



Turbulent Magnetic Dynamos with Halo Lags, Winds, and Jets

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Abstract

This paper presents scale-invariant/self-similar galactic magnetic dynamo models based on the classic equations and compares them qualitatively to recently observed magnetic fields in edge-on spiral galaxies. We classify the axially symmetric dynamo magnetic field by its separate sources, advected flux, and subscale turbulence. We ignore the diffusion term under plausible physical conditions. There is a time dependence determined by globally conserved quantities. We show that magnetic scale heights increase with radius and wind velocity. We suggest that active galactic nucleus (AGN) outflow is an important element of the large-scale galactic dynamo, based on the dynamo action of increasing subscale vorticity. This leads us to predict a correlation between the morphology of coherent galactic magnetic field (i.e., extended polarized flux) and the presence of an AGN.

Unified Astronomy Thesaurus concepts: [Disk galaxies \(391\)](#); [Extragalactic magnetic fields \(507\)](#); [Cosmic magnetic fields theory \(321\)](#)

1. Introduction and Background

Dynamo theory as an explanation for the magnetic fields in galaxies, including the Milky Way, has a long history both observationally and theoretically. Observational reviews and discussion of the relevant theory may be found in Beck (2015) and Krause (2015). The textbook by Klein & Fletcher (2015) discusses the theory and observation with abundant references. Blackman (2015) discusses modern theoretical developments in the theory, although these are not employed in our work.

More recently, a scale-invariant version of the classical theory (e.g., Steenbeck et al. 1966; Moffat 1978; Henriksen 2017; Henriksen et al. 2018a, 2018b; Woodfinden et al. 2019) has had some success. This approach succeeds in isolating and predicting the more common qualitative elements of a galactic magnetic field, much of which is compatible with previous work. Observational data have been enhanced by the CHANGES survey, which has been reviewed recently in Irwin et al. (2019). The presence of magnetic spiral “arms” or “ropes” in the radio halos of galaxies was predicted originally by this approach in Henriksen (2017) and subsequent work by this author and colleagues. A key prediction has been supported recently by the inferred magnetic field in the radio halo of the “Whale” galaxy NGC 4631, as described in Mora-Partiarroyo et al. (2019) and Woodfinden et al. (2019).¹

In parallel to the observational/analytic studies, numerical work has blossomed. This approach studies either the cosmological origin of galactic magnetic fields or the development from a “seed field” of a galactic dynamo. We do not pretend to review this literature fairly, but we can refer to some that have been brought to our attention.

In the cosmological category we find early work by Wang & Abel (2009) and Beck et al. (2012), and this has been continued into the present by Pakmor et al. (2014), Rieder & Teyssier (2016, 2017a, 2017b), and Pakmor et al. (2018), among others. The paper, Rieder & Teyssier (2016), is noteworthy for establishing that global, supernova-driven, turbulence during galaxy formation can amplify seed fields to necessary low-

redshift values. This is really a globally distributed α/Ω dynamo, in which one finds limiting exponential field growth with timescale $1/\Omega$. Our limiting time dependence found below under the assumption of scale invariance agrees with this behavior. In Katz et al. (2019) and Martin-Alvarez et al. (2020), the authors introduce the notion of tracking the cosmological history of a given low- Z magnetic field.

The cosmological environment, beyond clusters, is not suitable for the assumption of asymptotic scale invariance because of large-scale continuing evolution. The extended infall onto a forming galaxy is a possible exception. However, the numerical work that focuses on the origin and structure of galactic fields should, when smoothed appropriately, correspond to our assumption of self-similarity (including similarity between galaxies). Recent papers in this category by Butsky et al. (2017) and Pakmor et al. (2020) are particularly relevant. We will compare our results with theirs in a later discussion after we have presented our calculations and observations.

There is a growing consensus that galactic winds are an essential component of the long-lived galactic dynamo. In fact, throughout this series of papers on scale invariance, we have found it necessary to include such winds. This was already the case in our original suggestion (Henriksen & Irwin 2016), which required halo lag and disk wind to develop an X magnetic field. In the current paper we also call attention to active galactic nucleus (AGN) outflows as being possibly a part of the dynamo (see also Predehl et al. 2020). The numerical work attempts to reveal the detailed origins of such winds, whereas this paper does not make statements about the wind origin. Rather, we take the wind to be a necessary part of the self-similar, scale-invariant development, which we present below.

Recently, several relevant numerical/analytic works have been submitted in draft form to arxiv.org. Two papers, Quataert et al. (2021, 2021), have examined galactic winds driven by cosmic rays, either by random diffusion or by streaming along the magnetic field. It appears that diffusion is more effective, but it is not clear that such winds can produce the observed X field at all heights above the disk. An alternate approach in a recent draft, Steinwandel et al. (2021), suggests magnetic pressure can drive a disk outflow after the field has been

¹ For a visualization, see also <https://public.nrao.edu/news/giant-magnetic-ropes>.

amplified by the α/Ω dynamo. The amplification in the disk is much like that proposed by Butsky et al. (2017). This paper includes the outflow as an integral part of the dynamo. Moreover, the paper presents a biconical outflow from the galaxy that might coincide with the large-scale soft X-ray emission found in Predehl et al. (2020).

The more traditional wind from the disk is driven by vigorous star formation with the consequent supernova explosions. This has not seemed to be efficient enough, but there is a recent, very relevant paper, Su et al. (2018). This paper shows in numerical detail how discrete, localized supernovae enhance the outflow over what is achieved by smoothly averaging the supernova energy input over the star-forming disk.

Recent numerical papers by Garaldi et al. (2021) and Attia et al. (2021) have addressed the problem of primordial magneto genesis. These study the development of primordial magnetic fields at ionization fronts during the recombination due to the well-known Biermann battery and a recent mechanism called the Durive battery. They follow the field and galaxy evolution to the current epoch and find (Garaldi et al. 2021) that the different initial field seedings have a small ultimate effect. This may be due to the turbulent dynamo occurring naturally and becoming dominant, as found in Rieder & Teyssier (2016).

The application of scale invariance/self-similarity to such complicated physical problems relies on the whole system of interest attaining an internal asymptotic behavior. The philosophy is similar to applying thermodynamic laws to the beginning and end of a string of chemical reactions without the intervening details. The numerical work furnishes the physical details at a cost of complexity, while the scale invariance/self-similarity yields only ultimate behavior, although it is frequently exact.

Our theoretical technique is the same as always in this series, but we have ignored intermediate-scale diffusion in favor of other parts of the dynamo, namely small-scale (“fast”) turbulence and outflow/inflow. This allows simpler analytic results for the most part. Halo lag and “fast turbulence” throughout the disk and halo are new parts of the assumed self-similarity, as is the possible contribution of an AGN to the dynamo.

A slightly different approach to understanding the magnetic field in edge-on galaxies has been pioneered, for example, in Ferrière & Terral (2014) and Terral & Ferrière (2017). In this approach, different topological classes of magnetic fields are assigned to a galaxy, and then the consequent pattern of Faraday rotation is carefully calculated. This allows comparison with observations, particularly for the Milky Way. These fields are somewhat arbitrary, but in fact they correspond quite closely to our scale-invariant dynamo fields.

1.1. Qualitative Model and Philosophy

It helps when interpreting the magnetic field topology that we find to recall that scale-invariant fields must appear self-similar at all resolutions and so they normally only close at infinity. This is true even though the divergence of the field is everywhere zero, so that a field line is never broken. Exceptional cases do arise wherein the field lines close through a singularity, but we do not study these here.

In this paper we study the structure of scale-invariant (or self-similar) magnetic dynamo fields, which contain, for the first time, both a rotating halo that lags behind the disk rotation

and an accelerating outflow/inflow. Observational examples of halos that are best fit with accelerating outflows are NGC 3556 (Miskolczi et al. 2019) NGC 891 (Schmidt et al. 2019), and NGC 5775 (G. Heald et al. 2021, in preparation). We also allow the (scaled) subscale (also called “fast” elsewhere) turbulent vorticity (sometimes described as hydrodynamic helicity) to increase with latitude in an attempt to imitate an AGN. The usual scale-invariant variation of this quantity is proportional to cylindrical radius and hence it decreases near the axis.

The “small-”scale (i.e., sub-resolution-scale) turbulence must have net hydrodynamic vorticity for the α effect to be present, so we often write subscale vorticity as synonymous with the α effect. Because of a notational clash with our temporal scale factor α , we refer to this effect as the α_h effect.

The inclusion of a rotating galactic halo that lags the disk is an important observed feature that we include in the dynamo action. Rotational velocity in the galactic halo gas significantly declines with height above the disk of spiral galaxies (e.g., Rand 2000; Heald et al. 2007). This “halo lag” inspired a model (Henriksen & Irwin 2016, = **H-I**) wherein the halo magnetic field is generated through coupling to the distant halo and/or the intergalactic medium. This model accounts for both the lagging halo rotation and a global magnetic field. The model is dynamical in the sense that pressure and gravity also contribute to the steady state, but it did not include the subscale turbulent dynamo.

The present formalism follows that in Henriksen et al. (2018a) so the formalism is only briefly introduced in the next section. The scaled lag and wind/accretion are allowed to be variable with the latitude in the halo. However, we do not solve for the dynamics of the halo gas. Rather, according to our basic assumption, the halo lag, outflow/inflow, and subscale helicity are determined in functional form by the scaling symmetry. The time dependence is determined by the global invariants under the scaling symmetry, much as the scale-invariant dissipation rate determines the Kolmogorov scaling law in hydrodynamic turbulence.

The **H-I** model led to a rather coherent field structure. The celebrated “X-type” projected magnetic field (Beck 2015; Krause 2015) was present, as were also spiraling field lines (axially symmetric) rising and expanding into the galactic halo. In this paper we wish to determine whether there are characteristic observable galactic magnetic topologies that may distinguish between a dominant α_h effect or a dominant macroscopic dynamo. We compare some of our results to the recent catalog of magnetic fields in the CHANG-ES survey given in Krause et al. (2020). All of these images are freely available for download in FITS format at queensu.ca/changes.

2. Basic Theory

We refer to Henriksen et al. (2018a) and references therein for a detailed presentation of scale-invariant or self-similar classical dynamo theory, including time dependence and axial symmetry. However, in this study we set the diffusivity equal to zero, but include a lag in the rotational halo velocity as well as accelerating/decelerating outflow or accretion, from or onto the disk. An innovation is that the subscale turbulent helicity is allowed to vary with height in addition to the usual variation with radius. This produces an axial magnetic field that may in reality be generated by jets or nuclear winds.

It is worth discussing briefly the conditions under which we may justifiably ignore the diffusion term in the dynamo equation (Equation (3)) below. If we compare in order of magnitude the terms in the diffusion coefficient η and in the subscale helicity α_h to the convection term, we obtain respectively the ratios

$$\frac{\eta}{LV}, \quad \frac{\alpha_h}{V}, \quad (1)$$

where L is a macroscopic spatial scale and V is a macroscopic velocity. One estimate for the diffusion coefficient, η , is the turbulent value, $u\ell$, where ℓ is the subscale and u is the corresponding subscale turbulent velocity. We will take this to be $u \approx \alpha_h$. Then the first ratio is small in proportion to $(\alpha_h/V)(\ell/L)$. It is small compared to the second α_h term insofar as ℓ/L is small. In the CHANG-ES data, a linear resolution scale on a galaxy is typically several hundred parsecs, which may be regarded as a minimum L . The subscale is on the scale of star formation so that ℓ/L is $O(10^{-2}) - O(10^{-1})$. Similarly, u/V will be small, if u is a turbulent velocity and V is macroscopic.

Another estimate of η is as a Bohm-type value due to gyrating relativistic electrons in a magnetic field. This suggests

$$\eta \approx \frac{cE_e}{3eB}, \quad (2)$$

where E_e is the electron total energy and e is the electronic charge in esu. This gives $\eta \approx 1.7 \times 10^{13} \gamma_e/B$ cgs, where γ_e is the electron Lorentz factor. For $B \approx 10^{-6}$ G and $\gamma_e \approx 10^5$, we obtain 1.7×10^{24} cgs. This is to be compared to LV , which is 10^{27} cgs for $L \approx 10^{20}$ cm and $V \approx 10^7$ cm s $^{-1}$. We conclude that diffusion is a higher-order effect in the general dynamo in the presence of convection.

Moreover, in previous works, Henriksen (2017) and Henriksen et al. (2018a), we have included the intermediate-scale diffusion in our self-similar solutions. These solutions are generally more complicated to obtain, but they do not show qualitative differences from the solutions in this paper. We believe that this is because the mean outflow or inflow dominates the intermediate-scale diffusion.

It is difficult to reconcile the kinematic scale-invariant symmetry that we assume with the actual observed halo lag (although see H–I). Strictly, a full treatment would require multidimensional self-similarity and so plunge the problem back into partial differential equations (although without the time dimension; Carter & Henriksen 1991). We achieve a reasonable form in this kinematical study with one self-similar variable by assuming that the dynamo equation holds in a pattern reference frame rotating with the velocity $v_p = v_p(r)\hat{e}_\phi$. Here e_ϕ is the azimuthal unit vector around the rotation axis of the galaxy.

A valid classical dynamo equation in the pattern frame requires Faraday’s law to hold plus the usual dynamo expression for the electric field (e.g., Henriksen 2017). One finds, starting with Faraday’s law in the systemic frame (assumed inertial), that to first order in v/c^2 Faraday’s equation does hold in the pattern frame provided that the pattern speed is solid-body rotation. A solid-body rotation does not itself

contribute to dynamo action. This permits the magnetic field to be invariant on transforming to this frame of reference. Unfortunately, such a uniform rotation does not correspond to the principal rotation of the galaxy. Rather it corresponds to a spiral arm pattern rotation or perhaps the pattern rotation of magnetic spiral arms. Nevertheless, the invariance between the pattern frame and the inertial systemic frame means that the observed field should correspond to that found in the pattern frame.

Our total rotational speed will be $v_p(r) + v_s(z)$, where $v_s(z)$ is the halo rotational velocity in the pattern frame, normally a lag relative to v_p . It would be ideal to have v_p constant if we identify the pattern speed with the galactic thin disk rotation. Unfortunately, this choice adds a spurious first-order term to the pattern frame Faraday’s equation, namely $-v_p B_r/(r)$ added to the time derivative. This term represents the dynamo action of a constant rotation velocity. It may only be ignored compared to the time derivative if our vertical lag rate (that is δ below) is larger than v_p/r .

We proceed to solve the classical dynamo equation (Steenbeck et al. 1966; Moffat 1978) in the pattern frame for the magnetic vector potential (we set $\eta = 0$, which is included in the following equation for reference only):

$$\partial_t A = \alpha_h \nabla \times A + v \times \nabla \times A - \eta \nabla \times \nabla \times A, \quad (3)$$

where a scaled magnetic field and the scaled vector potential are related by

$$b = \nabla \times A, \quad (4)$$

α_h is the subscale turbulent “helicity,” and v is the global vector velocity of the halo gas.

For Dimensional³ simplicity, the scaled magnetic field is taken as

$$b \equiv \frac{B}{\sqrt{4\pi\rho}}, \quad (5)$$

where ρ is some arbitrary constant with the Dimension of density. This gives b the Dimension of velocity and so A has the Dimension of specific angular momentum. The formula for the vector potential requires ignoring an electrostatic field that would result from large-scale charge separation after integration of Faraday’s law. It seems unlikely that a large-scale electrostatic field should be important. Such an integration has the additional benefit of allowing the subscale helicity α_h (and the diffusion coefficient η where applicable) to vary with position without adding extra terms.

The search for axially symmetric (we use components in cylindrical coordinates),⁴ scale-invariant solutions of Equation (3) requires assuming that

$$A = \bar{A}(R, Z)e^{(2\delta - \alpha)T}, \quad (6)$$

where the scale invariants (equivalent to the self-similar variables) in space and time are

$$R \equiv re^{-\delta T}, \quad Z \equiv ze^{-\delta T}, \quad e^{\alpha T} \equiv (1 + \alpha t). \quad (7)$$

As in Henriksen et al. (2018a), the reciprocal scales in space and time are δ and α , respectively. If there were a fixed spatial

² Hence, to this order, the magnetic field is invariant between the systemic frame and the rotating pattern frame.

