Part I

The Sun

Activity 1

The Sun's Distance I: The Method of Aristarchus

Preview

The ancient Greek astronomer Aristarchus used an observation of the Moon to deduce the distance to the Sun. Although he greatly underestimated the solar distance, his methodology was valid and represents one of the earliest efforts to apply geometry to cosmic measurement.

1.1 Aristarchus's Distance to the Sun

Sometime around 250 BCE, the Greek astronomer Aristarchus conceived a way he might deduce the Sun's distance from a particular observation of the Moon. He envisioned how the Earth, Moon, and Sun are arranged in space at the instant when the Moon appears precisely half-illuminated—its first-quarter or last-quarter phase—as pictured in Figure 1.1. At that time, Aristarchus reasoned, the Sun rays strike the Moon perpendicular to our viewing direction from Earth. That is, the Earth, Moon and Sun together form a right triangle, one of whose angles measures 90°, as in Figure 1.2. In this figure, the little square indicates the "right," or 90°, angle; the side labeled $\boldsymbol{r}$ represents the distance from the Earth to the Moon; and the side labeled $\boldsymbol{d}$—the triangle's hypotenuse—is the distance between the Earth and the Sun. A similar right-triangle arrangement would occur if you held an illuminated ball at arm's length along a direction perpendicular to the direction of the incoming light.

Basic trigonometry provides the means to estimate the Sun's distance. In a right triangle, the cosine of an angle (abbreviated cos) is defined as the length of the side adjacent to the angle divided by the length of the hypotenuse. For the Earth–Moon–Sun triangle in Figure 1.2, this means that $\cos(\boldsymbol{E}) = \boldsymbol{r}/\boldsymbol{d}$, where $\boldsymbol{E}$ is the angle between the Moon and Sun as viewed from the Earth. To envision angle $\boldsymbol{E}$, imagine standing

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Figure 1.1. The Moon in its first-quarter phase. (Credit: NASA/GSFC/USRA.)

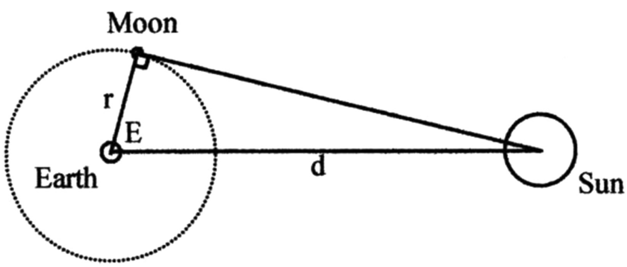

Figure 1.2. Right-triangle configuration of the Earth, Moon, and Sun during the Moon's first-quarter phase.

outside and extending your right arm toward the Sun and your left arm toward the Moon; the angle formed by your outstretched arms is $\boldsymbol{E}$.

A bit of algebra makes the cosine equation look like this:

$$\boldsymbol{d} = \boldsymbol{r}/\cos(\boldsymbol{E}). \tag{1.1}$$

Having already determined the Moon's distance $\boldsymbol{r}$, all Aristarchus needed to do to find the Sun's distance was to measure the angle $\boldsymbol{E}$ during the Moon's quarter phase. Easier said than done! If Figure 1.1 were drawn to actual scale, with the Sun very far away compared to the Moon—that is, with $\boldsymbol{d}$ many times larger than $\boldsymbol{r}$— angle $\boldsymbol{E}$ would be very close to, although not quite, a right angle. (In fact, trigonometry had not been invented in Aristarchus's time; he used analogous geometrical methods to accomplish the procedure described here.)

1. According to an ancient source, Aristarchus estimated—guessed?—that, during the quarter-moon phase, the angle $\boldsymbol{E}$ was 87°. Use this estimate in Equation (1.1) to determine how many times farther away the Sun is than the Moon. Your answer should take the form $\boldsymbol{d} = \boldsymbol{n} \times \boldsymbol{r}$, where $\boldsymbol{n}$ is a number. For now, leave $\boldsymbol{r}$ unspecified; it's merely a symbol that represents the Moon's distance. For example, if angle $\boldsymbol{E}$ were 30°, then cos ($\boldsymbol{E}$) = 0.866; plugging this into Equation (1.1), the Sun's distance $\boldsymbol{d}$ equals $\boldsymbol{r}$/0.866, or 1.15 × $\boldsymbol{r}$.
2. The solar distance $\boldsymbol{d}$ derived from Aristarchus's method is exquisitely sensitive to the value of angle $\boldsymbol{E}$. In other words, even a slight change in angle $\boldsymbol{E}$ produces a substantial change in the solar distance $\boldsymbol{d}$. To illustrate, recompute the solar distance if Aristarchus had assumed an angle $\boldsymbol{E}$ merely one degree larger, that is, 88° instead of 87°. Again express your answer in the form $\boldsymbol{d} = \boldsymbol{n} \times \boldsymbol{r}$.
3. Modern measurement reveals that Aristarchus was way off in his estimation of the angle $\boldsymbol{E}$ (so far off, in fact, that we suspect he just plucked a value "out of the hat"). The true value of $\boldsymbol{E}$ is 89.85°. Recompute the solar distance now, again in the form $\boldsymbol{d} = \boldsymbol{n} \times \boldsymbol{r}$.

