A Modern Course in Quantum Field Theory, Volume 2

Advanced topics

A Modern Course in Quantum Field Theory, Volume 2

Advanced topics

Badis Ydri

Department of Physics, Faculty of Sciences, Annaba University, Annaba, Algeria

IOP Publishing, Bristol, UK

ISBN 978-0-7503-1483-1 (ebook)
ISBN 978-0-7503-1484-8 (print)
ISBN 978-0-7503-1482-4 (mobi)

DOI 10.1088/2053-2563/ab0548

Version: 20190501

IOP Expanding Physics
ISSN 2053-2563 (online)
ISSN 2054-7315 (print)

British Library Cataloguing-in-Publication Data: A catalogue record for this book is available from the British Library.

Published by IOP Publishing, wholly owned by The Institute of Physics, London

IOP Publishing, Temple Circus, Temple Way, Bristol, BS1 6HG, UK

US Office: IOP Publishing, Inc., 190 North Independence Mall West, Suite 601, Philadelphia, PA 19106, USA

To my father for his continuous support throughout his life …

Saad Ydri
1943–2015

Also to my …
Nour

Contents

Appendices

Preface

This two-volume book was accepted for publication by IOPP (Institute of Physics Publishing) on 20 February 2017, submitted on 14 December 2018 and will appear in its final form during the spring of 2019. It contains a comprehensive introduction to the fundamental topic of quantum field theory starting from free fields and their quantization, renormalizable interactions, critical phenomena, the standard model of elementary particle physics, lattice field theory, the functional renormalization group equation, non-commutative field theory, topological field configurations, exact solutions of quantum field theory, supersymmetry and finally the AdS/CFT correspondence. The emphasis throughout is put on the physical principle of symmetry (especially the local principle of gauge symmetry) and on the mathematical machinery of the renormalization group equation à la Wilson. This book is the fifth book published by the author[1] and it completes therefore his in-depth detailed and constructive study of all fundamental areas of theoretical physics which took several years to complete. The author would like to thank his IOPP editor John Navas for all his help in publishing three of his books.

[1] Together with two open and free books on fundamental physics in Arabic.

Author biography

Badis Ydri

Badis Ydri—currently a professor of theoretical particle physics, teaching at the Institute of Physics, Badji Mokhtar Annaba University, Algeria—received in 2001 his PhD from Syracuse University, New York, USA and in 2011 his Habilitation from Annaba University, Annaba, Algeria.

His doctoral work, titled 'Fuzzy Physics', was supervised by Professor A P Balachandran. Professor Ydri is a research associate at the Dublin Institute for Advanced Studies, Dublin, Ireland, and a regular ICTP associate at the Abdus Salam Center for Theoretical Physics, Trieste, Italy. His postdoctoral experience comprises a Marie Curie fellowship at Humboldt University Berlin, Germany, and a Hamilton fellowship at the Dublin Institute for Advanced Studies, Ireland.

His current research directions include: the gauge/gravity duality; the renormalization group method in matrix and noncommutative field theories; noncommutative and matrix field theory; emergent geometry, emergent gravity and emergent cosmology from matrix models.

Other interests include string theory, causal dynamical triangulation, Hořava–Lifshitz gravity, and supersymmetric gauge theory in four dimensions.

He has recently published three books. His hobbies include reading philosophic works and the history of science.

Introduction

The luminous matter in the Universe is constituted of elementary fermion particles of spin 1/2 (leptons and quarks) which interact via elementary boson particles of spin 1 (gauge vector bosons) mediating the three fundamental interactions of nature: the electromagnetic interaction, the strong nuclear force and the weak nuclear interaction. The fourth fundamental force of nature (the gravitational force) is mediated instead by a tensor particle of spin 2.

These particles are all massless and these forces obey a fundamental symmetry principle called the gauge principle which can only be broken spontaneously via the Higgs particle (the breaking of the electroweak force into the observed electromagnetic force and weak interactions) which is an (the only) elementary particle of spin 0 in nature. This process of spontaneous symmetry breaking is what gives all elementary particles their measured masses and all the forces their observed strengths.

