



A Model to Predict the Size of Regolith Clumps on Planetary Bodies

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Abstract

Prior investigations of the behavior of regolith on the surface of planetary bodies has considered the motion and interactions of individual grains. Recent work has shown the significance of cohesion in understanding the behavior of planetary regolith, especially on small, airless bodies. Surficial regolith grains may detach from a planetary body due to a variety of phenomena, including aeolian effects, spacecraft operations, micrometeoroid bombardment, and electrostatic lofting. It is well known in terrestrial powder handling that cohesive powders tend to form clumps. We present a theory for the size of regolith clumps that are likely to form and be easier to detach from a surface than their constituent grains, assuming monodisperse, spherical grains. The model predictions are significant for our interpretation of the surface of asteroids, as well as understanding a variety of phenomena on planetary bodies and designing of sampling spacecraft.

Unified Astronomy Thesaurus concepts: [Asteroid surfaces](#) (2209)

1. Introduction

A regolith grain on the surface of a planetary body can be acted upon by gravity, cohesion, friction, air drag and an electrostatic force (the product of the grain's charge and local electric field strength). Grains can detach from the surface of a planetary body through a variety of mechanisms: for example, micrometeoroid bombardment, saltation, spacecraft operations, and electrostatic lofting. Cohesion dominates the behavior of subcentimeter regolith on small airless bodies (Scheeres et al. 2010), and research into electrostatic lofting of individual grains revealed the preferential lofting of intermediately sized grains (Hartzell & Scheeres 2011; Hartzell et al. 2013). Terrestrially, it is experimentally observed (in a variety of fields dealing with cohesive powders) that clumps of small grains detach more readily than individual small grains (Marshall et al. 2011). Close-up observations of the lunar regolith from the Apollo era have previously been described as “clumpy” (Helfenstein & Shepard 1999). Additionally, the detachment of clumps rather than individual grains from a bed of regolith has previously been hypothesized by Marshall et al. (2011). However, there is currently no model to predict when clumps will detach from the surface of a planetary body rather than individual grains, or the size of those clumps.

The possible formation and preferential detachment of clumps of grains have significant implications for our understanding of various planetary processes and the design of spacecraft to interact with planetary surfaces. It has been proposed (Durda et al. 2014) that “boulders” on asteroids may in fact be clumps (aggregates of smaller particles), since nongravitational forces will dominate in the literal microgravity asteroid surface environment. The idea that “boulders” on asteroid surfaces may not be coherent would also explain the recent OSIRIS-REx observations of very low thermal inertia boulders on the asteroid Bennu (Rozitis et al. 2020). Rozitis et al. (2020) have proposed that these low thermal inertia boulders may have porosities up to 50%. The porosity and

corresponding internal strength of surface particles (from boulders down to gravels) on asteroids will influence the design of spacecraft seeking to land on, anchor to, sample, or traverse these surfaces.

Prior experimental work has shown that airborne particles may clump together due to electrostatic charging (Greeley 1979) and that the formation of surficial regolith clumps may influence the erosion rate of Martian surface features (Greeley et al. 1982). This work quantifies the cohesive and gravitational forces acting on particles and predicts the size of agglomerates that are easier to detach from the surface than individual grains. The lofting of aggregates rather than individual grains due to aeolian stresses agrees with experimental observations by Marshall et al. (2011). The motion of clumps of grains, rather than individual grains, influences predictions of formation times of aeolian features, on Mars and other planetary bodies (e.g., comets and Titan).

The preferential motion of clumps, rather than individual grains, also influences modeling and theoretical investigations of planetary avalanches and electrostatic lofting. The detachment, motion, and breakup of clumps will change the dynamics of these phenomena. For example, the process of pouring sugar (an avalanche where cohesion is not significant) as compared to baking flour (highly cohesive and clumpy) is quite different. By analogy, theoretical predictions of planetary avalanches that neglect the formation of clumps may not properly predict the characteristics of avalanches on planetary surfaces. Similarly, the significance of electrostatic lofting in redistributing regolith across a planetary surface will be increased if clumps, rather than individual grains, are lofted.

Considering the gravitational and cohesive forces acting on regolith grains, we present a model to predict the size of regolith clumps that require less force to detach than their constituent grains, as a function of grain size, packing fraction, and central body size. We assume that all forces are collinear and act perpendicular to the surface. Additionally, we predict the size of clumps likely to be present in situ by modeling the bulk regolith porosity as the result of less porous clumps separated by voids. We primarily consider airless planetary bodies, but the results presented are also applicable to planets with atmospheres (e.g., Earth and Mars).



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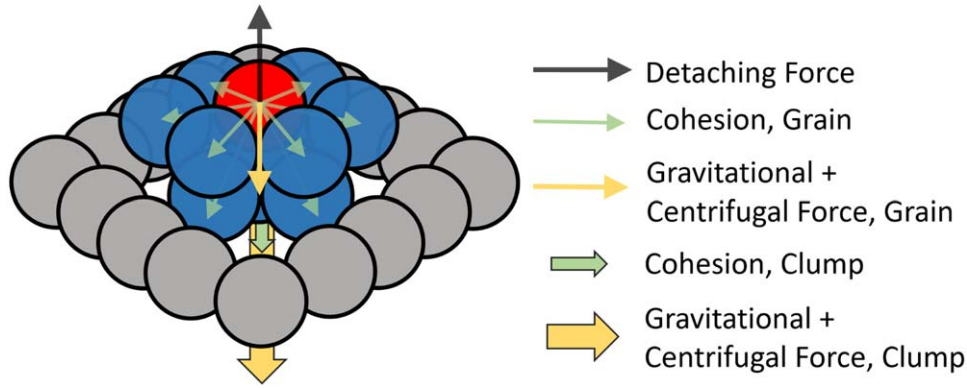


Figure 1. An idealized diagram showing the cohesion, gravitational, and centrifugal forces acting on a clump (red and blue grains) and grain (red grain) that oppose some force acting to detach from the surface, all within a rotating body frame. The bulk powder is denoted by gray grains, and there exist gaps between the clump and bulk powder that signify void space in packing. When considering the detachment of the single red grain, the blue neighboring grains exert cohesive forces on the red grain being detached.

This investigation is agnostic to the mechanism through which particles or clumps are detached from the surface. We compare the magnitudes of the forces required to remove individual particles and clumps. Any clump or particle removal mechanism must overcome the gravity and cohesion (and possibly other forces) holding a grain on the surface. This investigation does not consider additional interparticle forces active in certain detachment mechanisms (e.g., interparticle repulsion in electrostatic lofting, or small-scale recirculation in aeolian effects). Instead, we provide a first-order prediction of whether an individual particle or clump requires less force to detach.

2. Force Model for Constraining Clump Size

We assume that the net force adhering a regolith grain to a bed of grains is the sum of the gravitational force and the cohesive force acting on the grain. In order to detach a grain from the surface, the net force away from the surface (“upward”) must be greater than the net force toward the surface (“downward”). The net downward force varies with the size of the regolith grain. For small, fast-rotating bodies, centrifugal and gravitational accelerations can be the same order of magnitude, and the centrifugal force pulling grains upward should be subtracted from the gravitational force, lessening the net downward force on grains. In order for a clump to detach rather than an individual grain, the net downward force on an individual grain must be greater than the net downward force on a clump of grains. This section derives an expression for maximum clump size that satisfies this condition (i.e., the largest clump that requires less force to detach than an individual grain).