³ A capital D is used to distinguish a degree of freedom in parameter space from a geometrical dimension.

⁴ There is no consequent axial symmetry in the Cartesian field components. Absolute axial symmetry requires a coordinate invariant definition.

scale in the problem, then $\delta = 0$, while a fixed timescale requires $\alpha = 0$. In the latter case, our time, T , would be defined with δ replacing α . The limit $\alpha = 0$ is not strictly equivalent to an exact steady state (e.g., Henriksen 2017; Henriksen et al. 2018a, and below). Extensive discussion of the use of such scales may be found in Henriksen (2015), but in a given problem they may be assigned to give numerical quantities.

The other quantities must have the following forms relative to the pattern frame

$$v = \tilde{\alpha}_d \delta R e^{(\delta-\alpha)T} (0, v, w), \quad (8)$$

$$\alpha_h = \tilde{\alpha}_h \delta R e^{(\delta-\alpha)T}, \quad (9)$$

$$b = \tilde{b}(R, Z) e^{(\delta-\alpha)T}, \quad (10)$$

where we have allowed only an azimuthal velocity and an outflow/inflow wind component in this study. At this stage, the auxiliary functions $\tilde{\alpha}_h$ and v, w may be arbitrary functions of R and Z .

The similarity class (a ; see e.g., Carter & Henriksen 1991; Henriksen 2015) is sometimes used in what follows, and it is given in terms of the ratio of characteristic temporal and spatial scales according to

$$a \equiv \frac{\alpha}{\delta}. \quad (11)$$

To this point we have described a multivariable formulation of scale invariance (the invariant scaling follows by holding the self-similar variables constant), wherein the time dependence has been reduced to either a multiplication by a power law or by an exponential, depending on the class a . Proceeding with this analysis requires the solution of a partial differential equation in R and Z , once the auxiliary functions are given in the same variables. However, inspired mainly by the X-type field structure (e.g., Krause et al. 2020), we reduce the problem further in this series of papers by looking for a solution that depends only on the tangent of the *halo latitude*, in the form

$$\zeta \equiv \frac{Z}{R} \equiv \frac{z}{r}. \quad (12)$$

Such a solution is found to exist provided that $\tilde{\alpha}_h$ and v and w are at most functions of ζ and now

$$\tilde{A}(R, Z) = \tilde{A}(\zeta), \quad \tilde{b}(R, Z) = \frac{\tilde{b}(\zeta)}{R}. \quad (13)$$

For our halo lag description we take the Dimensionless velocity in the pattern frame (referred to as v_s earlier)

$$v(\zeta) = -v_1 |\zeta| \quad (14)$$

for constant v_1 . With this choice the explicit rotational velocity lag and wind/accretion velocity in this frame, by Equations (8) and (7), become, respectively,

$$\begin{aligned} v_\phi &= -\tilde{\alpha}_h \delta (v_1 |z|) e^{-\alpha T}, \\ v_z &= \tilde{\alpha}_h w(\zeta) \delta R e^{-\delta T}. \end{aligned} \quad (15)$$

When $\tilde{\alpha}_h$ is constant, the halo lag, v_ϕ , is linear in z but should $\tilde{\alpha}_h$ be linear in ζ , then the lag is proportional to z^2/r (see also Section 3.6). The consequent systemic form after adding a constant pattern velocity, v_p , is similar to that found dynamically in Henriksen & Irwin (2016). The expression derived there was found to be a reasonable fit to a well-measured halo lag galaxy NGC 891 (Oosterloo et al. 2007). We

discuss the form of the outflow/inflow factor $w(\zeta)$ in Section 3.4, below.

2.1. Time Dependence

In this section, we use the logarithmic time, T , as defined in Equation (7)⁵. When we recall that $R = r e^{-\delta T}$, we see from Equations (10), (13), and (7) that

$$b \propto e^{(2-a)\delta T} \propto (1 + \alpha t)^{(2/a-1)}. \quad (16)$$

It is now clear that $1/\alpha$ (or $1/\tilde{\alpha}_h \alpha$ when $\tilde{\alpha}_h$ multiplies α in the definition of T in Equation (18)) may be identified with a characteristic time (growth or decay) for the self-similar dynamo, but that a determines the form of the growth function.

This is where global conserved quantities enter the scale-invariant picture. If for example $a = 1$, corresponding to a constant global velocity or initial constant “seed” magnetic field, the field increases linearly in time at a rate given by α or $\tilde{\alpha}_h \alpha$. If we suppose instead that the usual magnetic helicity integrated over a volume V is conserved, that is

$$\int A \cdot b \, dV = \text{constant}, \quad (17)$$

then the global constant has Dimensions L^6/T^2 . This requires $6\delta - 2\alpha = 0$ for invariance in time, and hence $a = \alpha/\delta = 3$. We conclude from Equation (16) that the magnetic field must decline as $(1 + \alpha t)^{-1/3}$. Such a decline agrees with previous suggestions (e.g., Shukurov et al. 2006; Blackman 2015).

Conserving the local magnetic helicity density (Moffat 1978) itself suggests L^3/T^2 as the physical dimensions of a conserved quantity everywhere (gauge-invariant over a closed volume however small), hence $a = 3/2$ (a Keplerian value) from $3\delta - 2\alpha = 0$ and so the field grows slowly as $(1 + \alpha t)^{1/3}$.

If the macroscopic “current” helicity $b \cdot \nabla \times b$ is conserved locally, then the local constant has Dimensions L/T^2 so that $a = 1/2$, which gives rapid growth $\propto (1 + \alpha t)^3$. The volume-integrated version of this helicity implies L^4/T^2 whence $a = 2$, which implies a steady state. In our numerical studies, we normally use $a = 1$, which corresponds to either an invariant global velocity or a constant initial “seed” magnetic field. Such a velocity may be set by a flat rotation curve, or possibly by a typical constant outflow velocity. An initial seed field must be determined by cosmology and a theory of galaxy formation (e.g., Wang & Abel 2009; Beck et al. 2012; Pakmor et al. 2014).

A limiting exponential growth in the dynamo field is given by supposing that there is a global constant with a fixed dimension of reciprocal time. This might be a fixed angular velocity Ω or possibly a constant rotational lag rate, but in any case the assumption requires $\alpha \rightarrow 0$ so that the relevant quantity is conserved in time. In that case the limit of Equation (16) as $\alpha \rightarrow 0$ gives exponential growth as $b \propto e^{\delta t}$. Here, δ is a reciprocal time characteristic of the magnetized galaxy. If this is r/v_ϕ , then the e -folding time is essentially a rotation period, as might be expected.

⁵ When $\tilde{\alpha}_h$ is constant it is convenient to multiply it into α in the log time equation of Equation (7) because this removes it from the equations to be solved.

3. Separate Physical Parts of the Galactic Magnetic Dynamo

In this section, we study the magnetic field produced when either the subscale turbulent dynamo (α_h term) or the convective flux freezing in Equations (3) is dominant. There are various examples in each case depending on the parameters. We also consider some important combinations of these effects. The objective is to isolate magnetic topology that may reveal which mechanism is important in a magnetized edge-on galaxy. Such data have been reported in the recent CHANG-ES catalog (Krause et al. 2020). Face-on galaxies have already revealed their magnetic arm structure (e.g., Beck 2015).

In order to compare the various effects, we will be referring to a parameter vector that is a subset of the form, $\{v_1, w_0, w_1, \alpha_{h1}, K, r, \phi, z, C_1, C_2\}$, where r, ϕ , and z are coordinates. The exact description of these (scaled) parameters are found elsewhere in this paper. However, for simplicity, it is helpful to keep in mind the following, with larger numbers representing a stronger effect:

- v_1 : rotational lag rate of the halo (Equation (30)).
- w_0 : initial in-disk velocity of an outflowing wind (Equation (29)).
- w_1 : wind acceleration (Equation (29)).
- K : a parameter describing the similarity class, which is always $\equiv 1$ in this paper (Equation (21)).
- C_1, C_2 : describes whether the solution is a dipole (1, 0) or a quadrupole (0, 1).
- α_{h1} : a measure of the strength of the subscale turbulence (Equation (36)).

3.1. Subscale Turbulent Dynamo with Halo Rotational Lag

The auxiliary function $\tilde{\alpha}_h$ is first taken constant and in this section, $w = 0$. Because of the radial dependence, the turbulent dynamo vanishes at small radius. This explicitly avoids any effect of an AGN.

It is convenient to write the logarithmic time T according to

$$e^{\alpha T} \equiv (1 + \tilde{\alpha}_h \alpha t), \quad (18)$$

rather than the usual choice taken in Equation (7).

This choice of logarithmic time removes $\tilde{\alpha}_h$ from the equation to be solved for the azimuthal vector potential, which becomes simply (the prime indicates differentiation with respect to ζ)

$$(1 + \zeta^2)\tilde{A}_\phi'' + (K^2 - v')\tilde{A}_\phi = 0. \quad (19)$$

The constant $\tilde{\alpha}_h$ is constrained not to be strictly zero with this temporal scaling. Its value has an effect only on the time dependence of the field, once the reciprocal scale α ($a = 1 \geq \alpha = \delta$) has been chosen for macroscopic reasons. We treat the pure flux conservation (sometimes referred to as “flux freezing”) dynamo with $\tilde{\alpha}_h = 0$ separately below.

Subsequently the other components of the vector potential follow as (the middle expression restates Equation (19) when combined with the first and third)

$$\begin{aligned} K\tilde{A}_R &= v\tilde{A}_\phi - (1 + v\zeta)\tilde{A}_\phi' \\ K\tilde{A}_\phi &= \tilde{A}_R' + \zeta\tilde{A}_z', \\ K\tilde{A}_z &= \tilde{A}_\phi + (v - \zeta)\tilde{A}_\phi'. \end{aligned} \quad (20)$$

We have set

$$K \equiv 2 - a. \quad (21)$$

The scaled magnetic field can be shown to have the simple form

$$\tilde{b}_R = -\tilde{A}_\phi', \quad (22)$$

$$\tilde{b}_\phi = \tilde{A}_R' + \zeta\tilde{A}_z', \quad (23)$$

$$\tilde{b}_z = \tilde{A}_\phi - \zeta\tilde{A}_\phi'. \quad (24)$$

These equations agree with those in Henriksen et al. (2018a; the combined Equations (27) and (28) of that paper) when $\Delta = 1$, $u = w = 0$, and all diffusion terms are removed.

3.2. Pure α_h Dynamo

If we set $v = 0$ in the equations of Section 3.1, we obtain the pure α_h dynamo with a radially increasing subscale turbulent vorticity. Equation (19) becomes quite simple and is manifestly invariant under a change of sign of ζ , although the derivative will change sign on crossing zero if $\tilde{A}_\phi$ does not change sign. This occurs in what we call the dipolar solution below. The vector potential $\tilde{A}_\phi$ can also change sign, which implies, by Equation (24), that $\tilde{b}_z$ passes through zero at the disk. This occurs for the quadrupole solution. The equations for the magnetic field (Equations (22)–(24)) show that consequently only the radial field component changes sign on crossing the equator for the dipole solution, but that both b_z and b_ϕ change sign for the quadrupole case.

The solution of Equation (19) is given in terms of hypergeometric functions (as described below for $v \neq 0$ in Section 3.3), although at small ζ there is a simple oscillatory limit. In terms of the independent solution amplitudes $\{C_1, C_2\}$, the solution $\{1, 0\}$ is the “dipolar solution” (b_z continuous across the disk) and $\{0, 1\}$ is the “quadrupole solution” (b_z zero at the disk). In the quadrupole case, $\tilde{A}_\phi$ passes through zero and changes sign at the disk. Hence, b_R does not change sign on crossing the disk but b_z and b_ϕ do.

Conical symmetry is a characteristic of our version of self-similarity/scale invariance within a given galaxy.⁶ Hence, we should not be surprised to find evidence of this symmetry in each base solution of Equation (3). Nevertheless, the behavior can be unexpectedly complex. We show some of the possibilities in Figures 1 and 2, together with the projections in Figure 3.

As a general comment on our figures, when field lines are shown they are accurate in 3D given the point at which we start them. In 3D vector plots, each vector shows the local direction of the magnetic field where the local point is either at the tail or the head of the vector. When looking at 2D planar projections, the vectors are the component of the local magnetic field in that plane. Components perpendicular to that plane do not appear. Because the plotting routine plots the relative strength of vectors, these projections can give a false impression of the strength of the projected field. This must be taken into account when one tries to reconcile the planar field with a 3D representation of the same field. The problem is not present when considering different views of the same 3D field lines.

Figure 1 demonstrates that, unless one is very near the galactic axis, the spirally rising field lines are very open. The dipole solution shows some interesting structure with height

⁶ This becomes a similarity symmetry between different galaxies, which implies scale invariance in the family of galaxies when the length scale is the radius of the disk.

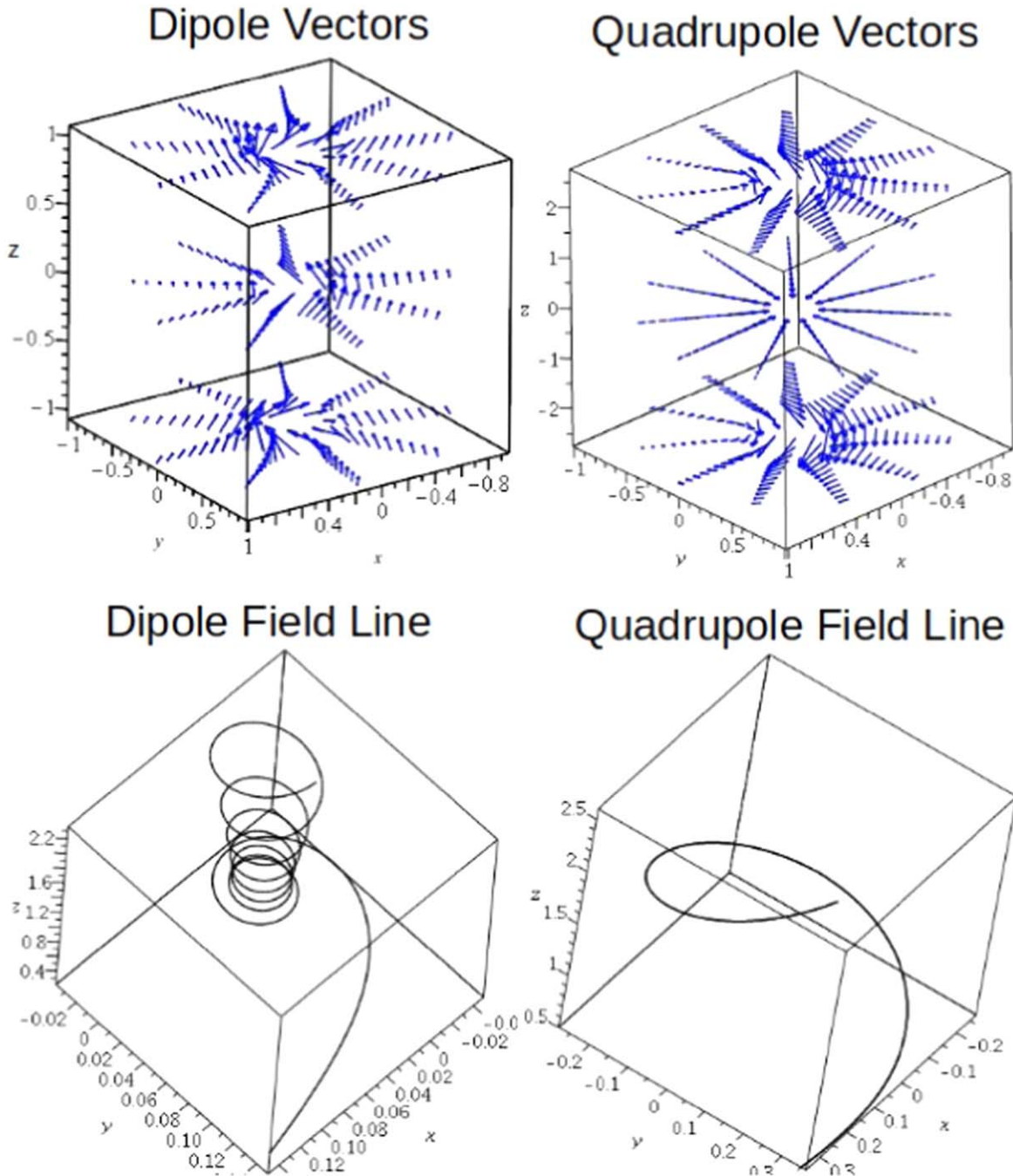


Figure 1. We show the dipole at left and quadrupole at right in this figure, both above and below the plane. On the upper left, the local field direction is shown in 3D for the dipole solution with only the subscale turbulent helicity of Equation (9) present. The similarity class $a = 1$, and the length unit is the radius of the disk. The spiral twist is very small except near the axis. The field line at the lower left is for the dipole solution. It begins very near the axis at a large height above the disk and descends while spiraling rapidly to the point $\{r, \phi, z\} = \{0.22, 0, 0.5\}$. Note that the azimuthal field changes direction at large height. On the right, the quadrupole solution shows less structure. The spiraling is very slow even at small radius (note the scale in the lower right-hand image) and the azimuthal field component does not change sign even at large heights. The quadrupole field line shown, starts at $\{r, \phi, z\} = \{0.5, 0, 0.5\}$ and rises. Both of these field lines are shown in clusters in Figure 2.

because near the axis, the vertical and azimuthal field components go through zero and reverse direction on the same side of the disk, normally referred to as a “parity change.” The field line at the lower left is shown descending from a large height, but this is not necessary. The dipole field line ensemble in Figure 2 begins at $z = 1.8$. The quadrupole solution at the upper right of Figure 1 is a slowly rising spiral. The single field line is assembled in a cluster spaced around a circle in Figure 2, the different lines corresponding to different colors. Only close to the axis does the quadrupole show the “X-field” behavior in

the $\{r, z\}$ plane. However, the dipole solution shows a strong “X field” throughout each quadrant. These poloidal projections are shown in Figure 3 below.