Quantum field theory is a relativistic quantum theory which describes precisely this luminous matter and its interactions. In fact, it is widely believed that quantum field theory should also describe dark matter and perhaps even dark energy (in terms of vacuum energy). This quantum field theory is perturbatively renormalizable. However, quantum field theory enjoys also non-perturbative formulation either directly (through lattice field theory, the renormalization group equation and conformal field theory) or indirectly by admitting exact solutions (especially in two dimensions but also in four dimensions via the supersymmetric gauge principle).

Furthermore, the 'modern' or 'new' quantum field theory includes also gravity via the AdS/CFT correspondence which is the most celebrated paradigm of gauge/gravity holographic duality. Hence, modern quantum field theory which governs all elementary particles and their interactions as well as gravity can be summarized in three major sub-theories:

1. The standard model of elementary particles: This provides a unified scheme of the electromagnetic force, the weak interaction, and the strong nuclear force, and is due historically to the work of Weinberg, Abdu Salam and Glashow among many other physicists. The standard model is the most successful (experimentally) quantum field theory to date and perhaps the most successful theory ever (especially its quantum electrodynamics (QED) component). It accounts for a large body of phenomenological effects and observations seen in nature in terms of only a finite (but still relatively large =19) number of parameters such as the gauge coupling constants, the Higgs vacuum expectation value, the CKM angles and the theta angle governing CP violation. The standard model is however, mostly perturbative and it includes in a fundamental way the phenomena of spontaneous symmetry breaking and is based entirely on the meta-theory of the renormalization group equation.

   The standard model consists of two parts. The first part is the electroweak force which unifies quantum electrodynamics and quantum flavordynamics which describes the weak force. The second part of the standard model consists of quantum chromodynamics (QCD) which describes the strong force. QCD admits a non-perturbative definition given typically in terms of a lattice formulation, and lattice QCD is arguably the most sophisticated discipline in computational physics.

2. Supersymmetric gauge theory in four dimensions: This allows us a non-perturbative formulation (one in which we do not need a small parameter of expansion) of the gauge principle which can be solved exactly (like the harmonic oscillator in quantum mechanics) in many instances by means of supersymmetry and holomorphy among other things (Witten–Seiberg–Nekrasov theory). This is of paramount importance to strongly coupled systems such as quantum chromodynamics since the strong force is a highly non-perturbative interaction. However, supersymmetric gauge theory also gives a profound understanding of the phenomena of spontaneous gauge symmetry breaking and the associated phenomena of renormalization.
3. AdS/CFT duality: As stated above, the gravitational force is not mediated via a vector gauge boson but via a tensor particle of spin 2 called the graviton. The AdS/CFT duality is the theory which allows us to bring gravity and black holes into the realm of unitary quantum field theory. Although this theory emerged historically from string theory it is intrinsically a quantum field theory. It relies heavily on conformal field theory, supersymmetry and renormalization. It states simply that supergravity theory (string theory in general) in an anti-de Sitter (AdS) spacetime which is five dimensional is given precisely by a superconformal gauge field theory (CFT) living on the boundary of AdS which is an ordinary four dimensional Minkowski spacetime (a concrete realization of the holographic principle). The AdS/CFT correspondence generalizes to the so-called gauge/gravity duality.

In this book we will mainly focus on the first axis (gauge interactions and the standard model of elementary particle physics). However, we will also prepare the ground for the second axis (chapters 14–16 on exact solutions of quantum field theory, monopoles and instantons and supersymmetry) and for the third axis (in chapter 17 we give a systematic overview of the AdS/CFT correspondence and then show how Einstein's gravity emerges from quantum entanglement).

The main emphasis throughout this book will be on the physical principle of symmetry (especially the role of symmetry groups in the quantum theory, their representation theory and conservation laws) but also on the mathematical machinery of the renormalization group equation (chapters 6–9, 12 and 13).

The renormalization group equation will allow us to study, beside the usual problems of quantum field theory relevant to particle physics (found in chapters 6–8), two more interesting physical problems: critical exponents of second order phase transitions in statistical physics (chapter 9) and renormalizability of non-commutative field theory (in chapter 13). Chapter 12 contains a systematic presentation of the functional renormalization group equation.

We will start the book in the usual way with canonical quantization of free fields (scalar field of spin 0 and spinor field of spin 1/2) in chapters 2 and 3. Then we will consider in chapter 4 perturbation theory of phi-four theory where the S-matrix structure of quantum field theory is exhibited explicitly. This is our first fundamental interaction in this book.

