does not undergo photon–photon interactions in linear optics.

The property of photons being *immune to EM noise* makes a light beam a special tool as compared to electrical circuits that are prone to picking up stray EM signals. Similarly, photon–photon interactions cannot take place under normal light levels. In the same way that *electronics* deals with the flow of electrons in electrical circuits, photon-related studies address *photonic circuit*s that fall within the domain of *photonics*. This means that the two light rays can cross without interacting with each other. The photon–electron interaction, which falls within the domain of *light–matter interactions*, is an important area of interdisciplinary research. Chapter 2 introduces this aspect, along with spectroscopy.

1.2 Branches of photonics

This term photonics is used to describe the control of photons and the photon nature of light. This is one of the modern area of optics that deals with the technologies used to generate and harness light. 'Laser applications' is an interdisciplinary field that uses the principles of conventional optics, electromagnetism, spectroscopy, and quantum optics. Therefore, several subcategories (shown in figure 1.1) are encompassed by the term *photonics*. In addition, electrodynamics, which is a self-sufficient theory, is used in various areas of optical technologies.

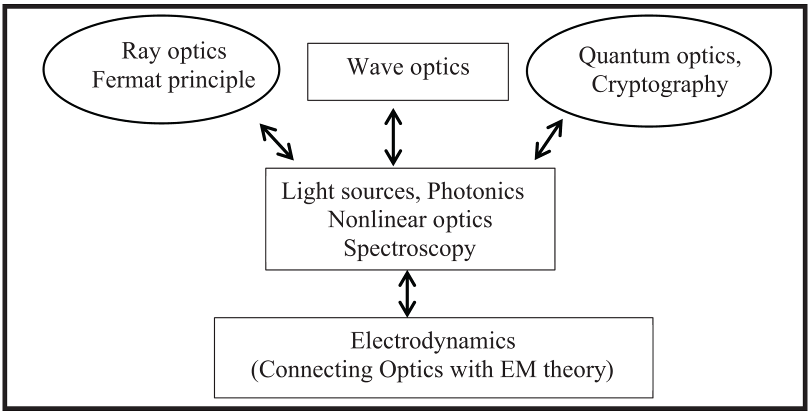

Figure 1.1. Interdisciplinary nature of photonics, illustrating some application areas.

1.2.1 Conventional optics

According to Fermat's principle, light rays travel along the path that can be traversed in the least time. When light propagates through large objects in which the wavelength of light is smaller than the dimensions of the object, its behavior can be explained by drawing light rays. We experience this in the form of reflection or refraction of light rays from a surface. Under such circumstances, the light rays follow the rules of geometrical optics. This method of understanding light falls under *ray optics*.

1.2.2 Electromagnetism and wave optics

Light is an electromagnetic wave and, as such, its electric and magnetic fields are represented in their vector forms (see section 1.3). However, in *wave optics*, the scalar representation of EM fields is sufficient. In Young's double-slit interference experiment, for example, Huygens' principle of secondary wavelets is used to explain wave interference. This can be achieved without taking the components of the EM field into account.

1.2.3 Quantum optics

Certain phenomena cannot be explained using EM theory. Optical phenomena that can only be explained by treating light as a stream of photons, such as coherent states and photon entanglement (♠ see chapter 15) fall within this category. The Mach–Zehnder interferometer (♠ see chapter 9) is used to experimentally test the basic theoretical proposals in quantum optics, such as the entanglement of photons and Bell's inequalities. Quantum cryptography also is the subject of *quantum optics*.

1.2.4 Light–matter interaction or quantum electronics

This topic addresses the interactions of light with matter. The phenomena of absorption and spontaneous and stimulated emission (i.e. the spectroscopy of atoms and molecules) are studied here. Nonlinear optics, bulk spectroscopy, and the spectroscopy of low-dimensional materials in the form of monolayers, quantum wires, or quantum dots fall into this category.

1.2.5 Optoelectronics

This is an area in which both electrical current and light are required for the operation of a device. The presence of electronics in the device controls the optical character of the device. Devices that fall into this category are electronic in nature but evolve light. Examples include the light-emitting diodes, solar cells, and display devices in modern equipment, including smartphones. Edison's bulb may fall into this category as well. Specialized centers are dedicated to this topic worldwide. This research area has immense applications in the photonics industry.

1.2.6 Electro-optics

An optical switch that operates the automated door of an elevator or an office falls into this category. In this field, the electronics in an item of equipment is used to control the device in combination with an optical effect. Electro-optic shutters fall into this category. In addition, the change in the optical response (absorption/transmission) of a material due to AC or DC electric/magnetic fields falls within this area. Examples of devices based on electro-optics include those based on the Kerr effect or Faraday rotation (♠ see chapter 3).