Figure 3 shows the $\{r, z\}$ cuts through the upper half of the upper row images of Figure 1. These figures correspond to the images in Figure 2. Only the dipole shows a convincing high-latitude “X field” in projection. The axial behavior of the magnetic fields shows the field coming down at $z \gtrsim 0.3$, and then rising at larger radii. At the same radius below $z \sim 0.3$, the field is rising as part of a field that descended at smaller radii.

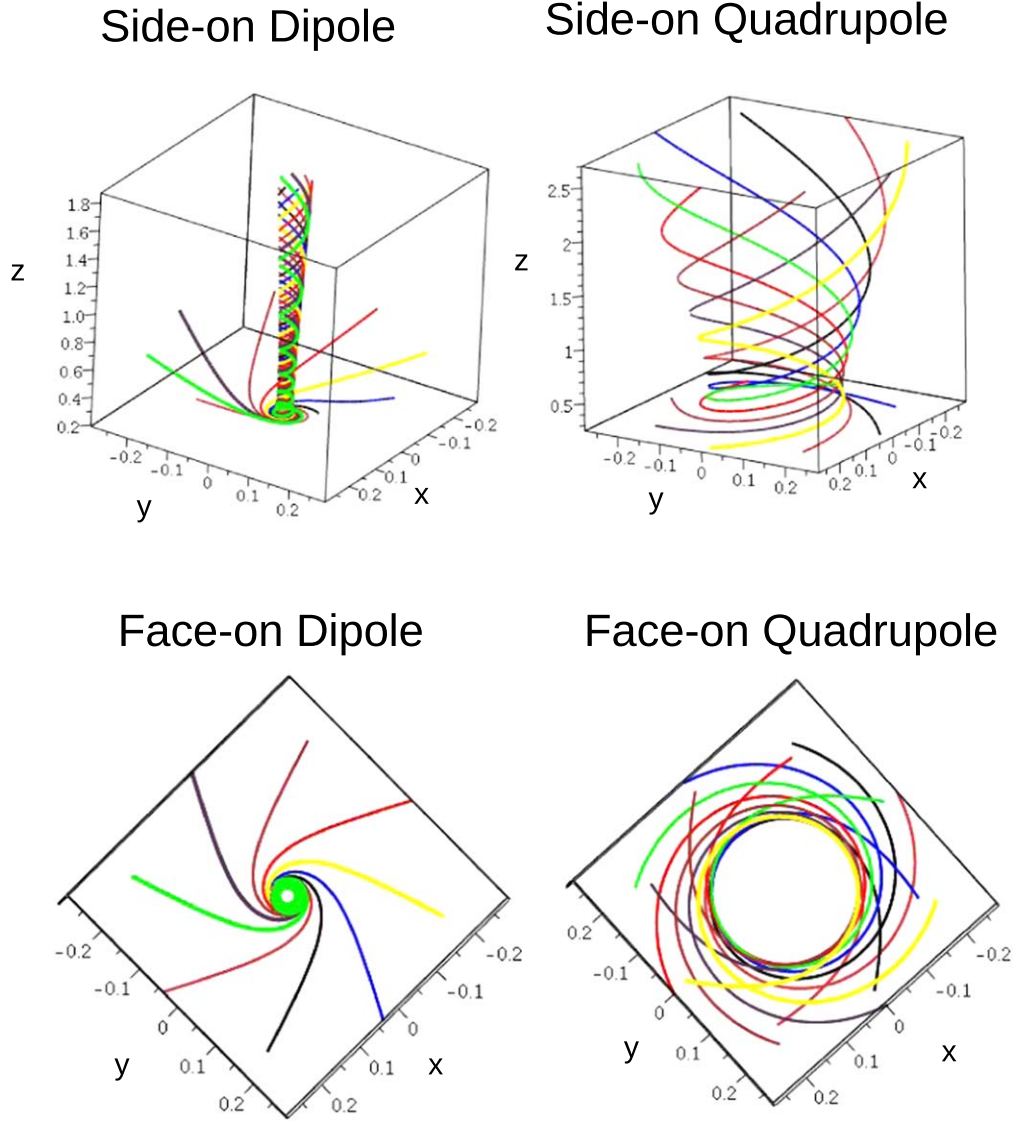


Figure 2. The left column shows a collection of field lines for the dipole and the right column shows the quadrupole. The top row shows a side view and the bottom row shows a face-on view (the $\{r, \phi\}$ projection). For the dipole, the lines begin on a circle of radius $r = 0.22$ at height $z = 0.5$ and they are spaced at 45° around the circle. The field lines are descending on the central “funnel” and rising at large radius. One of these field lines (top left) is shown at the lower left in Figure 1 for clarity. The rising field lines indicate the connections between the outer vectors at the upper left in Figure 1. For the quadrupole, the field lines stand on radius $r = 0.25$ at a height of 0.25. They are spaced at 45° around the circle and are rising.

The quadrupole image on the right shows a magnetic field parallel to the disk plus an axial rising (inside $r \sim 0.15$) magnetic field. We should recall Equation (9) at this point, which indicates that the true subscale turbulent vorticity goes as $\tilde{\alpha}_h \delta r e^{-\alpha T}$. When $K = a = 1$ and $\alpha = \delta$, and the quantity δr is generally set so as to give a macroscopic velocity in order of magnitude. That leaves $\tilde{\alpha}_h$ to determine the magnitude of the alpha effect.

One can only expect the pure α_h dynamo in galaxies for which there is little evidence of macroscopic jets or winds (unless the flow is parallel to the magnetic field). The macroscopic (α/Ω) dynamo in our model occurs either due to the lagging rotation in the halo or because of a gradient in the outflow/inflow velocities. There are essentially no parameters in the magnetic field of this section beyond the choice of the self-similar class, a , which we have set equal to 1, reflecting a constant global velocity. This encourages us to search for this coherent magnetic field behavior in the observations. One might expect this behavior to be correlated with vigorous star formation.

Referring to the observed fields displayed in the CHANG-ES catalog of Krause et al. (2020), NGC 3044 may be a candidate for a pure star formation (i.e., subscale turbulence resulting in α_h) dynamo (see the top of Figure 4 and the figures of this section). The base function would be predominantly dipole. The characteristic magnetic behavior is observed to be predominantly high-latitude “X field” with no parallel equatorial component. Moreover, the field falls off rapidly radially but extends far into the halo. The very strong axial field is a formal feature of Equation (3) when $\alpha_h \rightarrow 0$ as $r \rightarrow 0$, because the spatial gradient times α_h is required to match the time derivative. There is also the $1/r$ field dependence that reflects the self-similar global symmetry. Figures 1–3 all illustrate these properties for the α_h dipole. The rotation measure (RM) given in Krause et al. (2020) and displayed in rudimentary form by contours in Figure 4 varies as might be expected from rising, spiraling, field lines. In between the double lines in the galaxy figures (solid and dashed curves close to each other) is where the RM sign changes. On

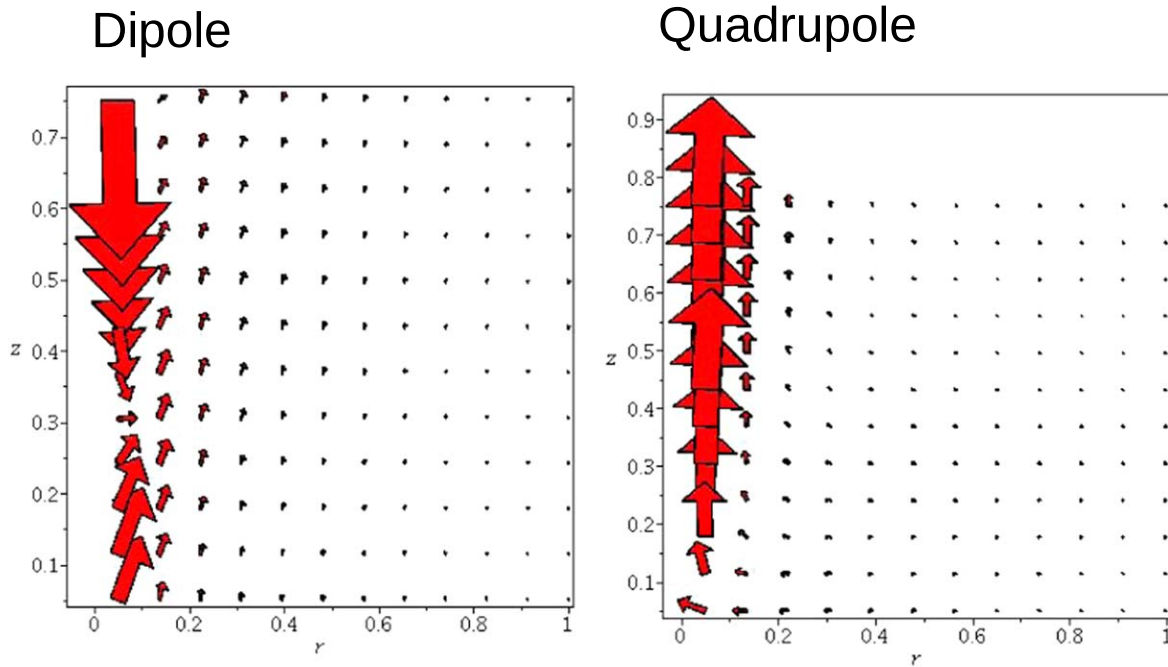


Figure 3. The left figure shows the pure α_h dynamo dipole in an $\{r, z\}$ cut. The right figure shows the pure $\{\alpha_h\}$ quadrupole in an $\{r, z\}$ cut. The only parameter is $\tilde{\alpha}_h$, which appears only in the evolution timescale. It does not affect the field topology.

the solid curve side, the RM is positive (field lines pointing toward us) and on the dashed curve side, the RM is negative (field lines pointing away from us). The RM also shows a sign change across the disk in some regions which can correspond to a change in sign of radial field. Figures 9 and 10 below show that the α_h dynamo with wind or even pure flux freezing with wind may also fit the data. NGC 3448 (not shown) may be a more depolarized second example.

NGC 3556 is another extreme where a coherent galactic field is scarcely established. It is likely that it is dominated by a series of local dynamos driven by local star formation, so we are actually seeing the turbulent nature of subscale vorticity. That is, there are multiple α_h dynamos acting. The data are shown for comparison in the bottom image of Figure 4.

3.3. Halo Rotational Lag and the α_h Dynamo

The general solution to Equation (19) with $v' = -v_1$ for $\tilde{A}_\Phi(\zeta)$ becomes

$$\tilde{A}_\Phi(\zeta) = C1 (1 + \zeta^2) F(\alpha_1, \beta_1, \gamma_1, -\zeta^2) + C2 \zeta (1 + \zeta^2) F(\alpha_2, \beta_2, \gamma_2, -\zeta^2), \quad (25)$$

where $F(\alpha, \beta, \gamma, z)$ is the hypergeometric function. The arguments are (with $n = 1, 2$)

$$\begin{aligned} \alpha_n &= \left(\frac{3}{4} + \frac{n-1}{2}\right) + \frac{\sqrt{1 - 4(K^2 + v_1)}}{4}, \\ \beta_n &= \left(\frac{3}{4} + \frac{n-1}{2}\right) - \frac{\sqrt{1 - 4(K^2 + v_1)}}{4}, \\ \gamma_n &= \frac{1}{2} + (n-1). \end{aligned} \quad (26)$$

In our examples, the solution $\{C1, C2\} = \{1, 0\}$ is “dipolar” in form with b_z nonzero at the disk, while the solution $\{C1, C2\} = \{0, 1\}$ is quadrupolar and has zero b_z at the disk.

From Equation (9), we note that when $\tilde{\alpha}_h$ is constant we may use its value to define the subscale turbulent velocity < 1 and so let the velocity scale δr be macroscopic. That is, δr reflects mainly lagging rotational velocity v or wind velocity w (although we do not use a “wind” velocity in this section). Consequently, Equation (8) shows that the magnitude of v and w should be chosen $O(1/\tilde{\alpha}_h)$ so as to reflect the larger macroscopic rotation and wind magnitudes. It is likely that $\tilde{\alpha}_h = O(0.1)$ (e.g., Tamburro et al. 2009) so that $v_1 = O(1)$ (Heald et al. 2007)⁷ and w_o and w_1 may be $O(10)$ in extreme cases (e.g., Heesen et al. 2018). Extensive supernova activity may lead to larger subscale velocities locally.

In Figure 5, we show a dipole solution and a quadrupole solution in 3D on the top row and a cluster of corresponding field lines for each case in the bottom row. We show only the upper half-plane for clarity, but b_Φ and b_z change sign on crossing the disk in the quadrupole solution, while only b_R does so in the dipole solution.

We see that the quadrupole solution produces a projected “X-type” geometry (recall the axial symmetry) at a small polar angle that includes the axis, while the dipole solution produces the X geometry at much larger polar angles. The dipole field lines appear to be straight in this representation because the poloidal field is much larger than the toroidal field. However, they do turn in azimuth as is shown in Figure 6.

The dipole has an interesting structure at a large height where the field lines become tightly wound near the axis. The winding sense is opposite to the sense of the exterior winding, which implies that the azimuthal field passes through zero at some height and radius. This leads to a sign change in the RM with latitude that can be seen in an RM plot (Henriksen et al. 2018a), but we do not show this here for brevity.

The addition of rotational halo lag has not greatly changed the topology from that of the pure α_h dynamo as seen in

⁷ The halo rotational lag rate was found to vary from 40 to 15 km s⁻¹ kpc⁻¹ in Oosterloo et al. (2007).

