# Experimental Radiative Lifetimes, Branching Fractions, and Oscillator Strengths of Some Levels in Eri and Er II 

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#### Abstract

Radiative lifetimes for 104 levels of Erbium (Er) I in the energy range between $31,926.003$ and $44,525.705 \mathrm{~cm}^{-1}$ and 51 levels of Er II from $31,381.779$ to $47,840.962 \mathrm{~cm}^{-1}$ were measured by a time-resolved laser-induced fluorescence method. Branching fraction (BF) determinations for 356 lines from 47 out of 104 Er I levels and 122 lines related to 19 out of 51 Er II levels were completed based on the emission spectra of hollow cathode lamps recorded using the 1.0 m Fourier transform spectrometer and are available from the digital library of National Solar Observatory on Kitt Peak, USA (http://diglib.nso.edu/). By combining these BFs and the lifetime results measured in this work, absolute transition probabilities and oscillator strengths were determined for 352 lines of Er I and 92 lines of Er II for the first time, increasing the total number of lines with experimental transition probabilities to over 910 for Er I and over 540 for Er II.


Key words: atomic data - atomic processes - methods: laboratory: atomic - techniques: spectroscopic
Supporting material: machine-readable tables

## 1. Introduction

Accurate measurements of radiative parameters for rare-earth (RE) atoms and ions are of particular importance in many fields, such as in astronomy, in the lighting-research community, and in theoretical work. In the past few decades, a number of spectral lines of RE atoms and ions have been observed in stellar spectra (Cowley \& Mathys 1998; Cowley et al. 2000; Sneden et al. 2003; Johnson \& Bolte 2004; Ryabchikova et al. 2006; Ryabchikova \& Romanovskaya 2017), which require accurate radiative parameters for the line identifications and abundance determinations. Rich emission spectra in the visible region make RE elements increasingly being used in many commercial metal halide high-intensity discharge lamps. Radiative parameters are needed for modeling and diagnosing these lamps (Biémont \& Quinet 2003). RE elements characterized with open $4 f$ shells have strong relativistic effects and configuration interaction, making the calculations very difficult. Thus, reliable experimental parameters will be valuable for benchmarking theoretical calculations (Lawler et al. 2008).

Erbium ( $\mathrm{Er}, Z=68$ ) is an RE element observed in the spectra of many stars, such as in AP stars, in AM stars, and in S-type stars (Cowley 1976; Cowley et al. 2000). Over the years, some studies have been focused on acquisition of the radiative parameters for ErI and Er II. For ErI, using the Hartree-plus-statistical exchange approximation, Cowan (1973) computed transition probability values for nine transitions. By means of the absorption method with the atomic beam acting as an absorbent, Dohnalik et al. (1979) measured the absolute values of the oscillator strength for three spectral lines $\left(\lambda_{1}=\right.$ $386.285 \mathrm{~nm}, \lambda_{2}=400.797 \mathrm{~nm}$, and $\lambda_{3}=415.111 \mathrm{~nm}$ ). Using the delayed-coincidence method, the lifetimes of 13 excited states in the region $17,074-26,237 \mathrm{~cm}^{-1}$ belonging to the $4 f^{12} 6 s 6 p$ and $4 f^{11} 5 d 6 s^{2}$ configurations were measured by Marek \& Stahnke (1980). Employing the same method, Gorshkov \& Komarovskii (1981) reported 25 lifetimes with energies in the range $19,200-33,485 \mathrm{~cm}^{-1}$. With the modelpotential method, which includes both exchange and core polarization, Migdalek \& Marcinek (1984) calculated oscillator
strengths for $4 \mathrm{f}^{12} 6 \mathrm{~s}^{2}-4 \mathrm{f}^{12} 6 \mathrm{~s} 6 \mathrm{p}$ transitions. With the timeresolved laser-induced fluorescence (TR-LIF) method, Xu et al. (2003a) reported 18 lifetimes for the odd-parity $4 \mathrm{f}^{12} 6 \mathrm{~s} 6 \mathrm{p}, 4 \mathrm{f}^{11} 5 \mathrm{~d} 6 \mathrm{~s}^{2}, 4 \mathrm{f}^{11} 5 \mathrm{~d}^{2} 6 \mathrm{~s}$, and $4 \mathrm{f}^{11} 5 \mathrm{~d} 6 \mathrm{~s} 7 \mathrm{~s}$ configurations in the range $26,237-42,154 \mathrm{~cm}^{-1}$. With the same method, the lifetimes of 56 even-parity and 67 odd-parity levels in the range $16,070-40,440 \mathrm{~cm}^{-1}$ were measured by Den Hartog et al. (2010). Later, Lawler et al. (2010) reported atomic transition probabilities for 562 lines of Er I with the method of combining branching fractions (BFs) measured using a Fourier transform spectrometer (FTS) with radiative lifetimes reported by Den Hartog et al. (2010).

For Er II, with the beam-foil method, lifetimes of the levels 26,099 and $29,973 \mathrm{~cm}^{-1}$ were first reported by Engman et al. (1976). Employing the delayed-coincidence method with crossing atom and electron beams, Gorshkov \& Komarovskii (1981) measured lifetimes of 11 levels in the energy range $26,099-31,386 \mathrm{~cm}^{-1}$. Using the same method but with excitation by a collinear tunable dye laser, lifetimes of 11 levels lying in the region $16,643-35,671 \mathrm{~cm}^{-1}$ were reported by Bentzen et al. (1982). Using argon-erbium arc plasma, Musiol \& Labuz (1983) measured transition probability values for 101 lines. By the means of TR-LIF method, Xu et al. (2003b, 2004) determined lifetimes of 30 odd-parity levels in the energy region $25,592-36,148 \mathrm{~cm}^{-1}$ and 8 even-parity levels in the range $33,753-55,317 \mathrm{~cm}^{-1}$, respectively. With the same method, Stockett et al. (2007) measured lifetimes for eight even-parity and 72 odd-parity levels. Later, by combining the BFs measured using an FTS and the lifetime values reported by Stockett et al. (2007), Lawler et al. (2008) determined transition probabilities for 418 lines.