2.1. Forces Acting on a Clump

We derive an expression for the forces acting on a clump of grains. Regolith grains are approximated as monodisperse spheres of uniform density. We assume that grains in a clump have a local packing fraction ϕ . The maximum value of ϕ is 0.74, corresponding to face-centered cubic and hexagonal close packing, and the minimum value is ϕ_b , where ϕ_b is the bulk packing fraction of the regolith. Figure 1 shows a sketch of the forces acting on a clump and grain in our model. Assuming that a clump consists of grains packed into a three-dimensional volume of height h , the equation for cohesion on a clump is

(based on Equations (7) and (8) in Sánchez & Scheeres 2014)

$$F_{c,clump} = \frac{A_h A_b \phi C_{avg}}{8R}, \quad (1)$$

where R is the radius of the grains, A_b is the clump cross-sectional area, C_{avg} is the mean coordination number (the average number of contacts between a clump and its neighbors), and A_h is the reduced Hamaker constant for the regolith. The reduced Hamaker constant for lunar regolith is 0.036 N m^{-1} (Sánchez & Scheeres 2014), which assumes a Hamaker constant of $5.14 \times 10^{-20} \text{ J}$ and a separation between grains due to adsorbed molecules of $1.17 \times 10^{-10} \text{ m}$ (about the diameter of an oxygen ion). C_{avg} is taken to be 4.5 (Sánchez & Scheeres 2014). Sánchez & Scheeres (2014) note that C_{avg} will decrease with realistic (noncrystalline) granular packings. The sensitivity of our results to C_{avg} will be discussed in Section 4.1. An alternate method to characterize cohesion involves modeling the capillary force and quantifying the strength of this force through its relation to the change in surface energy during particle contact formation (Starukhina 2000). However, we follow the implementations of the van der Waals force used in prior simulations of regolith particles (Hartzell & Scheeres 2011; Scheeres et al. 2010; Sánchez & Scheeres 2014).

The gravitational force acting on a clump (Equation (2)) is derived for a known grain radius R and surface gravity g . We approximate asteroid regolith properties based on recent measurements of Bennu by the OSIRIS-REx mission. Since the bulk porosity of Bennu is 50% and its bulk density is 1200 kg m^{-3} (Lauretta et al. 2019), the density of a regolith grain ρ is assumed to be 2400 kg m^{-3} . The clump’s volume V_{clump} is calculated by assuming that the clump has a height h :

$$F_{g,clump} = V_{clump} \phi \rho g = A_b h \phi \rho g. \quad (2)$$

Note that sliding friction forces are not active because, due to the uniformly spherical shape of the grains, the surface of contact between any two grains is a point (not a plane). Additionally, there is no rolling friction because rotation of the grains is not considered. Furthermore, “friction” as typically considered is caused by surface asphericities (not relevant to our perfectly spherical grains) and chemical attractions (captured by our explicit definition of van der Waals attraction).

2.2. Forces Acting on a Grain

The cohesive force on a single grain is described by Hartzell & Scheeres (2011):

$$F_{c,\text{grain}} = A_h R C_{\text{FCC}}, \quad (3)$$

where C_{FCC} is the mean coordination number for face-centered cubic (FCC) packing ($C_{\text{FCC}} = 12$). Crystalline packings of spherical “grains” (or molecules) are commonly studied in materials science, and the maximum possible coordination number of a crystalline packing is 12. Cohesion is maximized when the coordination number is maximized (i.e., when a grain has the theoretical maximum of 12 contacts). An individual grain that would be detached from a regolith surface would have fewer than 12 contacts, since there will be no crystalline matrix on the “top side” of the grain. The number of contacts on an individual regolith grain strongly depends on the packing of the surface. If the regolith surface has a fractal structure, a single regolith grain could have as few as one contact. For simplicity, we assume that $C_{\text{FCC}} = 12$. The effect of assuming a reduced coordination number is discussed in Section 4.1. Additionally, we note that $C_{\text{FCC}} \neq C_{\text{avg}}$. C_{FCC} considers the contacts to an individual grain from all directions, whereas C_{avg} considers the contacts to a clump that cross a single plane that must be severed for the clump to detach.

The gravitational force acting on an individual grain is

$$F_{g,\text{grain}} = \frac{4}{3}\pi R^3 \rho g. \quad (4)$$

2.3. Largest Clump Constrained by Net Downward Force

In order for a clump of grains to detach from the bulk regolith bed rather than a single grain, the net downward force on a single grain must exceed the net downward force on a clump. Thus, in order for a clump to be detached rather than an individual grain, the following expression must be satisfied:

$$F_{c,\text{grain}} + F_{g,\text{grain}} - F_{\text{cent},\text{grain}} \geq F_{c,\text{clump}} + F_{g,\text{clump}} - F_{\text{cent},\text{clump}} \quad (5)$$

$$\begin{aligned} F_{\text{cent}} &= m r_s \omega^2 \\ m_{\text{grain}} &= \frac{4}{3}\pi R^3 \rho \\ m_{\text{clump}} &= A_b h \phi \rho. \end{aligned}$$

The centrifugal force F_{cent} is defined for clumps and grains based on their respective volumes, regolith density ρ , the body’s radius at the surface r_s , and the body’s spin rate ω . Substituting the expressions for the forces given by Equations (1)–(4) into Equation (5) and solving for the clump cross-sectional area (A_b) gives the maximum clump cross-sectional area ($A_{b,\text{max}}$) that is easier to detach than an individual grain:

$$\begin{aligned} A_h R C_{\text{FCC}} + \frac{4}{3}\pi R^3 \rho (g - r_s \omega^2) &\geq \frac{A_h A_b \phi C_{\text{avg}}}{8R} + A_b h \phi \rho (g - r_s \omega^2) \\ A_{b,\text{max}} &= \frac{8A_h R^2 C_{\text{FCC}} + \frac{32}{3}\pi R^4 \rho (g - r_s \omega^2)}{A_h \phi C_{\text{avg}} + 8R \phi h \rho (g - r_s \omega^2)}. \end{aligned} \quad (6)$$

Equation (6) gives the largest clump that is easier to detach than a single grain, as a function of grain radius R and local

packing fraction ϕ . All other variables in Equation (6) are constant for a given system as they are a property of either the regolith (A_h , C_{avg} , ρ) or the body (r_s , ω , g) or are based on modeling assumptions (h , C_{FCC}). For small, fast-rotating bodies the centrifugal acceleration can be comparable to the gravitational acceleration. When gravitational and centrifugal accelerations are comparable (reducing the effective surface gravity), the role of cohesion in dictating clump size increases, resulting in larger clumps being easier to detach than individual grains.

Equation (6) gives the maximum cross-sectional clump size that is easier to detach than an individual grain. Ideally, we would model clumps as cubic volumes (e.g., $V_{\text{clump}} \propto s^3$) with uniform side length (s) given as a function of R , ϕ , and g . Note that in Equation (6) the clump cross-sectional area A_b , $\max(\propto s^2)$ is found on the left-hand side of the equation and h ($\propto s$) is found on the right-hand side of the equation. Solving Equation (6) for clump side length s does not yield an analytical solution. In Figure 2, we show that the cubic clump volume (black line) is bounded by clump volumes assuming height h as a function of grain radius R . We relate h to grain size through a , the discrete side length of a face-centered cubic or hexagonal close packed cube, where $a = 2\sqrt{2}R$. Figure 2 compares the volumes of clumps with heights of $2a$, $3a$, and $4a$ to cubic clumps of 5 mm grains on Bennu (assuming a surface gravity of $8.16 \times 10^{-5} \text{ m s}^{-2}$, 245 m mean radius, and 4.296 hr rotation period; Lauretta et al. 2019) for a variety of clump packing fractions. Clumps with heights $2a$ and $4a$ bound the volume of the cubic clump. We find that $s < 4a$ for other grain sizes and gravities as well (including 5 mm grains on Earth and 70 μm grains on Earth). A clump height of $4a$ presents an upper limit on the clump size over the range of grain sizes (micron–centimeter) and gravities (Earth–Bennu) that we consider in this study. The volume of a cubic clump approaches the volume of a clump where $h = 4a$ as local packing fraction decreases in Figure 2. Note that our model’s assumption of spherical, monodisperse grains requires $\phi_b < 0.74$, and the packing fraction observed on asteroids is much less than 0.74. Equation (6) provides an upper limit on clump size. If it is possible to produce and detach clumps with height $4a$, then it must be possible to produce and detach cubic clumps, since $h = 4a$ is an upper bound on cubic clump volume. As a result, we will assume a clump height of $4a$, or $8\sqrt{2}R$, moving forward.