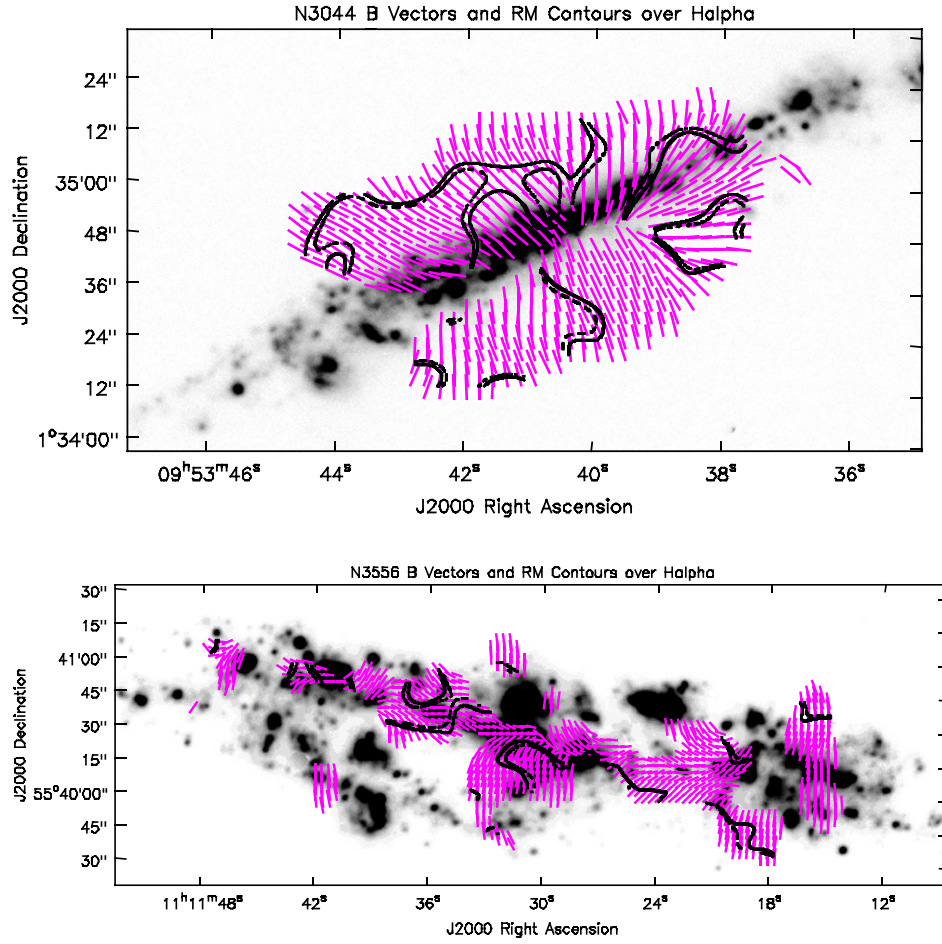


Figure 4. NGC 3044 (top) and NGC 3556 (bottom) with $H\alpha$ emission shown in grayscale. B vectors, corrected for Faraday rotation, are shown in magenta. RM contours have been set only for the purpose of indicating where there is a sign change from positive to negative or vice versa. The contours are set to $+10$ (solid curves) and -10 (dashed curves) rad m^{-2} for NGC 3044 and at $+15$ (solid curves) and -15 (dashed curves) rad m^{-2} for NGC 3556 to help guide the eye. That is, the sign change occurs in the narrow gap between the solid and dashed contours. See Krause et al. (2020) for more explanation of the polarization data and Vargas et al. (2018) for information on the $H\alpha$ image. All data have been downloaded from queensu.ca/changes.

Figures 2 and 3. The dipole solution changes the rotation sense at height near the axis but remains similar at a large radius, showing a strong X field. The quadrupole field is less tightly wound in the presence of halo lag and shows an “X field” only in a very limited range of latitudes. At this point, X-field magnetic topology favors very much a dipole symmetry across the galactic equator.

Figure 6 shows the poloidal and toroidal cuts of the same fields shown in Figure 5. The quadrupole solution shows very similar behavior to the pure alpha solution although it is less tightly wound. The dipole solution has very tight winding near the galactic axis as does the pure turbulent field and has a very strong X-field behavior generally. The sign of the pitch angle of the dipole magnetic field changes sign with height and radius. Such behavior is more in accordance with the average CHANG-ES behavior, as shown in Krause et al. (2020).

In terms of the observations reported in Krause et al. (2020), Figures 6 and 3 show that it would be difficult to detect the effect of moderate halo rotational lag on the α_h dynamo for the dipole, except near the axis. However, near

the axis, tight winding can lead to beam depolarization as for the dipole in Figure 5. This often seems to be the case observationally. The quadrupole does not deliver a convincing X field.

We turn in the next section to a similar study of the magnetic effects of a combined accelerating wind and halo rotational lag.

3.4. Accelerating Disk Wind Plus Halo Rotational Lag Acting on the Turbulent Dynamo

Given the (subscale) turbulent dynamo with constant $\tilde{\alpha}_h$ plus either outflow or accretion and rotational halo lag, Equations (3) plus scale invariance lead to the equations for the scaled vector potential as

$$\begin{aligned} K\tilde{A}_R &= -\tilde{A}'_\Phi(1 + \zeta v(\zeta)) + v(\zeta)\tilde{A}_\Phi - w(\zeta)(\tilde{A}'_R + \zeta\tilde{A}'_Z), \\ K\tilde{A}_\Phi &= \tilde{A}'_R + \zeta\tilde{A}'_Z - w(\zeta)\tilde{A}'_\Phi, \\ K\tilde{A}_Z &= \tilde{A}_\Phi + (v(\zeta) - \zeta)\tilde{A}'_\Phi. \end{aligned} \quad (27)$$

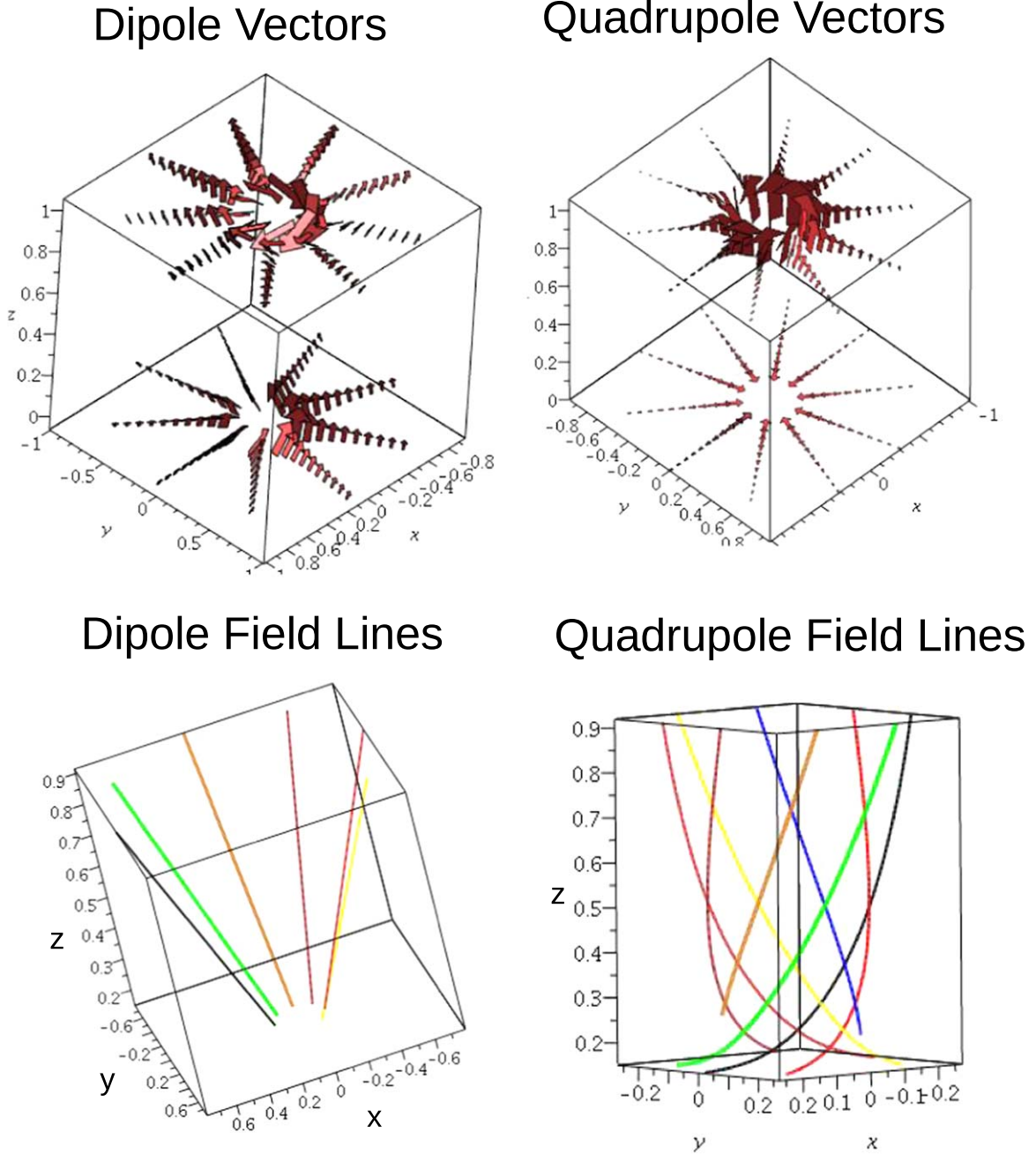


Figure 5. The top row shows the magnetic vectors in 3D for the dipole on the left and the quadrupole on the right. The bottom row shows clusters of field lines for the dipole on the left and the quadrupole on the right. The field lines of the lower row start at $z = 0.15$, $r = 0.25$, and then in steps of $\pi/4$ around the circle indicated by different colors. Only half the circle is shown for clarity. The rotational lag rate is $v_1 = 1$, $K = a = 1$ for all of these examples. For the magnetic field vectors of the top row, only two levels in z are shown for clarity, but they may be connected by referring to the 3D images of the second row.

These may be combined to give the equation for $\tilde{A}_\Phi$ as

$$(1 + w(\zeta)^2 + \zeta^2)\tilde{A}_\Phi'' + 2w(\zeta)(K + w(\zeta)')\tilde{A}_\Phi' + (K(K + w(\zeta)') - v(\zeta)')\tilde{A}_\Phi = 0. \quad (28)$$

The scaled magnetic field components are given as in Section 3.1. These can be expressed solely in terms of the azimuthal vector potential and its derivative plus the auxiliary velocities, by using Equations (27).

The forms for the auxiliary scaled vertical velocity and the scaled rotational lag (the physical velocity is given in Equation (8)) are taken as

$$w(\zeta) = w_o + w_1\zeta, \quad (29)$$

$$v(\zeta) = -v_1\zeta, \quad (30)$$

where w_o gives the scaled vertical velocity at the disk, w_1 is the scaled rate of acceleration with ζ and v_1 gives the scaled halo lag. If w_o and w_1 are negative, then we have a decelerating flow onto the disk. If $w_1 > 0$ with negative w_o then we may have an accelerating inflow. The rotational lag rate v_1 is generally positive. One should note that near the axis of the galaxy $\zeta \rightarrow \infty$. Either scaled velocity is then singular above the nucleus, but the physical velocity (8) simply increases with height.

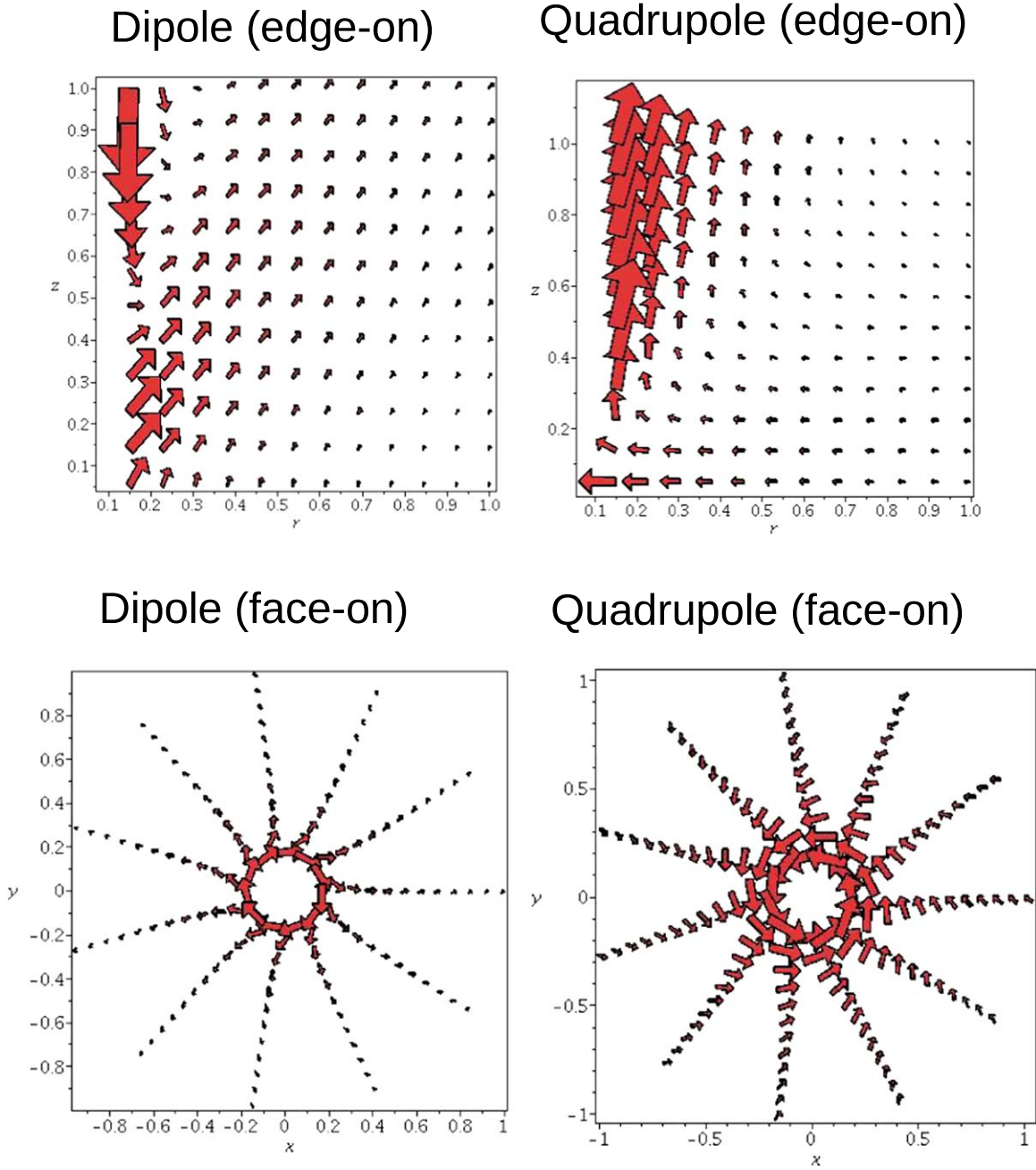


Figure 6. In the upper row, the left column shows the dipole magnetic field for the case $\{v_1, K, C1, C2\} = \{1, 1, 1, 0\}$ projected into the $\{r, z\}$ poloidal plane, integrated from $r = 0.15$ and $z = 0.05$. This contrasts with the X-field behavior. The right column shows the same projection for the quadrupole solution with parameters $\{1, 1, 0, 1\}$. The lower row shows the corresponding projections in the toroidal plane at $z = 0.5$.

Equation (28) may now be solved in terms of hypergeometric functions with complicated arguments, according to MAPLE 2019. We proceed to express a few results for “typical” parameters in Figures 7 and 8.

In Figure 7, we show sample “dipole” and “quadrupole” magnetic fields with and without a strong wind acceleration. The cuts are in the $\{r, z\}$ plane and because the field is axially symmetric, the 3D topology is easily inferred. The macroscopic motions act on the subscale magnetic field that is present due to turbulence (α_h dynamo). The constant auxiliary function $\bar{\alpha}_h$ does not appear in the equations except in the temporal evolutionary Equation (18). The magnetic topology is very sensitive to an accelerating wind velocity. Such a gradient

generates an α_h/Ω (macroscopic) dynamo just as a halo rotational lag or a gradient of angular velocity in the disk.

