IOP Publishing

Principles of Biophotonics, Volume 2
Light emission, detection, and statistics
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Chapter 6

Black body radiation

The most commonly encountered electromagnetic radiation is of thermal origin. A *black body* is an idealized model of a physical object that absorbs *all incident* electromagnetic radiation. Because it is a perfect *absorber* at all wavelengths, a black body is also an ideal *emitter* of thermal radiation. This *black body radiation* has a certain frequency (or wavelength) distribution, which is characterized by a maximum. This spectral distribution of radiation by bodies at thermal equilibrium was a problem of critical interest at the turn of the 20th century, and led to the development of quantum mechanics.

Thermal radiation is emitted by a body that exists at a temperature higher than absolute zero. In essence, this radiation is generated by converting the internal energy of the body at thermal equilibrium and represents the reverse process to absorption. The spectral content of the radiation is determined by the *mode distribution*, that is, the spatial frequency content of the electromagnetic field within a certain bounded space, or *cavity* (figure 6.1). A mode of the electromagnetic field in the cavity satisfies the condition of vanishing electric field at the wall. Clearly, as the wavelength decreases, there are increasingly more ways of 'fitting' the modes in the cavity. The formula that correctly predicts the thermal radiation by a black body was derived by Planck in 1900 [1], as described below.

6.1 Planck's radiation formula

Let us consider a radiating cavity, with its dimensions much larger than the wavelength of light (see, e.g., [2]). The problem of finding the spectral distribution, $du(\nu)/d\nu$ (energy per unit frequency, ν), of the radiation emitted by this cavity, approximated by a black body, comes down to calculating the number of modes, $d\nu$, into the volume V, that exist within a certain frequency range $d\nu$. The radiated energy per mode, per unit volume is

doi:10.1088/978-0-7503-1644-6ch6

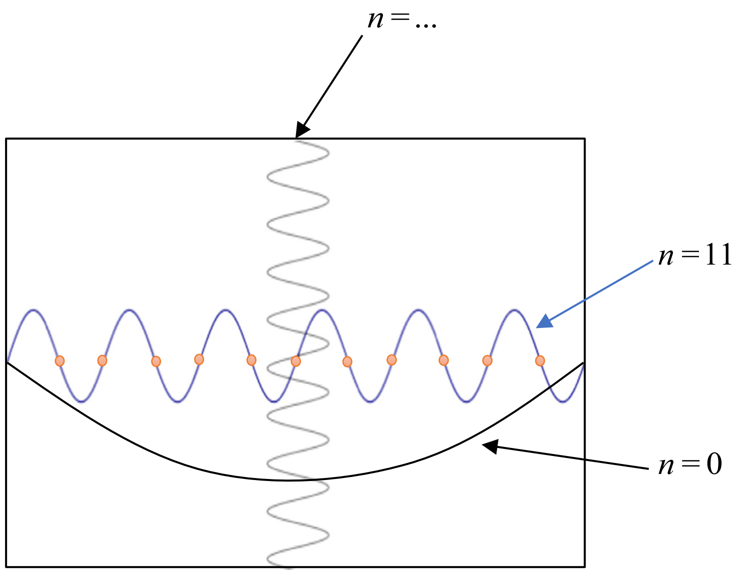

Figure 6.1. Cavity modes: the sinusoids represent the real part of the electric field. All surviving field modes have zero values at the boundary. The modes are indexed by n, the number of zeros along an axis (orange dots).

$$du(\nu) = 2f\frac{h\nu}{V}dN. \tag{6.1}$$

In equation (6.1), f is the *probability of occupancy* associated with a given mode, h is Planck's constant ($h = 6.6 \cdot 10^{-34}$ J s), and the factor two accounts for the two polarization modes that can exist in the cavity. The expression for f is obtained from the *Bose–Einstein statistics* that apply to indistinguishable particles with an unlimited state of occupancy, i.e. not obeying Pauli's exclusion principle [2],

$$f = \frac{1}{e^{\frac{h\nu}{k_B T}} - 1}, \tag{6.2}$$

where k_B is Boltzmann's constant, $k_B = 1.38 \cdot 10^{-23}$J/K. The average energy per mode is

$$\begin{aligned} \langle E \rangle &= fh\nu \\ &= \frac{h\nu}{e^{\frac{h\nu}{k_B T}} - 1}. \end{aligned} \tag{6.3}$$