Although many studies on radiative parameters for Er I and Er II have been done in previous work, sufficient data are not available for highly excited levels. Considering the needs in different fields, in the present work, radiative lifetimes of 104 levels for Er I lying in the energy range $31,926.003-44,525.705 \mathrm{~cm}^{-1}$ and 51 levels of Er II in the range $31,381.779-47,840.962 \mathrm{~cm}^{-1}$ were measured by the TR-LIF method. Meanwhile, the lifetimes of 47 Er I levels and

Table 1
Parameters of the FTS Spectra Used in This Work

| Index | Date | Serial <br> Number | Buffer Gas | Lamp Current (mA) | Wavenumber Range $\left(\mathrm{cm}^{-1}\right)$ | Limit of Resolution $\left(\mathrm{cm}^{-1}\right)$ | Beam Splitter | Filter |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1988 Nov 10 | 3 | Ar | 500 | 7346-42640 | 0.055 | UV | $\ldots$ |
| 2 | 1988 Nov 11 | 4 | Ar | 500 | 13437-4264 | 0.055 | UV | $\mathrm{CuSO}_{4}$ |
| 3 | 1987 Jan 13 | 5 | Ar | 500 | 3488-15077 | 0.029 | Vis | $\ldots$ |
| 4 | 2000 Feb 28 | 32 | Ar | 26.5 | 7929-34998 | 0.053 | UV | $\ldots$ |
| 5 | 2002 Feb 26 | 10 | Ar | 27 | 7929-34998 | 0.050 | UV | $\ldots$ |
| 6 | 2000 Feb 28 | 27 | Ar | 26.5 | 7929-34998 | 0.053 | UV | $\ldots$ |
| 7 | 2000 Feb 28 | 26 | Ar | 23 | 7929-34998 | 0.053 | UV | $\ldots$ |
| 8 | 2000 Feb 28 | 28 | Ar | 17 | 7929-34998 | 0.053 | UV | $\cdots$ |

Note. All spectra are publicly available from the digital library of the NSO on Kitt Peak, USA (http://diglib.nso.edu/).


Figure 1. Fluorescence decay curve of the Er I level $35,601.377 \mathrm{~cm}^{-1}$ with an exponential fit.

19 Er II levels reported in this work were combined with BFs determined based on the spectra recorded using an FTS to derive transition probabilities $(g A)$ and oscillator strengths $(\log (g f))$ for 356 lines in ErI levels and 122 lines related to Er II levels.

## 2. Lifetime Measurements

Since the experimental setup was recently presented in detail by Tian et al. (2016), here we only give a brief description. Free neutral and singly ionized Er in their ground and low-lying metastable levels were produced by focusing laser pulses vertically onto the surface of a rotating Er target placed inside a vacuum chamber with the pressure of about $10^{-4}-10^{-3} \mathrm{~Pa}$. The laser pulses are emitted from a 532 nm Q-switched Nd: YAG laser operating at 10 Hz with an energy of $5-10 \mathrm{~mJ}$. In order to excite Er I and Er II to the investigated states, another Nd:YAG 532 nm laser was used to pump a dye laser (DCM or Rhodamine 6G dyes), which was followed by different nonlinear processes to obtain tunable excitation source. Then the excitation laser was horizontally sent into the vacuum chamber to intersect with the Er atoms and ions at the distance of about 8 mm above the target surface. The two pump lasers were triggered by a digital delay generator (SRS DG535), which can also freely adjust the delay time between them. Fluorescence from the measured levels was collected by a


Figure 2. Typical fluorescence decay curve of the Er I level $38,667.799 \mathrm{~cm}^{-1}$ with the fitted convolution curve between the laser pulse and an exponential.


Figure 3. Comparisons between the Er II $g A$ values in this work, by Lawler et al. (2008), and by Musiol \& Labuz (1983). The dashed line corresponds to the equality between our results and the others.

