Turbulent Flows: an Introduction

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To Lucy, my wife, for her unending patience and support.

Ian Castro

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Preface

Turbulence is a subject of fundamental interest and importance within the field of fluid mechanics and has attracted wide and detailed attention for well over 150 years, not least from physicists and engineers in industry, academia, and government research laboratories of many kinds. Numerous books on the subject of turbulence have been variously aimed at late undergraduates, postgraduates, or full-time researchers. Some of these have become almost classical texts, containing much of the received wisdom that is of foundational importance to any students of the subject; such texts are often used as the basis for, or at least as adjuncts to, university courses in the subject. Other texts are essentially monographs, useful for deeper exploration of more specialised topics. However, although the number of research papers discussing turbulent flows in the contexts of aerodynamics, hydrodynamics, meteorology, oceanography, and even astronomy has grown exponentially over the last twenty years or so, hardly any modern books that can serve as general introductions to the topic have appeared since the very early years of this century. In this text, we have sought to provide such an introduction whilst, at the same time, giving students embarking on the subject a flavour of some of the more recent ideas. Like many earlier texts, we have unashamedly concentrated on what might be called canonical turbulent flows. This is not simply because they provide the necessary foundations for a basic understanding of turbulence, but also because it is largely those canonical flows which have attracted the most attention over the last few decades, particularly because of the increasing ability of computational approaches to yield accurate solutions to them under certain circumstances, thus allowing early hypotheses to be tested even more comprehensively than by physical experiments.

We have aimed our text at typical physics or engineering students who have already studied basic courses in fluid mechanics and are thus probably in their third or fourth year of undergraduate study, or perhaps starting postgraduate research work involving some kind of turbulent flow. In either case, such students might be engaging with their first formal course on turbulence. The flow of the book is similar to a typical order of presentation used in such courses (as, indeed, used by the authors in their own courses developed and taught at the University of Southampton). The level of mathematical understanding required is no more than the student would have acquired during their earlier studies. We purposely do not discuss experimental or computational methods. There are texts that cover both these topics, but the techniques involved generally move significantly faster than our level of understanding of turbulence so, in our view, the student is better served by keeping up to date with these techniques through the journal literature. However, this book does include a substantial section on the tools commonly used to interpret the data (which are usually time records of fluctuating quantities) produced by numerical or physical experiments; these tools are undoubtedly longer-lasting. We have also included short biographies of some of the 'Giants of Turbulence' at the end of each chapter, highlighting some of the important figures on whose shoulders so many subsequent researchers have stood in order to expand the understanding of turbulence over the last 150 years. Material for some of these sections has been partly culled from the splendid 2011 (CUP) book *A Voyage Through Turbulence* edited by Davidson *et al*, which provides much more extensive and fascinating reading for those particularly interested in the history of the subject and the development of the pivotal ideas.

Each chapter concludes with exercises which are designed to enhance the understanding of the chapter's subject matter. Many of these require the use of one or more of the data sets supplied in the associated 'GitHub' website freely available online at

https://cvanderwel.github.io/TurbulentFlows/

This online repository contains a copy of the exercises, all necessary data files, and partial worked solutions (mainly in MATLAB and Python). It will be maintained by CV and readers are encouraged to contact her with any queries or potential updates they would like to see included. The programming scripts in the repository are provided under the MIT License, which means that readers are free to use, copy, and modify the content to help them learn about turbulent flows. Our hope is that this will be useful not just to students but also to teachers who might like to expand the set of examples, use the codes provided as a basis to help with their own work, generate full worked solutions, etc.

Ian P Castro
Christina Vanderwel
August 2021

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Our interest in and understanding of turbulent flows has naturally been influenced by numerous teachers and colleagues over many years. It would be impossible to acknowledge them all, but we would like to mention the few who were seminal in sparking our initial interest in the subject and setting us on our own paths of turbulence discovery: Peter Bradshaw and (the late) Les Bradbury (for IPC) and Stavros Tavoularis (for CV). We owe them an incalculable debt. We are also grateful to more recent colleagues, particularly Bharathram Ganapathisubramani and John Shrimpton, who, despite the restrictions of 'lockdowns' driven by the COVID-19 pandemic, during which the book was written, have provided helpful comments and encouragement.