Then canonical quantization of the free abelian vector field of spin 1 is considered in chapter 5 where pure Yang–Mills gauge interactions with $SU(N)$ groups are also introduced. In chapter 6 perturbation theory of quantum electrodynamics (which describes the gauge interaction of a spinor field with a vector field) and its renormalization is considered in great detail. For example, we derive explicitly from the renormalization properties of the theory measurable physical effects such as the electron anomalous magnetic moment. Furthermore, the links to particle physics, i.e. the relations between quantum field theory correlation functions and particle physics cross sections and decay rates, are established explicitly in this chapter which shows more clearly the S-matrix structure of quantum field theory.

The path integral formalism is introduced in chapters 7 (for scalar fields) and 8 (for spinor and vector fields). In chapter 7 perturbative renormalizability of phi-four theory is considered at the two-loop order using the effective action formalism, whereas in chapter 8 the Faddeev–Popov quantization of the abelian and non-abelian vector fields is considered. Perturbative renormalizability of $SU(N)$ gauge theory coupled to matter, transforming in some representation of the gauge group, is then discussed (asymptotic freedom, anomalies, BRST and background field methods, etc).

In chapter 10 we discuss phenomenology of particle physics, then provide an explicit and detailed construction of the standard model Lagrangian and explain the phenomena of spontaneous symmetry breaking via the Higgs mechanism. In chapter 11 we give an explicit construction of scalar, spinor and vector lattice actions, then discuss the main Monte Carlo algorithms used and some sample numerical simulations.

In more detail, this book is then organized into chapters as follows:

1. **Relativistic quantum mechanics:** This chapter contains standard preparatory material. We will present an overview of special relativity [1], relativistic Klein–Gordon and Dirac wave equations and the convention in this book for Dirac spinors [2], and a self-contained discussion of representation theory of the rotation and Lorentz groups [3].
2. **Canonical quantization of free fields:** After a brief excursion in classical mechanics [4] we present in this chapter the canonical quantization of free scalar and Dirac fields with a detailed calculation of the corresponding propagators [2, 5]. Then we give a thorough discussion of symmetries starting with discrete symmetries [2], the Poincaré group and its representation theory [3, 5], symmetries in the quantum theory, internal symmetries and the role of Noether's theorem in conservation principles [5, 6].
3. **The phi-four theory:** A detailed discussion of the S-matrix, the Gell-Mann–Low formula, the LSZ reduction formulas, Wick's theorem, Green's functions, Feynman diagrams and the corresponding Feynman rules of

quantum Φ^4-theory is presented following [5]. This is our first non-trivial example of an interacting field theory and its canonical quantization.

4. **The electromagnetic field and Yang–Mills gauge interactions:** In this chapter we discuss in great detail the canonical quantization of the electromagnetic gauge field with emphasis on $U(1)$ gauge invariance and the Gupta–Bleuler method. Then a pedagogical introduction to Yang–Mills gauge interactions with $SU(2)$ and $SU(N)$ gauge groups (and even for general gauge groups) is presented. These gauge fields describe in Nature spin 1 particles (the so-called vector bosons) which encompass the carriers of the electromagnetic force (the photon γ), the nuclear strong color force (the gluons g) and the nuclear weak radioactive force (the W and Z^0 vector bosons).

   Good pedagogical references for the canonical quantization of the electromagnetic field are [5, 6].
5. **Quantum electrodynamics:** The goal in this chapter is to develop canonical perturbation theory beyond the free field approximation of QED which is an interacting (local gauge) theory of the Dirac field (electrons and positrons) and the gauge vector field (photons). The formalism of canonical quantization of QED is found in [5], whereas radiative corrections and renormalization are found in [2].
6. **Path integral quantization of scalar fields:** In this chapter we will present the path integral method which is a central tool in quantum field theory and then give a detailed account of the effective action in the case of a scalar field theory. A brief discussion of spontaneous symmetry breaking is also given. These are very standard topics and we have benefited here from the books [2, 7, 8] and the lecture notes [9].
7. **Path integral quantization of Dirac and vector fields:** We develop the powerful and elegant path integral method for spinor fields (Grassmann variables) and gauge fields (gauge fixing, Faddeev–Popov method, ghosts). Then we give two important applications based on the path integral formalism. Firstly, we present a detailed derivation of the one-loop beta function of QCD with $SU(N)$ gauge theory and matter fields in the fundamental representation and discuss the resulting phenomena of asymptotic freedom. Secondly, we present the one-loop (and in fact exact) axial or chiral anomaly in QED and the Fujikawa path integral method. We also discuss briefly the background field method and symmetries within the path integral method (Schwinger–Dyson equations and Ward identities).
8. **The Callan–Symanzik renormalization group equation:** All second-order phase transitions in Nature are described by the Callan–Symanzik renormalization group equations of Euclidean scalar field theory. In this chapter, after a detailed discussion of renormalizability of quantum field theories, in particular the scalar ϕ^4 theory, we present an explicit construction of the Callan–Symanzik renormalization group equations. Then, a detailed calculation of the critical exponents of second-order phase transitions starting from the renormalization properties of scalar ϕ^4 field theory at the two-loop order is carried out explicitly. We follow closely the book [10].

