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Introduction to Nonlinear Optics of Photonic Crystals and Metamaterials (Second Edition)

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Chapter 1

Introduction

In this book an introduction and discussion of some of the basic principles of linear and nonlinear optical nano-systems are given. The focus is on engineered optical systems that have been of recent interest in physics, engineering, and applied mathematics for their opto-electronic applications. These include photonic crystals and metamaterials, and in the following discussions the operating principles of photonic crystals and metamaterials are outlined. Photonic crystals, which have been of great interest in opto-electronic designs, are materials that exhibit a periodic dielectric variation in space and are designed to manipulate light with wavelengths of order of the length scales of the periodicity of the photonic crystal lattice [1–5]. The manipulation of light is accomplished through the use of the Bragg scattering properties of the periodic lattice of the dielectric which may be periodic in one, two, or three dimensions. Metamaterials often have periodic dielectric and magnetic properties but are designed to manipulate light of wavelengths much greater than the periodicity of the dielectric and magnetic lattice [1–5]. Metamaterials are designed to exhibit homogeneous properties to the light traveling within them. While photonic crystals are of interested for the diffractive effects they have on light propagating within them, metamaterials are of interest for the permittivity and permeability that they exhibit when they are treated as a uniform medium.

For both systems a particular goal of our presentation is to describe their behaviors as nonlinear optical systems [6–9]. To accomplish this, a general review is also given of the properties of these systems considered as linear optical systems. The treatments assume an undergraduate background in electrodynamics and are developed starting from an elementary level. The presentation provides an exposition of the basic principles of the photonic crystal and metamaterial systems. The wave excitations found in linear medium systems and their elementary properties are treated in detail. A detailed development, requiring no previous background in

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nonlinear materials, of the excitations in nonlinear systems consisting of plane waves and bright, dark, and gray solitons is also given.

1.1 Photonic crystals

Photonic crystals are periodic arrays of different dielectric materials [1–5]. They are literally crystals formed from dielectric materials rather than the individual atoms of materials such as crystalline NaCl, $CaCl_2$, etc The interest in the dielectric array is that it exhibits a band structure for the propagation of electromagnetic modes similar to the electronic band structure exhibited in metallic and semiconductor materials used in electronic designs [10]. As with the electron systems, the interest in the optical systems is in modes that have wavelengths of order of the length scale of the periodicity of the system. At these length scales the system displays a variety of diffraction effects on the excitations in the system. Various pass and stop frequency bands are opened in the frequency spectrum of excitations that can exist in the photonic crystal. Light at frequencies in a pass band will propagate through the photonic crystal while light at frequencies in a stop band will not propagate in a photonic crystal and are typically reflected from the system.

The existence of pass and stop frequency bands is the basis for engineering applications of photonic crystals. A cavity resonator, for example, can be formed as a cavity within the bulk of a photonic crystal. An electromagnetic wave with a frequency in the stop band of the photonic crystal cannot propagate out of the cavity through the bulk photonic crystal so it will be confined to the cavity. Such cavities offer high Q resonance cavities for laser applications that are not available through other technologies [11]. Another application of photonic crystal cavities is in the suppression and enhancement of atomic transitions. If the excited state of an atom which radiates at a stop band frequency is put in the cavity it will be suspended in the excited state due to its inability to radiate the excitation energy though the bulk of the photonic crystal. Similarly, the frequency mode density of states in a photonic crystal pass band can be enhanced from those of free space. This enhancement of the density of states increases the rate of decay of an excited atom within a photonic crystal cavity into enhanced pass band frequency modes over its free space decay rate [11].

The stop band effect can also be used in the design of waveguides meant to channel the flow of light through space [1–5]. A channel cut through the bulk of a photonic crystal will restrict light at frequencies in the stop bands of the bulk photonic crystal from propagating away from the waveguide, i.e., light at stop band frequencies will only move along the channel of the waveguide. Waveguides based on such photonic crystal designs can be more effective than fiber optic technologies in forming optical circuits. For example photonic crystal waveguides afford the possibility of sharper bends in their guiding channels and lower losses in general than are found in traditional fiber optics approaches [12]. In addition, photonic crystals can be designed from a wider variety of materials than are typically used in fiber optics.