Planck's formula predicts the spectral density of the radiation emitted by a black body, at thermal equilibrium, as a function of temperature. Let us assume a cavity of size much larger than the wavelength of light. Black body radiation applies to an object that absorbs all radiation incident to it, and re-radiates energy that depends

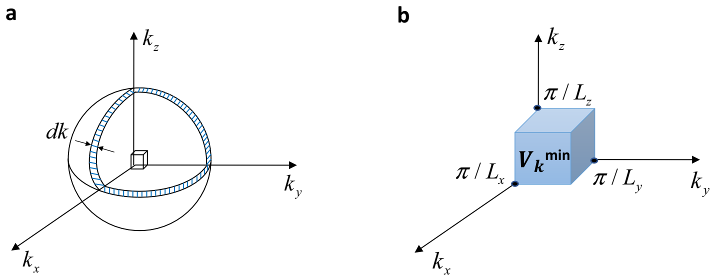

Figure 6.2. a) Mode distribution in a cavity. b) The cube at the origin in a) is the smallest volume in **k**-space, defined by the inverse dimensions of the cavity.

only on its temperature and not the incident radiation. The radiated field can be considered as consisting of the *resonant modes* of the cavity.

In order to find the number of modes per frequency interval, let us consider the wavevector space in figure 6.2(a). The spherical shell in the first octant of the **k**-space is

$$dV_k = \frac{1}{8} 4\pi k^2 \, dk = \frac{\pi}{2} k^2 \, dk. \tag{6.4}$$

The number of modes within this interval is

$$dN = \frac{dV_k}{V_k^{\min}}, \tag{6.5}$$

where $V_k^{\min}$ is the volume in **k**-space formed by the smallest spatial frequencies, $V_k^{\min} = \frac{\pi^3}{L_x L_y L_z}$, with $L_x L_y L_z = V$ the volume of the cavity (figure 6.2(b)). Equation (6.5) can now be expressed as

$$dN = \frac{\pi}{2} \frac{V}{\pi^3} k^2 \, dk. \tag{6.6}$$

In order to find the number of modes dN per frequency interval, $d\nu$, rather than dk, we use the *dispersion relation*, namely, that the magnitude of the wavevector, $k = |\mathbf{k}|$ equals the wave number in vacuum,

$$k = 2\pi \frac{\nu}{c}, \tag{6.7}$$

Combining equations (6.6) and (6.7), we obtain

$$\begin{aligned} dN &= \frac{\pi}{2}\frac{V}{\pi^3}\frac{4\pi^2\nu^2}{c^2}\frac{2\pi}{c}d\nu \\ &= \frac{4\pi V}{c^3}\nu^2\, d\nu. \end{aligned} \tag{6.8}$$

Finally, plugging equation (6.8) into equation (6.1), we obtain the *radiated energy per unit frequency range and unit volume*, $\rho(\nu)$, that is, Planck's formula derived in 1900 (for a review of Planck's work on the theory of heat radiation, see [1]):

$$\begin{aligned} \rho(v) &= \frac{du(v)}{d\nu} \\ &= \frac{8\pi h}{c^3}\frac{\nu^3}{e^{\frac{h\nu}{k_B T}} - 1}. \end{aligned} \tag{6.9}$$

Equation (6.9) is fundamental to calculating any quantity related to black body radiation. For example, from equation (6.9), we can calculate the power flowing from the cavity, through a surface A and element of solid angle $d\Omega$, per unit of frequency (see figure 6.3),

$$\begin{aligned} d^2P(\nu) &= \frac{c}{2}du(\nu)\frac{A\cos\theta\, d\Omega}{2\pi} \\ &= \frac{c}{2}\rho(\nu)d\nu A\cos\theta\frac{d\Omega}{2\pi} \end{aligned} \tag{6.10}$$

where A is the area, θ is the angle with respect to the surface normal, and the solid angle element, $d\Omega = 2\pi\sin\theta\, d\theta$. Integrating on the hemisphere, we obtain the total power

$$\begin{aligned} dP(\nu) &= \frac{c}{2}du(\nu)A\int_0^{\pi}\cos\theta\sin\theta\, d\theta \\ &= \frac{1}{4}cA\, du(\nu) \\ &= \frac{1}{4}cA\rho(\nu)d\nu. \end{aligned} \tag{6.11}$$