An expression for the maximum number of grains per clump as a function of surface gravity and grain size can be derived from the force constraint (Equation (6)) as follows:

$$V_{b,\text{max}} = A_{b,\text{max}} h \phi = A_{b,\text{max}} (8\sqrt{2}R) \phi \quad (7)$$

$$N_{\text{grains},\text{max}} = \frac{V_{b,\text{max}}}{V_g} = \frac{A_{b,\text{max}} (8\sqrt{2}R \phi)}{\frac{4}{3}\pi R^3}$$

$$N_{\text{grains},\text{max}} = \frac{48\sqrt{2}}{\pi} \frac{A_h C_{\text{FCC}} + \frac{4}{3}\pi R^2 \rho g}{A_h C_{\text{avg}} + 64\sqrt{2}R^2 \rho g}. \quad (8)$$

Note that while Figure 2 shows a decreasing clump size as the local packing fraction increases, ϕ does not appear in Equation (8). Thus, the decreasing clump size in Figure 2 is due to the tighter packing of the grains at higher local packing

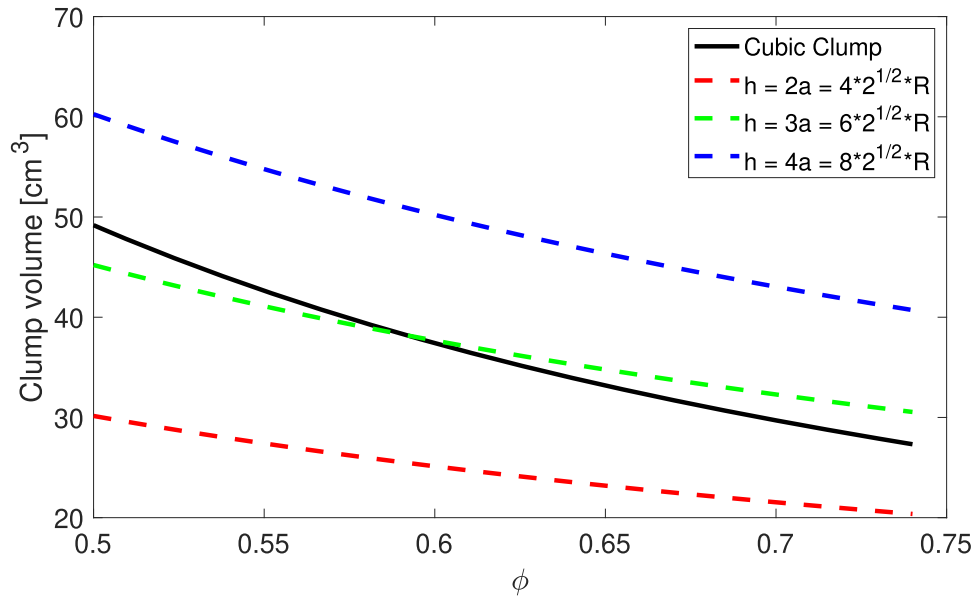


Figure 2. Clump volume as a function of local packing fraction and assumed clump height h are compared for cubic clumps (black, uniform side length) and clumps with height $2a$, $3a$, and $4a$. The plot is generated for 5 mm grains on Bennu (assuming Bennu's spin rate). The cubic clump volume is less than $4a$ for other gravities and particle size of interest.

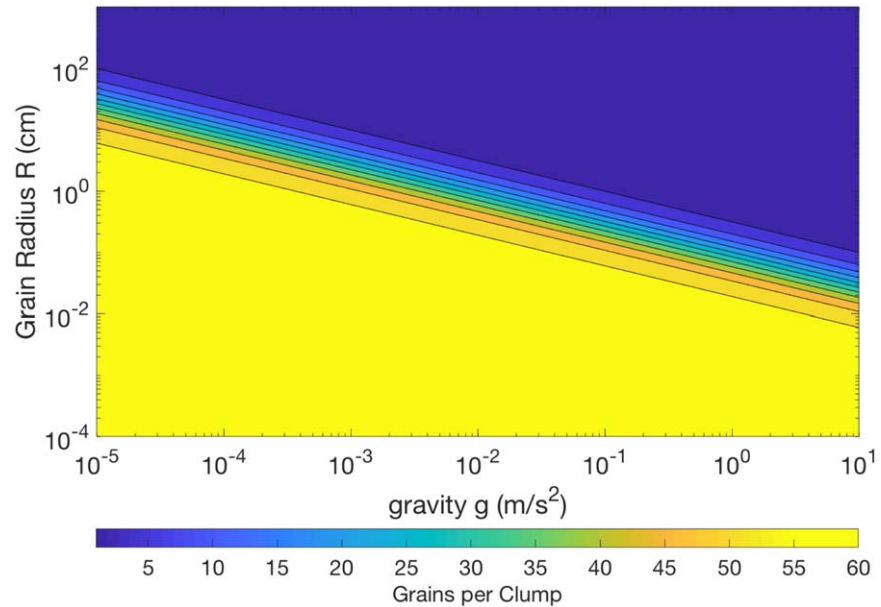


Figure 3. The maximum number of grains per clump as a function of gravity and grain size, assuming negligible centrifugal acceleration. The packing fraction is assumed to be 50%, and the grain density is assumed to be 2400 kg m^{-3} .

fractions, not necessarily a change in the number of grains in the clump.

Equation (8) is plotted for several gravities and grain radii in Figure 3. We note that the centrifugal force is dropped in Equation (8) because Figure 3 assumes no central body spin, since the spin rate can vary widely between bodies. For a given gravity and grain size, Figure 3 indicates the number of grains in the largest clump capable of being detached. Figure 3 shows that a clump composed of fifty-seven 2 cm radius grains can detach on Bennu, but only single 2 cm grains can detach under the substantially greater gravities of Earth and the Moon. Additionally, Figure 3 shows that 57-grain clumps of $25 \mu\text{m}$ radius grains can detach on Earth.

3. Geometric Model for Constraining Clump Size

In addition to the force constraint on the detachment of a clump, the bulk and local packing fractions place a geometric constraint on clump size. A given bulk packing fraction can be produced via clumps (with a higher local packing fraction) separated by defects. We define a defect to be a void in the packing lattice that serves as the structural weak point from which a clump can detach from the bulk powder.