9. **Standard model:** The standard model of elementary particle physics describes all known particles and their interactions which are observed in Nature. It is based on the following grand theoretical principles:
   - Relativistic invariance.
   - It is a local gauge theory based on the gauge group $SU(3) \times SU(2)_L \times U(1)_Y$.
   - The gauge group is spontaneously broken down to $SU(3) \times U(1)_{\rm em}$. This generates mass in a gauge-invariant way.
   - It consists of a lepton sector, a quark sector, a Higgs term and a gauge sector. The matter sector (leptons, quarks and Higgs) are coupled minimally to the gauge sector (which ensures renormalizability). The mechanism by which the symmetry is spontaneously broken is the Higgs mechanism. The Higgs field is coupled to the quarks and leptons via gauge invariant renormalizable Yukawa couplings.
   - It is a chiral gauge theory, i.e. left-handed quarks and leptons couple to the gauge field differently (in the fundamental representation) than right-handed quarks and leptons (singlet representation).
   - Renormalizability: The standard model is a renormalizable theory (interaction terms between the gauge fields and the matter fields are given by minimal coupling). The requirement of gauge invariance guarantees renormalizability and unitarity.
   - The standard model is not invariant under parity P (nor under CP where C is charge conjugation). But it is invariant under CPT where T is time reversal. This holds in the lepton sector.
   - Anomaly cancellation: This is the second quantum consistency check (after renormalizability) which states that any local symmetry like gauge symmetry cannot be allowed to be anomalous. This is satisfied in the standard model since the number of lepton families is equal to the lepton of quark families.

   Extensions of the standard model include grand unified theories GUT's (such as $SU(5)$ or $SO(10)$ or any other group which contains the standard model gauge group as a subgroup), supersymmetry (minimal supersymmetric standard model), non-commutative geometry (Connes' standard model) and stringy extensions. Unification of the three forces (color strong, electromagnetic and weak) described by the standard model with gravity is however only achieved in string theory.

   In this chapter, and after a brief excursion into the phenomenology of particle physics (isospin symmetry, quark model, neutrino oscillations, etc), we give a detailed construction of the standard model Lagrangian starting with the Glashow, Weinberg, Salam electroweak theory, then we discuss the Higgs mechanism and spontaneous symmetry breaking, Majorana fermions, neutrino mass and the seesaw mechanism, and then finally we provide an extension to the quark sector and quantum chromodynamics as well as a summary of anomaly cancellation. We will follow the general presentations of [3, 9, 11, 12].