One method of introducing a waveguide into a two-dimensional photonic crystal formed as a periodic array of identical parallel axis dielectric cylinders will be a focus in later considerations of guided modes. For this system the interest is in light propagating in the plane perpendicular the axes of the cylinders. A waveguide is introduced into the cylinder array by replacing a periodic array of dielectric cylinders of the photonic crystals by a set of identical impurity cylinders with different dielectric properties from those of the photonic crystal. This is done along a crystal axis of the two-dimensional array of the photonic crystals, and by choosing the impurity dielectric correctly a set of waveguide modes bound to and propagating along the array of waveguide impurity cylinders can be made to exist at stop band frequencies of the original photonic crystal. Later it is shown that the modes of this kind of waveguide system can be represented by a set of difference equations. This representation provides a helpful means of understanding the physics of photonic crystal waveguides and circuits.

One of the earliest applications of the ideas of photonic crystals, occurring in the initial stages of the developments of laser technology, was the application of one-dimensional or layered photonic crystals in the design of laser mirrors [1–5, 13]. Here the periodic layering of different dielectric media can be used to create low loss, highly reflective mirrors [13]. Similar layering effects are observed in insects in which the metallic, mirrored appearance of the shell of the animal is due to the layering of dielectrics rather than to a presence of metallic reflecting elements in the shell of the insect. In both of these systems the presence of pass and stop bands of the layering leads to the functioning of the reflecting surfaces. In addition to the insect example of photonic crystals developed in nature, a number of periodic one-, two-, and three-dimensional nano-systems that occur naturally in plant and animal materials have been suggested as a basis for the design of photonic crystals [1–5, 13]. The interest in these comes from the pre-existing nano-scale periodicity which is hard to create in a laboratory. A number of laboratory techniques have been developed for creating photonic crystals, but these are not discussed here.

In addition to photonic crystals formed as arrays of slabs, or cylinders, or three-dimensional periodic ordering of dielectric features, surfaces with periodic surface profiles are also found to exhibit important photonic crystal properties [1–5]. Periodic surfaces have long been studied for their applications as diffraction gratings, and a brief review of their basic theory is presented. In addition, under certain well defined conditions periodic surfaces can support surface electromagnetic waves. These are electromagnetic modes which are bound to the interface between two media, representing excitations localized about the interface and propagating parallel to the interface. These surface waves are important excitations with applications in electromagnetic scattering from the surface, in the design of various sensors, and in surface enhanced Raman scattering (SER) [1–5]. They feature prominently in the diffuse scattering of light at rough surfaces and in the diffuse generation of second harmonics of light at rough surfaces.

In regards to nonlinear effects in photonic crystals and waveguides, a treatment of the basic properties of parametric oscillators and parametric amplifiers is given [6, 7]. These are discussed in both bulk optical media and in the context of nonlinear fiber optical waveguides [12]. Bragg grating technology in fiber optics is also a topic. These systems are represented in many important technological applications.

1.2 Metamaterials

Metamaterials are artificial materials that are designed as arrays of nano-circuits known as split ring resonators (SRRs) [14]. SRR nano-circuits are inductor–capacitor resonator circuits which in metamaterial designs may be arrayed on a periodic nano-lattice or in a more complex array arrangement. The resonator circuits are known as SRRs because in the basic form of their design they are a metal ring with a split gap interrupting the ring. The loop of the ring provides self-inductance to the SRR, and the gap of the SRR is filled with a dielectric material to form a capacitor. Composed in this way the SRR is found to resonate at the frequency $\omega_{\text{SRR}} = \frac{1}{\sqrt{LC}}$, where L is the SRR self-inductance and C is its capacitance [15–21]. For practical applications more complex SRR designs are used but all of the various designs operate on the basic principles that are outlined above. Within a particular metamaterial array the SRRs are coupled together by mutual inductive couplings between the various SRRs. This gives rise to many-body effects which also affect the optics of the material.