It is not shown here but we find that a strong disk wind ($w_o \lesssim \sim 10$) without strong acceleration only convects the boundary conditions at the disk to large heights, with little change in the topology. For this reason, we have focused on the acceleration in the examples.

What this figure emphasizes overall is the strong dependence on the wind gradient of the magnetic topology in the halo with a moderate initial disk wind. The “quadrupole” solution shows an “X-field” structure only with the wind acceleration. The dipole field gives a strong “X field,” particularly without wind acceleration.

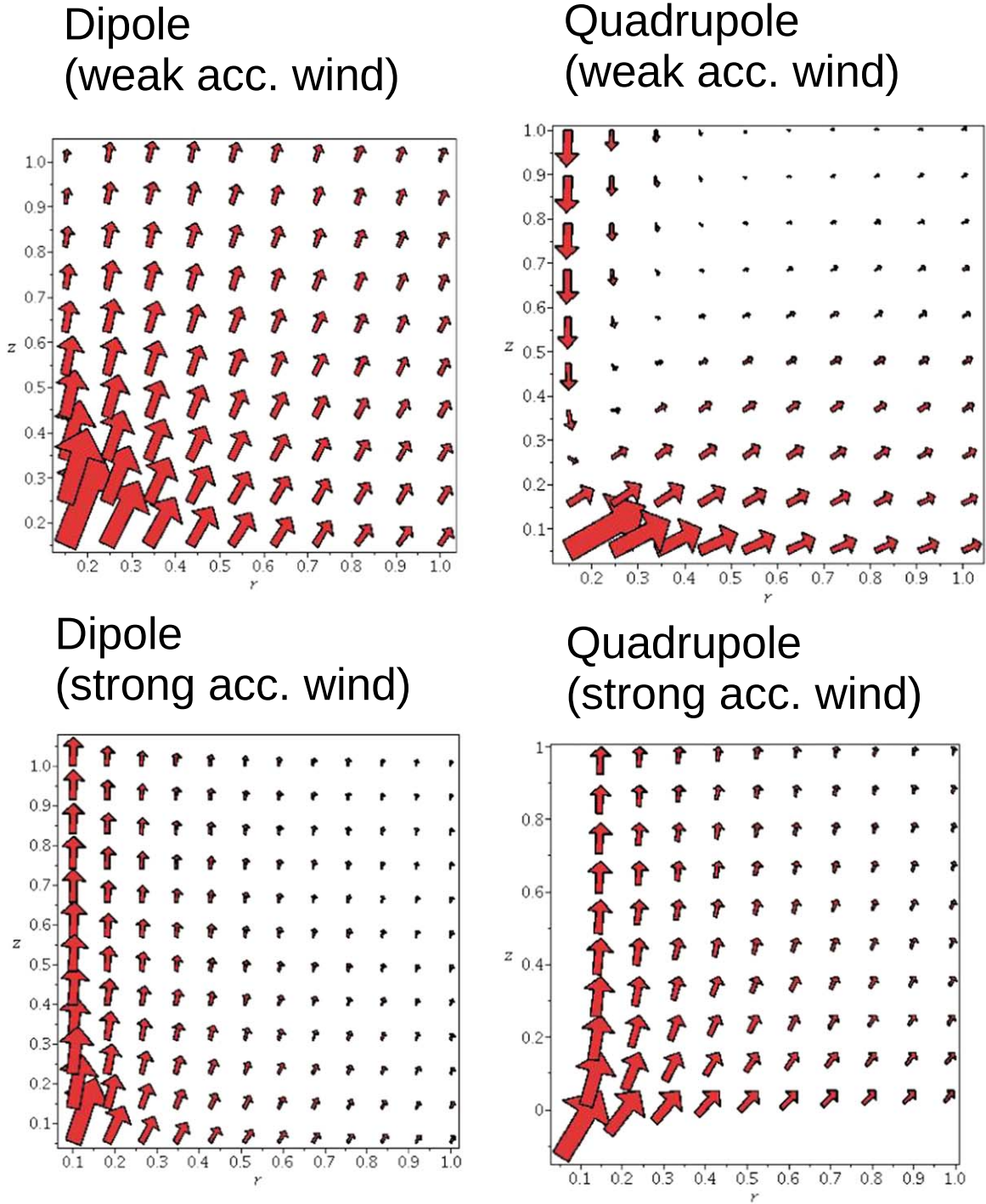


Figure 7. Here, the magnetic field is subject to a disk wind and vertical acceleration in an outflow in addition to subscale turbulence. The left-hand column shows the dipole field in the (r, z) plane with parameter vector $\{v_1, w_0, w_1, K, r, z, C_1, C_2\}$ equal to $\{0, 1, 1, 1, r, z, 1, 0\}$ at the upper left (a weaker wind) and $\{0, 1, 3, 1, r, z, 1, 0\}$ at the lower left (a strong wind). The strong accelerating accretion at the lower left has pulled the field nearly parallel to the axis. The right column shows the quadrupole solution with parameter vector $\{0, 1, 1, 1, r, z, 0, 1\}$ at the upper right and $\{0, 1, 3, 1, r, z, 0, 1\}$ at the lower right. At the lower right, the magnetic field is rendered almost entirely vertical by a gradient in the vertical wind velocity.

By varying the initial disk wind w_0 and the acceleration rate w_1 , intermediate cases can be found. The field is also combed out by accelerating accretion because there is no barrier at the disk in the model. Decelerating accretion can be arranged with both w_0 and w_1 negative.

The quadrupole column in Figure 8 compares a moderate accelerating wind with no halo rotational lag (upper panel) to one with a halo rotational lag, comparable to the accelerating

wind velocity at the disk. The flatter spiral generation with the presence of a halo lag is quite marked because the lines in the lower panel rise only to a height of 0.7 rather than ~ 1 . We recall that the unit of length is the radius of the galactic disk.

A similar flattening effect is observed with the same change in rotational halo lag for the dipole, as shown in the left-hand column panels. Because it appears (Figure 7) that the dipole is the principal source of strong X fields, we can recognize the

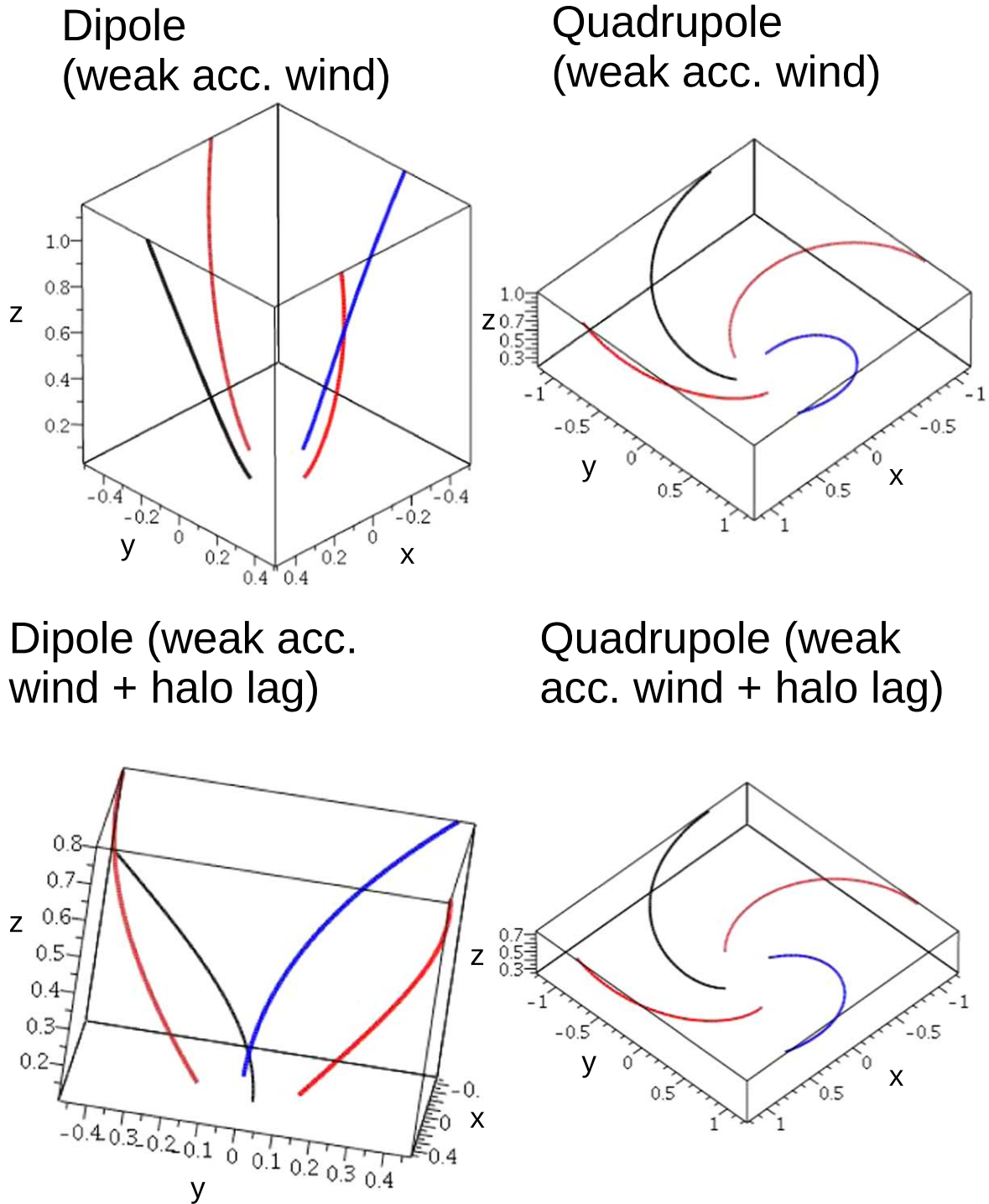


Figure 8. All the field lines in different panels of this figure are started at $r = z = 0.25$ and $\phi = 0$. At the upper left, a cluster of field lines for a dipole case is shown taken from the image at the upper left in figure 7. The parameter set is $\{v_1, w_o, w_1, K, r, \phi, z, C_1, C_2\} = \{0, 1, 1, 1, r, \phi, z, 1, 0\}$. The lower-left panel is for a dipole with parameter set $\{1, 1, 1, 1, r, \phi, z, 1, 0\}$. The field lines are placed at cardinal points around the initial circle and height. At the upper right, four quadrupole field lines that are taken from the $\{r, x\}$ image at the upper right in Figure 7 are shown. The parameter list is $\{0, 1, 1, 1, r, \phi, z, 0, 1\}$ for the quadrupole at the upper right. The lower-right cluster is for a quadrupole magnetic field with the parameter set $\{1, 1, 1, 1, r, \phi, z, 0, 1\}$.

effects of halo lag in the increasing polar angle (decreasing halo latitude) of the magnetic polarization with increasing lag rate.

Adding a halo rotational lag adds to the complexity of the magnetic topology. In particular, it can lead most noticeably to a reversal of the magnetic helicity with height, which leads to the observed reversal in “parity” in the RM. This is similar to the behavior found in Henriksen & Irwin (2016). The lag also increases the extent of the projected “X field” near the disk.

Figure 9 shows the CHANG-ES polarization (with the Faraday depth sign change) data for the galaxy NGC 4013. The X magnetic field is wound very tightly and is really only visible very close to the disk. This might suggest, according to Figures 7 and 8, that the coherent magnetic field in NGC 4013 is a quadrupole with considerable halo rotational lag. However, close inspection of Figure 9 reveals that the field is more like an inverted V than an X. That is, the field converges on the axis

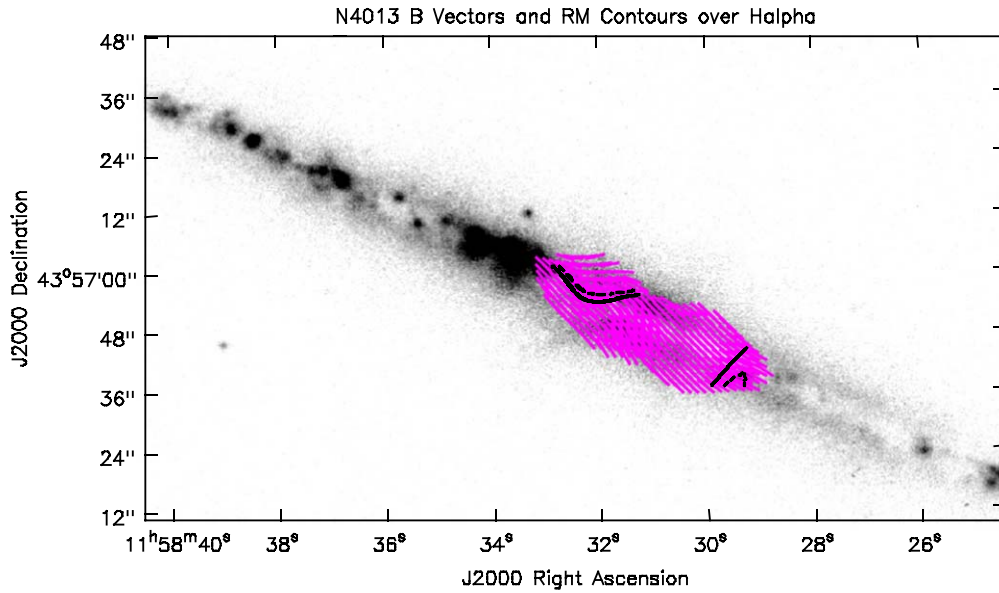


Figure 9. The image shows NGC 4013 with H α emission shown in grayscale. $\mathbf{B}$ vectors, corrected for Faraday rotation, are shown in magenta. RM contours have been set only for the purpose of indicating where there is a sign change from positive to negative or vice versa, and are present to help guide the eye. The contours are set to +10 (solid curves) and -10 (dashed curves) rad m $^{-2}$. See Krause et al. (2020) for more explanation of the polarization data and Vargas et al. (2018) for information on the H α image. All data have been downloaded from queensu.ca/changes.

above the disk. There is a weak indication of this for the quadrupole in Figure 6.

We conclude from this section that a vigorous constant wind can carry equatorial boundary conditions to large heights. However, a gradient in the wind velocity is more important for the magnetic complexity such as strong deviations from a pure α_h dynamo. The main effect of a strong wind is to transport the magnetic field to greater heights and to remove the “X field” in favor of a poloidal field perpendicular to the galactic disk. The behavior is particularly pronounced near the axis. The rotational halo lag, along with the wind acceleration, acts as an α/Ω dynamo given the α_h subscale turbulent average dynamo. The rotational lag gives a field strongly wound near the disk while the wind acceleration stretches the field vertically.

Reviewing the observations in the CHANG-ES catalog, we note that NGC 3044 (top of Figure 4) may be compatible with a moderate disk wind and acceleration as seen for the dipole in Figure 7 at the upper left. This is more consistent with the extended halo in NGC 3044 than is the pure α_h dynamo of Section 3.2.

Quadrupole behavior seems rarer but may be illustrated in NGC 4013 (Figure 9). We refer to both the CHANG-ES catalog (Krause et al. 2020) and Figure 12 in Stein et al. (2019). At the upper right in Figure 7, we see a quadrupole with a field largely parallel to the disk (i.e., a very low-latitude X field) that falls off rapidly into the halo. The model therefore predicts a large-angle but weak X field, which corresponds to Figure 12 in Stein et al. (2019). The data from Krause et al. (2020) is shown in Figure 9. The sign change in the RM across the disk in NGC 4013 can also be due to a quadrupole.

3.5. A Pure Macroscopic “Dynamo”

We consider the special case when there is no active α_h effect combined with macroscopic motion, as there has been in all previous cases. It should be described nevertheless as an α/Ω dynamo because there must be an initial field for the velocity to act on. With $a = 1$ and $K = 1$, flux density is conserved (equivalent Dimensionally to a magnetic field or

velocity constant) and a local field grows linearly according to Section 2.1.

