



Nonlinear Damping of Standing Kink Waves Computed With Elsässer Variables

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Abstract

In a previous paper, we computed the energy density and the nonlinear energy cascade rate for transverse kink waves using Elsässer variables. In this paper, we focus on the standing kink waves, which are impulsively excited in coronal loops by external perturbations. We present an analytical calculation to compute the damping time due to the nonlinear development of the Kelvin–Helmholtz instability. The main result is that the damping time is inversely proportional to the oscillation amplitude. We compare the damping times from our formula with the results of numerical simulations and observations. In both cases we find a reasonably good match. The comparison with the simulations shows that the nonlinear damping dominates in the high amplitude regime, while the low amplitude regime shows damping by resonant absorption. In the comparison with the observations, we find a power law inversely proportional to the amplitude η^{-1} as an outer envelope for our Monte Carlo data points.

Unified Astronomy Thesaurus concepts: Solar physics (1476); Solar coronal waves (1995); Solar corona (1483); Magnetohydrodynamics (1964); Solar active regions (1974); Solar coronal loops (1485); Solar coronal heating (1989)

1. Introduction

Magnetohydrodynamic (MHD) waves are ubiquitously present in the solar atmosphere and magnetosphere (Nakariakov et al. 2016). A particular kind of MHD waves are the transverse kink waves in coronal loops. They have been observed for more than two decades (Nakariakov et al. 1999; Schrijver et al. 1999; Aschwanden et al. 1999). These transverse waves are attractive to observe and model for two reasons: (1) they can be used for coronal seismology with the aim of estimating physical parameters in coronal loops (e.g., Pascoe & De Moortel 2014; Magyar & Van Doorsselaere 2018; Pascoe et al. 2018, see also the review by Nakariakov & Kolotkov 2020), and (2) they could play a role in heating the corona (e.g., De Moortel & Pascoe 2012; Terradas & Arregui 2018; Karampelas et al. 2019; Pagano & De Moortel 2019; Hillier et al. 2020, see also the reviews by Arregui 2015; Van Doorsselaere et al. 2020b).

In this work, we focus on standing transverse waves in coronal loops. Nowadays, we understand that these come in two flavors. On the one hand, there are decayless oscillations, as discovered by Wang (2011), Tian et al. (2012), and Nisticò et al. (2013). These have no apparent external excitation source and many researchers presume they are driven (possibly at the footpoints; Karampelas et al. 2017; Karampelas & Van Doorsselaere 2018; Afanasyev et al. 2020) to maintain the quasi-steady amplitude. It was shown observationally that their period scales with the loop length (Anfinogentov et al. 2015), conclusively showing that these are also standing waves. On the other hand, there are transverse waves in coronal loops that are impulsively excited by a flare (Aschwanden & Schrijver 2011) or low coronal eruption (Zimovets & Nakariakov 2015). These oscillations show an initial displacement from their equilibrium position by the external exciter, after which the coronal loop apparently freely oscillates with its natural frequency showing a strong damping.

In this work, we will consider the damping of the impulsively excited transverse waves, and leave the decayless waves aside for now.

For the early observations of impulsively excited standing kink waves, their strong damping was interpreted as damping in terms of resonant absorption (Ruderman & Roberts 2002; Goossens et al. 2002). This phenomenon is an ideal damping mechanism (Terradas et al. 2006) that converts the observed, large-scale, coherent transverse motions over time into localized, incoherent motions around the resonant layer (Soler et al. 2013; Goossens et al. 2014), which are hard to observe, resulting in an apparent damping. Resonant absorption is a damping that works in linearized MHD, leading to damping with exponential behavior that is independent of the amplitude of the wave (see, e.g., Goossens et al. 1992).

In recent years, it was found that the situation is more complicated than simple exponential damping. In simulations, it was shown that the damping starts with a Gaussian phase first (Pascoe et al. 2012), which was later confirmed with analytical calculations (Hood et al. 2013; Ruderman & Terradas 2013). Gaussian damping was also found in observations (Pascoe et al. 2016) and used for Bayesian seismology (Arregui et al. 2013b; Pascoe et al. 2017). Nowadays, the general damping profile has been characterized with numerical simulations (Pascoe et al. 2019).

Despite all this progress, the damping is independent of the amplitude, and nonlinear effects are not considered.

It was already speculated in the 80s that transverse kink waves are susceptible to the Kelvin–Helmholtz instability (KHI; Hollweg & Yang 1988). More recently, it has been confirmed numerically that standing transverse oscillation lead to the nonlinear development of the KHI (Terradas et al. 2008), resulting in the formation of so-called transverse wave induced Kelvin–Helmholtz rolls (or TWIKH rolls for short; Antolin et al. 2014, 2016; Van Doorsselaere et al. 2018). Magyar & Van Doorsselaere (2016)

constructed similar numerical models and investigated in a parametric study the damping time as a function of the initial amplitude. They found that low amplitude oscillations indeed follow the damping by resonant absorption, but that at high amplitudes the nonlinear damping takes over and a strong amplitude dependence was found. Physically this can be understood as the cascade of wave energy at large scales to the smaller scale TWIKH rolls, once again leading to apparent damping.