The purpose behind the composition of metamaterials as arrays of SRRs is to use their flexibility of design to create materials with enlarged sets of permittivity, ε, and permeability, μ, from those of naturally occurring materials [15–24]. In naturally occurring molecular solids the permittivity and permeability of the bulk solid arises from those of the molecules forming the systems. These properties are fundamentally limited by the nature and interactions of the molecular constituents of the materials. For example, in no naturally occurring material is it found that $\varepsilon < 0$ and $\mu < 0$ simultaneously at a single frequency of an applied electromagnetic plane wave. Regions of frequency are often found for which $\varepsilon < 0$ and regions of frequency are often found for which $\mu < 0$, but no natural materials have been found for which these two conditions are simultaneously satisfied at the same frequency. This limitation on ε and μ comes from the size restrictions of the molecular units and their effect on the magnetic response of molecular solids. The development of an artificial nano-circuit array allows these restrictions to be overcome.

At microwave and terahertz frequencies engineered SRR materials with $\varepsilon < 0$ and $\mu < 0$ simultaneously can be formulated at these frequencies through the use of an SRR array to customize the magnetic response of the material. The SRR ring geometry is arranged so that the resonant frequency $\omega_{\text{SRR}} = \frac{1}{\sqrt{LC}}$ of the ring is in the microwave or terahertz region. Under this condition it is found that the size of the SRRs needed is much greater than the molecules available in molecular solids. In addition, at frequencies near ω_{SRR} the wavelength of the externally applied electromagnetic waves is slowly varying over a volume containing many SRRs so that the SRR material appears to the external wave to be a continuous medium. As the frequency, ω, of an externally applied field is changed so as to pass through the SRR's resonance at ω_{SRR}, the magnetic moment of the SRR changes sign. Consequently, about the resonance of the SRR system the SRR can exhibit a frequency region in which $\mu < 0$. This artificially induced $\mu < 0$ is easily correlated with a region of $\varepsilon < 0$ for the dielectric media in which the rings are embedded. The resulting metamaterial displays a set of frequencies for which $\varepsilon < 0$ and $\mu < 0$ at the same time.

1.3 Negative index of refraction

With the introduction of metamaterials a full range of ε and μ values become available for optical design. In particular, the case of $\varepsilon < 0$ and $\mu < 0$ is now a practical possibility. This is very important for refractive optics as materials with $\varepsilon < 0$ and $\mu < 0$ will exhibit negative refractive indices [18, 22, 25, 26]. The possibility of negative indexed materials was theoretically considered early in the twentieth century but was only treated as a curiosity [25, 26]. In naturally occurring positive index materials figure 1.1 shows that light incident on a planar interface from a positive indexed medium to an optically rarer or denser positive indexed medium is refracted from the second to the fourth quadrants. In negative index materials figure 1.2 shows that light incident on a planar interface from a positive indexed medium to a negative indexed material is refracted from the second to the third quadrants. Consequently, light incident in the second quadrant can now be guided though any angle in the forward direction of the third and fourth quadrants. This has found applications in cloaking protocols in which light directed at an object can be guided around it by using a continuous variation of positive and negative refractive indices [14, 18, 27]. Once around the object the light is sent along a path set to give the appearance that the object is not present. The object is made to appear not present even in its presence. Other such variations of positive and negative indices have found applications in simulating optical effects from general relativity. In noting these interesting properties of metamaterials, it should also be pointed out that under special circumstances the Bragg reflection properties of photonic crystals,