The existence of a global magnetic flux constant requires $a = 3$ and hence $K = -1$. Global flux conservation can only produce topological enhancements in the magnetic field which will decline slowly in time. We recall the discussion of time dependence in Section 2.1, which shows the magnetic field to decrease in time as $(1 + \alpha t)^{-1/3}$ with $a = 3$.

We set $\tilde{\alpha}_h = 0$ in this section and so Equation (7) defines the logarithmic time. Because we also suppress the diffusion in this paper, we are left with the pure flux freezing during the macroscopic motion as described by the second term in Equation (3). We continue with our scale-invariant formulation, which prescribes the velocity field according to Equation (8) and the magnetic field according to Equations (10) and (13). We proceed with only rotational and outflow/inflow velocities so that the radial velocity $u = 0$. This limit is related to the study performed by Henriksen & Irwin (2016) using dynamical equipartition and a Mestel disk, but here the assumption of scale invariance dictates the velocity field, rather than a dynamical model.

The reason for studying this limiting case is due to the predictive nature of the analytic magnetic fields with regard to scale heights of the magnetic field. This is related to the recent results for CHANG-ES galaxies reported in Krause et al. (2018).

The surviving term in Equation (3) requires solving a simple set of equations for the scale-invariant vector potential namely

$$\begin{aligned} K\tilde{A}_R &= v(\tilde{A}_\Phi - \zeta\tilde{A}'_\Phi) - w(\tilde{A}'_R + \zeta\tilde{A}'_z), \\ K\tilde{A}_\Phi &= -w\tilde{A}'_\Phi, \\ K\tilde{A}_z &= v\tilde{A}'_\Phi. \end{aligned} \quad (31)$$

Note that it is the outflow/accretion that drives the whole system. Setting $w = 0$ renders the system trivial.

Equations (31) are easily solved analytically when we take the halo rotational lag in the pattern frame and the outflow

velocity in the familiar forms

$$\begin{aligned} v &= -v_1 \zeta, \\ w &= w_o + w_1 \zeta. \end{aligned} \quad (32)$$

However, even the case where v and w are both constant (so that physical velocities are proportional to r from Equation (8)) is of interest as it is both analytic and simple. The field components follow ultimately from Equations (22) to (24). These imply with v and w constant and Equations (31) that

$$\begin{aligned} b_R &= -\frac{e^{(2\delta-\alpha)T}}{rw} K C_1 e^{-\frac{K\zeta}{w}}, \\ b_\Phi &= \frac{e^{(2\delta-\alpha)T}}{rw} e^{-\frac{K\zeta}{w}} (C_1 v - C_2 K), \\ b_Z &= \frac{e^{(2\delta-\alpha)T}}{r} C_1 e^{-\frac{K\zeta}{w}} \left(1 + \frac{K}{w} \zeta\right). \end{aligned} \quad (33)$$

From these relations we observe that the exponential decline of the magnetic field in z at fixed r is slower with larger wind velocity, so that the scale height increases. The exponential decline is also slower at larger r for a constant outflow, which gives the magnetic (or radio) halo a “butterfly” shape. The azimuthal field increases with increasing v and decreases with increasing w at the disk. The radial field also decreases with increasing w at the disk. Boundary conditions at the disk are “lifted” into the halo. This behavior has also been found in the previous section.

In general, an individual field line always extends in the same sense in radius, azimuth, and height so that it appears as a rising, expanding helix, much as shown in Henriksen & Irwin (2016). It differs from the pure α_h dynamo of Section 3.2 in that the spirals are detectable at much larger radii. This topology already gives an X -type field near the disk when projected into the $\{r, z\}$ plane.

We note from the solution that b_R changes sign with w on crossing the disk. However, there is no halo rotational lag in z with v constant, but there is in radius by Equation (8). Therefore, there is no need for v to change sign on crossing the disk. Moreover, b_Φ also changes sign on crossing the disk. The component b_Z does not change sign because ζ also changes sign. Thus the field obeys a dipole symmetry, but with a change in sign of both tangential field components.

As a result of these symmetries, there is a change in the magnetic helicity crossing the disk. The term in C_2 comes from the homogeneous part of the equation so we normally set it equal to zero. It would be necessary in order to confront a specified boundary condition. The only quadrupole-type solution is with $C_1 = 0$, which gives only a purely azimuthal field, possibly discontinuous at the disk.

When the outflow and lag functions have the forms of Equations (32), rather than being constant, the solution for the field becomes

$$\begin{aligned} b_R &= \frac{e^{(2\delta-\alpha)T}}{rw(\zeta)} K \tilde{A}_\Phi(\zeta), \\ b_\Phi &= -\frac{e^{(2\delta-\alpha)T}}{rw(\zeta)} \left(\zeta v_1 \left(1 + \frac{K\zeta}{w(\zeta)}\right) - \frac{Kv_1}{w_1} \left(\zeta + \frac{w_o}{w_1} \ln w(\zeta)\right) + KC_2/C_1 \right) \tilde{A}_\Phi(\zeta), \\ b_Z &= \frac{e^{(2\delta-\alpha)T}}{r} \left(1 + \frac{K}{w} \zeta\right) \tilde{A}_\Phi(\zeta). \end{aligned} \quad (34)$$

The azimuthal vector potential has the power-law form

$$\tilde{A}_\Phi = C_1 e^{-\frac{K}{w_1} \ln w(\zeta)}. \quad (35)$$

In these formulae $w_1 \neq 0$ and, if $w_o = 0$, there is a logarithmic singularity at $\zeta = 0$. Once again the general behavior is that of a rising, widening spiral as was first seen in Henriksen & Irwin (2016). On crossing the equator, we might expect w_o and v_1 to change sign, but not w_1 so that w changes sign. Hence, b_R and b_Φ change sign but b_Z does not, implying dipole behavior as in the constant-velocity case. We intend the real part of the logarithm in these expressions.

We append a relatively generic example in Figure 10 in order to fix ideas. Depending mainly on the relative size of the halo lag v_1 relative to the wind acceleration, w_1 , the spirals are flatter when the ratio is large and more extended when the ratio is small. Recall that the units in the figures are the disk radius. The constant wind velocity merely convects boundary conditions as we have seen before. This is because it does not contribute to a differential twisting of the field.

The “ X -type” field behavior is found in projection as a characteristic of rising, spiraling fields. It will be at an angle close to the disk when the halo lag rate is dominant and closer to the galactic axis when the wind acceleration is stronger. A strong wind acceleration case (not shown) has almost straight field lines in the halo rather than flatly spiraling field lines. This is very similar to the quadrupole α_h behavior.

We summarize this section by comparing it to the pure α_h dynamo of Section 3.2. We have found a widening, spiraling halo structure in both limits as elaborated below.

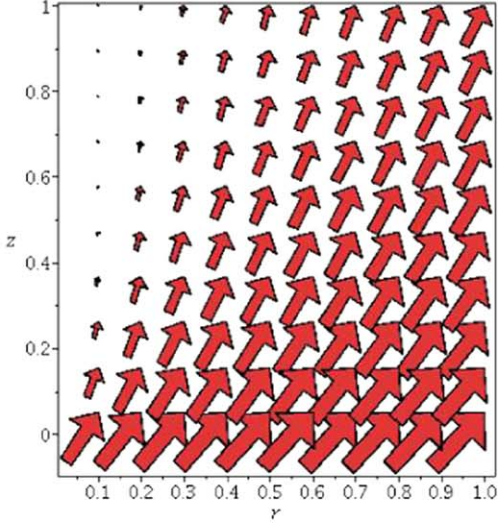
The pure α_h subscale turbulent average dynamo produces tightly wound spirals near the axis of the galaxy, where they are unlikely to be observed. The large radius spirals are very slow and the projections into the $\{r, z\}$ plane show an “ X -type” topology only near the disk for the quadrupole solution. The dipole α_h solution shows a rapid decline in field strength with height in the azimuthal and vertical field components when the lag and wind are weak.

The macroscopic flux-freezing scale-invariant dynamo of the present section produces rising spiral magnetic fields with a large polar angle when the halo rotational lag velocity dominates the wind velocity. The X -type topology is then projected into the $\{r, z\}$ plane in a well-defined low-latitude sector. The wind-dominated case shown at the upper left gives a more typical X -field topology. There is also scope for varying the sign of the Faraday depth above the disk when the spirals are rapidly turning inside the galactic halo. This is especially likely when the halo rotational lag dominates.

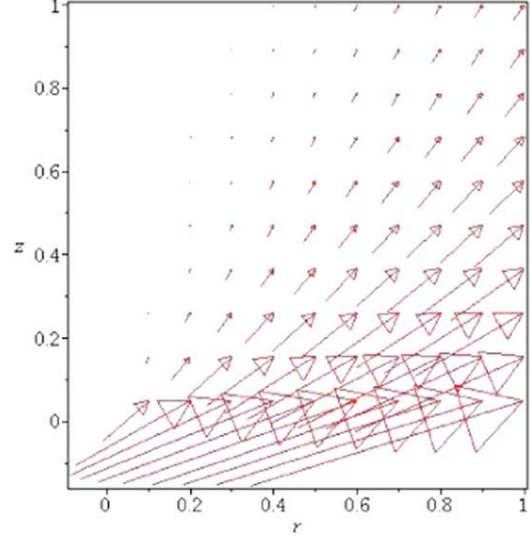
Referring once more to Krause et al. (2020), the flux-freezing “dynamo” alone cannot give both a high-latitude X field and a parallel field in the disk. However, it shows clearly the association between the X -field topology and rising magnetic spirals. The wind-dominated field is a natural explanation for the observed magnetic topology in NGC 3044 (see Figure 4).

The C -band image of NGC 4631 indicates similar poloidal behavior, but the multiple sign changes in RM in this galaxy suggest the presence of non-axially symmetric spiral arms (see e.g., Woodfinden et al. 2019) in the halo.

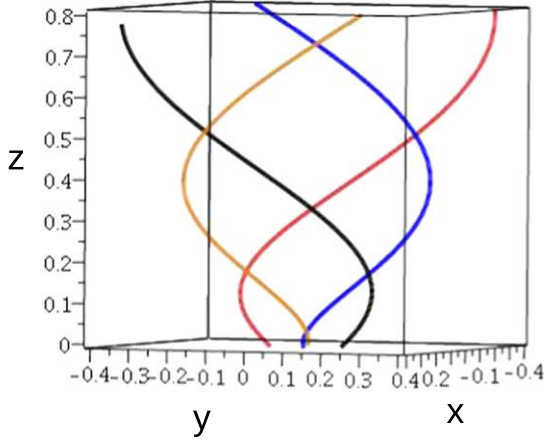
Dipole (acc. wind +
weak halo lag)



Dipole (acc. wind
+ strong halo lag)



Dipole (acc. wind +
weak halo lag)



Dipole (acc. wind +
weak halo lag)

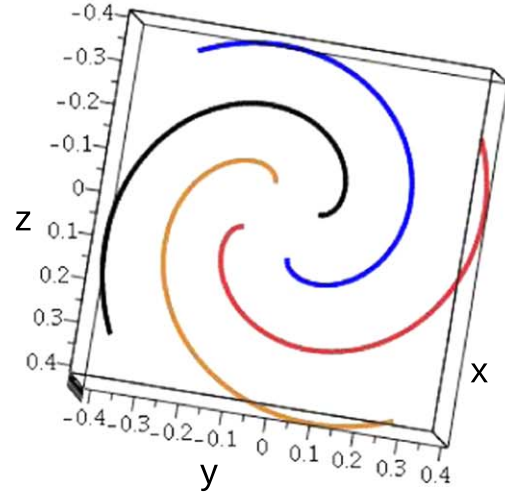


Figure 10. In the bottom row we show a cluster of 3D field lines for the parameters $\{v_1, w_o, w_1, K, C_1, C_2\} = \{0.15, 1.0, 0.1, 1, 1, 0\}$. The field lines all pass through $r_o = 0.25, z_o = 0.25$, and through the cardinal points on the circle of radius r_o . The side view is on the left and the top view is on the right. There is no true quadrupole as can be seen from Equations (34) and (35). The plot in the $\{r, z\}$ plane at the upper left is for the same parameters. The plot at the upper right shows the effect of a strong halo rotational lag with parameters $\{1, 0.25, 0.25, 1, 1, 0\}$. The arrows correspond to values at their head.

3.6. Latitude Dependence of Subscale Turbulence and Rotational Halo Lag

We turn to consider a subscale turbulent dynamo varying with latitude and acted on by a rotationally lagging halo. That is, we allow the auxiliary turbulent quantity $\tilde{\alpha}_h$ to be a function of $\zeta \equiv z/r$. By concentrating the scaled subscale turbulent helicity on the galactic axis in this way, we imitate the effect

that an AGN may have on the global galactic magnetic field. The effects of an axial wind must be left to another work.

The suggestion of an axially symmetric “AGN dynamo” assumes that the AGN axis is approximately aligned with the axis of the galaxy. In galaxies where this is not the case, we can expect more asymmetric galactic magnetic fields, at least near the nucleus. It is not necessary for a galaxy to have an axial concentration of magnetic fields, such as may be produced by

an AGN, in order to display a large-scale coherent magnetic field. We have seen this in previous sections, especially with the quadrupole mode where the field may be tightly wound about the axis.

If we choose $\tilde{\alpha}_h$ to be linear in the tangent of the latitude, ζ , according to

$$\tilde{\alpha}_h = \alpha_{h1}\zeta, \quad (36)$$

then we must use T as defined in Equation (7). Consequently, the constant $\tilde{\alpha}_{h1}$ no longer vanishes from the equations for the vector potential.

We have, with Equation (36), assumed (recalling Equation (9)) that the physical subscale turbulent helicity increases linearly with height in the halo. However, the scale-invariant solution may be expected to react to the latitude dependence in $\tilde{\alpha}_h$. The linear form satisfies the antisymmetry across the disk (e.g., Klein & Fletcher 2015).

This form for $\tilde{\alpha}_h$ parallels the assumption regarding the form of the halo rotational lag (see Equation (14)) relative to a pattern frame. In fact, if we assume that our calculation is done in a uniformly rotating (Ω) pattern frame, we can associate $2\Omega r$ with the base hydrodynamic vorticity to which we add the value (Equation (36)).

Equations (3) for the components of the vector potential become, when $\tilde{\alpha}_h$ varies generally with ζ ,

$$\begin{aligned} K\tilde{A}_R &= -\tilde{\alpha}_h(\zeta)\tilde{A}'_\Phi + v\tilde{A}_\Phi - \zeta v\tilde{A}'_\Phi, \\ K\tilde{A}_\Phi &= \tilde{\alpha}_h(\zeta)(\tilde{A}'_R + \zeta\tilde{A}'_Z), \\ K\tilde{A}_Z &= \tilde{\alpha}_h(\zeta)(\tilde{A}_\Phi - \zeta\tilde{A}'_\Phi) + v\tilde{A}'_\Phi, \end{aligned} \quad (37)$$

and these combine to give the equation for the azimuthal vector component. Once this equation is solved, the other components and the magnetic field components follow. The equation to be solved takes the general form

$$\begin{aligned} (1 + \zeta^2)\tilde{A}_\Phi'' + \frac{d \ln \tilde{\alpha}_h(\zeta)}{d\zeta}(1 + \zeta^2)\tilde{A}'_\Phi \\ + \left(\frac{K^2}{\tilde{\alpha}_h(\zeta)^2} - \zeta \frac{d \ln \tilde{\alpha}_h(\zeta)}{d\zeta} - v' \right) \tilde{A}_\Phi = 0. \end{aligned} \quad (38)$$

Using Equations (36) and (14), this last equation simplifies to

$$(1 + \zeta^2)\tilde{A}_\Phi'' + \left(\zeta + \frac{1}{\zeta} \right) \tilde{A}'_\Phi + \left(\frac{K^2}{\alpha_{h1}^2 \zeta^2} - 1 + v_1 \right) \tilde{A}_\Phi = 0. \quad (39)$$

This equation has a logarithmic singular point at $\zeta=0$. However, in the limit $\zeta \rightarrow 0$, the solution is proportional to ζ^p , where $p^2 = -K^2/\alpha_{h1}^2$. Consequently, the singularity is an oscillation in $\ln \zeta$ and is easily recognized as $\alpha_{h1} \rightarrow 0$. We observe in our examples below that this oscillation is associated with the subscale turbulence becoming dominant, as it extends to smaller scales.