1.2.7 Light-wave technology

The whole of modern-day communication is based on data exchange with a large frequency bandwidth. This includes the communication used by television, the internet, and the telephone. Devices and systems that are used in optical communication and optical signal processing fall into this category. Fiber-optic communication is the key example in this field.

1.3 Maxwell's equations and their connection to optics

Optics is connected to EM theory through Maxwell's equations (MEs). The basic laws of reflection and refraction can be derived from the electric ($\vec{E}$) and magnetic ($\vec{B}$) field vectors via solution of the plane-wave equation. The set of Maxwell's four equations for EM fields in vacuum from classical electrodynamics are written as follows:

$$\text{(i). } \vec{\nabla}.\,\vec{E} = \frac{\rho}{\varepsilon_0} \quad \text{(Gauss's law)} \qquad \text{(ii). } \vec{\nabla}.\,\vec{B} = 0 \quad \text{(No name)}$$
$$\text{(iii). } \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \text{(Faraday's Law)} \qquad \text{(iv). } \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} \quad \text{(Ampère's law fixed by Maxwell)} \tag{1.1}$$

Here, ϵ_0 and μ_0, are known as the permittivity (in farad m^{-1}) and permeability (in Henry m^{-1}) of free space, respectively; ρ is the charge density in Cm^{-3}, and $\vec{J}$ is the current density in A m^{-2} in the region. It should be noted that these equations are of the *first order* in time and space. The equations do not have symmetry in either the $\vec{E}$ or $\vec{B}$ fields. By working on these equations a little, we can obtain symmetric equations for either of the fields, as follows. For instance, on taking the curl of the Faraday's law (equation (iii)),

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla} \times (-)\frac{\partial \vec{B}}{\partial t} \equiv -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B}).$$

Using the vector identity $\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla}(\vec{\nabla}.\ \vec{E}) - \vec{\nabla}^2 E$, the above equation can be rewritten as

$$\vec{\nabla}(\vec{\nabla}.\ \vec{E}) - \vec{\nabla}^2 E = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B}).$$

For charge-free ($\rho = 0$) and current-free ($\vec{J} = 0$)regions, we can use equations (i) and (ii) to write the following wave equation:

$$\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \nabla^2 \vec{E} = 0. \tag{1.2}$$

Here, c is the speed of light $\left(c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}\right)$ in m s^{-1}. We have obtained equation (1.2) for one variable ($\vec{E}$) at the cost of using a *second-order* differential equation. Similarly, one can write the wave equation for the $\vec{B}$ field as well.

We can assume a general solution of equation (1.2) for $\vec{E}(\vec{r},t)$ in units of V m^{-1} as

$$\vec{E}(\vec{r},\ t) = |\ E_0\ |\ \hat{n} \cos(\vec{k}.\ \vec{r} - \omega t + \phi). \tag{1.3}$$

Here, E_0 is the amplitude of the wave. For a simple case, we take a wave propagating in the z direction (i.e. $\vec{k}.\ \vec{r} = kz$), as shown in the diagram below (figure 1.2). This is known as the plane-wave solution. The unit vector $\hat{n}$ indicates the direction of

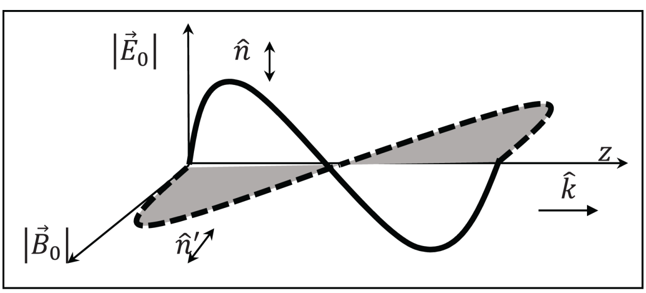

Figure 1.2. The electric field ($\vec{E}_0$), magnetic field ($\vec{B}_0$), and the direction of propagation of an EM wave ($\hat{k}$) make a triad. The polarizations of the field vectors are indicated by $\hat{n}$ and $\hat{n}'$, respectively.

oscillation of $\vec{E}$ and is known as the *polarization* of the field vector; k is the propagation constant or the wavevector, i.e. the number of waves per unit length; ω is the frequency; and ϕ is the phase factor.