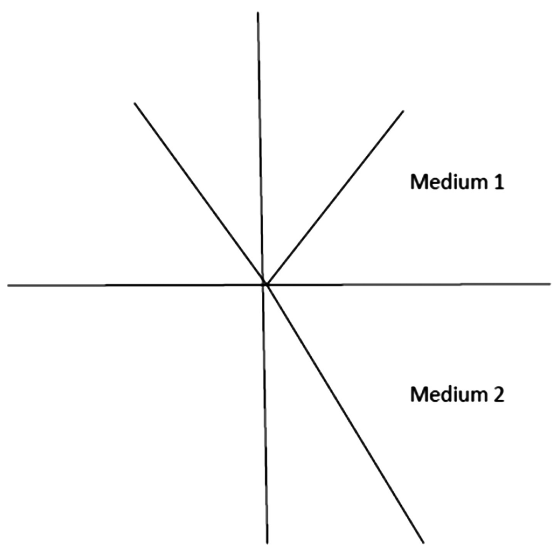

Figure 1.1. Schematic for refraction of light at the interface between two positive indexed media. The incident wave in medium 1 is transformed at the surface into a reflected wave in medium 1 and into a refracted wave in medium 2. The incident wave is in the second quadrant and for the positive media the refracted wave is in the fourth quadrant.

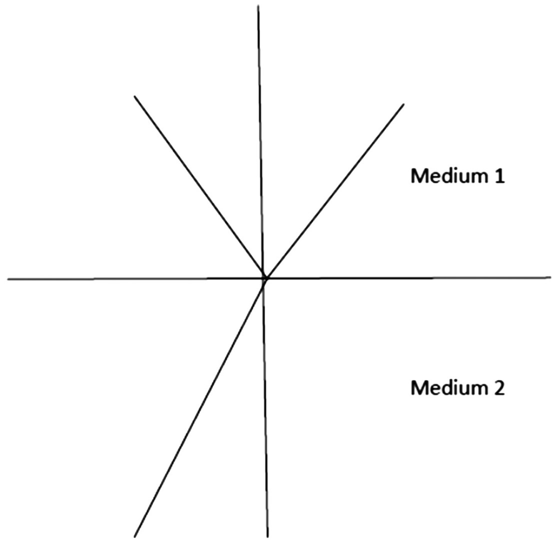

Figure 1.2. Schematic for refraction of light at the interface between positive indexed medium 1 and negative indexed medium 2. The incident wave in medium 1 is transformed at the surface into a reflected wave in medium 1 and into a refracted wave in medium 2. The incident wave is in the second quadrant and for the negative index medium the refracted wave is in the third quadrant.

related to the periodicity of the dielectric medium, have been found to mimic some of the properties of metamaterials.

The positive and negative indexed materials are alternatively referred to as right- and left-handed materials due to the different relationship of the three orthogonal vectors of the electric vector, magnetic vector, and wave vector between the two types of systems [26]. Indeed, it can be shown that waves in right-hand materials propagate electromagnetic energy parallel to the wave vector while waves in left-hand materials propagate electromagnetic energy antiparallel to the wave vector. This can have effects for example on the Cherenkov radiation in right- and left-handed materials, respectively, as well as of course for the refractive properties and the properties of antenna radiation [14, 24, 27].

1.4 Perfect lenses

One particularly interesting applications of metamaterials is in the design of so-called perfect lenses [22]. The functioning of the perfect lens is based on the new refractive property of negative index of refraction. The bending of light at the interface of a positive and negative indexed material from the second to the third quadrant allows a planar surface to form a focused image as light passes through planar surfaces. This is due to the increased bending of the light as it goes between the two media over that found between any two positive indexed media.

In the absence of negative refractive indexed materials, a curved surface is needed between two positive indexed media to focus light as it passes between the two media. This is the operating principle in the design of telescope and microscope lenses.

The design of lenses with curved surfaces limits their size. Consequently, only light passed through the aperture of the lens can be focused, and this limits the wavelength of light which will pass through the lens and become part of the image located at the focus of the lens [28]. Light must have a wavelength less than the aperture of the lens to reach the focus and, consequently, the image formed by the lens is not perfect as the loss of some wavelength components of the image decreases the resolution of the image.