The magnetic field is no longer invariant under a change in sign of ζ according to Equation (39) because of the term in v_1 , which changes sign on crossing the disk. The sign of α_{h1} is irrelevant as one might expect because only the magnitude of the vorticity should produce the alpha dynamo.

A solution of Equation (39) at positive ζ can be pasted into the negative region (keeping the sign of $\tilde{A}_\Phi$ unchanged, but remembering to change the sign of v_1) to establish symmetry.

This yields, by Equation (37) and the equations for b , a change in sign of b_R and b_Φ but not in b_Z . This would construct local dipole boundary conditions, but a quadrupole boundary follows by changing the sign of $\tilde{A}_\Phi$. However, the oscillations in sign near the disk due to small-scale turbulence confuse the issue. These fluctuations become smoothed to a coherent quadrupole for the $\{0, 1\}$ solution and to a coherent dipole for the $\{1, 0\}$ solution when $v_1 = \alpha_{h1} = O(1)$.

We observe, from Equations (8) and (9), that if indeed $\tilde{\alpha}_h = \alpha_{h1}\zeta$, and $v = -v_1\zeta$, then

$$v_s = -\delta r v_1 \zeta e^{(-\alpha)T} \quad \alpha_h = \alpha_{h1} \zeta e^{(-\alpha)T}. \quad (40)$$

It is convenient to choose δr to be the actual lag rate so that $v_1 = O(1)$. This means that the value of α_{h1} is likely to be also $O(1)$.

The magnetic field components take the same form as in Section 3.1 except for $\tilde{b}_\Phi$. Equation (23) becomes, using the expression for $\tilde{A}_\Phi$ in Equation (37) divided by $\tilde{\alpha}_h(\zeta)$,

$$\tilde{b}_\Phi = \frac{K\tilde{A}_\Phi}{\tilde{\alpha}_h(\zeta)}. \quad (41)$$

Recalling the linear form $\tilde{\alpha}_h$ (Equation (36)), we see that the sign of α_{h1} affects the sign of $\tilde{b}_\Phi$ as one expects from helical (vortical) turbulence, but it does not affect the magnetic topology.

3.6.1. Examples

The two independent solutions of Equation (39) ($\{C_1, C_2\} = \{0, 1\}$ or $\{1, 0\}$) are complex conjugates. We choose the real part of the $\{0, 1\}$ solution and the imaginary part of the $\{1, 0\}$ solution. This is because these choices best fit the expected quadrupole and dipole boundary conditions.

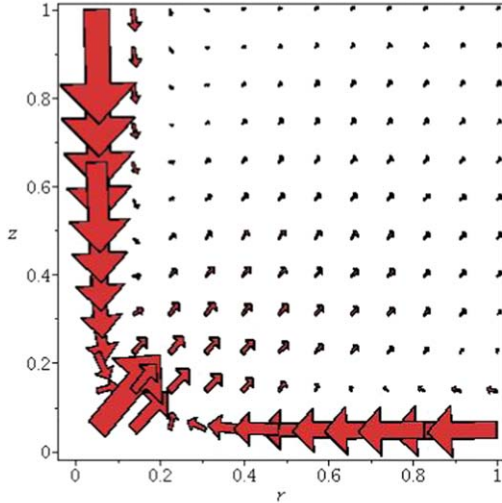
The oscillatory logarithmic singularity at small ζ is quite obvious in the solutions for small α_{h1} near the disk for small values of α_{h1} . This is formally because whenever K^2/α_{h1}^2 dominates the factor of $\tilde{A}_\Phi$ in Equation (39), the oscillatory approximation extends to larger ζ . However, ζ must still be less than 1 (polar angle greater than 45°), so that the oscillations occur near the galactic disk. Physically, this is of interest because the α_h term in Equation (3) is due to subscale turbulence. As this term becomes dominant at smaller scales (i.e., smaller z at fixed r), we can expect turbulence to manifest itself in the magnetic field.

The $\{r, z\}$ plane for the $\{0, 1\}$ (quadrupole) solution is shown at the upper right in Figure 11 and some corresponding field lines are shown at the lower right. The field lines are constrained to the same scales on each axis. The images in the left column of the figure are for the $\{1, 0\}$ (dipole) solution. Strong field lines descend rapidly near the axis and then rise. The “X-type” projected field lines are confined to a range of latitudes near 45° latitude.

One should remember that in all images of the $\{r, z\}$ plane, the diagonal, apparently rectilinear magnetic vectors, are actually turning in the $\{r, \phi\}$ plane. Thus, they represent rising or falling spirals as confirmed in Figure 12. The twist rate differs depending on the parameters and frequently varies with height above the disk and radius in the disk.

The quadrupole behavior is strongest parallel to the axis and also parallel to the equator. The transition in field direction occurs near 45° and removes any X-type topology in favor of a nearly vertical field at large latitude. The corresponding field

Dipole (Latitude dependent Turb. + halo lag)



Quadrupole (Latitude dependent Turb. + halo lag)

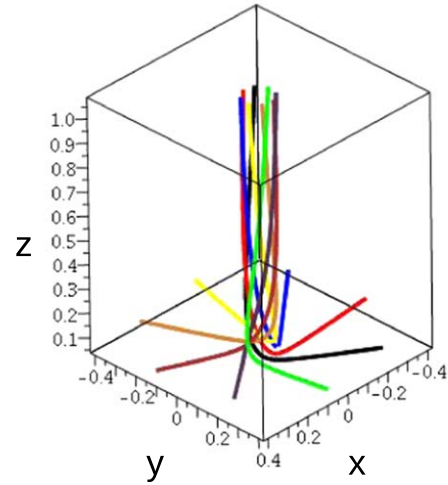
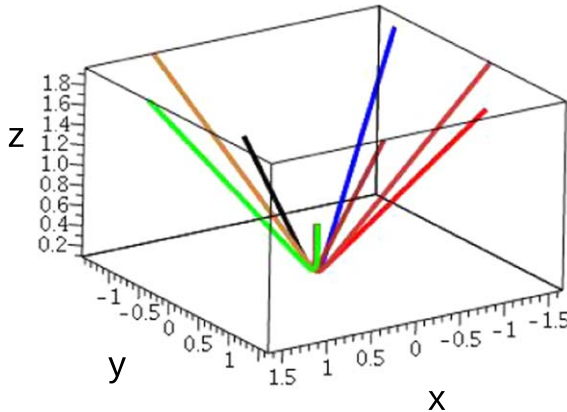
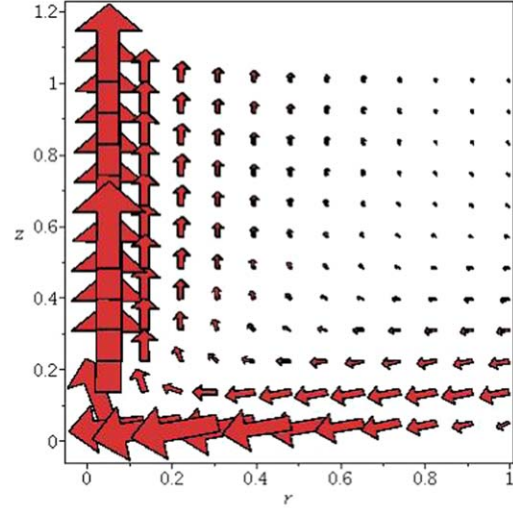


Figure 11. In the left column, we show the dipole with parameters $\{v_1, \alpha_{h1}, K, r, z, C_1, C_2\} = \{1, 1, 1, r, z, 1, 0\}$. The $\{r, z\}$ plane is on the top and a cluster of field lines is on the bottom. All field lines in this figure pass through $r = 0.05$ and $z = 0.1$ and are distributed in steps of $\pi/4$ in azimuth. The $\{r, z\}$ cuts must be imagined rotated about the axis and the field line plots are filled in by similar field lines. The right column illustrates, in the same manner, the quadrupole with parameters $\{1, 1, 1, r, z, 0, 1\}$.

lines in the lower-right panel illustrate that the field near the axis is nearly vertical with a small toroidal field. The upper-right panel shows how field lines starting at higher latitudes fill the space of the lower panel.

The dipole behavior in the left column of Figure 11 has a descending strong vertical field near the axis that rises almost rectilinearly, as seen in the field lines in the lower panel. These produce the X-field topology near 45° . The toroidal plot in Figure 12 indicates only a very small azimuthal field compared to the quadrupole. These figures are for nearly equal lag velocity and subscale turbulent velocity.

Figure 12 shows, at the upper right, the 3D version of the $\{r, z\}$ cut at the upper right in Figure 11. The toroidal cut of the same field is shown at the lower right. The field lines descend

from the halo at a large radius and then rise again at latitudes greater than $\pi/4$. The very slight toroidal field near the axis compared to the vertical field is shown in the lower-right panel.

At the upper left in Figure 12, we have the 3D image of the dipole with the same parameters. The field lines descend while turning slowly and subsequently rise rectilinearly at larger radius. This develops in a self-similar fashion at all heights. The transition region contains a projected “X field” present mainly near 45° . The azimuthal components of the vectors change sign with radius.

In Figure 13, we show the effects of reducing the scale and strength of the subscale turbulence. The halo rotational lag has been kept constant but $\tilde{\alpha}_{h1} = 0.25$ in both the quadrupole and dipole panels. Both modes now show X-field topology in two

Dipole (Latitude dependent Turb. + halo lag)

Quadrupole (Latitude dependent Turb. + halo lag)

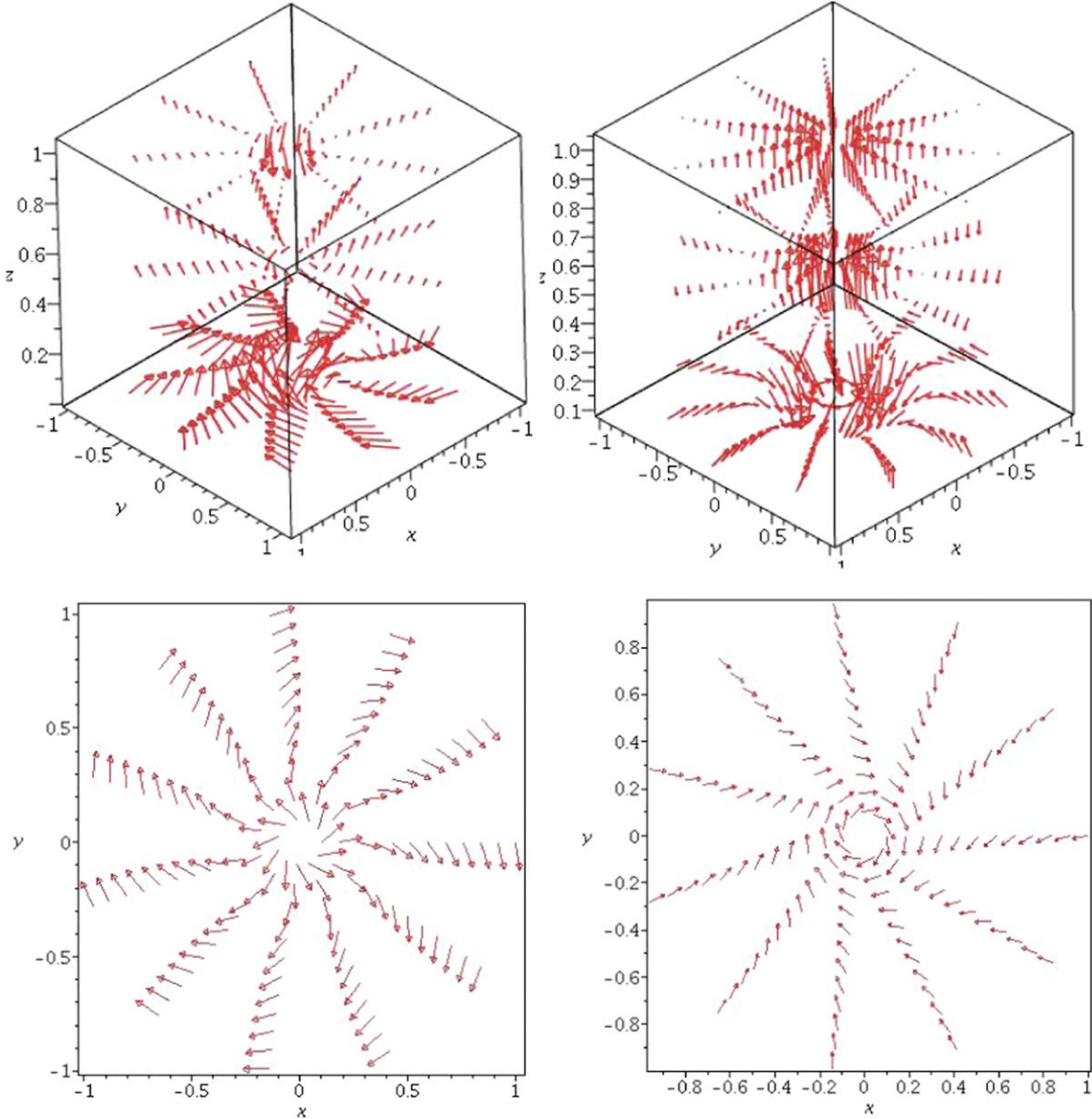


Figure 12. The figure at the upper left is the 3D version of the case shown in the left column of Figure 11. The parameters are the same, $\{v_1, \alpha_{h1}, K, r, \phi, z, C_1, C_2\} = \{1, 1, 1, r, \phi, z, 1, 0\}$. The right column has the parameter set $\{1, 1, 1, r, \phi, z, 0, 1\}$ and corresponds to the right column in Figure 11. The lower panels show an $\{r, \phi\}$ toroidal projection at height $z = 0.25$ in each case for the same parameters.

narrow but separated sectors. The quadrupole actually shows three X-field sectors, but one is very close to the equator of the galaxy. The more interesting development is the increasing turbulence in the disk of the galaxy for both modes. This tendency increases in latitude extent as $\tilde{\alpha}_{h1}$ decreases further.

This small-scale turbulence is to be expected when the alpha dynamo dominates. Its appearance actually represents a confirmation of the form for this subscale dynamo, as used in the classical Equation (3).

An interesting result of this section is the appearance in the dipole of a strong axial magnetic field, together with the X-field topology that is intermediate between the axis and the equator. Although we have not included a “jet” or a nuclear “wind” directly, the dipole produces an axial field that we know from previous sections would not be changed by a constant vertical outflow velocity required in a jet or wind. This is achieved by assuming a strong source of scaled subscale vorticity on the galactic axis. This can be produced by an AGN, that is, by a

Dipole (reduced sub-scale turbulence)

Quadrupole (reduced subscale turbulence)

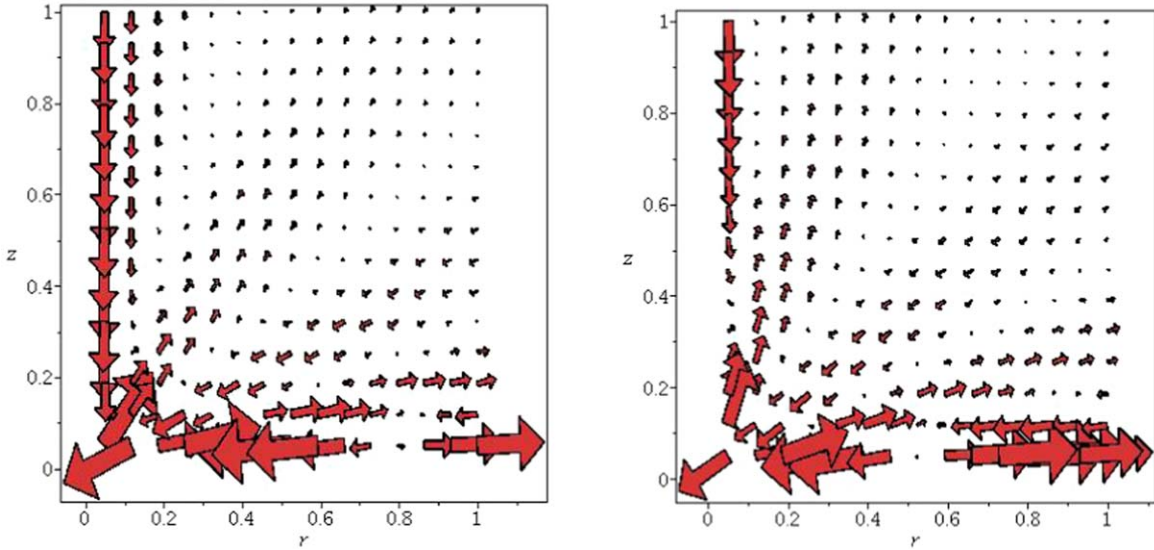


Figure 13. The image at left shows the $\{r, z\}$ field line vectors for the dipole case $\{v_1, \alpha_{h1}, K, r, \phi, z, C_1, C_2\} = \{1, 0.25, 1, r, \phi, z, 1, 0\}$. At right we have the quadrupole field line vectors with parameter set $\{1, 0.25, 1, r, \phi, z, 0, 1\}$. These reveal the onset of galactic disk small-scale magnetic turbulence.

supermassive black hole or a burst of star formation. Such nuclear flows may therefore be an important contributor to the global galactic magnetic field.