Table 2
Measured Lifetimes of Er I Levels and Comparison with Previous Results

| Upper Level ${ }^{\text {a }}$ |  |  | Lower Level ${ }^{\text {a }}$ |  | $\lambda_{\text {Exc. }}$ <br> (nm) | $\lambda_{\text {Obs. }}$ <br> (nm) | Lifetime (ns) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Config. | $J$ | $E\left(\mathrm{~cm}^{-1}\right)$ | $J$ | $E\left(\mathrm{~cm}^{-1}\right)$ |  |  | This Work | Previous |
| $4 \mathrm{f}^{11}\left({ }^{4} \mathrm{~F}^{\circ}\right) 5 \mathrm{~d} 6 \mathrm{~s}^{2}$ | 6 | 31926.003 | 6 | 0.000 | 313.224 | 400 | 109(6) | 106(5) ${ }^{\text {b }}, 105(10)^{\text {c }}$ |
|  | 3 | 31988.938 | 4 | 5035.193 | 371.006 | 371 | 493(14) |  |
|  | 3 | 32695.370 | 4 | 5035.193 | 361.531 | 456 | 111(2) |  |
|  | 8 | 34288.437 | 7 | 7696.956 | 376.060 | 401 | 80.0(56) |  |
|  | 7 | 34596.396 | 7 | 7696.956 | 371.755 | 434 | 86.3(78) |  |
|  | 5 | 34684.329 | 6 | 0.000 | 288.315 | 361 | 44.6(21) | 42.6(21) ${ }^{\text {b }}, 45(4)^{\text {c }}$ |
|  | 8 | 34756.580 | 7 | 7696.956 | 369.554 | 394 | 102(4) |  |
|  | 7 | 35191.459 | 7 | 7696.956 | 363.709 | 497 | 130(6) |  |
|  | 6 | 35218.609 | 7 | 7696.956 | 363.350 | 420 | 66.9(51) |  |
|  | 8 | 35300.412 | 7 | 7696.956 | 362.273 | 404 | 42.1(40) |  |
|  | 5 | 35328.671 | 6 | 7176.503 | 355.212 | 418 | 213(13) |  |
|  | 9 | 35402.733 | 9 | 8620.565 | 373.383 | 419 | 201(13) |  |
|  | 8 | 35493.301 | 9 | 8620.565 | 372.124 | 401 | 58.5(47) |  |
| $\begin{aligned} & 4 \mathrm{f}^{12}\left({ }^{1} \mathrm{G}\right) 6 \mathrm{~s} 6 \mathrm{p}\left({ }^{3} \mathrm{P}^{\circ}\right) \\ & 4 \mathrm{f}^{11}\left({ }^{2} \mathrm{~K}^{\circ}\right) 5 \mathrm{~d} 6 \mathrm{~s}^{2} \end{aligned}$ | 3 | 35585.434 | 4 | 5035.193 | 327.330 | 403 | 17.7(3) |  |
|  | 4 | 35601.377 | 4 | 5035.193 | 327.159 | 402 | 226(4) |  |
|  | 6 | 35632.524 | 6 | 0.000 | 280.642 | 349 | 194(2) | 190(10) ${ }^{\text {c }}$ |
|  | 4 | 35783.707 | 4 | 5035.193 | 325.219 | 399 | 153(4) |  |
| $4 \mathrm{f}^{12}\left({ }^{1} \mathrm{G}\right) 6 \mathrm{~s} 6 \mathrm{p}\left({ }^{3} \mathrm{P}^{\circ}\right)$ | 3 | 35818.334 | 4 | 5035.193 | 324.853 | 399 | 158(2) |  |
|  | 7 | 35833.788 | 8 | 9350.106 | 377.591 | 482 | 22.7(6) |  |
|  | 6 | 35918.454 | 7 | 7696.956 | 354.340 | 482 | 75.8(53) |  |
|  | 6 | 35963.366 | 7 | 7696.956 | 353.777 | 414 | 59.1(37) |  |
|  | 6 | 36026.864 | 7 | 7696.956 | 352.984 | 480 | 70.6(38) |  |
|  | 4 | 36280.029 | 4 | 5035.193 | 320.053 | 392 | 31.5(17) |  |
|  | 5 | 36287.081 | 4 | 5035.193 | 319.981 | 392 | 484(6) |  |
|  | 7 | 36418.402 | 8 | 9350.106 | 369.436 | 469 | 30.3(6) |  |
|  | 4 | 36431.169 | 4 | 5035.193 | 318.512 | 389 | 31.4(15) |  |
|  | 5 | 36465.938 | 4 | 5035.193 | 318.160 | 389 | 201(4) |  |
|  | 4 | 36747.783 | 4 | 5035.193 | 315.332 | 385 | 264(5) |  |
| $4 \mathrm{f}^{11}\left({ }^{4} \mathrm{G}^{\circ}\right) 5 \mathrm{~d} 6 \mathrm{~s}^{2}$ | 3 | 36930.659 | 4 | 5035.193 | 313.524 | 382 | 14.0(3) |  |
| $4 \mathrm{f}^{11}\left({ }^{4} \mathrm{I}^{\circ}\right) 5 \mathrm{~d}^{2}\left({ }^{3} \mathrm{~F}\right)\left({ }^{4} \mathrm{~F}^{\circ}\right) 6 \mathrm{~s}$ | 3 | 37075.570 | 4 | 5035.193 | 312.106 | 380 | 178(4) |  |
|  | 4 | 37083.198 | 4 | 5035.193 | 312.032 | 380 | 123(3) |  |
| $4 f^{12}\left({ }^{1} \mathrm{G}\right) 6 \mathrm{~s} 6 \mathrm{p}\left({ }^{3} \mathrm{P}^{\circ}\right)$ | 3 | 37125.206 | 4 | 5035.193 | 311.623 | 404 | 36.8(8) |  |
|  | 4 | 37210.947 | 4 | 5035.193 | 310.793 | 378 | 73.9(32) |  |
|  | 4 | 37265.129 | 4 | 5035.193 | 310.271 | 402 | 115(2) |  |
|  | 3 | 37289.592 | 4 | 5035.193 | 310.035 | 401 | 17.8(4) |  |
|  | 4 | 37319.434 | 4 | 5035.193 | 309.749 | 401 | 268(2) |  |
|  | 5 | 37398.878 | 5 | 6958.329 | 328.509 | 375 | 157(7) |  |
|  | 5 | 37535.325 | 4 | 5035.193 | 307.691 | 373 | 95.3(9) |  |
|  | 6 | 37542.792 | 6 | 7176.503 | 329.313 | 383 | 84.3(26) |  |
|  | 4 | 37582.298 | 4 | 5035.193 | 307.247 | 373 | 127(3) |  |
|  | 3 | 37649.344 | 4 | 5035.193 | 306.615 | 396 | 107(2) |  |
|  | 7 | 37714.613 | 6 | 7176.503 | 327.460 | 353 | 11.0(3) |  |
|  | 5 | 37778.256 | 4 | 5035.193 | 305.408 | 370 | 251(5) |  |
|  | 7 | 37806.983 | 6 | 7176.503 | 326.472 | 351 | 93.0(22) |  |
|  | 4 | 37853.673 | 5 | 6958.329 | 323.673 | 393 | 73.1(72) |  |
|  | 4 | 37970.621 | 5 | 6958.329 | 322.453 | 391 | 14.2(9) |  |
|  | 7 | 38195.326 | 6 | 7176.503 | 322.385 | 347 | 96.9(45) |  |
|  | 8 | 38253.685 | 7 | 7696.956 | 327.260 | 361 | 27.9(17) |  |
|  | 7 | 38301.613 | 6 | 7176.503 | 321.284 | 345 | 29.1(19) |  |
| $4 \mathrm{f}^{12}\left({ }^{3} \mathrm{H}\right) 6 \mathrm{~s} 6 \mathrm{~d}$ ? | 6 | 38407.470 | 7 | 7696.956 | 325.621 | 453 | 16.2(8) |  |
|  | 6 | 38547.832 | 6 | 7176.503 | 319.680 | 375 | 63.7(40) |  |
|  | 6 | 38604.588 | 7 | 7696.956 | 323.545 | 368 | 110(3) |  |
|  | 7 | 38664.249 | 6 | 7176.503 | 317.584 | 369 | 37.7(9) |  |
| $4 \mathrm{f}^{12}\left({ }^{3} \mathrm{H}\right) 6 \mathrm{~s} 6 \mathrm{~d} ?$ | 6 | 38667.799 | 6 | 7176.503 | 317.548 | 372 | 22.4(13) |  |
|  | 5 | 38703.037 | 6 | 7176.503 | 317.193 | 372 | 21.1(13) |  |
|  | 8 | 38734.243 | 7 | 7696.956 | 322.193 | 355 | 20.9(11) |  |
| $4 \mathrm{f}^{12}\left({ }^{3} \mathrm{H}\right) 6 \mathrm{~s} 6 \mathrm{~d}$ ? | 7 | 38751.622 | 6 | 7176.503 | 316.705 | 461 | 19.9(8) |  |
|  | 6 | 38786.542 | 6 | 7176.503 | 316.355 | 371 | 50.7(33) |  |
| $4 \mathrm{f}^{12}\left({ }^{3} \mathrm{H}\right) 6 \mathrm{~s} 6 \mathrm{~d}$ ? | 5 | 38795.477 | 6 | 7176.503 | 316.266 | 365 | 19.2(4) |  |
| $4 \mathrm{f}^{12}\left({ }^{3} \mathrm{H}\right) 6 \mathrm{~s} 6 \mathrm{~d}$ ? | 7 | 38797.084 | 6 | 7176.503 | 316.250 | 436 | 18.1(6) |  |
|  | 7 | 38857.639 | 7 | 7696.956 | 320.917 | 370 | 146(5) |  |
| $4 \mathrm{f}^{12}\left({ }^{3} \mathrm{H}_{5}\right) 6 \mathrm{~s} 7 \mathrm{~s}\left({ }^{3} \mathrm{~S}_{1}\right)$ | 6 | 38870.770 | 7 | 7696.956 | 320.782 | 371 | 41.2(27) |  |
| $4 \mathrm{f}^{12}\left({ }^{3} \mathrm{H}\right) 6 \mathrm{~s} 6 \mathrm{~d}$ ? | 6 | 38923.586 | 6 | 7176.503 | 314.990 | 363 | 62.2(32) |  |