With a perfect lens formed of negative refractive indexed material only planar surfaces are needed to make a focusing lens. In addition, the permittivity and permeability can be adjusted so that even the evanescent components of light from the object are reassembled at the image, giving a complete characterization in the image of the object. In principle an image with perfect resolution can be formed by imaging with an infinite slab of negative index medium. There are, however, a number of technical difficulties in the practical application of the ideas of a perfect lens. Due to the resonant nature of the materials designed to exhibit negative refractive index, the frequencies over which these properties are exhibited are limited. In addition, resonant structures tend to exhibit losses, well known from the Kramers–Kronig relations.

1.5 Periodicity

Both photonic crystals and metamaterials can be formed by the repetition of mesoscopic features on a periodic lattice. In the case of photonic crystals the interest is in the manipulation of electromagnetic radiation with wavelengths of order of the lattice constant, while metamaterials are used in the management of radiation with wavelengths much larger than the lattice constant. Metamaterials appear to be homogeneous while the important properties of the photonic crystals arise from their detailed periodic structure. For both photonic crystal and metamaterial systems formed from linear dielectric media the light propagating in the systems is electromagnetic waves satisfying linear wave equations. In metamaterials the excitations are simple plane waves and the effect of the media on the propagating waves is characterized by the effective ε and μ of the metamaterial. For the photonic crystal, on the other hand, a full account of the periodicity of the dielectric media must be given so that the excitations are more complex than simple plane waves, and a variety of methods for the calculation of the electromagnetic band structure of these materials are available. These methods are well known, having initially been developed for the treatment of the motion of electrons in the periodic ion background of semi-conductors and metals. Among those methods discussed later are the plane wave expansion method and the method of Wannier functions [2, 4, 10]. In addition, computer simulations based on finite difference time domain methods [29] and the method of moments [30] are useful in determining the system properties.

The plane wave expansion technique for photonic crystals is based on Fourier transforming the equations of motion of the systems and studying their properties in frequency–wave vector space. These methods give a good account of the properties of linear photonic crystals and are quick and easy to implement. Computer simulation methods in space and time or space and frequency can also be applied to determine the dispersion relation and field properties of the modes of the photonic crystal system. Such computer simulations are particularly important in applications treating waveguides and impurities in both photonic crystal and metamaterial systems. The treatment of waveguides and impurities is also a strength of the more analytic methods of Wannier functions [10, 31, 32]. In the method of Wannier functions, an orthogonal set of basis functions which are localized in space are generated. This set is then used to expand the modes of photonic crystals and study their properties. All of these methods will be discussed later.

1.6 Excitations in nonlinear media

While linear systems are easily handled numerically, nonlinear systems in which the dielectric properties depend on the amplitude of the excitations they support are much more challenging. In nonlinear systems, a linear combination of the wave functions of two separate solutions of the system is not a wave function solution of the system, i.e., the principle of linear superposition of modes is no longer valid. The violation of the principle of linear superposition is easily seen in the wave-like modes of nonlinear systems [7]. Due to the amplitude dependence of the dielectric, wave solutions of nonlinear systems have dispersion relations that depend on the amplitude of the wave. Consequently, increasing or decreasing the amplitude of a wave changes the frequency of its oscillation, and this change in frequency is not accounted for in a simple linear combination of plane wave solutions.

In addition, nonlinear systems exhibit new types of excitations not found in linear systems. Examples of these are solitons [9, 33–36]. Solitons occur as pulses and kinks that propagate through nonlinear optical systems. They exist due to the dependence of the dielectric properties of the systems on the amplitude of the excitations. This can be understood as follows: in a system composed solely of a linear dielectric it is possible to bind a localized electromagnetic mode to a finite region of space by appropriately changing the value of the linear dielectric in that finite region. In line with these ideas, introducing a localized electromagnetic pulse into a nonlinear dielectric system can be done in such a way that the change in the nonlinear dielectric constant caused by the changing pulse intensity supports the localized pulse. The localized pulse in turn then supports the change in the nonlinear dielectric. Such a pulse can be stationary or it can travel through the system with a constant velocity. The pulse is known as a bright soliton because of its intensity maximum. These ideas also apply to kinks. For the case of kink excitations the electric field intensity exhibits a dip rather than an intensity peak. The kink intensity shapes the dielectric response of the system which in turn supports the kink wave function. As in the case of bright solitons, kinks can also be either stationary or move through the system with constant velocities. If the intensity of the dip goes to zero the kink is known as a dark soliton, and if the intensity dip does not go to zero the kink is known as a gray soliton. In addition to the single soliton solutions there are solutions involving a number of solitons scattering from one another.