If jets or nuclear starbursts do contribute to the coherent galactic magnetic field, we can expect a correlation between the presence of a galactic AGN and the strength of the magnetic field. In fact, the disk field is also strong and turbulent (see Figure 13). Both the correlation and the turbulence would indicate an important subscale dynamo. Similar global magnetic topology can be produced without an AGN but the strong axial field will be absent. This prediction may be readily tested for galaxies where the AGN axis and galactic axis coincide.

Perhaps the best candidates from CHANG-ES results are NGC 4388 and NGC 3079. Recent images of these two galaxies are shown in Figure 14. There is indeed a correlation between the strength of the coherent galactic polarization and the presence of AGN activity as a nuclear wind. The RM sign changes in NGC 4388 tend to indicate spiraling fields above the disk. This is less clear in NGC 3079 but also present. NGC 4388 shows rapid changes in field direction in the disk region, which may be turbulent.

4. Discussion

Our intention in this paper has been to explore scale-invariant, axially symmetric, time-dependent magnetic dynamos. We have proceeded by isolating and combining the turbulent α/Ω and flux-freezing “dynamos.” Diffusion has been ignored under defined conditions. Formally, this assumes infinite conductivity. Excess flux can be removed from the galaxy by a disk wind with a gradient.

4.1. Subscale Turbulent Dynamo Only

In Section 3.2, we studied the α_h dynamo alone and displayed the results for the quadrupole and dipole in Figure 1. There are very few parameters in this part because we chose $\tilde{\alpha}_h$ to be constant with latitude. This allowed us to remove it from the equations by placing it in the time dependence and as a factor in the velocity.

In Figure 1, one sees in the dipole a very slow spiral twist except near the axis, as is shown in the lower row by an axial field line. There is also a characteristic oscillation in the direction of the azimuthal field with height in the dipole. The quadrupole on the right has slowly spiraling field lines as indicated in the lower row of the figure. Clusters of these field lines would project only weakly as an “X-type” topology above the plane as indicated in Figures 2 and 4. Notice that no strong vertical field is produced near the axis of the galaxy and the turbulent dynamo produces only a weak, nearly vertical, X field.

It should be remarked that our model generally has no absolute measure of the strength of the magnetic field unless we have a measure of such a field at a given time. We can say that we expect $\tilde{\alpha}_h \delta r \leq \approx 0.1$ if δr is comparable to the rotation speed of the disk. The growth timescale is then $\approx 1/(\delta \tilde{\alpha}_{h1})$, that is, perhaps equal to 10 rotation periods. The period would be measured at the start of the flat rotation curve of the galaxy.

4.2. Turbulent Dynamo Plus Rotational Lag

In Section 3.3, we added a halo lag to the α_h dynamo. The lag is a form of the macroscopic α/Ω dynamo. The rotational lag is linear with height, and we take the lag rate in our examples to be typically $dv_\phi/dz = -\alpha_d \delta \approx 10\text{--}20 \text{ km s}^{-1} \text{ kpc}^{-1}$. In Figure 5, we show clusters of field lines and 3D vector plots for the quadrupole and dipole. Figure 6 shows the $\{r, z\}$ and $\{r, \phi\}$

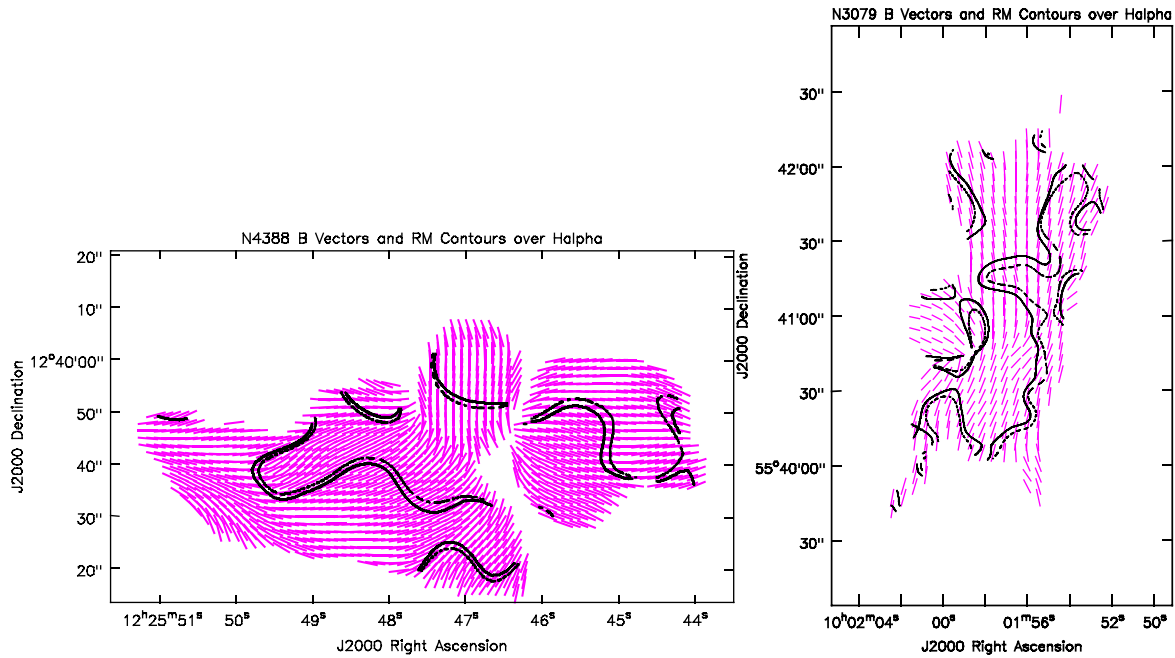


Figure 14. NGC 4388 (left) and NGC 3079 (right) with $H\alpha$ emission shown in grayscale. $\mathbf{B}$ vectors, corrected for Faraday rotation, are shown in magenta. RM contours have been set only for the purpose of indicating where there is a sign change from positive to negative or vice versa and are present to help guide the eye. The contours are set to $+10$ (solid curves) and -10 (dashed curves) rad m^{-2} for NGC 4388 and $+15$ (solid curves) and -15 (dashed curves) rad m^{-2} for NGC 3079. See Krause et al. (2020) for more explanation of the polarization data and Vargas et al. (2018) for information on the $H\alpha$ image. All data have been downloaded from quensu.ca/changes.

cuts. The rotational lag increases the twist of the quadrupole relative to the zero-lag case, but the poloidal field is not greatly changed. The dipole has been more strongly affected. It shows a stronger X -field topology and continues to show what would be a sign change in the RM with height.

4.3. Turbulent Dynamo Plus Wind and Lag

In Section 3.4, we have added an outflow/inflow to the turbulent dynamo with rotational lag. Figure 7 shows, for both the quadrupole and the dipole, the strong and weak wind, zero rotational halo lag, $\{r, z\}$ vectors. It is clear that the strong wind gradient “straightens” (more vertical) the spirals in both cases and migrates the field to greater heights. The dipole field lines are rendered almost vertical. Moderate wind disk velocity and gradient enhance the X -field topology.

Figure 8 shows sample field lines for the quadrupole on the left and the dipole on the right. The upper row has no rotational halo lag but moderate wind parameters corresponding to the upper row in Figure 7. The lower row adds a typical rotational halo lag, and we see the field lines spiraling at lower heights.

In Figure 9, we have shown the peculiar polarization of the galaxy NGC 4013 as a possible example of a quadrupole turbulent field distorted by a lagging halo.

4.4. Flux Freezing

In Section 3.5, we have focused on the effect of pure flux-freezing scale invariance. There is no current turbulent dynamo so that at some earlier time, there must be a seed field. The field will eventually decay according to the discussion in Section 2.1, depending on global constants. The reason for dwelling on this rather restricted “dynamo” is that (a) it is wholly analytic, and (b) it creates at least a temporary magnetic field similar to that of a real dynamo. An analytic solution also

exists when the magnetic field is not axially symmetric but rather forms magnetic spiral arms. We do not pursue this in this paper.

The results of this section are summarized in Equations (33) and (34). However, Figure 10 shows a cluster of field lines for the dipole with moderate acceleration and lag compared to the outflow velocity. This shows beautifully the rising spiral topology of the magnetic field. The $\{r, z\}$ cuts in the upper row are for the same parameters on the left. The X field is strongly present. When there is strong rotational lag and weak wind, the low-latitude X field at the upper right is found.

The forms of the explicit equations for the magnetic field are noteworthy for their description of the $\{r, z\}$ dependence. In the constant scaled velocity (there is a linear radial dependence for the physical velocity) case we find an exponential decline as a function of $-z/(rw)$. The scale height is therefore wr and increases both with w and r . This produces a higher magnetic halo at a larger radius and with a stronger wind. This can be compared to the CHANG-ES paper conclusions in this regard (Krause et al. 2018).

The behavior found here affords an explanation of the correlation between scale height and diameter found in Krause et al. (2018), provided that galaxies scale similarly as well as self-similarly. A superposition of the coherent magnetic fields in CHANG-ES galaxies (Krause et al. 2020) encourages this view.

The solution with variable wind and halo rotational lag is more complicated but varies in the halo essentially with the power $(w_0 + w_1 z/r)^{-K/w_1}$. For accelerating outflow, when w_0 and w_1 are positive, the magnetic halo has a similar structure to that given by constant velocity although it is not exponential. The halo structure in z evidently can be more subtle with $w_1 < 0$.

4.5. High-latitude Concentration of Turbulence Plus Rotational Lag

Section 3.6 illustrates how the subscale turbulent dynamo might appear when the turbulence is strong near the galactic axis. Figure 11 shows an $\{r, z\}$ cut at the upper right for the quadrupole and corresponding field lines at the lower right. The concentration of field to the axis is evident, but there is no X field. The left column shows the dipole for the same parameters. The field is again very strong at height and there is an X field at 45° .

Almost unique to this case is the dominant axial poloidal field. Something similar is found with constant $\tilde{\alpha}_h$ but it is not as pronounced. The “ X -field” region is extensive in the dipole. This morphology of the X field and “jet” axial field is quite distinct from the other examples in this paper.

The upper 3D and $\{r, z\}$ images in Figure 12 have the same parameters as the upper panels in Figure 11. These confirm the structure found in the $\{r, z\}$ plane. The lower images show the field lines for the same parameters. The quadrupole shows the extreme axial field.

Figure 13 shows an unexpected result of reducing the amplitude and vertical scale of the subscale turbulence while it becomes dominant. The turbulence becomes visible in the disk of the galaxy.

This section led us to suggest that the AGN (jet or nuclear wind) may have a part to play in the large-scale coherent galactic magnetic field. We would predict an isolated X field, a strong parallel equatorial field, possibly turbulent, and a strong field parallel to the axis as indicative of an “AGN dynamo” with halo rotational lag.

The only element missing in this study of the “parts” that make up an axially symmetric scale-invariant galactic dynamo is diffusion. The removal of the increasing magnetic flux is accomplished here by either outflow or implicitly by a globally conserved quantity. This was discussed in Section 2.1. We believe that this catalog can help in identifying some of the processes at work in galactic magnetism. Magnetic arms have been discussed elsewhere and lead to quite recognizable RM structures in the halo.

4.6. Relevant Numerical Work

The recent numerical study, Butsky et al. (2017), is remarkable for its detailed simulation of what we refer to as the subscale turbulent dynamo. A widely distributed, naturally occurring, set of supernovae are used to pump energy and magnetic field at stellar scale into the interstellar medium. A numerically established, unmagnetized, spiral galaxy is the initial condition. In their Figure 2, the evolved magnetic field is shown as a spiral in the disk but with no convincing “ X field” in the halo. The field does not generally extend into the halo as far as indicated by observations. We conclude that additional lag or wind effects are required as we have indicated in this paper. There is a remarkable convergence in the conclusion stemming from two quite different methods.

In the paper Pakmor et al. (2020), the authors examine a suite of cosmological simulations to determine the outflow of magnetic flux from a disk galaxy into its halo and more distant surroundings. Their conclusion is similar to our own, in that coherent fields require coherent gas flows. However, they also require subscale turbulence to amplify an initial seed field, to

the point where coherent flow can operate. This turbulence operates even in the circumgalactic gas, just as we assume in Equation (36).

Their Figure 8 shows the turbulence actually present in the gas and magnetic field. In our approach, this turbulence must produce an electric field parallel to the magnetic field for amplification to occur Steenbeck et al. (1966). We have found such turbulence to appear at small ζ , but with the turbulent velocity of Equation (36), the energy spectrum is $\propto \zeta^3$, that is, k^{-3} with $k \propto \zeta^{-1}$. A Kolmogorov spectrum requires $\alpha_h \propto \zeta^{1/3}$ which changes Equation (38) slightly. This does not change the qualitative amplification ability of the turbulence.

More recent papers, already cited in the introduction, study the driving of the galactic wind and the evolution of the magnetic field starting from early magneto genesis. The important conclusion from our perspective is the successful amplification of plausible primordial fields to those galactic fields observed today. This seems to require the action of a distributed turbulent (α/Ω) dynamo during galaxy formation. Wind-driving has been shown to be associated with vigorous star formation and the resulting α_h turbulence provided that the supernovae are inserted discretely. None of these results conflict fundamentally with the results of this and our earlier papers that take the scale-invariant asymptotic view, i.e., the limiting consequence of all the physical interactions in the numerical developments is the scale-invariant solution.

There is a kind of consensus that the “ X -field” topology is due to the action of galactic outflow on a preexisting field of stellar scale origin. It is easy to show that an extended wind in spherical geometry (e.g., Chevalier 1985), centered on the nucleus of a galaxy and acting on a preexisting radial field component, will produce an “ X field” globally given a realistic rotation law. This is most readily shown using a Lagrangian approach to the field evolution, but we must leave the demonstration to other work.

5. Summary

In this paper we have not attempted detailed modeling of any galaxy. We hope rather to motivate subsequent work in this direction by encouraging the simplifying assumption of scale invariance. To this end, we have compared the contribution of various elements of the scale-invariant dynamo to the polarization morphology of the CHANG-ES edge-on galaxies. Most axially symmetric observational structures can be reproduced (for nonaxial symmetry, see Woodfinden et al. 2019), and the effects of different elements are sufficiently different that observations may eventually distinguish them. Scale invariance also finds support in the stacked image of 28 rather different galaxies shown in Figure 1 in Krause et al. (2020). When each individual galaxy is spatially scaled to the largest galaxy size, the stacked image still reveals coherent magnetic field structures; this would not be present without a certain scale invariance.

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