Also in the meta-analysis of Goddard et al. (2016), which considered 25 observed transverse loop oscillation events, it was found that the damping time depends on the oscillation amplitude. Their initial estimate was that the damping time τ (normalized to the period P) would have an upper limit of $\eta^{-1/2}$, with η being the displacement amplitude. The study was later extended by Nechaeva et al. (2019), who collected data over the entire solar cycle, resulting in more than 200 cases. They found once again a strong observational dependence of the damping on the amplitude, and fitted an upper limit for the damping time of $\tau/P \sim \eta^{-0.68}$.

The nonlinear aspects of kink modes have been considered in the past, but they were mostly focused on the calculation of eigenfunction modifications and period changes (Ruderman & Goossens 2014). For example, Ruderman (2017) found that the nonlinear effect of kink waves is to generate fluting modes with azimuthal wavenumber $m=2$. This was later confirmed numerically by Terradas et al. (2018). Moreover, some models studied the KHI instability criterion in oscillating loops (Barbulescu et al. 2019; Hillier et al. 2019).

Previously, the evolution of propagating waves and their energy was modeled by Ruderman et al. (2010). They found extra damping compared to the linear regime, as a consequence of the small scale development by the nonlinear effects. Motivated by the discovery of the uniturbulent regime (Magyar et al. 2017), the nonlinear evolution of (propagating and standing) kink waves was recently revisited by Van Doorselaere et al. (2020a). They used a formulation in terms of Elsässer variables to estimate the wave energy density and turbulent energy cascade rate (similar to common practices in solar wind modeling; Bruno & Carbone 2013; van der Holst et al. 2014). They found a damping time for the propagating wave that was inversely proportional to the amplitude, i.e., η^{-1} .

Here we investigate how the results in Van Doorselaere et al. (2020a) extend to standing waves. We compare our results to the numerical parametric study of Magyar & Van Doorselaere (2016) and the observational results of Nechaeva et al. (2019).

2. Earlier Results

In Van Doorselaere et al. (2020a), the simplest model for a magnetic field-aligned, overdense cylinder was considered (uniform magnetic field B , internal/external density $\rho_{i/e}$). There the transverse kink waves are mathematically described by Bessel functions, leading to the usual dispersion relation (Zaitsev & Stepanov 1975; Wentzel 1979; Edwin & Roberts 1983). Van Doorselaere et al. (2020a) found the expressions for the energy density w in standing or propagating kink waves in the thin-tube limit $\delta = k_z R \ll 1$, where k_z is the longitudinal wavenumber and R

is the radius of the loop. In that approximation, the radial Bessel eigenfunctions $\mathcal{R}(r)$ for the total pressure perturbation P' reduce to

$$\lim_{\delta \rightarrow 0} \mathcal{R}(r) = \begin{cases} A \frac{r}{R} & \text{for } r \leq R \\ A \frac{R}{r} & \text{for } r > R, \end{cases} \quad (1)$$

with oscillation amplitude A . They found expressions for the Elsässer variables

$$\mathbf{z}^\pm = \mathbf{v} \pm \frac{\mathbf{b}}{\mu\rho} \quad (2)$$

for the perturbations, where $\mathbf{v}$ and $\mathbf{b}$ are the velocity and magnetic field perturbation and the $+$ ($-$) represent the downward (upward) traveling Alfvén wave in a uniform medium. Their expressions for the associated energy density to these Elsässer variables for kink waves are given in their Equations (52) and (53) as

$$w_i^\pm = \frac{1}{4} \frac{1}{\rho_i (\omega^2 - \omega_{Ai}^2)^2} \frac{A^2}{R^2} \times \begin{cases} (\omega \cos k_z z \sin \omega t \pm \omega_{Ai} \sin k_z z \cos \omega t)^2 & \text{(standing)} \\ (\omega \mp \omega_{Ai})^2 \sin^2(k_z z - \omega t) & \text{(propagating)}, \end{cases} \quad (3)$$

$$w_e^\pm = \frac{1}{4} \frac{1}{\rho_e (\omega^2 - \omega_{Ae}^2)^2} \frac{A^2 R^2}{r^4} \times \begin{cases} (\omega \cos k_z z \sin \omega t \pm \omega_{Ae} \sin k_z z \cos \omega t)^2 & \text{(standing)} \\ (\omega \mp \omega_{Ae})^2 \sin^2(k_z z - \omega t) & \text{(propagating)}. \end{cases} \quad (4)$$

In these expressions, subscripts i/e correspond to the internal and exterior region, the oscillation frequency is ω , the Alfvén frequency ω_A is related to the Alfvén speed V_A through $\omega_A = k_z V_A$. In these formulae, the top line corresponds to the standing wave, while the bottom line corresponds to propagating waves, as indicated.

