# Elementary Cosmology (Second Edition)

From Aristotle's universe to the Big Bang and beyond

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#### James J Kolata

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Cover image: Eta Carinae Nebula (NGC 3372), optical image. This nebula (a vast cloud of dust and gas) surrounds the variable star Eta Carinae. This massive star, over 100 times the mass of the Sun, is the most luminous star known in our Galaxy. It radiates its energy at a rate that is 5 million times that of the Sun. Because of its huge mass, it is extremely unstable. It is in the constellation Carina, about 9000 light years away from Earth. This image includes filters for the light from hydrogen (H-alpha), oxygen (OIII) and sulphur (SII) ions. Image credit: Dr Luke Dodd/Science Photo Library. "Isn't that what it means to be a scientist? To push the boundaries of the unknown? To bravely, actively explore the enormity of our universe?"

#### -Robyn Mundell, Brainwalker

"The more clearly we can focus our attention on the wonders and realities of the universe about us, the less taste we shall have for destruction."

#### -Rachel Carson

"One of the basic rules of the universe is that nothing is perfect. Perfection simply doesn't exist. Without imperfection, neither you nor I would exist."

#### -Stephen Hawking

"Tune your television to any channel it doesn't receive and about 1 percent of the dancing static you see is accounted for by this ancient remnant of the Big Bang. The next time you complain that there is nothing on, remember that you can always watch the birth of the universe."

-Bill Bryson, A Short History of Nearly Everything

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## Preface

Cosmology is the study of the origin, size, and evolution of the entire universe. Every culture has developed a cosmology, whether it be based on religious, philosophical, or scientific principles. In this book, the evolution of the scientific understanding of the universe in the Western tradition is traced from the early Greek philosophers to the most modern 21st century view.

This book began as a series of lecture notes for a one-semester course at the University of Notre Dame called "Elementary Cosmology." An elective for non-science majors, it was designed to acquaint the non-mathematically-inclined student with the most important discoveries in cosmology up to the present day and how they have constantly altered our perceptions of the origin and structure of the universe. It examined such questions as: "Where did the universe come from?" or "Why do scientists now feel sure that its birth was in a great cosmic fireball called the Big Bang?" and "Where did the Big Bang itself come from?" The emphasis was on class discussion of readings from "science popularizations" for the curious and intelligent layperson, focusing eventually on the many interesting and exciting new discoveries in cosmology in the late 20th and early 21st century.

After a brief introduction to the concept of the "scientific method," which underpins all scientific approaches to the study of the universe, the first part of the book describes the way in which detailed observations of the universe, first with the naked eye and later with increasingly complex modern instruments, ultimately led to the development of the "Big Bang" theory which supplies the framework for our current understanding of cosmology. The key to this theory is the realization that our universe, far from being static and eternal, has instead been expanding in size since its origin some 13.5 billion years ago. While the fact of this expansion was accepted rather soon after it was first proposed in the 1920s and 1930s, the more radical idea that the universe had a birthday was more difficult for scientists to accept. It was only with the development of modern, satellite-based communication devices in the 1960s that instruments sensitive enough to detect the "cosmic microwave background" (CMB) were produced. It is now understood that the CMB consists of ancient light, emitted at a time near to the formation of the universe. As such it's the "smoking gun" of the Big Bang, and detailed studies of its properties have led to many interesting and fascinating new discoveries in cosmology.

The second part of the book traces the evolution of the Big Bang theory itself, including the very recent observation that the expansion of the universe is itself accelerating with time. In addition, the contribution of modern physics to our understanding of the mechanism of the Big Bang is discussed, and the state of the universe at various eras throughout its history is described. Finally, some speculations beyond current knowledge that bear on cosmology are introduced and the implications for future developments in our understanding of the universe are described.

The text contains many links to websites that clarify and extend the discussion. By following these links (some to images and videos), the reader can attain a much more in-depth understanding of many of the concepts introduced in this book. This is especially true for those seeking a more mathematical discussion of these topics, which is beyond the level of the current text.

# Acknowledgements

To my wife Ann who encouraged me throughout this book project, and to the many Notre Dame students whose questions and comments over the years contributed immensely to the development of the Elementary Cosmology course. I also acknowledge with gratitude the assistance of Robert Trevelyan of IOP Publishing in the production of this manuscript in its final form.

# Author Biography

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James J. Kolata is an emeritus professor of physics at the University of Notre Dame in the USA and a Fellow of the American Physical Society. He is the author of over 300 research publications in nuclear physics.