Assume that a given bulk volume V has a known average porosity P . Porosity is related to the bulk packing fraction ϕ_b as follows:

$$\phi_b = 1 - P. \quad (9)$$

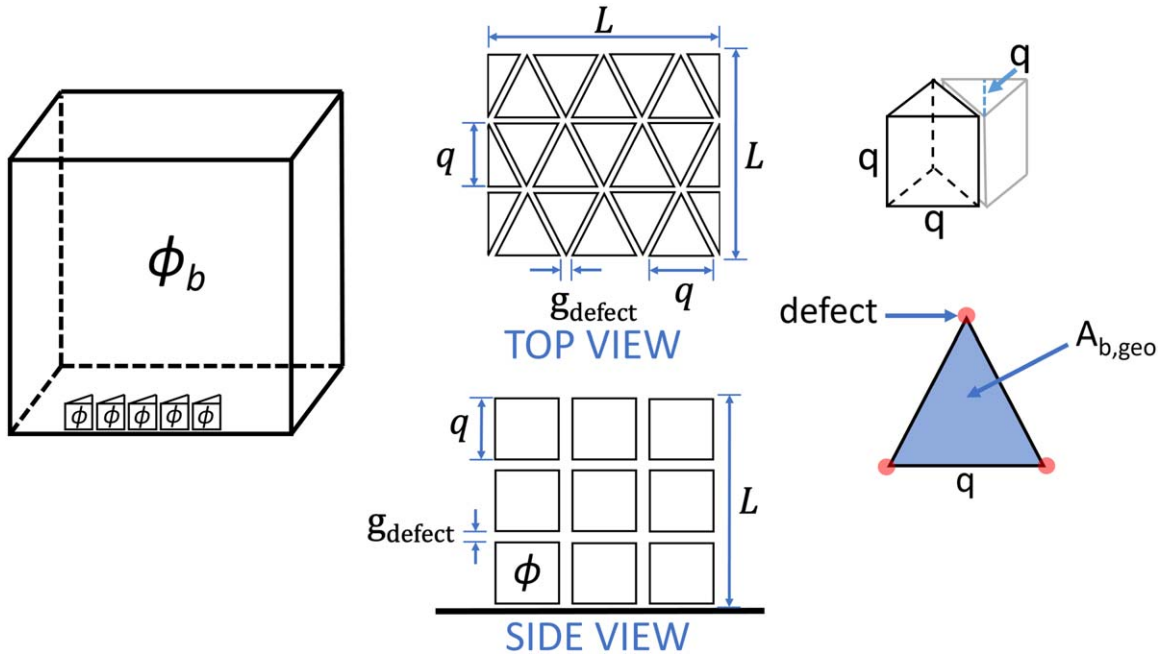


Figure 4. Diagram of the geometric model. The total volume V with bulk packing fraction ϕ_b contains clumps locally packed with packing fraction ϕ (where $\phi \geq \phi_b$) with a side length q . Clumps are separated by defects of length g_{defect} . Three defects are minimally required to form the cross section of a clump $A_{b,\text{geo}}$.

Assuming N monodisperse spheres of radius R , the volume occupied by grains is $V_{g,\text{tot}}$:

$$V_{g,\text{tot}} = \frac{4}{3}\pi R^3 N.$$

Porosity can be described by the fraction of empty space (V_e) in V :

$$P = \frac{V - V_{g,\text{tot}}}{V} = \frac{V_e}{V}. \quad (10)$$

We assume that the total volume V , occupied by grains packed at bulk porosity P , is evenly discretized into uniform volumes of height q (see Figure 4). The cross section of these volumes is a triangle of height and base q . The volumes will be referred to as clumps, each containing grains packed with the local packing fraction ϕ (where $\phi \geq \phi_b$). Since the clumps can be more tightly packed than the bulk, empty regions (defects) of length g_{defect} form between these clumps. The volume of grains is given by Equation (11), where V_q is the total volume of the clumps.

$$V_{g,\text{tot}} = \phi V_q \quad (11)$$

The void volume V_e is the summation of the defect volumes in V and the empty space in all the clumps. The total volume can be described by the sum of its empty and occupied space:

$$V = V_e + \phi V_q. \quad (12)$$

Since the regolith grains are approximated by uniform spheres, the local packing fraction ϕ must be between ϕ_b and $\phi_{\text{FCC}} = 0.74$. We assume that the defects between ϕ -packed clumps are less than $2R$ in length, since having a defect smaller than the diameter of a single grain eliminates the possibility of a grain falling into one of these spaces.

Combining Equations (9), (10), and (12) yields the expression for V_q in terms of the known bulk packing fraction ϕ_b , total

volume V , and local packing fraction ϕ :

$$V_q = \frac{V\phi_b}{\phi}. \quad (13)$$

Along the total volume's side length L , there exist n clumps and n defects:

$$L = nq + ng_{\text{defect}}, \quad (14)$$

where q is the length of the base of one of the triangle clumps along the length L . Figure 4 shows a top view of the total volume V . Assume that the height of V is defined by n layers of the cross section shown in Figure 4. The space between these layers shown by the side view in Figure 4 is also g_{defect} , and this is equivalent to the separation distance between clumps in the top view. The clump side length q determines clump size. From Equation (14), the expression for q is

$$q = \frac{L}{n} - g_{\text{defect}}. \quad (15)$$

Substituting $V = L^3$, $V_q = (nq)^3$, and Equation (13) into Equation (15), we find q as a function of the local packing fraction ϕ and bulk packing fraction:

$$q = (g_{\text{defect}}\phi_b^{1/3})/(\phi^{1/3} - \phi_b^{1/3}) \quad (16)$$

We assume that clumps are defined by three defects (as shown in Figure 4), which are weak points in the granular structure. Because three points are necessary to define a plane and a planar surface is the simplest boundary for a clump, we assume that three defects define a clump. An arbitrarily complicated clump boundary could be defined, but this would increase the clump surface area, thereby increasing its cohesive attraction to the surrounding regolith. Thus, we assume that the base of the clump is an isosceles triangle and $A_{b,\text{geo}}$ is the cross-sectional area of the clump derived from the geometric

constraints:

$$A_{b,geo} = \frac{1}{2}q^2. \quad (17)$$

To calculate the geometrically constrained clump volume, we must define the height of the clump. Following our development of the force-constrained clump volume, we define the height of the geometrically constrained clump as $4a$. The volume occupied by the grains in a geometrically constrained clump is given by Equation (18), and the number of grains in the clump is given by Equation (19):

$$V_{b,geo} = 8\sqrt{2}R\phi A_{b,geo} \quad (18)$$

$$N_{grains} = \frac{V_{b,geo}}{V_g}$$

$$N_{grains} = \frac{3\sqrt{2}g_{defect}^2\phi_b^{\frac{2}{3}}\phi}{\pi R^2(\phi_b^{\frac{1}{3}} - \phi_b^{\frac{1}{3}})^2}. \quad (19)$$

In standard crystalline packings, a single cube of volume will contain portions of grains in addition to whole grains (see Figure 5(a)). A real, physical clump of grains will only include whole grains. Imagine that the height of the clump defines the size of a box superimposed on the crystalline grain structure as illustrated in Figure 5. The clump will be defined by grains fully encompassed in this box, so removing the grains only partially inside the box will decrease the clump's true packing fraction ϕ_t from the nominal local packing fraction ϕ . Figure 5 shows how our definition of the clump height changes the ratio of the true packing fraction to the nominal packing fraction (ϕ_t/ϕ). When ϕ_t/ϕ approaches 1, the clump volume given by the geometric model accurately predicts the clump volume calculated using the true local packing fraction. $h = 4a$ yields a ratio $\phi_t/\phi = 0.96$ (Figure 5(f)). Thus, the true clump volume will be slightly less than predicted by our model. For the FCC or hexagonal close packed (HCP) packing ($\phi = 0.74$), $\phi_t = 0.710$. ϕ_t/ϕ will change with different local packing fractions. Applying $\phi_t/\phi = 0.96$ as a correction factor for all $\phi < 0.74$ will lead to artificially larger clumps being predicted to detach on various bodies at looser local packing. Since we wish to be conservative in our prediction of largest detachable clump volume, we do not apply a correction factor to ϕ . Because our model treats the nominal packing fraction as equivalent to the true packing fraction, it leads to an underestimation of the detachable clump size.

4. Clump Size Predictions

A clump must satisfy both the force constraint (Equation (6)) and the geometric constraint (Equation (17)). If the clump size dictated by the geometric constraint (Equation (18)) is smaller than the maximum clump size satisfying the force constraint (Equation (7)),

$$V_{b,geo} \leq V_{b,max}, \quad (20)$$

then a clump, rather than an individual grain, will detach.