The general types of solitons we will look at are bright, dark, and gray solitons. The bright, dark, and gray solitons that are found in our photonic crystals and metamaterials are further classified by the nature of their wave functions [33, 35] into simple pulsed solitons and envelope solitons. Simple pulsed solitons occur as smooth, slowly varying pulses or a smooth, slowly varying twisting of the amplitudes of the electromagnetic fields in space. Simple pulses do not exhibit any sinusoidal modulations of the basic wave function envelopes of the excitations. Envelope solitons, on the other hand, are more complex, being composed of plane waves modulated by an envelope in the form of a pulse or kink. For these cases the solitons may be obtained in the continuum limit of the system or in the discrete lattice of the system. In discrete lattice systems they are often referred to as intrinsic localized modes or discrete breathers in the limit where they are stationary in the system. These distinctions will be discussed later.

Later the theory of soliton-like modes is discussed for both the discrete and continuum limits of photonic crystal and metamaterial systems [35]. The solutions in the discrete lattice systems are shown to be easily accessible to a variational treatment in which the wave functions are expressed as linear expansions in an appropriate set of basis states defined over the discrete lattice. Solutions of the form of bright, dark, and gray soliton-like excitations are obtained [33, 35]. In the continuum limit of the lattice systems descriptions in terms of the nonlinear Schrödinger and Klein–Gordon equations are given for photonic crystal and metamaterial systems. Both systems are found to exhibit closed form soliton-like solutions, and the properties of the bright, dark, and the gray solitons found in these systems are discussed. The relationships between the soliton-like excitations in the nonlinear Schrödinger and Klein–Gordon equations are also demonstrated.

1.7 Systems with defects and disorder

An important factor entering the study of optical components is the effects on the system arising from the introduction of disorder into the problem [37–49]. Disorder can enter the problem in the form of a mild or small renormalization of properties or with increasing disorder as a transition of the system to a whole new range of characteristic behaviors [37–40]. This change of behavior is essentially a phase transition similar to other magnetic or chemical phase transitions [10]. In the present case, however, the phase transition is observed in a plot of the system transport properties as a function of the intensity of disorder in the materials.

With the introduction into a system of mild disorder the studies of its properties can be made using analytical or computer simulation methods which treat the renormalized forms of the modal excitations in the materials. As the disorder increases a point is reached in the increasing degree of disorder of the system at which a fundamental change occurs in the modal excitations of the materials [10, 37–40]. In this transition the functional forms of the wave function are transformed to new types. An example of this is the introduction of disorder into a homogeneous medium which supports plane wave modal excitations.

With weak disorder the excitations are renormalized plane waves which extend throughout the material. At a point of increased disorder in the medium, however, a transition is made so that the modes in the material become localized modes.

Specifically, the localized modes are restricted to a finite region of space in the materials. This modal transition changes the transport properties of the material, and in electrical conductors is observed as a metal–insulator transition known as the Anderson transition [10, 37]. Similar transitions from conductive to non-conductive behaviors with the presence of increasing disorder are observed in optical and acoustic materials.

Related to random disorder, site impurities and lines or clusters of impurities can also be purposely introduced into systems in order to form various types of resonant structures and waveguides. These may have various technological applications and may exist in engineered materials in addition to design imperfections. They form an important topic of technological interest and are introduced in the context of the applications of group theory techniques in their study

Consequently, as a final topic in the book an introduction is given to the study of disorder in optical media [34–49]. This will include some basic studies of single site and multiple site impurities in photonic crystals and metamaterials. For this treatment, some elements of group theory are introduced into the discussion of cluster impurities. This is followed by an introduction of Anderson localization. First a treatment of localization is given in the context of conductivity in electronic systems [10, 37]. This is followed by the occurrence of the localization transition in the treatments of transport properties of optical systems. As a final point, we conclude with the discussion of weak Anderson localization in the diffuse reflection of light from randomly rough planar and periodic surfaces [44, 49], and in various effects in the presence of nonlinearity.