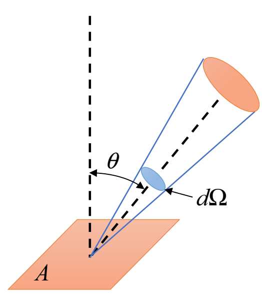

Figure 6.3. Power flow out of a black body surface.

Further, the *spectral exitance*, that is, the power per surface area per frequency (see section 2.11), $M_\nu = \dfrac{d^2P}{dA\,d\nu}$, has the form

$$\begin{aligned} M_\nu &= \frac{1}{4} c\rho(\nu) \\ &= \frac{2\pi h}{c^2} \frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1}. \end{aligned} \tag{6.12}$$

The spectral exitance can be expressed in terms of the wavelength as

$$\begin{aligned} M_\lambda &= M_\nu(\nu = c/\lambda) \frac{d\nu}{d\lambda} \\ &= \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}. \end{aligned} \tag{6.13}$$

Note that simply replacing ν with λ in equation (6.12) yields the wrong formula. The Jacobian factor, $\dfrac{d\nu}{d\lambda}$, is very important, as it ensures that the M_λ is a *distribution.* One way to remember the change from a frequency to a wavelength distribution is that the exitance in each infinitesimal range is constant, namely,

$$M_\lambda\, d\lambda = M_\nu\, d\nu. \tag{6.14}$$

Figure 6.4 illustrates M_λ for different temperatures.

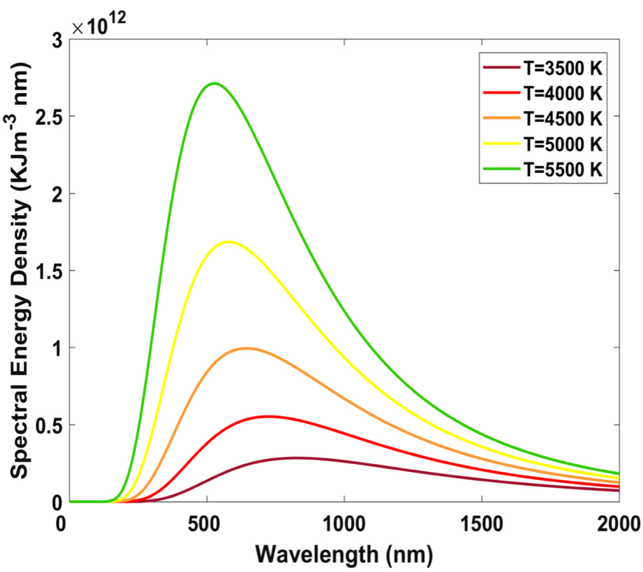

Figure 6.4. Spectral exitance of black bodies at different temperatures.

6.2 Wien's displacement law

We note immediately that both the energy per unit volume and unit frequency, $\rho(\nu)$, and the spectral exitance, M_ν, the power per surface area per frequency, exhibit a maximum at a particular frequency, $\nu_{\max}$, which is a function of temperature (see figure 6.5),

$$\left.\frac{d\rho(\nu)}{d\nu}\right|_{\nu=\nu_{\max}} = \left.\frac{dM_\nu}{d\nu}\right|_{\nu=\nu_{\max}} = 0. \tag{6.15}$$

The dependence of $\nu_{\max}$ on temperature is known as *Wien's displacement law* (see problem 6.2),

$$\nu_{\max} \propto T.$$

An equivalent way of expressing the displacement law is via the wavelength of the maximum emission,

$$\lambda_{\max} \propto \frac{a}{T},$$

where a is a constant, a = 2900 μm K. Note that $\lambda_{\max}$ is obtained by finding the maximum of the function M_λ with respect to λ (see problem 6.2). Simply substituting $\nu_{\max}$ yields the wrong result, in other words, $\lambda_{\max} \neq c/\nu_{\max}$.