In Euler's form, equation (1.3) is the real part of equation (1.4) as follows:

$$\vec{E}(\vec{r},\, t) = E_0\hat{n}e^{i(\vec{k}.\vec{r}-\omega t+\phi)}. \tag{1.4}$$

By using this solution for equation (1.2), we obtain

$$\frac{\partial}{\partial t}(e^{-i\omega t}) = -i\omega e^{-i\omega t};\ \text{and}\nabla(e^{i\vec{k}.\vec{r}}) = i\vec{k}(e^{i\vec{k}.\vec{r}}).$$

From these relations, we can effectively replace $\nabla$with $i\vec{k}$ and $\frac{\partial}{\partial t}$ with $-i\omega$ for plane-wave equations. Now, using ME (iii), we can write the relation between the electric and magnetic field vectors with $\hat{k}$ as follows:

$$\frac{E(\hat{k}\times\hat{n})}{c} = B\hat{n}'.$$

This explains the *transverse nature of the EM wave* indicated in the diagram. The oscillation of the field vectors is perpendicular to the propagation direction of the wave. We recall that this is in contrast to sound waves, in which the rarefaction and densification of the medium's particles takes place in the propagation direction of the wave—for this reason, sound waves are generally called *longitudinal waves.*

The unit vector ($\hat{k}$) indicates the direction of propagation of the wave. From ME (ii), we can see that that the $\vec{B}$ field of an EM wave that has $\hat{n}'$ as its direction of oscillation is perpendicular to $\hat{k}$ (i.e., $\hat{n}'\bullet\hat{k} = 0$). Similarly, $\hat{n}$ and $\hat{k}$ are mutually perpendicular to each other, as can be seen in the figure.

The corresponding Maxwell's equations in matter are written by introducing the displacement vector ($\vec{\mathrm{D}}$ in Cm^{-2}), the available charge and current densities, and the $\vec{H}$ field (in A m − 1). The $\vec{\mathrm{D}}$ field is defined as $\vec{D} = \epsilon\vec{E}$ and $\vec{H}$ is related to $\vec{B}$ according to $\vec{H} = \frac{\vec{B}}{\mu}$. Here, ϵ and μ are the permittivity and permeability of the *medium*. In metals, the current density ($\vec{J}$) is related to the conductivity (σ) by $\vec{J} = \sigma\vec{E}$. In metals, the square root of the product of the quantities ω, μ, and σ is defined as the inverse of the *skin depth* (s) according to $s = \sqrt{\frac{2}{\omega\mu\sigma}}$ (♠ see question 7). Measured in units of nm, s is the distance over which the amplitude of the EM waves decays to $1/e$ of its original value while propagating in the material with given parameters.

♣ The relation $\vec{J} = \sigma\vec{E}$ is known as 'Ohm's law'. The elementary form of this law is studied in high school. It states that the current (I) across a resistor is proportional to the potential difference (V) between the two ends of the resistor (R), according to $V = IR$. As an exercise, one can obtain this relation by expressing E in terms of V across a metal bar of length L with resistance R and conductivity σ. It is necessary to replace $|\vec{J}|$ with the current (I) per unit area (A) according to this definition.

Exercise 1.1. A linearly polarized plane EM wave is propagating in the z direction, and its plane of polarization is the x direction. The electric field of the wave has an amplitude given by $| E_0 |$. The frequency of the wave is ω, and its wave number is k. What are the electric and magnetic fields of the wave?

Solution: The electric field vector of the wave ($\vec{E}$) is

$\vec{E} = | E_0 | \hat{e}_x \cos(kz - \omega t)(Vm^{-1})$. The magnetic field vector is written as

$\vec{B} = \frac{(\vec{k} \times \vec{E})}{\omega} = \frac{1}{\omega} k | E_0 | (\hat{e}_z \times \hat{e}_x) \cos(kz - \omega t)$.

Therefore $\vec{B} = \frac{k}{\omega} | E_0 |(\hat{e}_y) \cos(kz - \omega t)$ tesla (T). Note that we have ignored the phase part.

Exercise 1.2. (a). Show that the dimension of skin depth (s) is that of length.

(b). What is the typical value of s for copper for the frequencies in the visible region (ω˜10^{15}Hz)?

Solution:

(a). This is easy to verify if we take the dimension of charge to be [Q]. To get the dimension of skin depth (s) as $[L]$, use the dimensions $[T^{-1}]$, $[M LQ^{-2}]$, and $[M^{-1}L^{-3}T Q^2]$ for ω, μ, and σ, respectively.

(b). For copper, the value of $\sigma = 10^7(\Omega\text{cm}^{-1})$. Using the value of $\mu = 10^{-6}N/A^2$ and an angular frequency for the visible region of $\omega = 10^{15}$Hz, we get the value of s˜ 10 nm. This indicates that visible radiation can access the surface of the copper for distances up to the order of 10 nm!

Exercise 1.3. While ME (iii) is responsible for the generation of electricity, which equation is used in the mechanism of cooking by induction stove?