The order of the book is to first begin with a treatment of the dielectric properties of materials. The properties and models of linear photonic crystals and metamaterials are next discussed along with the basic methods used to compute their properties. This is followed by discussions of nonlinear photonic crystal and metamaterial models and their theoretical treatments. Various soliton modes and discrete breathers found in these systems are presented along with some final comments on computer simulation studies. To conclude, a treatment of sites, clusters, and waveguide impurities are given, followed by a general discussion of Anderson localization.

References

[1] Joannopoulos J D, Vilenueve P R and Fan S 1995 *Photonic Crystals* (Princeton, NJ: Princeton University Press)

[2] Sakoda K 2001 *Optical Properties of Photonic Crystals* (Berlin: Springer)

[3] Favennec P N 2005 *Photonic Crystals: Towards Nanoscale Photonic Devices* (Berlin: Springer)

[4] McGurn A R 2002 *Survey of Semiconductor Physics* ed W Boer (New York: Wiley) ch 13

[5] Waser R (ed) 2005 *Nanoelectronics and Information Technology: Advanced Electronic Materials and Novel Devices* 2nd edn (Weinheim: Wiley)

[6] Boyd R W 2003 *Nonlinear Optics* 2nd edn (Amsterdam: Academic)

[7] Mills D L 1998 *Nonlinear Optics* (Berlin: Springer)

[8] Banerjee P P 2004 *Nonlinear Optics* (New York: Dekker)

[9] Kivshar Y S and Agrawal G P 2003 *Optical Solitons* (Amsterdam: Academic)
[10] Marder M P 2010 *Condensed Matter Physics.* 2nd edn (Hoboken, NJ: Wiley)
[11] Noda S, Fujita M and Asano T 2007 Spontaneous-emission control by photonic crystals and nanocavities *Nat. Photonics* **1** 449
[12] Agrawal G P 2012 *Fiber-Optics Communication Systems* 4th edn (New York: Wiley)
[13] Yeh P 2005 *Optical Waves in Layered Media* (New York: Wiley)
[14] Engheta N and Ziolkowski R W (ed) 2006 *Metamaterials: Physics and Engineering Explorations* (New York: Wiley-IEEE)
[15] Giri P, Choudhary K, Sen Gupta A, Bandyopadhyay A K and McGurn A R 2011 Klein–Gordon equation approach to nonlinear split-ring resonator based metamaterials: one-dimensional systems *Phys. Rev.* B **84** 155429
[16] Eleftheriou M, Lazarides N and Tsironis G P 2008 Magnetoinductive breathers in metamaterials *Phys. Rev.* E **77** 036608
[17] Eleftheriades G V 2009 EM transmission-line metamaterials *Mater. Today* **12** 30
[18] Kourakis I, Lazarides N and Tsironis G P 2007 Self-focusing and envelope pulse generation in nonlinear magnetic metamaterials *Phys. Rev.* E **75** 067601
[19] Lapine M, Gorkunov M and Ringhofer K H 2003 Nonlinearity of a metamaterial arising from diode inserts into resonant conductive elements *Phys. Rev.* E **67** 065601
[20] Shadrivov I V, Morrison S K and Kivshar Y 2006 Tunable split-ring resonators for nonlinear negative-index metamaterials *Opt. Express* **14** 9344
[21] Tamayama Y, Nakanishi T and Kitano M 2013 Observation of modulation instability in a nonlinear magnetoinductive waveguide *Phys. Rev.* B **87** 195123
[22] Pendry J B 2000 Negative refraction makes a perfect lens *Phys. Rev. Lett.* **85** 3966
[23] Agranovich V M and Gartstein Y N 2006 Spatial dispersion and negative refraction of light *Phys.–Usp.* **49** 1029
[24] Agranovich V M 2006 Hybrid organic–inorganic nanostructures and light–matter interaction *Problems of Condensed Matter Physics* ed A L Ivanov and S G Tikhodeev (Oxford: Clarendon) ch 2
[25] Ramakrishna S A 2005 Physics of negative index materials *Rep. Prog. Phys.* **68** 449
[26] Veselago V G 1968 The electrodynamics of substances with simultaneously negative values of ε and μ *Sov. Phys.–Usp.* **10** 509
[27] Cai W and Shalaev V 2001 *Optical Metamaterials: Fundamental and Applications* (New York: Springer)
[28] Smith W J 2007 *Modern Optical Engineering* 4th edn (New York: McGraw-Hill)
[29] Taflove A 1995 *Computational Electrodynamics: The Finite-Difference Time-Domain Method* (Boston, MA: Artech House)
[30] El Misilmani H M, Kabalan K Y, Abou-Shahine M Y and Al-Husseini M 2015 A method of moment approach in solving boundary value problems *J. Electromagn. Anal. Appl.* **7** 61
[31] McGurn A R 2005 Impurity mode techniques applied to the study of light sources *J. Phys.* **38** 2338
[32] McGurn A R 1996 Green's-function theory for row and periodic defect arrays in photonic band structures *Phys. Rev.* B **53** 7059
[33] Dauxois T and Peyrard M 2006 *Physics of Solitons* (Cambridge: Cambridge University Press)
[34] Sievers A J and Page J B 1995 *Dynamical Properties of Solids* ed G K Horton and A A Maradudin (Amsterdam: North-Holland)