This relationship between the temperature of the source and the peak wavelength of its emission led to some researchers expressing the 'color' of thermal light by the

Temperature	Source
1850 K	Candle flame, sunset/sunrise
2400 K	Standard incandescent lamps
2700 K	"Soft white" compact fluorescent and LED lamps
3000 K	"Warm white" compact fluorescent and LED lamps
3200 K	Studio lamps, photofloods, etc.
5000 K	Compact fluorescent lamps (CFL)
6200 K	Xenon short-arc lamp
6500 K	Daylight, overcast
6500 – 9500 K	LCD or CRT screen
15,000 – 27,000 K	Clear blue sky

Figure 6.5. Color temperature for various thermal sources.

temperature of the source. For example, *color temperature* is a typical measure on commercial microscopes when adjusting the power of the illuminating lamp. It has become customary to compare light sources (light bulbs, light emitting diodes, incandescent lamps, computer monitors, etc) in terms of their color temperatures. Thus, the color temperature defines which black body radiation would most closely match the light in question, from 'warm reddish' to 'cool blueish' (see figure 6.5). For example, the blue–white fluorescent light in most offices may have a color temperature of 5000 K, while an incandescent bulb has a temperature of 2000–2500 K, giving it a more reddish, 'warmer' appearance.

We experience Wien's displacement formula in our daily activities, as follows. (1) The Sun's effective temperature is 5800 K, which places its peak emission at ~500 nm (green), near the maximum sensitivity of our eye. This fact suggests that humans evolved to gain maximum sensitivity of their visual system at the most dominant wavelength emitted by the Sun. (2) Dimming the light on an incandescent light bulb will result in shifting the color toward red (longer wavelengths). (3) Heating a piece of metal will eventually produce radiation, first of red color and then blue–white when the temperature increases further. One can say that 'white-hot' is hotter than 'red-hot'. (4) Warm-blooded animals at, say, $T = 310$ K (37 °C), emit peak radiation at ~10 μm, in the infrared region of the spectrum, outside our eye sensitivity. Some reptiles and specialized cameras can sense these wavelengths and, thus, detect the presence of such animals. (5) Wood fire can have temperatures of 1500–2000 K, with peak radiation at ~2–2.5 μm. This means that most of the radiation is in the infrared spectrum, which we sense as heat, but only a small portion of the spectrum is visible.

6.3 Stefan–Boltzmann law

Another fundamental property of black body radiation is that the frequency-integrated spectral exitance, meaning, the *exitance* (in W/m^2), is proportional to the fourth power of temperature (proof left as an exercise, see problem 6.1),

$$\begin{aligned} M &= \int_0^\infty M_\nu \, d\nu \\ &= \sigma T^4. \end{aligned} \tag{6.16}$$

Equation (6.16) is known as the *Stefan–Boltzmann law*, and the proportionality constant, or the *Stefan–Boltzmann constant*, has the value $\sigma = 5.67 \cdot 10^{-8}\ \mathrm{Wm^{-2}\ K^{-4}}$.

The total power emitted by a black body is MA, where A is the area of the source. As a result, the Stefan–Boltzmann law becomes a practical means to estimate the size of other stars. Thus, the total power emitted by the Sun (S) and the star of interest (x), are, respectively

$$P_S = 4\pi {R_S}^2 \sigma {T_S}^4$$

and

$$P_x = 4\pi {R_x}^2 \sigma {T_x}^4.$$

The unknown radius can be easily obtained as

$$R_x = R_S\left(\frac{T_S}{T_x}\right)^2\sqrt{\frac{P_x}{P_S}}.$$

Note that the effective temperature of the star can be measured from the spectral distribution of the radiation fitted with Planck's formula.

A body that does not absorb all the incident radiation emits less total energy than a black body and is sometimes called a *gray* body. These bodies are characterized by an emissivity, $\varepsilon < 1$, such that the exitance is scaled down as

$$M = \varepsilon\sigma T^4.$$

6.4 Asymptotic behaviors of Planck's formula

Investigating Planck's formula we can readily find two asymptotic behaviors for the black body radiation, as follows. At *low temperatures*, $h\nu \gg k_B T$, we obtain

$$f \simeq e^{-\frac{h\nu}{k_B T}}$$
$$\rho(\nu) \simeq \frac{8\pi h}{c^3}\nu^3 e^{-\frac{h\nu}{k_B T}}. \tag{6.17}$$

This formula approximates well the high frequency portion of the curve (figure 6.6). This behavior is known as the Wien approximation.