Table 2
(Continued)

| Upper Level ${ }^{\text {a }}$ |  |  | Lower Level ${ }^{\text {a }}$ |  | $\lambda_{\text {Exc }}$. <br> (nm) | $\lambda_{\text {Obs. }}$ <br> (nm) | Lifetime (ns) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Config. | $J$ | $E\left(\mathrm{~cm}^{-1}\right)$ | $J$ | $E\left(\mathrm{~cm}^{-1}\right)$ |  |  | This Work | Previous |
|  | 4 | 38982.003 | 5 | 6958.329 | 312.269 | 257 | 90.9(14) |  |
|  | 7 | 38986.765 | 6 | 7176.503 | 314.364 | 368 | 32.2(10) |  |
|  | 6 | 39010.445 | 7 | 7696.956 | 319.351 | 455 | 22.2(4) |  |
| $4 \mathrm{f}^{12}\left({ }^{3} \mathrm{H}\right) 6 \mathrm{~s} 6 \mathrm{~d}$ ? | 7 | 39070.645 | 7 | 7696.956 | 318.738 | 368 | 33.5(6) |  |
|  | 6 | 39075.883 | 6 | 7176.503 | 313.486 | 361 | 19.4(7) |  |
|  | 7 | 39145.450 | 6 | 7176.503 | 312.804 | 362 | 22.0(4) |  |
|  | 6 | 39164.268 | 6 | 7176.503 | 312.620 | 365 | 124(3) |  |
| $4 \mathrm{f}^{12}\left({ }^{3} \mathrm{H}\right) 6 \mathrm{~s} 6 \mathrm{~d}$ ? | 6 | 39193.564 | 6 | 7176.503 | 312.333 | 365 | 23.6(14) |  |
|  | 6 | 39256.456 | 7 | 7696.956 | 316.862 | 364 | 33.9(12) |  |
|  | 6 | 39350.505 | 6 | 7176.503 | 310.810 | 363 | 8.2(3) |  |
|  | 7 | 39359.683 | 6 | 7176.503 | 310.721 | 360 | 4.8(2) |  |
|  | 8 | 39525.603 | 7 | 7696.956 | 314.182 | 362 | 18.5(5) |  |
|  | 6 | 39535.919 | 5 | 6958.329 | 306.959 | 253 | 37.1(7) |  |
|  | 3 | 39688.151 | 4 | 5035.193 | 288.576 | 376 | 121(2) |  |
|  | 5 | 39812.042 | 6 | 7176.503 | 306.414 | 357 | 55.6(41) |  |
|  | 8 | 39843.429 | 7 | 7696.956 | 311.076 | 354 | 17.7(5) |  |
|  | 7 | 40131.326 | 8 | 9350.106 | 324.873 | 354 | 29.7(29) |  |
|  | 7 | 40159.218 | 7 | 7696.956 | 308.050 | 354 | 20.8(16) |  |
|  | 6 | 40333.576 | 7 | 7696.956 | 306.404 | 352 | 29.4(23) |  |
|  | 7 | 41269.956 | 8 | 9350.106 | 313.285 | 393 | 14.0(2) |  |
|  | 7 | 41809.765 | 8 | 9350.106 | 308.075 | 385 | 24.5(16) |  |
|  | 6 | 41907.318 | 6 | 7176.503 | 287.929 | 374 | 29.6(25) |  |
|  | 7 | 41989.925 | 8 | 11557.670 | 328.599 | 383 | 48.3(36) |  |
|  | 5 | 42360.935 | 6 | 7176.503 | 284.217 | 384 | 21.0(10) |  |
|  | 8 | 42376.272 | 7 | 7696.956 | 288.356 | 459 | 12.7(11) |  |
|  | 5 | 42542.173 | 6 | 7176.503 | 282.760 | 366 | 15.7(7) |  |
|  | 6 | 42627.566 | 6 | 11799.778 | 324.383 | 373 | 46.7(26) |  |
| $4 \mathrm{f}^{12}\left({ }^{3} \mathrm{H}_{4}\right) 6 \mathrm{~s} 7 \mathrm{~s}\left({ }^{3} \mathrm{~S}_{1}\right)$ | 5 | 42736.803 | 6 | 7176.503 | $281.212$ | 375 | $19.7(29)$ |  |
|  | 8 | $42797.550$ | 7 | $7696.956$ | $284.895$ | $361$ | $6.3(4)$ |  |
|  | 6 | 42882.350 | 6 | 7176.503 | 280.066 | 388 | 10.6(4) |  |
|  | 6 | 43191.617 | 7 | 11887.503 | 319.447 | 366 | 18.6(6) |  |
|  | 6 | 43298.210 | 7 | 7696.956 | 280.889 | 356 | 6.3(3) |  |
|  | 8 | 43537.586 | 7 | 7696.956 | 279.013 | 370 | 11.0(4) |  |
|  | 8 | 43586.058 | 9 | 8620.565 | 285.996 | 351 | 8.9(3) |  |
|  | 9 | 43982.879 | 9 | 8620.565 | 282.787 | 364 | 13.8(11) |  |
|  | 7 | 44039.685 | 8 | 9350.106 | 288.271 | 355 | 14.7(4) |  |
|  | 8 | 44041.430 | 9 | 8620.565 | 282.319 | 372 | 8.9(9) |  |
|  | 10 | $44112.202$ | 9 | $8620.565$ | 281.757 | 362 | 11.7(4) |  |
|  | 8 | 44201.047 | 9 | 8620.565 | 281.053 | 387 | 13.2(4) |  |
|  | 7 | 44394.215 | 8 | 9350.106 | 285.355 | 384 | 12.3(3) |  |
|  | 9 | 44525.705 | 8 | 9350.106 | 284.288 | 357 | 11.2(5) |  |

Notes.
${ }^{\text {a }}$ Kramida et al. (2018).
${ }^{\mathrm{b}}$ Den Hartog et al. (2010).
${ }^{c}$ Xu et al. (2003a).
fused-silica lens in the direction perpendicular to the two laser beams and focused into a monochromator, equipped with a photomultiplier tube to convert the fluorescence into an electric signal. Then the signal was sent to a digital oscilloscope (Tektronix DPO7254) for recording.