Solution: We know that Faraday's law suggests that changing magnetic fields give rise to current; ME (iv) suggests that a changing electric field gives rise to a magnetic field. In an induction stove, when an AC current passes through the coil of a conducting wire, it induces a magnetic field within the skin depth of the ferromagnetic base of the cooking pan. The eddy currents induced by the magnetic field in the thick base of the cooking pan result in Joule heating due to its resistivity. This heat is responsible for cooking the food contents of the pan by heat conduction.

1.4 A few topics related to lasers and optics

1.4.1 Phase velocity and group velocity

The electric field of the wave (i.e. $\vec{E}(\vec{r}, t)$) is given in three dimensions by equation (1.3). For the one-dimensional case (i.e. along z), it can be rewritten as $\vec{E}(z, t) = | E_0 | \hat{n}\cos(\vec{k} . \vec{z} - \omega t + \phi)$. At a certain numerical value of the phase, the movement of the wave can be tracked so that the derivative $\frac{d\phi}{dt} = 0$. The plot of ω vs k can be used to define two important concepts regarding the velocity of the wave: (i) the phase velocity ($\vec{v}$) of the waveform is defined as the ratio of ω and k according to $| \vec{v} | = \frac{\omega}{k}$; (ii) the instantaneous derivative $\left(\frac{d\omega}{dk}\right)$, on the other hand, comes into the picture when the waveform consists of slightly varying frequencies and wave vectors.

In this case, the waveform propagates as a superposition of two or more modulated waves known as a *wave packet.* The speed of the modulated signal is defined as the *group velocity*, $\left(v_g = \frac{d\omega}{dk}\right)$. For a nondispersive medium, v_g remains equal to the magnitude of $\vec{v}$.

1.4.2 Power flow and Poynting vector

We know, for instance, that heat is also transported by sunlight to the Earth. An important aspect of electromagnetic (EM) wave transport is the propagation of energy. The energy densities in electric (u_E) and magnetic (u_B) fields are given by $\mathrm{u_E} = \frac{\epsilon 0}{2}E^2$, and $\mathrm{u_B} = \frac{1}{2\mu_0}B^2$, respectively. The energy density of the EM field is given by

$$\mathrm{u_{em}} = \frac{1}{2}\left(\frac{1}{\mu_0}B^2 + \epsilon_0 E^2\right).$$

Maxwell's equations help in the derivation of an energy conservation equation. The energy flow per unit area per unit time is given by the Poynting vector ($\vec{\mathrm{S}}$),

$$\vec{\mathrm{S}} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}). \tag{1.5}$$

1.4.3 Radiation pressure and angular momentum

The tail of a comet in orbit around the Sun always points away from the Sun. This happens due to the radiation pressure exerted by the momentum of the light. From a dimensional analysis, we can see that the magnitude of the linear momentum ($\vec{p} = \hbar\vec{k}$) imparted by a single photon of linearly polarized light is given by the ratio of the energy $\Delta U(=h\nu)$ absorbed to the speed of light, as follows:

$$\vec{\mathrm{p}} = (h\nu/c)\hat{k}. \tag{1.6}$$

Alternatively, the momentum of a photon can be written in terms of its wavelength λ as $p = h/\lambda$. Using equation (1.5), with $|\vec{E}| = c|\vec{B}|$, the density of the linear momentum in the EM field is given by

$$<\boldsymbol{P}_{em}> = \frac{\vec{\mathrm{S}}}{c^2} = \quad \epsilon_0(\vec{E} \times \vec{B}). \tag{1.7}$$

When a stream of N photons per unit area per second falls perpendicularly on a perfectly black surface, it is assumed that all the photons are absorbed, thereby completely transferring their momenta to the surface. The irradiance (I') in units of W m^{-2} and the energy (ΔU) absorbed by the area (A) in time (Δt) are related by $\Delta U = I' A \Delta t$. As the energy is completely absorbed, the gain in momentum based on equation (1.7) is

$$\Delta p = \Delta U/c \equiv I' A \Delta t/c.$$

The force (F) is defined as the rate of change of momentum, i.e., $F = \Delta p/\Delta t \equiv I'A/c$. Therefore, the radiation pressure (P_r) which is equivalent to the force per unit area is given by the following:

$$P_r = I'/c. \tag{1.8}$$

♣ The radiation pressure also has applications in the micromanipulation of particles using laser beams.

For a 100% reflecting surface, the photon undergoes a change of momentum equal to $2p$. Therefore, the pressure exerted on the surface is equal to $2I'/c$.