1.2 The Sun's Diameter

Once the Sun's distance is known, it's easy to find the Sun's true diameter, in units we'll call "Moon-diameters," that is, the number of Moons that would fit across the face of the Sun. During a total solar eclipse, the Moon almost precisely covers the Sun; in other words, the Moon and the Sun *appear* the same angular diameter in the sky. However, the Sun is much farther away than the Moon; therefore, to *appear* the same diameter as the Moon, the Sun must, in fact, be a much larger body than the Moon. For example, if the Sun were three times farther than the Moon, yet appeared the same size as the Moon in the sky, we conclude that the Sun must actually be three Moon-diameters wide. Or in general, if the Sun is $\boldsymbol{n}$ times farther than the Moon, yet they appear the same size, the Sun must be $\boldsymbol{n}$ Moon-diameters wide.

4. (a) Using your value for $\boldsymbol{n}$ from part 1, write down an expression for the Sun's diameter in units of Moon-diameters, according to Aristarchus. (b) Aristarchus went on to find the Sun's diameter in units we will call "Earth-diameters." To do this, he estimated, from lunar eclipse observations, that the Moon is 1/3 as wide as the Earth. Considering this information and your answer to part 4(a), write down an expression for the Sun's diameter in units of Earth-diameters, according to Aristarchus.

1.3 The Sun's Distance Revisited

Curiously, the extant evidence suggests that Aristarchus might not have carried out the next logical step: finding the Sun's distance in units of Earth-diameters. With such information, he could have formed a true scale model of the Sun–Earth system, in the same way that a globe shows a scaled-down version of our planet. The Sun's distance is obtained by answering the following question: given the Sun's true width,

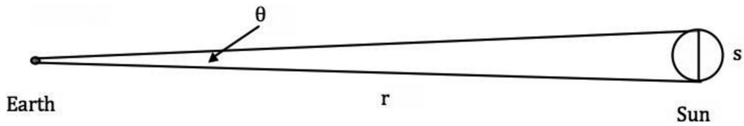

Figure 1.3. Located at distance r, the Sun's diameter s spans an angle θ in the sky.

expressed in Earth-diameters in part 4(b), how many Earth-diameters away must it be to appear as small as it does in the sky—a mere ½ degree across?

To answer this question, we apply a geometrical relation we will call the sector formula:

$$\mathbf{r} = 57.3 \times \mathbf{s}/\boldsymbol{\theta}, \tag{1.2}$$

as illustrated in Figure 1.3. Here r is the Sun's distance, s is the Sun's true diameter (expressed in Earth-diameters), and the Greek letter theta θ is the Sun's apparent diameter in the sky, ½ degree. For example, if an object is 10 Earth-diameters wide (s) and spans an angle of 4° in the sky (θ), its distance (r) is equivalent to about 143 Earth-diameters.

5. Using the sector formula Equation (1.2), with your answer to part 4(b) for the estimated diameter of the Sun, compute the Sun's distance in units of Earth-diameters, according to Aristarchus.

Modern measurements reveal that Aristarchus erred significantly in his estimates of the Sun's diameter and distance: the Sun is actually more than 100 Earth-diameters across and 11,000 Earth-diameters distant! There's nothing wrong with Aristarchus's method; it was just impossible to reliably carry out in his day. Nevertheless, Aristarchus's erroneous solar distance became the *de facto* standard into the Middle Ages. For the first time, someone had calculated the size and distance of a celestial body using observational data gathered from Earth.

Worksheet, Activity 1: The Sun's Distance I: The Method of Aristarchus

Name ________________________________

1.

2.

3.

4. (a)

   (b)

5.