In the measurements, by monitoring the wavelength of the dye laser through a high-precision wavemeter (HighFinesse WS6) and by checking the fluorescence wavelengths, we made sure only investigated Er I and Er II levels were excited. Beside that, all possible effects, such as the saturation, collision, radiation trapping, and flight-out-of-view effects, which can influence lifetime measurements, were minimized via modifications of experimental conditions (Wang et al. 2018). A
magnetic field, about 100 Gauss, was added over the plasma zone by a pair of Helmhotz coils to eliminate possible quantum beats caused by the Earth's magnetic field. During the measurements, an average of more than 1000 pulses was performed to improve the signal-to-noise ratio of each signal.

## 3. BF Determinations

For determination of BFs for all Er I and Er II levels whose lifetimes were measured in this work, we attempted to observe emission spectra of an Er hollow cathode lamp with our grating spectrometer (Acton SpectraPro500i), but too many of the investigated lines overlap with other Er I and Er II lines so that

Table 3
Measured Lifetimes of Er II Levels and Comparison with Previous Results

| Upper Level ${ }^{\text {a }}$ |  |  | Lower Level ${ }^{\text {a }}$ |  | $\lambda_{\text {Exc. }}$ <br> (nm) | $\lambda_{\text {Obs. }}$ <br> (nm) | Lifetime (ns) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Config. | $J$ | $E\left(\mathrm{~cm}^{-1}\right)$ | $J$ | $E\left(\mathrm{~cm}^{-1}\right)$ |  |  | This Work | Previous |
|  | 9/2 | 31381.779 | 11/2 | 440.434 | 323.192 | 381 | 298(11) |  |
|  | 11/2 | 31801.102 | 11/2 | 440.434 | 318.871 | 406 | 127(4) | 126(6) ${ }^{\text {b }}$ |
|  | 7/2 | 31902.682 | 9/2 | 5132.608 | 373.551 | 374 | 113(4) |  |
|  | 11/2 | 32073.360 | 11/2 | 440.434 | 316.126 | 401 | 105(5) | 101(5) ${ }^{\text {b }}, 102(10)^{\text {c }}, 104(10)^{\text {d }}$ |
|  | 9/2 | 32267.246 | 11/2 | 440.434 | 314.200 | 369 | 50.1(27) | $45.9(23){ }^{\text {b }}, 49(3)^{\text {c }}$ |
|  | 11/2 | 32618.753 | 11/2 | 440.434 | 310.768 | 364 | 75.7(38) | $73.4(37)^{\mathrm{b}}, 75(3)^{\mathrm{c}}$ |
|  | 9/2 | 32753.468 | 11/2 | 440.434 | 309.473 | 391 | 124(2) |  |
|  | 7/2 | 32790.085 | 9/2 | 5132.608 | 361.566 | 365 | 58.8(33) |  |
|  | 11/2 | 32811.006 | 11/2 | 440.434 | 308.923 | 361 | 74.0(53) | 72.4(36) ${ }^{\text {b }}$ |
| $4 \mathrm{f}^{11}\left({ }^{4} \mathrm{I}_{15 / 2}^{0}\right) 6 \mathrm{~s} 6 \mathrm{p}\left({ }^{3} \mathrm{P}_{1}^{0}\right)$ | 17/2 | 33217.200 | 15/2 | 6824.774 | 378.897 | 490 | 26.5(8) |  |
| $4 \mathrm{f}^{11}\left({ }^{4} \mathrm{I}_{15 / 2}^{\circ}\right) 6 \mathrm{~s} 6 \mathrm{p}\left({ }^{3} \mathrm{P}_{1}^{\circ}\right)$ | 15/2 | 33547.268 | 15/2 | 6824.774 | 374.217 | 487 | 30.0(6) |  |
|  | 7/2 | 34203.251 | 9/2 | 7195.355 | 370.262 | 370 | 154(8) |  |
|  | 13/2 | 35276.531 | 11/2 | 440.434 | 287.059 | 356 | 136(3) | 144(7) ${ }^{\text {b }}$ |
|  | 9/2 | 35877.083 | 7/2 | 5403.688 | 328.155 | 400 | 306(9) |  |
|  | 11/2 | 35885.232 | 11/2 | 440.434 | 282.129 | 348 | 174(6) |  |
|  | 9/2 | 36322.233 | 7/2 | 5403.688 | 323.430 | 393 | 111(3) |  |
|  | 7/2 | 36471.984 | 9/2 | 5132.608 | 319.087 | 391 | 121(9) |  |
|  | 11/2 | 36643.232 | 9/2 | 5132.608 | 317.353 | 339 | 29.1(8) |  |
|  | 7/2 | 36738.247 | 7/2 | 5403.688 | 319.136 | 389 | 14.4(8) |  |
|  | 11/2 | 36863.932 | 9/2 | 5132.608 | 315.146 | 337 | 78.6(41) |  |
|  | 9/2 | 37038.764 | 9/2 | 5132.608 | 313.419 | 382 | 69.0(45) |  |
|  | 7/2 | 37057.724 | 9/2 | 5132.608 | 313.233 | 384 | 32.6(19) |  |
|  | 11/2 | 37098.956 | 9/2 | 5132.608 | 312.829 | 334 | 90.0(40) |  |
|  | 5/2 | 37126.953 | 7/2 | 5403.688 | 315.226 | 408 | 56.3(25) |  |
|  | 7/2 | 37527.159 | 7/2 | 5403.688 | 311.299 | 375 | 130(5) |  |
|  | 7/2 | 37698.823 | 7/2 | 5403.688 | 309.644 | 373 | 27.7(18) |  |
|  | 5/2 | 37712.008 | 7/2 | 5403.688 | 309.518 | 398 | 64.4(52) |  |
|  | 11/2 | 38642.554 | 11/2 | 7149.630 | 317.532 | 362 | 56.0(33) |  |
|  | 7/2 | 38651.814 | 9/2 | 7195.355 | 317.900 | 362 | 20.3(7) |  |
|  | 9/2 | 38847.186 | 11/2 | 7149.630 | 315.482 | 358 | 53.5(44) |  |
| $4 \mathrm{f}^{11}\left({ }^{4} \mathrm{I}^{\circ}\right) 6 \mathrm{~s} 6 \mathrm{p}$ ? | 13/2 | 38847.378 | 15/2 | 6824.774 | 312.279 | 363 | 15.8(7) |  |
|  | 7/2 | 39140.561 | 9/2 | 7195.355 | 313.036 | 377 | 88.6(38) |  |
|  | 11/2 | 39242.082 | 11/2 | 7149.630 | 311.600 | 355 | 76.6(41) |  |
|  | 7/2 | 39304.952 | 9/2 | 7195.355 | 311.433 | 354 | 68.2(61) |  |
|  | 9/2 | 39392.224 | 11/2 | 7149.630 | 310.149 | 353 | 91.4(48) |  |
|  | 9/2 | 39509.332 | 11/2 | 7149.630 | 309.026 | 371 | 65.5(63) |  |
|  | 9/2 | 39653.816 | 11/2 | 7149.630 | 307.653 | 348 | 81.2(49) |  |
|  | 9/2 | 39845.276 | 9/2 | 5132.608 | 288.079 | 345 | 57.9(35) |  |
|  | 7/2 | 39975.403 | 9/2 | 5132.608 | 287.003 | 346 | 61.2(43) |  |
|  | 9/2 | 40000.774 | 9/2 | 5132.608 | 286.795 | 345 | 27.7(27) |  |
|  | 5/2 | 40123.804 | 7/2 | 5403.688 | 288.017 | 342 | 17.5(6) |  |
|  | 11/2 | 40435.615 | 9/2 | 5132.608 | 283.262 | 300 | 69.4(35) |  |
|  | 7/2 | 40747.134 | 7/2 | 5403.688 | 282.938 | 413 | 124(6) |  |
|  | 9/2 | 40905.867 | 7/2 | 5403.688 | 281.673 | 335 | 25.1(24) |  |
|  | 7/2 | 40943.527 | 9/2 | 5132.608 | 279.244 | 334 | 25.4(16) |  |
|  | 9/2 | 40959.837 | 9/2 | 5132.608 | 279.117 | 333 | 64.1(42) |  |
|  | 9/2 | 41067.403 | 7/2 | 5403.688 | 280.397 | 331 | 18.5(5) |  |
|  | 9/2 | 41170.514 | 7/2 | 5403.688 | 279.589 | 330 | 29.5(16) |  |
|  | 7/2 | 41206.903 | 7/2 | 5403.688 | 279.305 | 349 | 64.2(30) |  |
|  | 7/2 | 41876.127 | 9/2 | 7195.355 | 288.344 | 324 | 32.6(7) |  |
|  | 13/2 | 47840.962 | 11/2 | 12388.090 | 282.065 | 359 | 125(7) |  |