Author biography

Ian P Castro

Ian P Castro is Emeritus Professor of Fluid Dynamics at the University of Southampton, UK. After graduating from the University of Cambridge (1968), he completed an MSc and then a PhD at Imperial College London while employed as the first Donald Campbell Memorial Fund Fellow. There followed six years as a Research Officer with the Central Electricity Generating Board (CEGB), after which he returned to academia, joining the University of Surrey in 1978. Various posts there led to appointment as Professor of Fluid Dynamics in 1990, and in 1993 he was appointed as Founder-Director of the National Power Environmental Flow Research Centre. He moved to Southampton, taking the new Chair in Fluid Dynamics, in January 2000. After serving as Deputy Head of School, from 2008 until formal retirement in late 2010 he was Head of the Aerodynamics and Flight Mechanics Research Group. He has served on various national and international scientific committees and although now Emeritus (from 2017), he continues some research and publishing activities which, over the years, have been concentrated on the experimental and numerical study of turbulent flows of environmental and industrial significance.

Christina Vanderwel

Christina Vanderwel is an Associate Professor in Fluid Dynamics at the University of Southampton, UK, and has worked there since 2014. She moved to the United Kingdom after completing an MSc (2010) and a PhD (2014) in Mechanical Engineering at the University of Ottawa, Canada, under the supervision of Prof Stavros Tavoularis. Since working at Southampton, she has been awarded a Marie-Curie Fellowship (2015) as well as a UKRI Future Leaders Fellowship (2020) to fund her research on turbulence and mixing. She has published on the topics of turbulent shear flows, boundary layers, the structure of turbulence, and turbulent diffusion.

Symbols

Roman

A, A_1, B, B_1	Nondimensional constants (used variously) or (for B) the Loitsyanskii integral ($m^7\ s^{-2}$) (Equation (5.23))
a	Signal amplitude or nondimensional constant (used variously)
a_{ij}	The anisotropy tensor ($m^2\ s^{-2}$) (Equation (5.37))
b_{ij}	Normalised anisotropy tensor (Equation (5.38))
C	Autocovariance function ($m^2\ s^{-2}$), or Fourier integral of the fluctuating velocity (m), or C, C^*, additional log law constants
C_p	Heat capacity ($J\ kg^{-1}\ K^{-1}$)
C_ϵ	The Kolmogorov constant
C_c	Corrsin–Obukhov constant (Equation (9.32))
C_d	Total drag coefficient
c	Scalar concentration ($kg\ m^3$)
c_f	Local skin friction coefficient
D	Body drag
d	Jet nozzle, or cylinder, or sphere diameter (m), or zero plane displacement (m)
$E(\kappa)$	Energy spectrum function ($m^3\ s^{-2}$)
E_{11}	One-dimensional spectral functions for the u'_1 velocity component ($m^3\ s^{-2}$) and similarly for E_{22} and E_{33}
$E(f)$	Energy spectrum function ($m^2\ s^{-1}$)
F	Force ($kg\ m\ s^{-2}$)
f	Frequency (s^{-1}), or nondimensional longitudinal correlation function, or friction factor, or f', normalised mean velocity similarity variable
f_i, f_o	Mean velocity similarity variables
$\tilde{f}$	A normalised arbitrary flow variable
g	Gravitational acceleration ($m\ s^{-2}$), or nondimensional transverse correlation function, or Richardson's constant (Equation (9.25))
H	Boundary layer shape factor
h	Roughness depth (m), or elevation of mass source (m), or channel half-height (m)
i, j, k	Tensor indices for the x_1, x_2 and x_3 directions
I_1, I_2	Boundary layer integral parameters (equations (8.36) and (8.37))
K	A (nondimensional) function of the longitudinal triple correlation
l, L	General length scales (m)
L_x, L_y	Longitudinal and transverse length scales (m)
M	Mach number, or Turbulence grid mesh size (m), or a puff of material (kg)
M_o	Jet momentum flux ($m^4\ s^{-2}$)
m	Mass (kg)

