Inverse Modeling

An introduction to the theory and methods of inverse problems and data assimilation

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Inverse modeling is *universal* in the sciences and economics.
How could you do without it?

Mathematics combines the *beauty of the mind*
with the *practice of life*.

It is joy to use *simple principles*
to carry out *complex tasks*.

It is extremely powerful to use a *balance*
of *generalization* and *specialization*,
to step back and see the *basic principles*
and then focus on the *detail* of particular problems.

Science is and has always been
an interplay between *many ideas*,
between *many minds*,
between *many humans.*

Science is and has always been
the quest to *understand creation*,
—the work of a *great God*,
and our Saviour in *Christ Jesus*.

Contents

Preface

Simulation of natural phenomena has been in the focus of science now for many centuries. The simulations are based on observations and scientific reasoning, which leads to the qualitative and quantitative description of natural or industrial processes.

When you try to bring *measurement data or observations* into simulations, you come to the fields of *inverse problems* and *data assimilation*. The key task of inverse problems is to infer knowledge about the structure of some system from measurements. Usually it is based on equations which model the underlying physical, chemical or biological processes, and it reconstructs sources or structural information. When the processes are *dynamical*, an important part of any realistic simulation is the evaluation of *initial states* in addition to *structural information* and underlying *parameter functions*. Data assimilation algorithms calculate initial states on which forecasts can be based.

Today, inverse problems are universal in the sciences. Basically each and every field of research or technology has its own inverse problems which are to be solved. Different fields have their own terminology, ranging from *medical imaging* to *tomography*, from *seismic exploration* to *inverse scattering* and *radar*, from *neuroscience* to *nondestructive testing*. At the same time, all these fields share many questions and phenomena, such that there is a high need for a joint approach. Here, functional analysis provides a very adequate framework with the language of normed spaces and Hilbert spaces – which has been highly successful in the world of physics now for 100 years.

Data assimilation has grown from geophysical and meteorological communities, where the determination of initial states has been of crucial importance for applications from the very beginning, since *forecasts* are possible only when reliable estimators for the initial conditions are available. Today, operational centers for *numerical weather prediction* (NWP) and climate projection are using top-500 supercomputers to serve our societies day by day[1]. At the same time, many further scientific communities discover the need for *data assimilation*, since they are moving from static estimates and qualitative science to quantitative forecasting for technological or medical phenomena.

Inverse Problems and Data Assimilation touch quite different *mathematical communities*. Inversion algorithms have first been suggested in the framework of mathematical analysis, partial differential equations and applied mathematics. It links into optimization as well as analysis in function spaces. Data Assimilation has been strongly linked to the stochastic communities, estimation theory and filtering. At the same time, it has been influenced by linear algebra and deterministic optimization. Today, we observe some convergence of these fields driven by the

[1] For example the Met Office in the UK, the European Center for Medium Range Weather Prediction (ECMWF) at Reading, UK, or the German Meteorological Service (DWD) at Frankfurt/Offenbach, Germany.

need to communicate and understand each other when working on joint applications.

Clearly, one single book cannot introduce the whole of inverse problems and data assimilation today – the selection we present might appear biased to some colleagues. We feel that it provides a broad generic selection, which will be useful for many different related problems as well. Also, different scientists work on the background of their particular field and its language. Our book aims to be introductory in many of its parts, with some deeper sections and chapters. Our goal is to reach a broad range of scientists from mathematics and applications. To this end we have included a lot of material about functional analysis which we found to be helpful for our own research over time, and also basic material about stochastic estimation theory and Markov Chain Monte Carlo methods.

We have added about 170 scripts in OCTAVE or MATLAB to the book (available for download **here**), which serve as a first step towards simulations and more sophisticated inversion or data assimilation algorithms. Extended versions of these scripts have been successfully applied to real data for practical problems such as acoustic sound analysis, magnetic tomography for fuel cells and cognitive neuroscience.

Our book has the claim to be full of insight which is highly relevant to practical applications. The second author works with algorithms in an operational environment, which is linked to many millions of users worldwide, ranging from about 40 other states to the flight control in central Europe and German federal weather warnings for all parts of society. This work is linked to dozens of different inverse problems, integrating them all into core data assimilation methods.