## Notes.

${ }^{\text {a }}$ Kramida et al. (2018).
${ }^{\mathrm{b}}$ Stockett et al. (2007).
${ }^{c} \mathrm{Xu}$ et al. (2003b).

BF measurements cannot be performed. However, it is possible to determine BFs using the FTS spectra available from the digital library of the National Solar Observatory (NSO) on Kitt

Peak, USA (http://diglib.nso.edu/). The NSO FTS, characterized with very high resolving power and excellent absolute wavenumber accuracy, has been used in BF measurements for

Table 4
Branching Fractions $\left(R_{k i}\right)$, Transition Probabilities, Oscillator Strengths of Er I Levels, and Comparison with Previous Results

| Upper level ${ }^{\text {a }}$ |  |  | $\tau(\mathrm{ns})$ | Lower level ${ }^{\text {a }}$ |  | Transition$\lambda_{\text {air }}(\mathrm{nm})^{\mathrm{b}}$ | $R_{k i}$ | $g_{k} A_{k i}\left(10^{6} \mathrm{~s}^{-1}\right)$ |  | $\log \left(g_{i} f_{i k}\right)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Config. | $J$ | $E_{k}\left(\mathrm{~cm}^{-1}\right)$ |  | $J$ | $E_{i}\left(\mathrm{~cm}^{-1}\right)$ |  |  | This Work | Previous ${ }^{\text {c }}$ | This Work | Previous ${ }^{\text {c }}$ |
|  | 6 | 31926.003 | 109(6) | 5 | 6958.329 | 400.405 | 1.00(0) | 119(7) | 120(7) | -0.54(2) | -0.54 |
|  | 3 | 31988.938 | 493(14) | 4 | 5035.193 | 370.900 | 0.906(7) | 12.9(4) |  | -1.58(1) |  |
|  |  |  |  | 4 | 10750.982 | 470.723 | 0.033(4) | 0.467(56) |  | -2.81(5) |  |
|  |  |  |  | 3 | 12377.534 | 509.765 | 0.034(4) | 0.486(65) |  | -2.72(6) |  |
|  |  |  |  | 2 | 13097.906 | 529.205 | 0.027(4) | 0.378(62) |  | -2.80(7) |  |
| $4 \mathrm{f}^{11}\left({ }^{4} \mathrm{~F}^{\circ}\right) 5 \mathrm{~d} 6 \mathrm{~s}^{2}$ | 3 | 32695.370 | 111(2) | 4 | 5035.193 | 361.428 | 0.079(12) | 5.00(79) |  | -2.01(7) |  |
|  |  |  |  | 4 | 10750.982 | 455.569 | 0.842(13) | 53.5(13) |  | -0.78(1) |  |
|  |  |  |  | 3 | 12377.534 | 492.041 | 0.050(6) | 3.16 (37) |  | -1.94(5) |  |
|  |  |  |  | 2 | 13097.906 | 510.128 | 0.024(4) | 1.51(27) |  | -2.23(8) |  |
|  |  |  |  | 4 | 23300.042 | 1064.067 | $0.005(1)$ | 0.29(8) |  | -2.30(12) |  |

Notes.
${ }^{\text {a }}$ Kramida et al. (2018).
${ }^{\mathrm{b}}$ Wavelength values computed from energy levels using the five-parameter formular from Peck \& Reeder (1972).
${ }^{\text {c }}$ Lawler et al. (2010).
(This table is available in its entirety in machine-readable form.)