As mentioned previously, the local packing fraction is bounded by the bulk packing fraction and FCC packing:

$$\phi_b \leq \phi \leq \phi_{FCC} = 0.74. \quad (21)$$

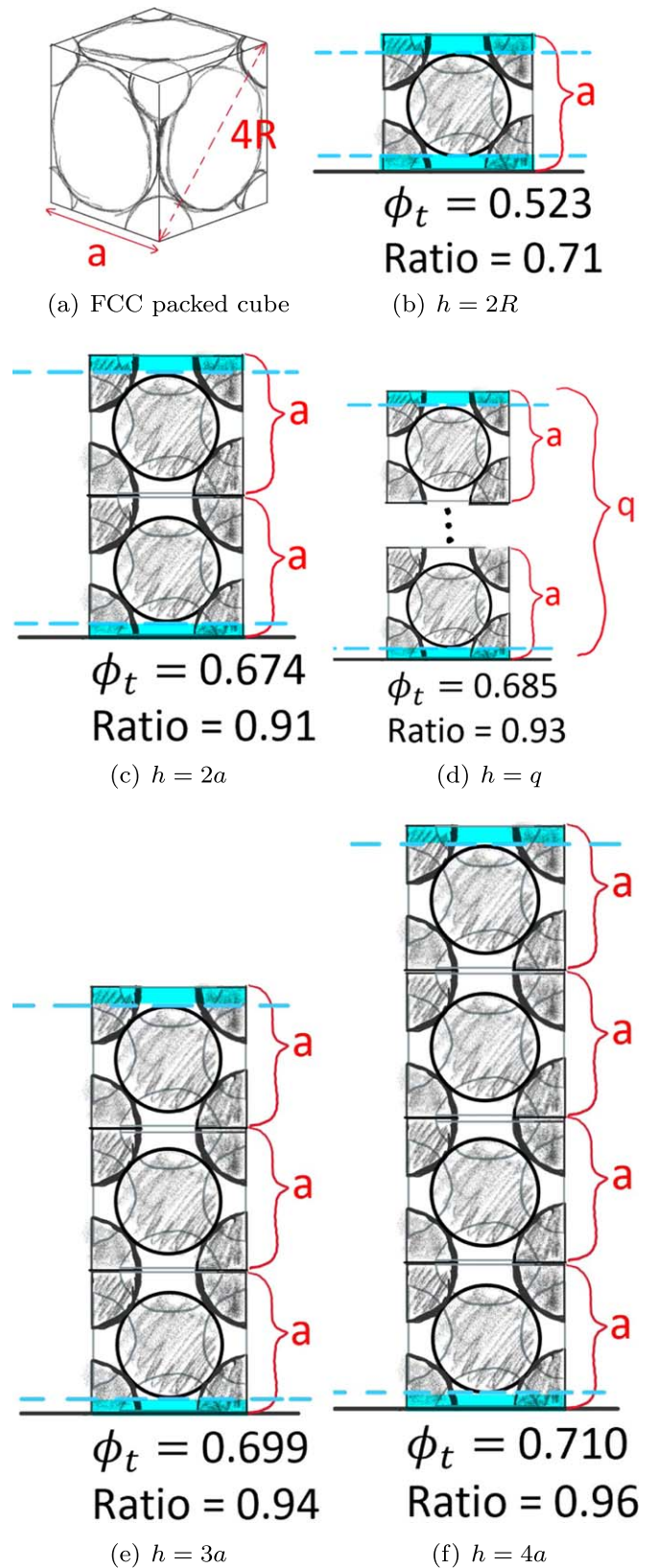


Figure 5. The true local packing fraction ϕ_t (neglecting partial grains) and ratio ϕ_t/ϕ as a function of assumed clump height for an FCC-packed clump ($\phi = 0.74$), where $a = 2\sqrt{2}R$. The cyan regions show the sections of the clump removed when discarding layers of partial grains from the top and bottom of the clump when assuming some height h .

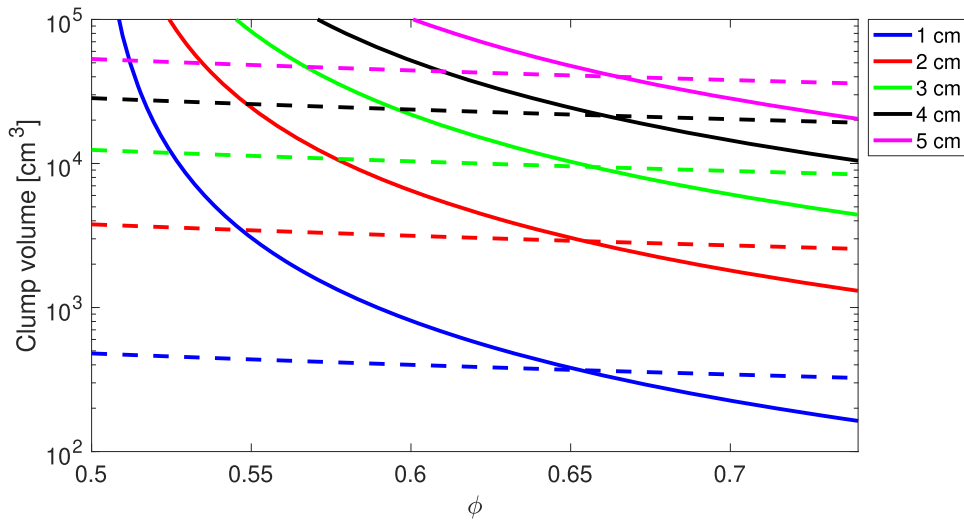


Figure 6. Clump volume as a function of grain size and local packing fraction on Bennu, assuming 0.5 bulk packing fraction. The dashed lines correspond to the maximum clump size from the force constraint, and the solid lines represent the clump size from the geometric constraint. Clump sizes where the solid line is below the dashed line will be preferentially detached rather than individual grains.

When $\phi = \phi_b$, the local packing fraction of clumps matches the bulk packing fraction. As a result, no defects will exist, and the bulk powder will be uniformly packed. The maximum ϕ corresponds to FCC packing, with a value of 0.74. When $\phi = \phi_{\text{FCC}}$, clump volume is minimized because ϕ_{FCC} produces the largest allowable clump density.

We assume $g_{\text{defect}} \leq 2R$, since defects larger than the diameter of grains may become filled in situ. The total volume's porosity is composed of the empty space within clumps, driven by ϕ , and the sum of all defect volumes. Therefore, for a fixed local packing fraction ϕ , increasing g_{defect} will increase geometric clump side length (Equation (16)), thus increasing clump size. Smaller values of g_{defect} may be more realistic owing to the size distribution of regolith in situ. Larger values of g_{defect} result in less frequent defects and, thus, larger clumps. Large values of g_{defect} may produce clumps that are too large to detach (i.e., that do not satisfy Equation 20). In the following results, the defect length is assumed to be $0.75R$. Since we assume $g_{\text{defect}} = 0.75R$, increasing the local packing fraction decreases the geometric clump side length, which leads to a smaller clump size.

The maximum detachable clump size on a given planetary body is calculated by solving Equation (20) for the local packing fraction that produces the equality constraint ($\phi_{\text{max,size}}$) and evaluating Equation (18) at $\phi_{\text{max,size}}$:

$$\frac{1}{2}q^2 \leq \frac{8A_h R^2 C_{\text{FCC}} + \frac{32}{3}\pi R^4 \rho g}{A_h \phi_{\text{max,size}} C_{\text{avg}} + 64\sqrt{2} R^2 \phi_{\text{max,size}} \rho g}. \quad (22)$$

We note that the centrifugal force has been dropped from Equation (22) for simplicity. At the equality of Equation (20), $\phi = \phi_{\text{max,size}}$, indicating the packing fraction corresponding to the maximum clump size where both the geometric and force constraints are satisfied. Note that Equation (16) defines q in terms of ϕ .

We will now present plots of clump size derived from the geometric ($V_{b,\text{geo}}$) and force ($V_{b,\text{max}}$) constraints as a function of local packing fraction, gravity, and grain size. These plots

show the range of clump sizes that can exist in situ and are easier to detach than individual grains.