10. **Introduction to lattice field theory:** In this chapter a quick excursion into the world of lattice field theory is taken. Scalar, fermion and gauge fields are constructed on the lattice explicitly. Then the two most used Monte Carlo algorithms in numerical simulations on the lattice (the Metropolis and the hybrid Monte Carlo algorithms) are explained within the context of very simple lattice models, namely the scalar phi-four in two dimensions and quenched electrodynamics. The classic textbooks on the subject of lattice field theory are [13–17].
11. **The Wilson and functional renormalization group equations:** The renormalization group equation is a central tool of perturbative and non-perturbative quantum field theory which is vital for a proper understanding of the renormalizability of the theory and its phase diagram. The Wilson approach [18] to the renormalization group equation is, in our opinion, the most profound description of the true nature and final goals of quantum field theory. In this chapter, and after a careful review of the original Wilson approach, we describe in great detail the functional renormalization group equation which is an exact non-perturbative formulation of the Wilson renormalization group equation. The original literature on the functional renormalization group equation includes Polchinski [19] (Polchinski's equation for the effective action) and Wetterich [20] (Wetterich's equation for the average action). See also [21].
12. **Non-commutative scalar field theory and its renormalizability:** In this chapter, and after an efficient introduction to non-commutative scalar field theory, we will apply the Wilson–Polchinski renormalization group equation, discussed in the previous chapter, to the problem of renormalizing non-commutative phi-four theory in 2 and 4 dimensions with and without the harmonic oscillator term. Non-commutative field theory is discussed in great detail in our book [22], whereas we follow closely the original programme of Grosse and Wulkenhaar [23–25] in the very difficult problem of renormalization of non-commutative phi-four theory on Moyal–Weyl spaces.
13. **Some exact solutions of quantum field theory:** The non-perturbative physics of a quantum field theory (as we have seen) can only be probed by means of Monte Carlo methods on lattices (which can become quite intricate technically and numerically) and/or by means of the exact renormalization group equation (which is always quite intricate analytically and mathematically). But sometimes exact solutions of the quantum field theory model presents themselves (lower dimensions and/or a high degree of symmetries) which allow us to access the sought-after non-perturbative physics of the theory directly. In this chapter we present as examples six models in dimension two which all enjoy exact solutions, allowing us an unprecedented look at the true heart, i.e. the non-perturbative reality, of a quantum field theory.
14. **The monopoles and instantons:** Monopoles are non-trivial topological gauge field configurations which appear in spontaneously broken gauge theory via

the Higgs mechanism. These are particle-like solitonic configurations characterized by stability and finite energy among other properties. Their stability is of a topological origin characterized by the so-called winding numbers or magnetic charges. For this reason monopoles are one of the best examples in which physics and topology become intertwined. The existence of the monopole requires the embedding of electromagnetism, i.e. the group $U(1)$, as a subgroup in a larger non-abelian group G with compact cover which then becomes broken spontaneously via the usual Higgs mechanism.

The original literature on the subject consists of ’t Hooft [26] and Polyakov [27]. Some of the pedagogical (from my perspective) lectures I can mention here: Lenz [28], ’t Hooft [29], Coleman [30] and Tong [31]. A comprehensive book is Shnir [32] and a comprehensive review is given by Weinberg and Yi [33].

Instantons are another fundamental topological gauge configuration, perhaps more fundamental than monopoles, which are given by events localized in spacetime and hence the other name given to them is pseudo-particles (in contrast with particles such as monopoles which are events localized in space). Instantons are also the gauge field configurations which dominate the path integral in the semi-classical limit with the trivial instanton identified precisely with the perturbative vacuum $A = 0$.

We will discuss here in great detail the theta term, the role of vacuum degeneracy, the quantization of the topological charge and the role of topology in instanton physics. More precisely, the instanton is defined as a solution of the self-duality equation with zero/finite energy which happens to saturate the Bogomolnyi bound. The BPST instanton solution is then derived explicitly. The original literature on the BPST instanton is the paper by Belavin, Polyakov, Schwartz and Tyupkin [34]. We then discuss in some detail the moduli space, the collective coordinates, the zero modes, the ADHM construction, the one-loop quantization in the background of instantons as well as the connection of instantons to quantum tunneling. We have benefited here greatly from the pedagogical presentations found in [31, 35, 36].

15. **Introducing supersymmetry:** In this chapter we introduce supersymmetry following mostly [37]. In particular, we will emphasize the formal quantum field theory aspects of the formalism of global $N = 1$ supersymmetry with a detailed calculation of the corresponding F- and d-terms following also [38]. A brief description of $N = 2$ is also given. The classic text on supersymmetry of Wess and Bagger [39] remains, in our view, one of the best books on quantum field theory. We have also benefited from [40, 41].
16. **The AdS/CFT correspondence:** The goal in this chapter is to provide a pedagogical presentation of the celebrated AdS/CFT correspondence adhering mostly to the language of quantum field theory (QFT). This is certainly possible, and perhaps even natural, if we recall that in this correspondence we are positing that quantum gravity in an anti-de Sitter

spacetime AdS_{d+1} is nothing else but a conformal field theory (CFT_d) at the boundary of AdS spacetime. Some of the reviews of the AdS/CFT correspondence which emphasize the QFT aspects and language include Kaplan [42], Zaffaroni [43] and Ramallo [44].