At *high temperatures*, $h\nu \ll k_B T$, the following approximations apply

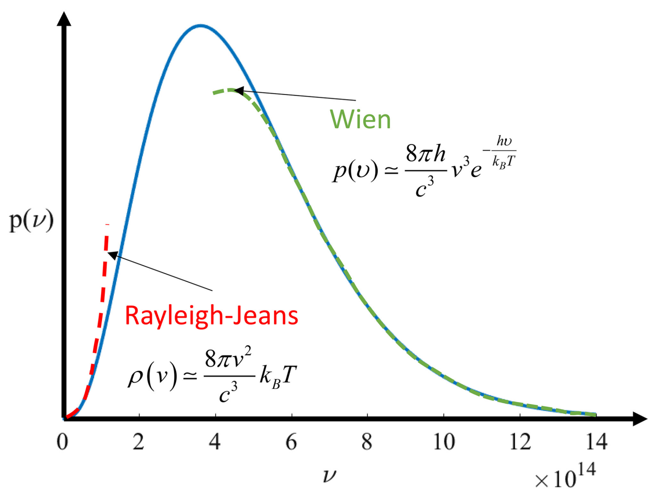

Figure 6.6. The asymptotic behavior of Planck's formula (blue curve) for high temperature (low-frequency, Rayleigh–Jeans, red curve) and low-temperature (high-frequency, Wien, green curve).

$$f \simeq \frac{1}{1 + \dfrac{h\nu}{k_B T} - 1} \tag{6.18a}$$

$$= \frac{k_B T}{h\nu}$$
$$\rho(\nu) \simeq \frac{8\pi\nu^2}{c^3} k_B T. \tag{6.18b}$$

Equation (6.18*a* and *b*) is known as the *Rayleigh–Jeans law* and describes well the low-frequency curve of Planck's equation (figure 6.6). Note that the Rayleigh–Jeans law is known as the *classic limit* formula. It strongly disagrees with Planck's law at high frequencies: one consequence of the Rayleigh–Jeans formula is that the amount of energy radiated over the entire spectral range diverges. This was known as the 'ultraviolet catastrophe' and was the main motivation behind developing a better understanding of black body radiation. Planck's work elucidated the problem and, at the same time, opened the door for quantum physics.

6.5 Einstein's derivation of Planck's formula

Einstein was able to arrive at the same solution for the energy density per frequency interval, $\rho(\nu)$, (equation (6.9)) by using the discrete energy levels of atomic systems. Thus, the quantum of energy $E = h\nu$ is assumed to be the difference between two atomic energy levels,

$$E_2 - E_1 = h\nu. \tag{6.19}$$

Furthermore, Einstein considered that there are only three fundamental processes by which the atomic system can exchange energy with the environment: absorption, spontaneous emission, and stimulated emission (figure 6.7).

Let us consider these three processes separately by denoting the number density of each level by N_1 and N_2. *Absorption* is the process by which an atom in state 1 absorbs an incident photon and is excited to energy level 2. The rate of increase of N_2 due to absorption is proportional to the number density of the atoms in level 1, N_1, and the incident energy density, ρ,

$$\frac{dN_2}{dt} = B_{12} N_1 \,\rho(\nu) \tag{6.20}$$

where B_{12} is the absorption rate, $[B_{12}] = s^{-1}$. As a result of absorption, incident energy is converted into the excitation of atoms from level 1 to 2.