Table 5
Branching Fractions $\left(R_{k i}\right)$, Transition Probabilities, Oscillator Strengths of Er II Levels, and Comparison with Previous Results

| Upper Level ${ }^{\text {a }}$ |  |  | $\tau$ (ns) | Lower Level ${ }^{\text {a }}$ |  | $\begin{gathered} \text { Transition }(\mathrm{nm})^{\mathrm{b}} \\ \lambda_{\text {air }} \end{gathered}$ | $R_{k i}$ | $g_{k} A_{k i}\left(10^{6} \mathrm{~s}^{-1}\right)$ |  | $\log \left(g_{i} f_{i k}\right)$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Config. | $J$ | $E_{k}\left(\mathrm{~cm}^{-1}\right)$ |  | $J$ | $E_{i}\left(\mathrm{~cm}^{-1}\right)$ |  |  | This Work | Previous ${ }^{\text {c }}$ | This Work | Previous |
|  | 11/2 | 31801.102 | 127(4) | 13/2 | 0.000 | 314.364 | 0.239(7) | 22.6(10) | 22.1(12) | -1.48(2) | -1.49 |
|  |  |  |  | 11/2 | 440.434 | 318.779 | $0.366(10)$ | 34.6(14) | 36.8(19),25(12) ${ }^{\text {d }}$ | -1.28(2) | -1.25 |
|  |  |  |  | 9/2 | 5132.608 | 374.867 | $0.079(5)$ | 7.49(51) | 7.32(60) | -1.80(3) | -1.81 |
|  |  |  |  | 11/2 | 7149.630 | 405.540 | 0.153(15) | 14.5(8) | 13.0(11) | -1.45(4) | -1.50 |
|  |  |  |  | 9/2 | 7195.355 | 406.294 | 0.111(8) | 10.5(8) | 10.7(10) | -1.58(3) | -1.58 |
|  |  |  |  | 9/2 | 11042.640 | 481.596 | 0.002(1) | 0.170(30) | 0.192(36) | -3.22(8) | -3.17 |
|  |  |  |  | 9/2 | 16552.871 | 655.633 | 0.029(6) | 2.73 (58) | 3.24(72) | -1.76(9) | -1.68 |
|  |  |  |  | 11/2 | 18889.101 | 774.260 | 0.004(1) | 0.340(80) | $0.516(132)$ | -2.51(10) | -2.33 |
|  |  |  |  | 11/2 | 21697.852 | 989.510 | 0.007(2) | 0.660(170) | 0.444(108) | -1.98(10) | -2.18 |
|  |  |  |  | 9/2 | 21894.055 | 1009.106 | 0.004(1) | 0.350(80) | 0.252(60) | -2.28(10) | -2.41 |

Notes.
${ }^{\text {a }}$ Kramida et al. (2018).
${ }^{\mathrm{b}}$ Wavelength values computed from energy levels using the five-parameter formula from Peck \& Reeder (1972).
${ }^{c}$ The values not labeled were measured by Lawler et al. (2008).
${ }^{\text {d }}$ Musiol \& Labuz (1983).
(This table is available in its entirety in machine-readable form.)
some atoms and ions (see e.g., Den Hartog et al. 2015; Lawler et al. 2017; Li et al. 2018). Table 1 is a list of eight FTS spectra from the NSO used in our BF study.
For the NSO FTS spectra, accurate calibration of spectral response is indispensable because the instrument system has different responses at different wavelengths. We made used of the ArI and ArII line calibrations, which is based on comparison of the high-precision branching ratios for ArI and Ar II lines (Hashiguchi \& Hasikuni 1985; Whaling et al. 1993) to the intensity ratios measured for the same lines to obtain the spectral response. As indicated in Table 1, the set of eight spectra used in this study including both high-current spectra to reveal extremely weak lines and low-current spectra, which can avoid the reabsorption effect characterized by a suppression of the strong emission line and the weak emission lines enhancements (Lawler et al. 2010). In the present work, by comparing BF results from spectra at different currents, we checked whether the reabsorption effect exists or not. If it exists, only the spectra at low currents were used. Considering
possible blends from spectral lines of Er and carrier Ar gas, we carefully distinguished the measured Er lines to make sure that the investigated lines were correctly assigned.

## 4. Results and Discussion

In the lifetime measurements, for the lifetimes longer than 40 ns , which correspond to five times the exciting pulse duration, the lifetime values were evaluated by fitting the recorded fluorescence curve to an exponential function with adjustable parameters. Otherwise, due to limitation of the excitation pulse duration and the response time of the detection system, a convolution fit of an exponential and an exciting pulse recorded by the same system to the decay curves is necessary for extracting lifetime values. As examples, a fluorescence curve of the ErI level $35,601.377 \mathrm{~cm}^{-1}$ with an exponential fit is shown in Figure 1, while the decay curve of the Er I level $38667.799 \mathrm{~cm}^{-1}$, the recorded excitation pulse and a convolution, are presented in Figure 2.

The radiative lifetimes of 104 levels in ErI lying in the energy range $31,926.003-44,525.705 \mathrm{~cm}^{-1}$ and of 51 levels of Er II lying in the range $31,381.779-47,840.962 \mathrm{~cm}^{-1}$ were determined and are listed in Tables 2 and 3, respectively, with quoted error bars, which consist of possible remaining systematic errors and statistical scattering errors from different recordings. The measured results are in a range of 4.8 to 493 ns , with uncertainties within $10 \%$, except for the ErI levels $42,736.803$ and $44,041.43 \mathrm{~cm}^{-1}$ that have uncertainties of $14.7 \%$ and $10.1 \%$, respectively. To our best knowledge, the lifetimes for 101 out of 104 levels in Er I and for 45 out of 51 levels in Er II were measured for the first time. Also included in the final columns of Tables 2 and 3 are some lifetime results from previous literature. We see good agreements between our and previous results with the differences, with ours as the reference ((ours-theirs)/ours), within $\pm 10 \%$.