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## Chapter 1

## The Scientific Method

In this chapter, we'll review the concepts of the scientific method and the representations of numbers large and small.

### 1.1 Introduction to the Scientific Method

The *scientific method* is a procedure for obtaining knowledge about the universe around us. It begins with the assumption that there is an objective reality that can be observed and measured, and provides a framework for systematizing and extending these observations. A key characteristic of the method is that it seeks to eliminate individual bias by insisting on reproducibility of results. In order to describe the method in more detail, it's necessary to define a number of terms, some of which have different meanings from the way they are understood in common usage. These are:

- Fact: The result of an experiment or observation. In the scientific context, facts are observations that have been repeatedly reproduced and confirmed so that they are regarded as "true" for all practical purposes. However it's important to recognize that truth in science is never final, and facts accepted today may be changed or even discarded tomorrow. This is quite different from the common usage in which a fact is often assumed to be an immutable truth.
- **Hypothesis**: A tentative explanation of a fact or set of facts that has not yet been extensively tested. It should lead to predictions that can either be verified or proved false.
- Law: A verified hypothesis about one specific aspect of the universe that describes what it does under a particular set of circumstances. An example from physics is Ohm's law, which describes the behavior of electrical currents in certain types of materials. A physical law is usually expressed in a mathematical equation that relates two or more quantities, as in Newton's law of gravity that relates gravitational force, masses, and distances.

• **Theory:** A conceptual framework that explains existing facts and correctly predicts the results of new observations. It can include facts, laws, and verified hypotheses, and in science it exists on a higher plane than any of these. This is undoubtedly the greatest difference in terminology from common usage, where one often hears that something is "only a theory and not a fact."

With this background, it's possible to describe the scientific method as a list of procedures to be followed:

- (1) **Observe** some aspect of the universe.
- (2) Construct a hypothesis that is consistent with your observations.
- (3) Use the hypothesis to make predictions.
- (4) Test those predictions by making further observations or experiments.
- (5) **Modify** your hypothesis in the light of your new results.

After repeated application of these procedures, the hypothesis can be considered to be verified and it may be possible to incorporate it into a scientific theory. Some characteristics of a successful scientific theory are:

- Generality: The ideal theory should explain as many facts as possible. A theory may at times serve as a fact explained by a more general theory having a greater range of applicability. An excellent example that will be discussed further is Kepler's laws of planetary motion, which were shown by Newton to be a direct consequence of his theory of universal gravitation. In turn, Newton's theory itself is an approximation to Einstein's general theory of relativity in cases where gravitational forces are weak.
- **Testability:** A theory should correctly predict new facts or observations that can be made. This principle has also been called "falsifiability." A theory with a limited range of applicability is at times called a model. It's especially important to guard against unwarranted extensions of a theory to areas where it has not yet been fully tested.
- **Beauty:** The simplest theory that can adequately account for a given set of facts will be preferred. In philosophy, this is known as Occam's razor.

It's also worthwhile to discuss the limits of the scientific method. Science seeks natural explanations for natural phenomena. At some point, current scientific knowledge becomes insufficient to provide such explanations. Scientists don't then stop trying to understand the phenomenon, but the "explanations" typically become increasingly speculative and enter the realm of "metaphysics," a branch of philosophy. Further work may ultimately reveal the "true" theory, and the history of cosmology is full of examples of this type of evolution.

### **1.2 Some Mathematics**

One of the more remarkable aspects of the study of cosmology is that the subject deals with both the largest and smallest objects in the physical universe. Because of this, it's important to review "scientific notation," which provides a compact

notation for numbers by expressing them in terms of powers of ten. Consider the quantity  $1.2 \times 10^3$  written in this scheme. The power of plus 3 implies that the decimal point is to be moved three places to the right to recover the number 1200. Similarly, small numbers are represented by negative powers. For example, the quantity  $1.2 \times 10^{-4}$  implies that the decimal point is to be moved four places to the left, yielding the number 0.00012. Note that the extra places were filled by zeros in both these examples. However numbers may be given with a higher degree of precision, as in the case of the number  $1.2567 \times 10^4$  which translates to 12,567.