Spontaneous emission is the process whereby an atom from level 2 decays radiatively (with emission of a photon) to the lower state,

$$\frac{dN_2}{dt} = -A_{21} N_2. \tag{6.21}$$

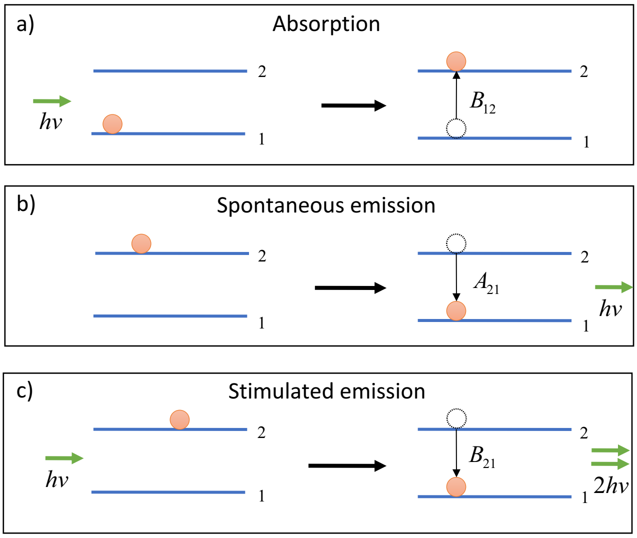

Figure 6.7. Radiative processes in a two-level atomic system: a) absorption, b) spontaneous emission, c) stimulated emission. Note how the stimulated rather than spontaneous emission is the reversed process to absorption.

The coefficient A_{21} is the spontaneous emission rate constant. Note that the inverse of A_{21} can be interpreted as the decay time constant, or *natural lifetime* (see section 5.5), $\tau = 1/A_{21}$.

Stimulated emission is the process by which, in the presence of an incident photon, an excited atom decays to level 1 and releases a photon of the same energy (frequency), direction of propagation, polarization, and phase. Stimulated emission can be regarded as the exact reverse of absorption. This process contrasts with *spontaneous emission*, where the emitted photon has no phase relationship with the stimulating photon, that is, it is emitted with equal probability in all directions of propagation, and with a random direction of the electric field vector (polarization). The rate depends on both the population density in state 2 and the strength of the stimulating light

$$\frac{dN_2}{dt} = -B_{21}N_2\rho(\nu). \tag{6.22}$$

Einstein combined all these processes to express the rate equations,

$$\begin{aligned}\frac{dN_2}{dt} &= -A_{21}N_2 + B_{12}N_1\rho(\nu) - B_{21}N_2\rho(\nu) \\ &= -\frac{dN_1}{dt}.\end{aligned} \tag{6.23}$$

Equation (6.23) also states that the rate of increase in the population of level 2, dN_2/dt, must be accompanied by an identical decrease (hence the negative sign) in the population of level 1, $-dN_1/dt$. Note that here all the nonradiative processes (resulting in loss by heat dissipation) have been ignored. At thermal equilibrium, the excitation and decay mechanisms must balance each other completely, such that

$$\frac{dN_2}{dt} = \frac{dN_1}{dt} = 0. \tag{6.24}$$

Combining equations (6.23) and (6.24), we obtain the ratio of the two population densities,

$$\frac{N_2}{N_1} = \frac{B_{12}\rho(\nu)}{A_{21} + B_{21}\rho(\nu)}. \tag{6.25}$$

Further, Einstein used the classic Boltzmann statistics, which gives the ratio of the two populations as

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} \cdot e^{-\frac{h\nu}{k_B T}}. \tag{6.26}$$

In equation (6.26), quantities g_1 and g_2 are the *degeneracy factors* for the two states, that is, the number of configurations in which a molecule can have the same energy. If we combine equations (6.25) and (6.26) to solve for $\rho(\nu)$, we obtain

$$\rho(\nu) = \frac{A_{21}}{B_{21}} \frac{1}{\frac{B_{12}g_1}{B_{21}g_2} e^{\frac{h\nu}{k_B T}} - 1}. \tag{6.27}$$

By comparing equation (6.27) with Planck's formula (equation (6.9)), Einstein realized that they are identical, provided two conditions are met

$$g_1 B_{12} = g_2 B_{21} \tag{6.28a}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}. \tag{6.28b}$$

Equations (6.28*a* and *b*) connect the three Einstein coefficients. Therefore, measurements on a certain radiative process, for example, measuring A_{21}, can inform about the other two coefficients as well as radiation.