This chapter contains, therefore, a thorough introduction to conformal symmetries, anti-de Sitter spacetimes, conformal field theories and the AdS/CFT correspondence. The primary goal however in this chapter is the holographic entanglement entropy. In other words, how spacetime geometry as encoded in Einstein's equations in the bulk of AdS spacetime can emerge from the quantum entanglement entropy of the CFT living on the boundary of AdS.

A sample of the original literature for the holographic entanglement entropy is [45–48]. However, a very good, concise and pedagogical review of the formalism relating spacetime geometry to quantum entanglement due to Van Raamsdonk and collaborators is found in [47] and [49].

This book also includes five appendices (two on classical physics, one on representation theory of Lie groups and Lie algebras, one on homotopy theory and one contains extra exercises given as examination problems throughout the years).

This book (especially the first volume) grew from a course of lectures delivered (five times) since 2010 at Annaba University (Algeria) to theoretical physics students at the Master level (first and second years).

All illustrations found in this book were created by Dr Khaled Ramda, Z Salem and L Bouraiou. The Monte Carlo results included in the chapter 'Introduction to lattice field theory' (chapter 11) are from our original numerical simulations.

References

[1] Griffiths D 1999 *Introduction to Electromagnetism* 3rd edn (Englewood Cliffs, NJ: Prentice Hall)

[2] Peskin M E and Schroeder D V 1995 *An Introduction to Quantum Field Theory* (Avalon Publishing)

[3] Boyarkin O M 2011 *Advanced Particle Physics: Vol I* (London: Taylor and Francis)

[4] Goldstein G 1980 *Classical Mechanics* 2nd edn (Reading, MA: Addison-Wesley)

[5] Strathdee J 1995 *Course on Quantum Electrodynamics, ICTP Lecture Notes*

[6] Greiner W and Reinhardt J 1996 *Field Quantization* (Berlin: Springer)

[7] Itzykson C and Drouffe J M 1989 *Statistical Field Theory: Volume 1, From Brownian Motion to Renormalization and Lattice Gauge Theory, Cambridge Monographs on Mathematical Physics* (Cambridge: Cambridge University Press)

[8] Polyakov A M 1987 Fields and Strings *Fields and Strings, Contemporary Concepts in Physics* (London: Harwood Academic Publishers)

[9] Randjbar-Daemi S *Course on Quantum Field Theory* (ICTP preprint of 1993-94 HEP-QFT (1))