If a beam of circularly polarized light (♠ see chapter 3) is incident on an absorbing medium, the surface experiences a torque as predicted by classical EM theory. The magnitude of the torque $|\,\zeta\,|$ per unit area is given by

$$\zeta \quad = \frac{I'}{\omega} \equiv \frac{Mh}{2\pi}.$$

Here, M is the total number of photons. This indicates that the angular momentum ($\vec{l}_{em}$) of the photon is $h/2\pi$. This value of intrinsic momentum of photon is known as the *spin angular momentum* (SAM) and its value is taken as 1 unit. While for right circularly polarized light, the spin of the photon is parallel to the direction of propagation, it is antiparallel for left circularly polarized light.

In addition, the angular momentum carried by the *phase part of the wavefront* (also known as the *phase front*) of the light is called the orbital angular momentum (OAM). This is included in the expression for the cross-product of the radius vector $\vec{r}(r, 0, z)$ of the beam and $\vec{p}_{\rm em}$ as follows:

$$\vec{l}_{\rm em} = \vec{r} \times \vec{p}_{\rm em} \equiv \epsilon_0[\vec{r} \times (\vec{E} \times \vec{B})]. \tag{1.9}$$

Equation (1.9) includes both the SAM and OAM contributions of light. Although the details are beyond the scope of this book, it is sufficient to mention that a light beam with an azimuthal phase dependence of $e^{i|l|\phi'}$ in its cross section has an OAM value that is higher by several factors than the SAM. Here, ϕ' is the azimuthal coordinate and l is an integer known as the *azimuthal mode index* or, the *order* of OAM.

♣ The transfer of SAM was first experimentally observed using a torque experienced by a suspended quarter-wave plate by Beth in 1936.

♣ For linearly polarized light, $<\vec{l}_{em}> =0$. The same is true for unpolarized light, as this is considered to be a mixture of right circularly and left circularly polarized lights. The only difference is that for linearly polarized light, the mixture is coherent (♠ see chapter 3).

♣ It is also of contemporary interest that light beams with different orders of OAM can be produced; these are known as *vortex beams* and find applications in super-resolution light microscopy and photonics.

♣ The magnitude of the OAM of the vortex beam is said to be related to the *topological charge* of the beam by $\pm|l|\hbar$.

1.4.4 Radiation emitted by the accelerated charge

For a single point charge q, the dipole moment can be given as $\vec{p}_{dip}(t) = q\vec{d}(t)$, where $\vec{d}$ is the position of the charge with respect to the origin. An oscillating dipole produces EM radiation in the perpendicular direction. The power (P) radiated by an accelerating charge with acceleration a is given by the generalized Larmor formula as follows:

$$P \propto \frac{\mu_0 q^2 \gamma^6 a^2}{6\pi c}, \tag{1.10}$$

with $\gamma = 1/\sqrt{1 - v^2/c^2}$, where v is the velocity of the relativistic particles. It can be seen that the power radiated by the charged particle is proportional to the square of the acceleration. The factor γ^6 indicates that the radiated power increases drastically for particles with speeds near to the speed of light.

Exercise 1.4. Write down the Poynting vector for the waves mentioned in exercise 1.1 by including the phase part, ϕ_1.

Solution: The electric field in V/m can be written as $\vec{E} = |E_0|\,\hat{e}_x \cos(kz - \omega t + \phi_1)$. By obtaining the corresponding $\vec{B} = \frac{(\vec{k} \times \vec{E})}{\omega}$ (T), the Poynting vector is given by

$$\vec{S} = \frac{1}{\mu_0}(\vec{E} \times \vec{B}) = \frac{k\,|E_0|^2}{\mu_0 \omega}(\hat{e}_z)\cos^2\!\left(kz - \omega t + \phi_1\right)(\mathrm{Wm^{-2}}).$$

1.4.5 Refractive index

The refractive index (n) of a medium is defined as the ratio of the speed of the light in vacuum (c) to that in the medium (v). It should be noted that a change in the speed of the light wave results in a change in its wavelength. However, the oscillation frequency of a wave remains unchanged in any medium. We can also write the expression for the refractive index as follows:

$$n^2 = \frac{\epsilon\mu}{\epsilon_0\mu_0} \text{ or } \epsilon_r\mu_r. \tag{1.11}$$

For a vacuum, the values of the relative quantities μ_r and ϵ_r are taken to be unity. For most optical media (viz. transparent glass) other than ferromagnetic materials, the value of μ is taken to be the same as that of vacuum. Therefore, the relative permittivity ϵ_r is given by n^2. The quantity ϵ_r often denotes the dielectric constant of the material, which is related to the electrical susceptibility as follows:

$$\epsilon_r \quad \equiv \quad n^2 = (1 + \chi_e).$$

In this formula, n is known as the high-frequency dielectric constant and is a function of the frequency. From here, an estimate of χ_e can be obtained, provided that the dielectric constant of a material is known.