We are convinced that both the insight given by mathematical analysis as well as the small-scale and large-scale numerical tests are important for applications. Development and progress today is only possible when theory and practice come together.

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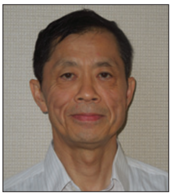
Last but not least we would like to express our thanks to our God, the creator of this world in Christ Jesus, for fascinating science, and for his deep grace and continuing guidance.

Author biographies

Gen Nakamura



Gen Nakamura received his Bachelor's Degree 1971 from International Christian University, his Master's Degree 1974 from the Graduate School of Tokyo Metropolitan University and Doctor of Science Degree 1977 from the Graduate School of Tokyo Metropolitan University.

He worked as Assistant Professor 1977-1991 at Josai University, 1981–1982 Lecturer of MIT, 1982–1983 Japan-US Exchange Fellow, 1989 Visiting Scientist of Brown University, 1991–1994 Professor of Josai University, 1994–1997 Professor of Science University of Tokyo, 1997–2001 Professor of Gunma University, 2001–2012 Professor of the Graduate School of Science, Hokkaido University, 2012–2013 Specially appointed Professor of the Graduate School of Science, Hokkaido University, 2013–2015 Professor of Inha University, since 2015 Harim Professor (special professor) Inha University, 2001- Visiting Professor of South East University. Professor Nakamura has served 2006–2009 as Program Officer of Japan Science for the Promotion of Science, 2009–2013 Board Member of Mathematical Society of Japan, 2011–2015 Expert Committee Member of Research Institute of Mathematical Science, Kyoto University and since 2011 Committee Member Based on the Fields of Specialties of Science Council of Japan.

He received several prizes, for example the Autumn Prize of Mathematical Society of Japan, 2000 and the Prize for Science and Technology by the Minister of Education, Culture, Sports, Science and Technology of Japan, 2009. He is Editor of Eurasian Journal of Mathematical and Computer Applications and Member of the steering committee of Eurasian Association for Inverse Problems.

Professor Nakamura has published 170 publications recorded in the MathScinet data base.

Roland Potthast

Roland Potthast received his Diplom 1993, a PhD in Mathematics 1994 and a Habilitation Degree in Mathematics 1999 at the University of Göttingen, Germany. Postdoctoral positions led him 1996–1998 to the University of Delaware, USA. After the Habilitation, Roland worked in a Consultancy Company in Cologne, Germany, 1999–2000, and at the Brunel University in London 2000–2001. From 2001 to 2006 he was leader of a *Young Researcher Group* on Inverse Problems in Göttingen, funded by the Volkswagen Foundation. He received an extraordinary professorship at the University of Göttingen in 2004. As a part-time activity, he was Owner and CEO of an

IT-company based in Göttingen for seven years 2005–2012, providing internet-based services to large-scale European companies.

Roland moved to the UK in 2006, with a lectureship at the University of Reading 2006, with promotions to Reader (Associate Professor) 2008 and Full Professor in 2010. Roland has been visiting professor at the University of Rennes, France, 2007–2009 and at the Research Center Jülich, Germany, 2009 and had a call to a W3 professorship in Munich 2010. Since 2010 he holds a post as 'Director and Professor' (B1) at the German *Federal Ministry of Transport and digital Infrastructure (BMVI),* leading the *Data Assimilation Department* of the German Weather Service (DWD) in Frankfurt/Offenbach, with a part-time involvement as full professor at the University of Reading, UK. He is supervising a group of about 25 scientists and coordinates a network of about 45 researchers in data assimilation and inverse problems working in collaboration with DWD.

Roland received several prizes and awards, among them 1994 the best PhD award of the German Mathematical Society, four years of full-time funding by the German Science foundation DFG and five years by the Volkswagen Foundation; in the UK an EPSRC Springboard fellowship in 2007 and a Leverhulme research fellowship in 2008, a 'Bridging-the-Gaps Award' by EPSRC in 2009–12; and the *Pichorides Distinguished Lectureship* of the University of Crete, Greece, in 2015.

Professor Potthast has published more than 65 mathematical research papers, which received more than 1000 citations, a book on inverse scattering theory 2001 and a book on neural field theory 2014.