[10] Zinn-Justin J 2002 *Quantum Field Theory and Critical Phenomena* (International Series of Monographs on Physics vol 113) (Oxford: Oxford University Press)
[11] Boyarkin O M 2011 *Advanced Particle Physics: Vol II* (London: Taylor and Francis)
[12] Dolan B 2004 *Particle Physics,* author's own web page
[13] Creutz M 1985 *Quarks, Gluons and Lattices Cambridge Monographs on Mathematical Physics* (Cambridge: Cambridge Univ. Press)
[14] Smit J 2002 *Introduction to Quantum Fields on a Lattice: A Robust Mate, Cambridge Lecture Notes in Physics* vol **15** (Cambridge: Cambridge University Press)
[15] Rothe H J 1992 *Lattice Gauge Theories: An Introduction, World Scientific Lecture Notes in Physics* vol **43** (Singapore: World Scientific)
[16] Montvay I and Munster G 1994 *Quantum Fields on a Lattice, Cambridge Monographs on Mathematical Physics* (Cambridge: Cambridge University Press) 491
[17] Gattringer C and Lang C B 2010 *Quantum Chromodynamics on the Lattice, Lecture Notes in Physics* vol 788 (Berlin: Springer)
[18] Wilson K G and Kogut J B 1974 The renormalization group and the epsilon expansion *Phys. Rep.* **12** 75
[19] Polchinski J 1984 Renormalization and effective Lagrangians *Nucl. Phys.* B **231** 269
[20] Wetterich C 1993 Exact evolution equation for the effective potential *Phys. Lett.* B **301** 90
[21] Kopietz P, Bartosch L and Schutz F 2010 *Introduction to the Functional Renormalization Group, Lecture Notes in Physics* vol **798** (Berlin: Springer)
[22] Ydri B 2017 *Lectures on Matrix Field Theory (Lecture Notes in Physics)* vol **929** (Berlin: Springer)
[23] Grosse H and Wulkenhaar R 2005 Power counting theorem for nonlocal matrix models and renormalization *Commun. Math. Phys.* **254** 91
[24] Grosse H and Wulkenhaar R 2005 Renormalization of φ^4 theory on noncommutative R^4 in the matrix base *Commun. Math. Phys.* **256** 305
[25] Grosse H and Wulkenhaar R 2003 Renormalization of φ^4 theory on noncommutative R^2 in the matrix base *J. High Energy Phys.* **0312** 019
[26] 't Hooft G 1974 Magnetic monopoles in unified gauge theories *Nucl. Phys.* B **79** 276
[27] Polyakov A M 1974 Particle spectrum in the quantum field theory *JETP Lett.* **20** 194
Polyakov A M 1974 *Pisma Zh. Eksp. Teor. Fiz.* **20** 430
[28] Lenz F 2005 *Topological Concepts in Gauge Theories, Lecture Notes in Physics* **659** 7
[29] 't Hooft G 2000 *Monopoles, Instantons and Confinement* arXiv:hep-th/0010225
[30] Coleman S R 1975 Classical lumps and their quantum descendents Lectures delivered at *Int. School of Subnuclear Physics (Ettore Majorana, Erice, Sicily, Jul 11-31, 1975)*
[31] Tong D 2005 *TASI Lectures on Solitons: Instantons, Monopoles, Vortices and Kinks* arXiv: hep-th/0509216
[32] Shnir Y M 2005 *Magnetic Monopoles* (Berlin: Springer)
[33] Weinberg E J and Yi P 2007 Magnetic monopole dynamics, supersymmetry, and duality *Phys. Rep.* **438** 65
[34] Belavin A A, Polyakov A M, Schwartz A S and Tyupkin Y S 1975 Pseudoparticle solutions of the Yang-Mills equations *Phys. Lett. B* **59** 85
[35] Vandoren S and van Nieuwenhuizen P 2008 *Lectures on Instantons* arXiv:0802.1862
[36] Dorey N, Hollowood T J, Khoze V V and Mattis M P 2002 The calculus of many instantons *Phys. Rep.* **371** 231
[37] Lykken J D 1997 *Introduction to Supersymmetry* arXiv:hep-th/9612114

[38] Weinberg S 2005 *The Quantum Theory of Fields Volume III: Supersymmetry* (Cambridge: Cambridge University Press)
[39] Wess J and Bagger J 1992 *Supersymmetry and Supergravity* (Princeton, NJ: Princeton University Press) pp 259
[40] West P C 1990 *Introduction to Supersymmetry and Supergravity* (Singapore: World Scientific) pp 425
[41] Bilal A 2001 *Introduction to Supersymmetry* arXiv:hep-th/0101055
[42] Kaplan J *Lectures on AdS/CFT from the Bottom Up*, author's own web page
[43] Zaffaroni A 2000 Introduction to the AdS-CFT correspondence *Class. Quant. Grav.* **17** 3571
[44] Ramallo A V 2015 Introduction to the AdS/CFT correspondence *Springer Proc. in Physics* vol **161** (Berlin: Springer) p 411
[45] Lashkari N, McDermott M B and Van Raamsdonk M 2014 Gravitational dynamics from entanglement 'thermodynamics' *J. High Energy Phys.* **1404** 195
[46] Faulkner T, Guica M, Hartman T, Myers R C and Van Raamsdonk M 2014 Gravitation from entanglement in holographic CFTs *J. High Energy Phys.* **1403** 051
[47] Van Raamsdonk M 2017 Lectures on Gravity and Entanglement *New Frontiers in Fields and Strings* (Singapore: World Scientific) pp 297–351
[48] Casini H, Huerta M and Myers R C 2011 Towards a derivation of holographic entanglement entropy *J. High Energy Phys.* **1105** 036
[49] Jaksland R 2017 *A Review of the Holographic Relation between Linearized Gravity and the First Law of Entanglement Entropy* arXiv:1711.10854