Numbers in science are associated with their corresponding "units." We will be using the "metric system" or MKS system in which the unit of length is the meter (m), the unit of mass is the kilogram (kg), and the unit of time is the second (s). The use of the kilogram illustrates another way to achieve a more compact notation, by the use of a prefix (in this case "kilo") in front of another unit (in this case the gram). Some of the most-used prefixes and their translations are:

kilo (k)	10 <sup>3</sup>	milli (m)	10 <sup>-3</sup>
Mega (M)	10 <sup>6</sup>	micro (µ)	10 <sup>-6</sup>
Giga (G)	10 <sup>9</sup>	nano (n)	10 <sup>-9</sup>
Tera (T)	10 <sup>12</sup>	pico (p)	$10^{-12}$
Peta (P)	10 <sup>15</sup>	femto (f)	$10^{-15}$
Exa (E)	10 <sup>18</sup>	atto (a)	$10^{-18}$
Zetta (Z)	10 <sup>21</sup>	zepto (z)	$10^{-21}$
Yotta (Y)	10 <sup>24</sup>	yocto (y)	$10^{-24}$

So 1 kilogram (kg) is equal to 1000 g. Notice that each prefix in this table is separated from the following one by a factor of  $10^3$ , or three *orders of magnitude* (factors of ten). Also, large units are associated with capitalized prefixes (with the sole exception of kilo). Finally, one commonly-used prefix that doesn't appear is "centi" (c), corresponding to  $10^{-2}$  as in the centimeter (cm) or 1/100 m. A good example of the concept of "orders of magnitude" as applied to a small fraction of the length scales we'll encounter in cosmology is contained here: Powers of Ten (https://micro.magnet.fsu.edu/primer/java/scienceopticsu/powersof10/index.html).

### 1.3 Exponential Growth

Later on in this book (Section 11.5), the fact that the early universe experienced a period of *exponential growth* will be discussed, so it's useful to review this concept in some detail. People unfamiliar with the properties of exponential growth are often surprised at how fast systems experiencing it can grow. A nice example is the Paal Payasam Legend of India:

The tale goes that the tradition of serving Paal Payasam (a sweet rice pudding) to visiting pilgrims started after a game of chess between a local king and the Lord Krishna himself.

The king was a big chess enthusiast and had the habit of challenging wise visitors to a game of chess. One day a traveling sage was challenged by the king. To motivate

his opponent the king offered any reward that the sage could name. The sage asked for a few grains of rice in the following manner: the king was to put a single grain of rice on the first chess square and double it on every subsequent one. The king marveled at the modesty of this request.

Having lost the game, and being a man of his word, the king ordered a bag of rice to be brought to the chess board. Then he started placing rice grains according to the arrangement: 1 grain on the first square, 2 on the second, 4 on the third, 8 on the fourth and so on.

The king soon realized that he was unable to fulfill his promise, because on the twentieth square he would have had to put  $2^{19} = 524,288$  grains of rice. On the fortieth square, the king would have had to put  $2^{39}$  (about 550 billion) grains of rice. And finally, on the sixty-fourth square, the king would have had to put  $2^{63}$  (or about 9 million trillion) grains of rice! At that point, the total weight of rice on the chess board would be equal to about 550 billion metric tons, or about 1000 times the total world rice production in 2018. Of course, that amount of rice wouldn't fit on the chess board. In fact, if placed in a layer two grains deep it would cover the entire surface area of the Earth, oceans included!

It was at that point that the Lord Krishna revealed his true identity to the king and told him that he didn't have to pay the debt immediately but could do so over time. That is why to this very day visiting pilgrims are still feasting on Paal Payasam and the king's debt to Lord Krishna is still being repaid.

Even more remarkably, what if the Lord Krishna had been greedy and asked for 3 grains on the second square, nine on the third square, and so on? It turns out that



Figure 1.1. Exponential growth plotted on a linear (upper) and log (lower) scale.

the total amount of rice would then have been about 200 billion times the immense amount mentioned above!

The very rapid increase in exponential decay leads to another misapprehension. Consider the upper of the two graphs above, on which the two cases mentioned above are plotted (the red curve actually extends far above the limit of the scale shown in the upper plot; Figure 1.1).

As is apparent, the number of grains on succeeding squares looks to be miniscule for a long time then suddenly appears to grow rapidly, at about square 60 for the  $y = 2^x$  plot. This is often interpreted as a *surge* or *spike* in the number, whereas in reality the value has been growing continuously but in an exponential fashion. A much better way of visualizing exponential growth is by way of a *logarithmic* (log) plot, the lower of the two graphs above. In such a plot, the y value is taken to be the logarithm of the actual number. A **logarithm** is the power to which a number (called the *base*) must be raised in order to get some other number. (The plot above uses base-10 logarithms.) As can be seen, exponential growth is a straight line on a log plot, which makes it much easier observe any deviation from pure exponential growth.

#### Full list of references

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