1.4.6 Dispersion curve

In a non-conducting (or dielectric) isotropic medium, the electrons are bound to atoms. If a dielectric is placed in in external electric field ($\vec{E}$), the electronic charge (q) is displaced from its equilibrium position. From classical electrodynamics, we know that the induced polarization $\vec{P}_{\text{ind}}$ for a number of oscillators per unit volume, N, is given by

$$\vec{P}_{\text{ind}} = N\vec{p}_{\text{ind}}.$$

We can write the expression for a damped oscillation forced by an external time-varying field (in the y direction) as

$$m\ddot{\vec{y}} + m\gamma\dot{\vec{y}} + K\vec{y} = q\vec{E}$$

or

$$\ddot{\vec{y}} + \gamma\dot{\vec{y}} + \omega_0^2\vec{y} = \frac{q\vec{E}}{m}, \tag{1.12}$$

where K is the force constant, m is the mass of the electron, ω_0 is the effective resonance frequency ($=\sqrt{K/m}$) of the bound electrons, and γ is the damping constant. Similar to the mass–spring system in mechanics, we assume a solution of equation (1.12) such as $\vec{y} = \vec{y}_0 \quad e^{i\omega t}$ to obtain the complex amplitude of oscillation as follows:

$$\vec{y}_0 = \frac{q\vec{E}}{m(\omega_0^2 - \omega^2 - i\gamma\omega)}.$$

As a result, an induced electric dipole moment $\vec{p}_{\text{ind}}$ is created in the y direction, as given by

$$\vec{p}_{\text{ind}} = \frac{q^2\vec{E}}{m(\omega_0^2 - \omega^2 - i\gamma\omega)}.$$

As a result of the incident $\vec{E}$, the $\vec{P}_{\text{ind}}$ in the dielectric is also given by $\vec{P}_{\text{ind}} = \epsilon_0\chi_e\vec{E}$, and we can obtain the value of χ_e as follows:

$$\chi_e = \frac{Nq^2}{\epsilon_0 m(\omega_0^2 - \omega^2 - i\gamma\omega)}.$$

The frequency dependence of the refractive index $n(\omega)$ leads to dispersion, which is represented as a complex quantity due to $n^2 = (1 + \chi_e)$ as follows:

$$n^2 = 1 + \frac{Nq^2}{\epsilon_0 m(\omega_0^2 - \omega^2 - i\gamma\omega)}. \tag{1.13}$$

For a large number of frequencies, this equation can be rewritten as

$$n^2 = 1 + \frac{Nq^2}{\epsilon_0 m}\Sigma_j\left(\frac{f_j}{\omega_0^2 - \omega_j^2 - i\gamma\omega}\right). \tag{1.14}$$

Here, f_j describes the relative potential strengths of the oscillation frequencies. For small values of γ (which are neglected), equation (1.14) can be written as follows:

$$n^2 = 1 + \frac{Nq^2}{\epsilon_0 m}\sum_j\left(\frac{f_j}{\omega_0^2 - \omega_j^2}\right). \tag{1.15}$$

This equation, when written in terms of the wavelength (λ), is known as Sellmeier's formula, an empirical relation proposed by Sellmeier in 1872 as follows:

$$n^2 = 1 + A_0\left(\frac{\lambda^2}{\lambda^2 - \lambda_0^2}\right), \tag{1.16}$$

where $A_0 = \frac{Nq^2\lambda_0}{4\pi^2\epsilon_0 mc^2}$ is known as Sellmeier's coefficient. Equations (1.15) and (1.16) are extremely useful formulae used to estimate the refractive index data for various wavelength regions for applications in linear and nonlinear optics. The original dispersion relation, which did not take account of anomalous dispersion, was proposed by Cauchy as early as 1836 (♠ see question 11).

1.4.7 Normal dispersion and anomalous dispersion

For simple systems such as gaseous media, $n(\omega) \cong 1$, we can write $(n^2 - 1)$ in equation (1.13) as $(n + 1)(n - 1) \approx 2n$. Near the resonant frequency, we can take $|\omega + \omega_0| \approx 2\omega_0$ and $|\omega - \omega_0| \ll \omega_0$. To separate $n(\omega)$ into refractive or real (n') and absorptive or imaginary (n'') parts, we take $n = n' + in''$. So, from equation (1.13),

$$(n' + in'')^2 = 1 + \frac{Nq^2}{\epsilon_0 m(\omega_0^2 - \omega^2 - i\gamma\omega)}.$$

By rationalizing the second term and separating it into real and imaginary parts, we get

$$n'^2 - n''^2 = 1 + \frac{Nq^2}{\epsilon_0 m}\left(\frac{(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}\right) \tag{1.17}$$

$$2n'n'' = \frac{Nq^2}{\epsilon_0 m}\left(\frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}\right). \tag{1.18}$$