4.1. Predicted Clumps on Bennu

Due to their low gravity, asteroids produce detachable clumps of relatively large grains. Figure 6 shows the clump size dictated by the force and geometric constraints for a variety of grain sizes, considering Bennu's surface gravity and spin rate (assuming a surface gravity of $8.16 \times 10^{-5} \text{ m s}^{-2}$, 245 m mean radius, bulk porosity of 50%, and 4.296 hr rotation period (Lauretta et al. 2019)).

Figure 6 shows predictions of the geometric and force constraint models and the resulting clumps that require less force to detach than individual grains. The intersection of a dashed line (force constraint) with the solid line (geometric constraint) of the same color indicates the maximum detachable clump size for that grain radius, occurring at $\phi_{\text{max,size}}$. The intersections occur at the lowest possible packing fraction for which clump detachment is possible. The largest detachable clump becomes more porous as grain size decreases. High packing fractions are difficult to achieve in nature, due to the polydispersity and asphericity of regolith in situ. Thus, the porous clumps (at packing fractions less than 0.74) are more likely to exist in nature. Figure 6 shows that centimeter-scale constituent grains produce decimeter-scale detachable clumps on Bennu. Similarly, Figure 7 demonstrates that millimeter-scale grains produce centimeter-sized clumps on Bennu.

Figure 8 shows the number of grains per clump dictated by force and geometric constraints for a variety of grain sizes on Bennu. In Figure 8, the intersection of a dashed line with the solid line indicates the number of grains in the maximum clump size. Note that the solid line representing grains per clump from the geometric constraint is independent of grain size (because g_{defect} in Equation (19) is a function of R). The grains per clump indicated by the dashed lines in Figure 8 are dependent on the surface gravity on Bennu and the assumed grain size through Equation (8). As grain radius decreases, the number of grains per clump increases from approximately 50 to 57 grains. As grain size decreases, the number of grains per clump asymptotes to a value between 57 and 58 grains.

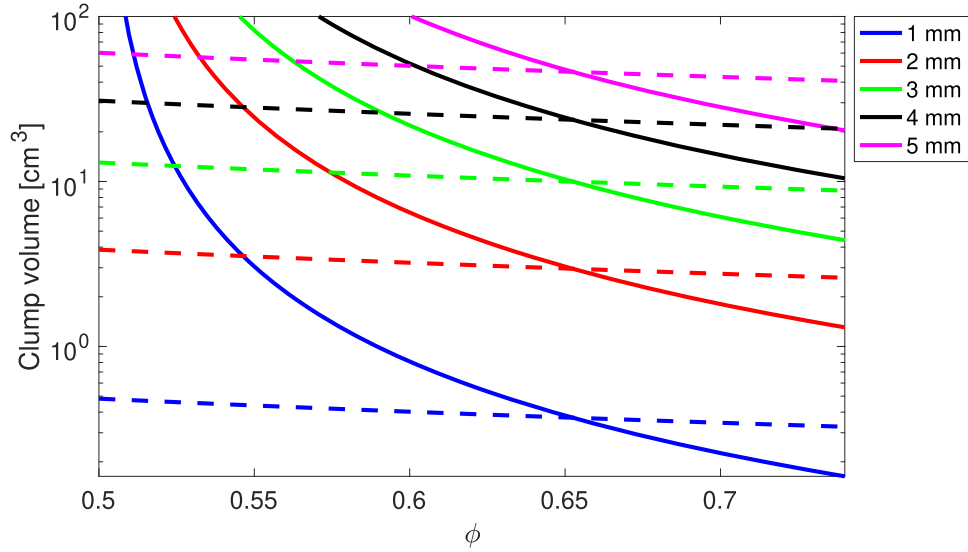


Figure 7. Clump volume as a function of grain size and local packing fraction on Bennu, assuming 0.5 bulk packing fraction. Grain radii on the millimeter scale are examined.

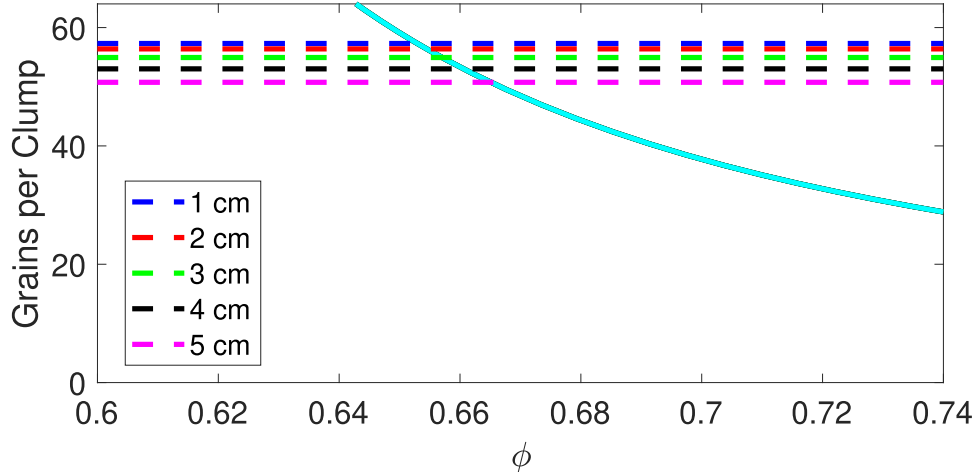


Figure 8. The number of grains per clump as a function of local packing fraction on Bennu, assuming 0.5 bulk packing fraction. The dashed lines correspond to the maximum number of grains per clump from the force constraint, and the single solid line represents the number of grains per clump from the geometric constraint, which is independent of grain size (Equation (19)).

In our investigation, we assume that C_{avg} , the mean coordination number, is 4.5 for clumps (Sánchez & Scheeres 2014) and the mean coordination number for grains is $C_{\text{FCC}} = 12$. Recall that C_{avg} controls the cohesion between the clump and the surrounding regolith. Assuming 300 μm radius monodisperse grains on Bennu, Figure 9(a) shows how the volume of a clump varies with the assumed C_{avg} . The geometrically constrained clump volume (solid cyan curve in Figure 9(a)) is independent of C_{avg} . Decreasing C_{avg} from the nominal value of 4.5 significantly increases clump volume and decreases $\phi_{\text{max, size}}$. The cohesion force on a clump is proportional to C_{avg} , so reducing this force allows clumps to grow in size. Increasing C_{avg} gradually decreases clump size and significantly increases $\phi_{\text{max, size}}$. Detachment is impossible once $\phi_{\text{max, size}}$ exceeds 0.74 and becomes more unrealistic in nature as $\phi_{\text{max, size}}$ increases since higher values of ϕ require more idealized, near-crystalline packings. Figure 9(b) shows how the results presented in Figure 9(a) change if we assume that the mean coordination number for individual grains (C_{FCC}) is 9, reducing the cohesion on a single grain

(Equation (3)). Reducing the cohesion on a grain makes the preferential detachment of a clump less likely in the force model. This is reflected in Figure 9(b) by a reduction in force-constrained clump size; all dashed lines shift downward, causing maximum clump size to decrease and $\phi_{\text{max, size}}$ to increase. The solid line remains unchanged since geometric clump size is independent of cohesion force on a grain.

4.2. Predicted Clumps on Earth

Since Earth's gravity is 10^5 times stronger than Bennu's gravity, clumps of centimeter-scale grains will not exist on Earth, as discussed in Section 2.3, which is in agreement with our everyday interactions with granular materials on Earth. However, clumps of micron-scale grains are detachable on Earth (see Figure 10).