6.6 Problems

1. Prove the Stefan–Boltzmann law (equation (6.16)).
2. Prove Wien's displacement law, that is, that the frequency maximum of Planck's curve is proportional to the temperature of the black body, $\nu_{\max} \propto T$ (equation (6.15)). Calculate $\lambda_{\max}$.
3. How many electromagnetic modes exist in a 1 m^3 cavity for light of central wavelength $\lambda_0 = 633$ nm and bandwidth $\Delta\lambda = 1$ nm? What are the central frequency and frequency bandwidth, ν_0, $\Delta\nu$? What are the central angular frequency and angular frequency bandwidth, ω_0, $\Delta\omega_0$?

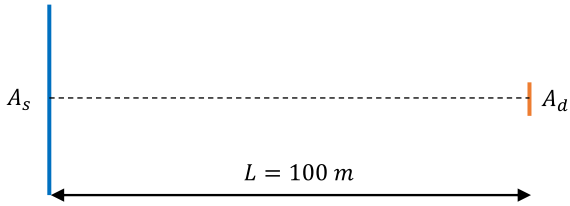

Figure 6.8. Problem 6.7.

4. What is the temperature of a black body whose maximum emission is at $\lambda = 600$ nm, $\lambda = 550$ nm, $\lambda = 400$ nm? Plot the three corresponding spectral emission curves as a function of wavelength, $I(\lambda)$, for all cases.
5. What is the frequency maximum of the Sun's radiation ($T = 5800$ K). If the Sun cooled by 1 F, how much lower would the total (frequency integrated) exitance be?
6. What is the temperature of the black body whose maximum spectral exitance corresponds to the maximum of the photopic curve?
7. A black body source of area $A_s = 10$ cm^2 and temperature $T = 1000$ K emits radiation, which is captured by a photodetector of area $A_D = 1$ cm^2, $L = 100$ m away, as depicted in figure 6.8.

   Consider the source and detector parallel and centered on the same optical axis.

   a) Calculate the power falling on the detector, within the wavelengths interval $\lambda \in [8,12]$ μm.
   b) The detector is moved 10 m farther from the source. What should be the new temperature that will yield the same power, in the same wavelength range, as before?
   c) The detector is moved to a new distance $L_1 = 1$ m ($T = 1000$ K) from the source. What should be the new wavelength interval, centered at $\lambda = 10$ μm, which will result in the same power at the detector as in a)?
8. A black body emits radiation with its maximum spectral exitance, $M_\nu(\nu)$, at frequency $\nu_{\max}$. Express the spectral exitance in terms of wavelength, $M_\lambda(\lambda)$, find the wavelength at maximum, $\lambda_{\max}$, and determine the relationship between $\nu_{\max}$ and $\lambda_{\max}$.
9. Express the spectral exitance of a black body (Planck's formula) in terms of inverse wavelengths, $1/\lambda$.
10. The wavelength-based spectral exitance of a black body is 1 W/(m^2 μm) at its peak. What is the temperature of the source in K?
11. A black body has a temperature of 1000 K and weighs 1 kg.
    a) Assuming constant temperature, how long will it take for the entire mass to be converted into radiation?
    b) The temperature is now $T = 10\,000$ K. What is the new time of total mass energy conversion?

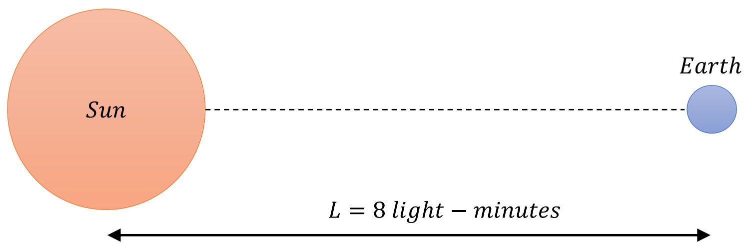

Figure 6.9. Problem 6.13.