Here, γ is related to the decay rate of the population. We can use these two relations to check the relation at the resonant frequency (i.e. $\omega = \omega_0$) as follows:

$$n'^2 - n''^2 = 1 \tag{1.19}$$

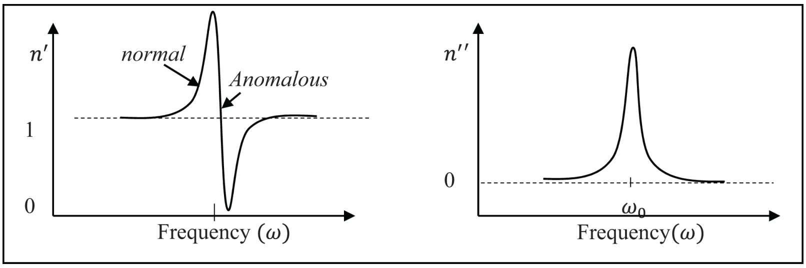

Figure 1.3. The real (n') and imaginary (n'') parts of the refractive index as a function of the frequency (ω). The *normal* and *anomalous* dispersion regimes are also indicated.

and

$$2n'n'' = \frac{Nq^2}{\epsilon_0 m\gamma\omega_0}. \tag{1.20}$$

Using equation (1.20), equation (1.19) can be rewritten as a quadratic equation, as follows:

$n'^4 - n'^2 - \left(\frac{Nq^2}{2m\epsilon_0\gamma\omega_0}\right)^2 = 0$ to give the roots of n'^2 as $n'^2 = \frac{1}{2} \pm \frac{1}{2}\sqrt{1 + \left(\frac{Nq^2}{m\epsilon_0\gamma\omega_0}\right)^2}$.

Only a positive root can represent real values. At very high frequencies, n' will approach unity. The variables n' and n'' are known as optical constants and are plotted in figure 1.3 based on equations (1.17) and (1.18). The plot of n' vs ω is known as the dispersion curve. The value of n' generally increases with frequency (known as normal dispersion), except for a narrow range where it falls sharply followed by a dip. This small region represents anomalous dispersion. The normal and anomalous dispersion regimes are indicated in the figure.

A plot of n'' against frequency shows a maximum at ω_0, indicating the resonant frequency dependence of the absorption coefficient. The anomalous or, negative dispersion regions find applications in the dispersion compensation of ultrafast laser pulses and optical communication, respectively (♠ see chapters 22 and 23 for more details).

1.5 Comparison of an electronic circuit and a photonic circuit

A basic electronics circuit used to charge and discharge a capacitor is shown in figure 1.4. Here, a capacitor (C) is charged by the battery through the resistor (R) when the switch (S_1) is closed (and switch S_2 is open). It is discharged through the resistor when S_1 is open (and S_2 is closed).

A simple photonic circuit can be produced with the help of two polarizers (P1 and P2) placed in the path of a polarized light beam. Depending on the orientation of P2 (i.e. whether it is parallel or perpendicular to the optical axis of P1), a photodetector will record changes in the signal. This photonic circuit can represent the output (i.e. the light sensed by the detector) in terms of binary numbers using one when P1 is parallel to P2 and zero when P1 is perpendicular to P2. (♠ see chapter 3 for detailed description of the phenomenon of polarization).

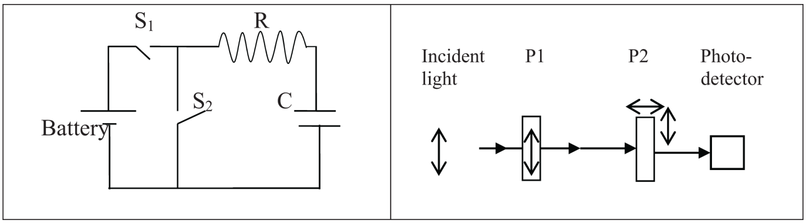

Figure 1.4. Typical electronic circuit used to *charge* and *discharge* a capacitor (C) through a resistor (R) and a battery (left panel). Representation of a photonic circuit (right panel) realized using polarized light (indicated by double arrows) with two polarizers (P1 and P2) and a photodetector.

Table 1.1. Notable Nobel prizes for laser-related work following the invention of the ruby laser in 1960 by Maiman.