The clumping of micron-scale grains on Earth follows the same trends as clumping of centimeter-scale grains on Bennu. Additionally, the detachment of millimeter-scale clumps consisting of micron-scale constituent grains demonstrated by

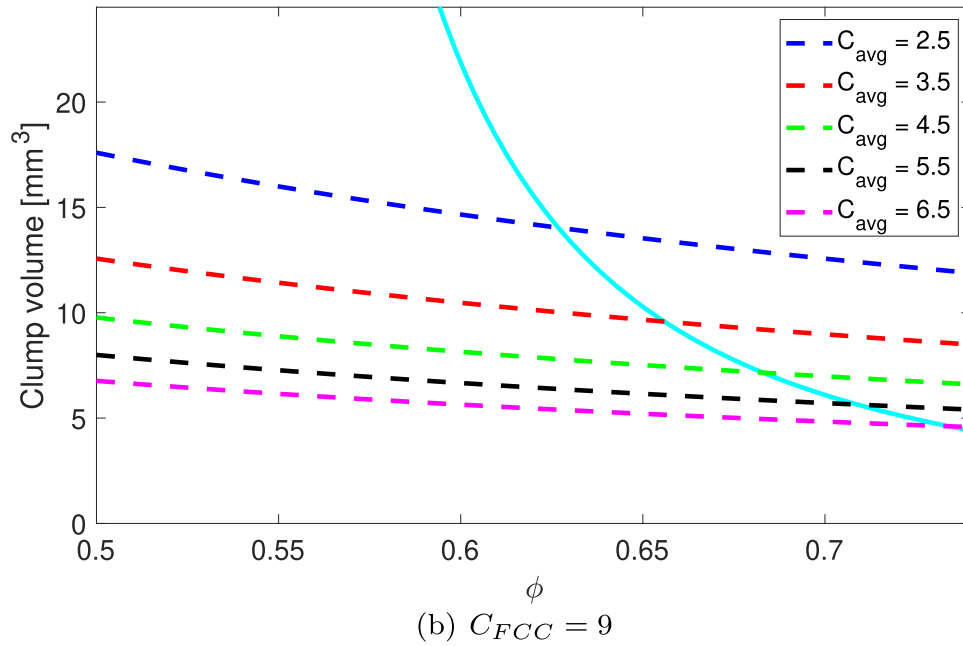
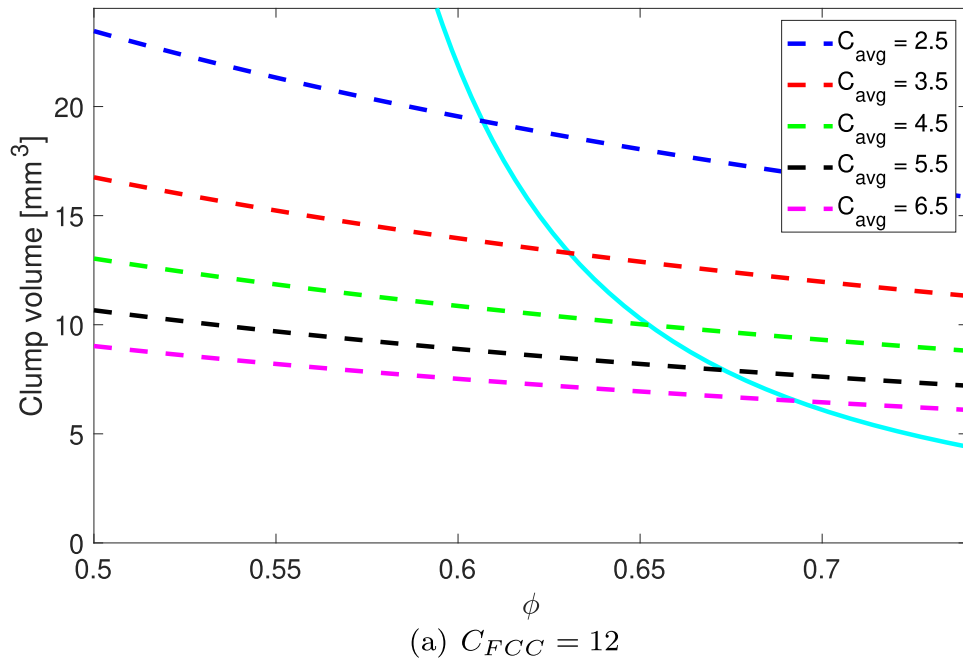


Figure 9. The volume of a clump on Bennu formed by $300\ \mu\text{m}$ radius grains as a function of local packing fraction. Dashed lines correspond to the maximum clump volume from the force constraint, for a given C_{avg} , and the solid line represents the clump volume from the geometric constraint. Two values of C_{FCC} (number of contacts per grain) are considered.

Figure 10 concurs with terrestrial studies of the size distribution of clumps formed in cohesive powder by micron-scale grains on Earth (Durda et al. 2014). The size distribution of clumps formed by cohesive powders on Earth has been experimentally measured for JSC-1A, $3\ \mu\text{m}$ glass microbeads, and ordinary unbleached flour (Durda et al. 2014). In the Durda et al. (2014) experiments, the granular material was sifted into a pile, and then the pile was tilted until the slope failed. Clump sizes were measured after the slope failure (i.e., avalanche). The size distribution of clumps formed from flour indicates that most of these clumps are approximately $0.65\ \text{mm}$ in diameter (Durda et al. 2014), with clumps up to $9\ \text{mm}$ observed. Approximating

these clumps as spherical, the volume of the average clump is $0.14\ \text{mm}^3$. Figure 10 indicates that $0.14\ \text{mm}^3$ clumps can detach on Earth for a constituent grain radius of approximately $80\ \mu\text{m}$. Flour ranges in diameter from 75 to $570\ \mu\text{m}$ (Patwa et al. 2014; Sakhare et al. 2014). Thus, our results agree with the average clump size observed in the Durda et al. (2014) experiments, in spite of the differences between the method of clump detachment we envision in our model (where a clump is detached normally from a bed) and the experiment (where clumps travel downslope during avalanching). Additionally, note that the model results were obtained using the Hamaker constant for lunar regolith. However, the Hamaker constant for

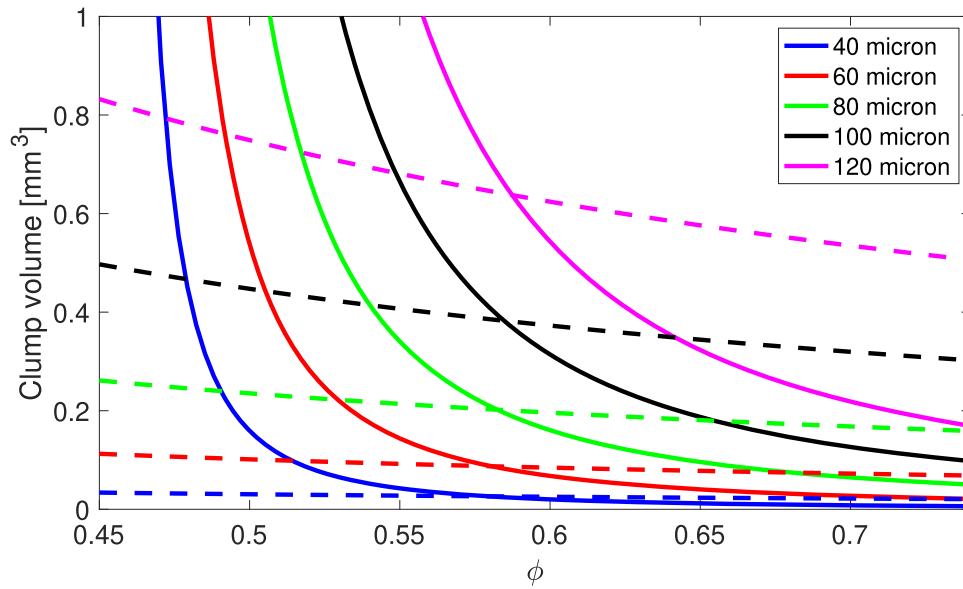


Figure 10. Clump volume as a function of grain size and local packing fraction on Earth, assuming 0.55 bulk packing fraction and a density of 1470 kg m^{-3} (Boukouvalas et al. 2006). The dashed lines correspond to the maximum clump size from the force constraint, and the solid lines represent the clump size from the geometric constraint.

baking flour ($\sim 6.5 \times 10^{-20}$; Siliveru 2016) is similar to that of lunar regolith ($\sim 5.14 \times 10^{-20}$). Our model does not predict clumps as large as 9 mm (as were observed much less frequently by Durda et al. 2014), which may be due to differences between the method of clump detachment in our model and that in the experiment or due to the conservative and simplifying assumptions made by our model (no atmosphere, spherical grains, uniformly sized grains).