The BFs were determined from line intensities divided by spectral response values. The BF uncertainties consist of two components: the line intensity errors and the calibration errors. The former were estimated using an analytical function of the signal-to-noise ratio, the FWHM of line, and the resolution interval of the spectrum (Sikström et al. 2002), while the latter were evaluated as $1 \%$ per 1000 wavenumber separation between the line of interest and the dominant line from the upper level (Wickliffe et al. 2000). The total uncertainty of the BF was calculated by the error-propagation theory (Sikström et al. 2002). We would like to deduce BFs for all Er levels measured in our lifetime experiment, but because the investigated levels have branches beyond the ultraviolet (UV) limit of the FTS spectra or they have one or more strong branches with a possibly severe blending problem, the BFs can be determined only for 47 Er I and 19 Er II levels.

BFs were combined with the measured lifetimes to determine $g A$ and $\log (g f)$ for which the uncertainties were evaluated by the error transfer theory (Sikström et al. 2002). The BFs, as well as the deduced $g A$ and $\log (g f)$, concerning Er I and Er II levels are listed in Tables 4 and 5, respectively. In these tables, we also present the previous results measured by Lawler et al. (2010), Musiol \& Labuz (1983), and Lawler et al. (2008) for comparisons. For the $g A$ values of ErI, good agreements can be seen between us and Lawler et al. (2010). The differences are less than $\pm 6.6 \%$ for three lines and $21.7 \%$ for the weak line 337.181 nm ( $\mathrm{BF} R_{k i}=0.038$ ).

Figure 3 is a comparison of our Er II $g A$ results to those reported by Musiol \& Labuz (1983) and by Lawler et al. (2008). We see, for the strong lines with $g A$ values greater than $10^{6} \mathrm{~s}^{-1}$, good agreements between our results and those by Lawler et al. (2008). A little larger discrepancies appear when comparing the $g A$ values by Musiol \& Labuz (1983) with our results. The similar situation also occurs in the comparisons between the $g A$ values by Lawler et al. (2008) and by Musiol \& Labuz (1983). Incorrect line identifications in the measurements by Musiol \& Labuz (1983) may be responsible for the deviations because the earlier grating spectrometer used by Musiol \& Labuz had a lower resolving power and cannot be competent to analyze the complex Er spectra.

In conclusion, radiative lifetimes for 104 levels of Er I in the energy range $31,926.003-44,525.705 \mathrm{~cm}^{-1}$ and 51 levels of Er II in the range $31,381.779-47,840.962 \mathrm{~cm}^{-1}$ were measured by the TR-LIF method. To the best of our knowledge, the lifetimes for 101 out of 104 levels in Er I and for 45 out of 51 levels in Er II were measured for the first time. BFs for 356 Er I lines and 122 Er II lines from the levels for which the lifetimes were studied in the present work were determined based on the FTS spectra from the digital library of the NSO on Kitt Peak, USA. By combining the BF and the lifetime results, the experimental $g A$ and $\log (g f)$ for these transitions were derived.

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## References

Bentzen, S. M., Nielsen, U., \& Poulsen, O. 1982, JOSA, 72, 1210
Biémont, E., \& Quinet, P. 2003, PhST, 105, 38
Cowan, R. D. 1973, NucIM, 110, 173
Cowley, C. R. 1976, ApJS, 32, 631
Cowley, C. R., \& Mathys, G. 1998, A\&A, 339, 165
Cowley, C. R., Ryabchikova, T., Kupka, F., et al. 2000, MNRAS, 317, 299
Den Hartog, E. A., Chisholm, J. P., \& Lawler, J. E. 2010, JPhB, 43, 155004
Den Hartog, E. A., Palmer, A. J., \& Lawler, J. E. 2015, JPhB, 48, 155001
Dohnalik, A., Dohnalik, T., \& Szynarowska, M. 1979, PhyS, 20, 39
Engman, B., Stoner, J. O., Martinson., I., \& Cerne, N. E. 1976, PhyS, 13, 363
Gorshkov, V. N., \& Komarovskii, V. A. 1981, OptSp, 50, 467
Hashiguchi, S., \& Hasikuni, M. 1985, JPSJ, 54, 1290
Johnson, J. A., \& Bolte, M. 2004, ApJ, 605, 462
Kramida, A., Ralchenko, Y., Reader, J. \& NIST ASD Team 2018, NIST
Atomic Spectra Database (ver. 5.5.6), https://physics.nist.gov/asd
Lawler, J. E., Sneden, C., Cowan, J. J., et al. 2008, ApJS, 178, 71
Lawler, J. E., Sneden, C., Nave, G., et al. 2017, ApJS, 228, 10
Lawler, J. E., Wyart, J. F., \& Den Hartog, E. A. 2010, JPhB, 43, 235001
Li, Q., Yu, Q., Wang, X. H., et al. 2018, JOSAB, 35, 2244
Marek, J., \& Stahnke, H. J. 1980, ZPhyA, 298, 81
Migdalek, J., \& Marcinek, R. 1984, JQSRT, 32, 269
Musiol, K., \& Labuz, S. 1983, PhyS, 27, 422
Peck, E. R., \& Reeder, K. 1972, JOSA, 62, 958
Ryabchikova, T. A., \& Romanovskaya, A. M. 2017, AstL, 43, 252
Ryabchikova, T. A., Ryabtsev, A., Kochukhov, O., \& Bagnulo, S. 2006, A\&A, 456, 329
Sikström, C. M., Nilsson, H., Litzén, U., Blom, A., \& Lundberg, H. 2002, JQSRT, 74, 355
Sneden, C., Cowan, J. J., Lawler, J. E., et al. 2003, ApJ, 591, 936
Stockett, M. H., Den Hartog, E. A., \& Lawler, J. E. 2007, JPhB, 40, 4529
Tian, Y. S., Wang, X. H., Yu, Q., et al. 2016, MNRAS, 457, 1393
Wang, X. H., Quinet, P., Li, Q., et al. 2018, JQSRT, 212, 112
Whaling, W., Carle, M. T., \& Pitt, M. L. 1993, JQSRT, 50, 7
Wickliffe, M. E., Lawler, J. E., \& Nave, G. 2000, JQSRT, 66, 363
Xu, H. L., Jiang, H. M., Liu, Q., Jiang, Z. K., \& Svanberg, S. 2004, ChPhL, 21, 1720
Xu, H. L., Jiang, Z., \& Svanberg, S. 2003a, PhyS, 67, 64
Xu, H. L., Jiang, Z. K., Zhang, Z. G., et al. 2003b, JPhB, 36, 1771