12. A black body is cooling at a rate $\alpha = 1\ \mathrm{K}/s$, starting at $T = 5000\ \mathrm{K}$. Plot the following versus time:
    a) frequency of maximum emission, $\nu_{\max}$
    b) total exitance, $M = \int_0^\infty M_\nu(\nu)d\nu$
    c) total exitance within the wavelength interval $\lambda \in (1{,}2)\ \mu\mathrm{m}$.
13. Assume the Sun is a black body that floods the Earth's surface with $1\,\mathrm{kW\,m^{-2}}$ of irradiance (assume isotropic radiation). If the radii of the Earth and Sun are $R_E = 6 \times 10^3\ \mathrm{Km}$ and $R_S = 7 \times 10^5\ \mathrm{Km}$, respectively, and the distance Sun–Earth is eight light-minutes (see figure 6.9):
    a) What is the total exitance of the Sun?
    b) What is the temperature of the Sun?
    c) What is the total power emitted by the Sun (assuming a spherical shape)?
    d) What is the total power falling on Earth?
    e) If Earth reflects off 20% of the light from the Sun and becomes a Lambertian source, what is the total power received back by the Sun?
14. Plot the following parameters versus temperature $T \in (0,\ \ 10\ 000)\ \mathrm{K}$ for a black body radiator's spectral exitance, $M_\nu(\nu)$.
    a) $\nu_{\max}$, $<\nu>$, and $<\nu> - \nu_{\max}$, where $<>$ denotes the ensemble average over the spectral distribution.
    b) σ_ν, the standard deviation and the spectral variance, $\sigma_\nu = <\nu^2> - <\nu>^2$
    c) skewness, $<\nu^3>$
    d) kurtosis, $<\nu^4>$.
15. Plot the temporal autocorrelation function associated with the black body radiation field at $T = 300\ \mathrm{K}$.
16. Plot the even and odd components of the black body radiation spectral exitance, respectively,

$$M_\nu^e(\nu) = \frac{M_\nu(\nu) + M_\nu(-\nu)}{2}$$
$$M_\nu^o(\nu) = \frac{M_\nu(\nu) + M_\nu(-\nu)}{2}.$$

Prove that, indeed, M_ν^e and M_ν^o are even and odd, respectively.

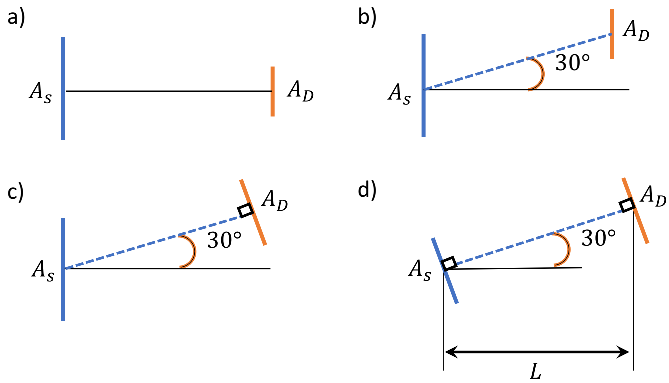

Figure 6.10. Problem 6.19.

17. A flat black body source, of area $A_S = 1\ \text{m}^2$, and temperature $T = 5000$ K, emits radiation that is collected by a detector, of area $A_D = 1\ \text{mm}^2$, at a distance $L = 1$ m.
    a) How much power falls on the detector?
    b) What percentage of the power at the detector is in the visible spectral range, $\lambda \in (380,\ 760)$ nm ?
    c) How many photons reach the detector?
18. Derive an expression for the spectral exitance change per unit temperature, assuming a black body source.
19. A Lambertian, black body source and photodetector are placed in four different configurations, as indicated (figure 6.10). If temperature is $T = 3000$ K, $A_s = 1\ \text{cm}^2$, $A_D = 1\ \text{mm}^2$, and $L = 1$ m, compute the power at the detector in all configurations.
20. A normal human body temperature is $T = 37$ °C.
    a) What is the wavelength of maximum spectral exitance, $\lambda_{\max}$?
    b) By how much does $\lambda_{\max}$ change if the person runs a high fever, $T = 40$ °C?
    c) What is the $\lambda_{\max}$ emitted by an alligator in a pond of temperature $T = 25$ °C?
21. In order to measure the temperature of an unknown object, one experimentalist performs measurements of the black body radiation power within narrow spectral ranges centered at $\lambda_1 = 10$ μm and $\lambda_2 = 20$ μm. It was found that $P_2/P_1 = 0.1$. What is the temperature of the body?

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