$\dot{m}_i$	Mass flux (kg s^{-1})
N	Number of samples
P	Mean pressure (kg m^{-2} s^{-2}),
p	Instantaneous pressure (kg m^{-2} s^{-2})
p'	Fluctuating pressure (kg m^{-2} s^{-2})
p, P	or probability density and cumulative probability density functions
P_k	Turbulence energy production rate (m^2 s^{-3})
Pe	Peclet number (Equation (9.5))
Pe_T	Turbulent Peclet number
Pr	Prandtl number (Equation (9.3))
Pr_T	Turbulent Prandtl number (Equation (9.28))
Q	A scalar source mass flow rate (kg s^{-1}), or an arbitrary flow variable, or the second invariant of the velocity gradient tensor (s^{-2})
q	An arbitrary flow variable
q_i	Heat flux (m^3 K s^{-1})
R	Correlation function (m^2 s^{-2}) or pipe radius (m)
R_T	Flow constant (entrainment parameter)
R_s	Scaling quantity for the Reynolds stress (m^2 s^{-2})
r	Spatial separation (m) or Radial coordinate (m)
Re	A general Reynolds number
S_{ij}	Rate of strain tensor (s^{-1}) (Equation (2.9))
Sc	Schmidt number (Equation (9.3))
Sc_T	Turbulent Schmidt number (Equation (9.28))
T	Energy transfer rate (m^3 s^{-3}), or Temperature (K), or timescale (s)
t	Time (s)
$\mathscr{T}$	Lagrangian integral turbulence timescale (s)
U, V, W	Mean velocities in the three coordinate directions (m s^{-1})
u, v, w	Instantaneous velocities in the three coordinate directions, usually x, y, z (m s^{-1})
u', v', w'	Fluctuating velocities in the three coordinate directions (m s^{-1})
U_B	Bulk velocity (m s^{-1})
U_c	Centreline velocity in a channel or pipe (m s^{-1})
U_m	Maximum velocity (m s^{-1})
U_s	A characteristic mean velocity scale (m s^{-1})
U_w	Wall velocity (m s^{-1})
U_∞	Freestream velocity (m s^{-1})
$\hat{u}$	A characteristic turbulence velocity scale (m s^{-1})
u_τ	Friction velocity (m s^{-1})
V	Volume (m^3) (see also the entry for U, V, W)
v	A Lagrangian velocity fluctuation (m s^{-1}) (see also the entry for u, v, w)
v_η	Kolmogorov velocity scale (m s^{-1}) (Equation (3.6))
w	Boundary layer wake function (Equation (8.28)) (see also the entry for u, v, w)
$\overline{X^2}$	Ensemble variance of particle displacements (m^2) (Equation (9.16))
x, y, z	Cartesian coordinate directions or, (for x in chapter 4), any random process
y_o	Roughness length (m) (Equation (8.51))

Greek

β	Mean flow shear rate (s^{-1}) (Equation (5.28)), or flow spreading parameter, or Dissipation spectrum constant
Γ	Circulation (m^2 s^{-1})
γ	Molecular diffusivity (m^2 s^{-1}) (Equation (9.2)), or intermittency factor (Equation (6.51)), or normalised central moment of a probability distribution (Equation (4.10))
γ_T	Turbulent diffusivity (m^2 s^{-1}) (Equation (9.13))
Δ	Rotta–Clauser boundary layer integral thickness (m) (Equation (8.24))
ΔU^+	Roughness function (Equation (8.49))
δ	Flow width (m)
δ_{ij}	Kronecker delta -1 if $i = j$, 0 otherwise (Equation (A.3))
δ^*	Boundary layer displacement thickness (m) (Equation (8.10))
ϵ	Turbulence kinetic energy dissipation rate (m^2 s^{-3})
ϵ_U, ϵ_{σ^2}	Standard errors
ϵ_c	Dissipation rate of the scalar concentration (kg^2 m^6 s^{-1}) (Equation (9.31))
η	Kolmogorov length scale (m) (Equation (3.6)), or similarity coordinate (y/δ), or a normalised cross-stream location
η_B	Batchelor length scale (m) (Equation (9.30))
θ	Geometric angle, or Boundary layer momentum thickness (m) (Equation (8.11)), or as θ', temperature fluctuation (K)
κ	Thermal diffusivity (m^2 s^{-1}) (Equation (9.1)), or wavenumber (m^{-1}), or von Kármán constant
κ_1	Streamwise wavenumber (m^{-1})
λ	Wavelength (m) or the Taylor (dissipation) microscale (Equation (3.4))
μ	Fluid viscosity (kg m^{-1} s^{-1})
ν	Kinematic viscosity (m^2 s^{-1})
ν_T	Turbulent (eddy) viscosity (m^2 s^{-1}) (Equation (2.25))
Π	Kármán measure (diagnostic function) (Equation (7.32)) or the Coles' wake strength parameter (Equation (8.27))
ρ	Density (kg m^{-3})
σ	Standard deviation
σ_{ij}	The general stress tensor (kg m^{-2} s^{-2})
τ	Shear stress (kg m^{-1} s^{-2}) or time lag (s)
τ_η	Dissipation timescale (s) (Equation (3.10))
τ_w	Wall shear stress (kg m^{-1} s^{-2}) (Equation (8.8))
Φ	Mean velocity gradient profile function (and Φ_i, Φ_o)
ψ	Kolmogorov spectrum function
Ψ	Compensated Kolmogorov spectrum function or pressure-gradient parameter (Equation (8.47))
$\Phi_{ij}(\kappa, t)$	Three-dimensional wavenumber spectrum (m^5 s^{-2})
$\boldsymbol{\omega}$	Vorticity (s^{-1})
Ω_i	ith mean vorticity component (s^{-1})
Ω_{ij}	Rate of rotation tensor (s^{-1}) (equation (2.10))
ζ	A Lagrangian time (s)