Year	Laser-related Nobel prizes
1964	Townes, Basov, and Prokhorov for the **maser–laser** principle.
1971	Gabor for the basic ideas of the **holographic** method.
1981	Nicolaas Bloembergen and Arthur Schawlow for **laser spectroscopy**.
1997	Claude Cohen-Tannoudji, Americans Steven Chu and William Phillips for the development of **methods to cool and trap atoms using laser light**.
1999	Ahmed Zewail for **ultrafast laser techniques**
2000	Zhores Alferov and Herbert Kroemer for **developing semiconductor heterostructures for continuous wave semiconductor diode lasers**.
2001	Eric Cornell, Wolfgang Ketterle, and Carl E. Wieman for **Bose–Einstein condensation**.
2005	Theodor Hansch and John Hall for the **development of laser-based precision spectroscopy**, known as the optical frequency comb technique.
2009	Charles K. Kao, Willard S. Boyle, and G. E. Smith for **fiber-optic communication and the CCD sensor**
2012	Serge Haroche and David J. Wineland for **experimental methods that enable the measurement and manipulation of individual quantum systems**.
2014	Isamu Akasaki, Hiroshi Amano, and Shuji Nakamura for the **invention of efficient blue light-emitting diodes** (Physics)
2014	Eric Betzig, Stefan W. Hell, and William E. Moerner for **stimulated-emission-depletion (STED)** microscopy using laser beams (Chemistry)
2017	Rainer Weiss, Kip S Thorne, and Barry C Barish for **the LIGO detector and the observation of gravitational waves**
2018	Arthur Ashkin, Gerald Mourou, and Donna Strickland for **optical trapping of particles and chirped pulse amplification of laser pulses**

1.6 Nobel prizes related to lasers

When the laser was invented by Maiman in 1960, it was '*an invention searching for an application*'. Since then, lasers have contributed to the fundamental discoveries and applications of modern technologies that could not have been achieved otherwise. Table 1.1 gives some of the notable Nobel prizes for laser-related work in last six decades.

Questions and problems

1. What is the difference between electro-optics and optoelectronics? Give one example of each.
2. (i) With reference to the properties of the photon, why is light immune to EM noise?
   (ii) How do we eliminate external EM noise in experiments that use electronic devices?
3. (i) Which of MEs is responsible for the generation of electricity, and which one describes the absence of magnetic monopoles?
   (ii) Estimate the speed of light using MEs in vacuum [Use $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2N^{-1}m^{-2}}$ and $\mu_0 = 4\pi \times 10^{-7}\mathrm{NA^{-2}}$].
4. Write down the *plane-wave solution* of a wave equation derived from MEs for charge-free, current-free regions. With reference to plane waves, what is the difference between the terms *wavefront* and *phase front*?
5. Using the solution of the wave equation in Euler's form, use ME (iii) to show that light is an EM wave and that $\vec{E}$ and $\vec{B}$ are perpendicular to each other.
6. If ρ_f and J_f are corresponding 'free' charge and current densities, respectively, $\vec{\mathrm{D}}$ and $\vec{H}$ are the displacement vectors and $\vec{H}$ the field (often referred to as the magnetic field), write down the ME for a medium for which the permittivity and permeability are ϵ and μ, respectively.
7. Assume that the displacement current term is too small to be neglected in ME (iv). For a charge-free and current-free region, let the wave equation be given by equation (1.2) and the speed of light in a medium be $=\sqrt{1/(\epsilon\mu)}$. Using the solution of the wave equation $\vec{E}(\vec{r}, t) = E_0\hat{n}e^{i(\vec{k}.\vec{r}-\omega t)}$, obtain the value of the skin depth for a metal of conductivity σ.
8. Differentiate between the phase velocity and the group velocity for a wave. Show that in the absence of dispersion, both velocities are equal.
9. Describe how an accelerated charge radiates.
10. Half of the Nobel prize was awarded for work on the optical trapping of microparticles in 2018. What is the principle of optical trapping? ♣You may like to read the original paper by *Arthur Ashkin (PNAS*, 94 (10), (1997), pp 4853–4860). You will know that the particle size matters with respect to the wavelength of the light used for optical levitation. The scattering and gradient forces balance to trap the particle on the focussed laser beam.
11. The general form of Sellmeier's empirical relation equation (1.16) is given by $n^2 = 1 + \sum_j \left(\frac{A_j\lambda^2}{\lambda^2 - \lambda_j^2}\right)$. For $\lambda \gg \lambda_j$ and $A_j = A$, prove that it approaches Cauchy's formula $n = C_1 + \frac{C_2}{\lambda^2} + ..$ for $C_1 = \sqrt{(1 + A)}$ and $C_2 = \frac{A\lambda_0^2}{2\sqrt{(1 + A)}}$.
12. How do you define the normal and anomalous dispersion regimes?
13. In the dispersion curve, at what frequency is the absorption coefficient maximized?

14. For a photonic circuit for a logic gate that uses polarisers, write down the truth table, which is similar to that for electronic gates.

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