[35] McGurn A R 2013 Barrier transmission map of one-dimensional nonlinear split-ring-resonator-based metamaterials: bright, dark, and gray soliton resonances *Phys. Rev.* B **88** 155110

[36] McGurn A R 2008 Transmission through nonlinear barriers *Phys. Rev.* B **77** 115105

[37] Lee P A and Ramakrishnan T V 1985 Disordered electron systems *Rev. Mod. Phys.* **57** 287

[38] Segev M, Silverberg Y and Christodoulides D 2013 Anderson localization of light *Nat. Photonics* **7** 197

[39] McGurn A 2020 *Introduction to Photonic and Phononic Crystals and Metamaterials* (San Rafael, CA: Morgan & Claypool)

[40] Shing P (ed) 1990 *Scattering and Localization of Classical Waves in Random Media* (Singapore: World Scientific)

[41] McGurn A R 2018 *Nanophotonics* (Cham: Springer)

[42] Cai W and Shalaev V 2009 *Optical Metamaterials* (Berlin: Springer)

[43] McGurn A R, Maradudin A A, Christensen K and Mueller F 1993 Anderson localization in one-dimensional randomly disordered optical systems that are periodic on average *Phys. Rev.* B **47** 13120

[44] McGurn A R 1990 Enhanced retroreflectance effects in the reflection of light from randomly rough surfaces *Surf. Sci. Rep.* **10** 357

[45] McGurn A R and Maradudin A A 1994 Localization of light in linear and non-linear random layered dielectric systems that are periodic on average in the proceeding of Third Int. Conf. on Electrical Transport and Optical Properties of Inhomogeneous Media *Physica* A **207** 435

[46] Devillard P and Souillard B 1986 Polynomial decaying transmission for the nonlinear Schrodinger equation in a random medium *J. Stat. Phys.* **43** 423

[47] McGurn A R, Sheng P and Maradudin 1992 Strong localization of light in two-dimensional disordered dielectric media *Opt. Commun.* **91** 175

[48] de Raedt H, Lagemdijk A and de Vries P 1989 Transverse localization of light *Phys. Rev. Lett.* **62** 47

[49] McGurn A R, Leskova T A and Agranovich V M 1991 Weak localization effects in the generation of second harmonics of light at a randomly rough vacuum-metal grating *Phys. Rev.* B **44** 11441