Clumping of micron-scale grains is relevant to a variety of processes on Earth. In the pharmaceutical industry, Dry Powder Inhalers (DPIs) use clumps of inert carrier particles and active, pharmaceutical particles as a drug delivery method (Donovan et al. 2012). Thus, variation in clump size as a function of the characteristics of the constituent particles is significant in controlling dosage. The grain sizes studied in Figure 10 also correspond to medium to large carrier particle sizes commonly found in dry powder inhalers (Donovan et al. 2012). Thus, the formation of clumps (as predicted by this model) can inform dosing control, as well as necessitate the shaking of inhalers (to break clumps) prior to use.

5. Discussion

The force constraint determines the clump size that is easier to detach than an individual, constituent grain. The geometric constraint predicts the clump size present in situ given the bulk packing fraction. Given these two constraints, we can predict the clump sizes that are easier to detach in situ than their constituent grains, for a range of grain sizes, packing fractions, and surface gravities.

On small asteroids like Bennu, our model predicts that centimeter-scale and smaller grains are capable of producing detachable clumps. As grain size decreases, the largest detachable clump becomes more porous. The largest clump that can be detached is a function of both the grain size and the local packing fraction. For a given grain size, as the packing fraction of the clump increases, the maximum clump size decreases. For a given grain size, the smallest clump will occur at a local packing fraction corresponding to FCC packing.

However, it is unlikely that high local packing fractions will occur in situ owing to the asphericity of regolith grains.

The OSIRIS-REx mission has observed particles ranging from centimeters to meters in size on the asteroid Bennu (Walsh et al. 2019). Our model predicts that decimeter-sized clumps can be formed from centimeter-scale constituent grains on Bennu (see Figure 6). Additionally, we predict that centimeter-scale clumps could be formed from millimeter-scale constituent grains. These predictions influence the interpretation of visual observations of the surface of small, rocky asteroids, challenging the assumption that an observed “rock” is coherent, and not, in fact, a clump, as has been proposed by Durda et al. (2014). Additionally, our prediction of the existence of porous decimeter clumps agrees with the Rozitis et al. (2020) observations of low thermal inertia, highly porous boulders on Bennu. The presence of clumps will influence the observed surface roughness of small planetary bodies. Further, our model predicts the preferential detachment of centimeter-scale clumps (rather than millimeter-scale constituent grains), which also concurs with observations of “airfall” from active regions on comet 67P/Churyumov-Gerasimenko (Thomas et al. 2015), and the ejection of decimeter-scale aggregates (Agarwal et al. 2016).

On Mars, the grain size in sand dunes was predicted to be $500 \mu\text{m}$ in diameter from thermal inertia data (Edgett & Christensen 1991), and observations conducted by the Mars Exploration Rover of the Eagle crater floor reveal basaltic grains down to even $50\text{--}125 \mu\text{m}$ in size (Sullivan et al. 2005). From Figure 3, we can see that, for Martian gravity, grains below $500 \mu\text{m}$ in size can form clumps consisting of multiple grains that are easier to detach from the surface. Thus, the size distribution of particles in Martian sand dunes derived from thermal observations could include clumps.

We have assumed that the force acting to detach a grain or clump from the surface is collinear with gravity and the net cohesive force. Depending on the grain detachment mechanism, this assumption may not be accurate (e.g., for particle entrainment in aeolian flows on Mars, as well as on asteroids,

where the local gravity may not be normal to the surface). This analysis could be revisited considering the appropriate components of the detaching and restraining forces. The analysis would not change significantly as long as there is some component of the detaching force that is normal to the surface. If the detaching force only has a tangential component, then a different approach should be taken. Nonetheless, the analysis presented here provides a first-order quantitative prediction of preferential clump detachment and the sizes of detachable clumps.

Planetary regolith grains are not spherical but highly angular (Tsuchiyama et al. 2011). The angularity of real regolith grains may produce local packing fractions that are larger or smaller than the spherical limit, depending on the history of the surface (e.g., compaction or expansion due to impact-induced vibration). For the force model, variation in the local packing fraction will influence the gravitational force. Accounting for aspherical grains in the geometric model is complex since the shape of the grains influences the size and frequency of defects between clumps, which are the driving factors for clump size. With increasingly large grain sizes, interparticle cohesion would be driven more by the local curvature radii at contact points than by grain size directly (Starukhina 2000).

Additionally, planetary regolith is polydisperse. A polydisperse mixture would influence the force model by increasing the local packing fraction, as smaller grains would fill in void spaces within clumps. This would lead to heavier clumps, which would result in smaller maximum clump sizes. Again, extension of the geometric model to the polydisperse mixture is more complicated than the modification of the force model. The total volume of the defects does not change for given bulk and local packing fractions, but the physical constraint on the length of the defects is removed since there are multiple grain sizes.

Detaching grains from the surface reduces their mean coordination number from 12 to 9. For monodisperse spherical grains, Figure 9(b) shows that assuming $C_{FCC} = 9$ leads to smaller clumps and tighter local packing. However, experimental work on terrestrial clumping using flour powder (Durda et al. 2014) observed that grains between 75 and 570 μm diameter produce millimeter-scale clumps on Earth, also predicted by our monodisperse terrestrial model for grain mean coordination number of 12 (Figure 10). Flour used in these experiments features polydisperse and aspherical grains that can increase the mean coordination number for a grain from 9 toward 12, causing a grain to experience more cohesion. Despite the simplified grain size and shape model used, the model presented here produces clump sizes that are in agreement with terrestrial experimental results with polydisperse, aspherical powders (Durda et al. 2014). Thus, this model provides a first-order estimate of clump sizes expected to be present and possible to detach on a variety of planetary bodies, including Earth.

Future refinements of this model should consider the polydispersity and asphericity of grains. Additionally, specific grain detachment mechanisms (e.g., aeolian, electrostatic, and impact-driven) and their possible effects on clump size should be evaluated. Finally, the predictions of this model should be tested with additional lab-based experiments and compared to observational data. Terrestrial experiments that more closely

match the conditions of the model (i.e., vertical clump detachment of spherical, monodisperse grains) would strengthen the predictions presented beyond the demonstrated agreement with Durda et al. (2014). Additionally, the current bevy of asteroid exploration missions provides a unique opportunity to search for evidence of regolith clumps.

6. Conclusions

Regolith grains adhere to the surface of airless bodies due to a net downward force: the summation of cohesion and gravitational forces. Detachment of a grain occurs when this net downward force is overcome by some mechanism. Cohesion dominates the behavior of small particles, and the formation of clumps is commonly observed on Earth. We have presented a model to predict the size of clumps that will be preferentially detached from the surface of body, rather than individual grains, as a function of grain size, local packing fraction, and the surface gravity. We see that, on asteroids, clumps of centimeter-scale and smaller grains are possible. Our predictions for clumps formed on Earth agree with extant experimental observations. These results are significant for understanding the motion of regolith and the structural and thermal inertia properties of the surfaces of planetary bodies, as well as predicting clump sizes in a range of terrestrial powder handling applications.

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