As explained in Van Doorselaere et al. (2020a), the energy cascade rate is given by

$$\epsilon^\mp = \frac{\rho}{2} \mathbf{z}^\mp \cdot (\mathbf{z}^\pm \cdot \nabla \mathbf{z}^\mp) = \mathbf{z}^\pm \cdot \nabla w^\mp, \quad (5)$$

with $w^\mp = \frac{\rho (\mathbf{z}^\mp)^2}{4}$.

They found that the dominant contribution to the energy cascade rate is due to

$$\epsilon^\mp = z_{re}^\pm \frac{\partial}{\partial r} w_e^\mp, \quad (6)$$

because these are the only terms that have a contribution at $\delta^3 = k_z^3 R^3$. In this expression, $z_{re}^\pm$ is the radial component of the Elsässer variable in the exterior plasma. Van Doorselaere et al. (2020a) computed

$$\epsilon^\mp = \frac{A^3 R^3}{r^7} \frac{1}{\rho_e^2 (\omega^2 - \omega_{Ae}^2)^3} \cos \varphi \times \begin{cases} (-\omega \cos k_z z \sin \omega t \mp \omega_{Ae} \sin k_z z \cos \omega t)(\omega \cos k_z z \sin \omega t \mp \omega_{Ae} \sin k_z z \cos \omega t)^2 & \text{(st.)} \\ (\omega \mp \omega_{Ae})(\omega \pm \omega_{Ae})^2 \sin^3(k_z z - \omega t) & \text{(pr.)} \end{cases} \quad (7)$$

in their Equation (55). To obtain the damping rate for propagating waves under uniturbulence (Magyar et al. 2017), Van Doorselaere et al. (2020a) divided the average energy density by the average energy cascade rate:

$$\tau = \frac{\langle w^+ + w^- \rangle}{\langle \epsilon^+ + \epsilon^- \rangle}, \quad (8)$$

where the averaging was over the cross-section and period:

$$\langle \epsilon \rangle = \int_0^\infty r dr \left(\int_0^{2\pi} d\varphi \frac{\omega}{2\pi} \int_0^{2\pi/\omega} dt \epsilon^2 \right)^{1/2}. \quad (9)$$

Intermediate results by Van Doorselaere et al. (2020a) are (their Equation (56)):

$$\begin{aligned} \epsilon &= \epsilon^+ + \epsilon^- = \frac{A^3 R^3}{r^7} \frac{1}{\rho_e^2 (\omega^2 - \omega_{\text{Ae}}^2)^3} \cos \varphi \\ &\begin{cases} -2\omega \cos k_z z \sin \omega t (\omega^2 \cos^2 k_z z \sin^2 \omega t - \omega_{\text{Ae}}^2 \sin^2 k_z z \cos^2 \omega t) & (\text{st.}) \\ 2\omega (\omega^2 - \omega_{\text{Ae}}^2) \sin^3(k_z z - \omega t) & (\text{pr.}) \end{cases} \end{aligned} \quad (10)$$

and (their Equations (57)–(58))

$$\begin{aligned} \langle \epsilon \rangle &= \frac{A^3 R^3}{\rho_e^2 (\omega^2 - \omega_{\text{Ae}}^2)^3} \frac{1}{5R^5} (\sqrt{\pi}) \\ &\begin{cases} \frac{1}{2} |\omega \cos k_z z| \sqrt{4\omega^4 \cos^4 k_z z + (\omega^2 \cos^2 k_z z - \omega_{\text{Ae}}^2 \sin^2 k_z z)^2} & (\text{st.}) \\ 2|\omega (\omega^2 - \omega_{\text{Ae}}^2)| \sqrt{\frac{5}{16}} & (\text{pr.}) \end{cases} \\ &= V^3 \frac{\sqrt{\pi} R}{10} \frac{\rho_e}{\omega^3} \\ &\begin{cases} |\omega \cos k_z z| \sqrt{4\omega^4 \cos^4 k_z z + (\omega^2 \cos^2 k_z z - \omega_{\text{Ae}}^2 \sin^2 k_z z)^2} & (\text{st.}) \\ \sqrt{5} |\omega (\omega^2 - \omega_{\text{Ae}}^2)| & (\text{pr.}) \end{cases} \end{aligned} \quad (11)$$

The quantity V denotes the velocity amplitude in the interior region of the loop.

We can compute from the previous expressions that

$$\begin{aligned} \langle \epsilon^\mp \rangle &= \frac{A^3 R^3}{\rho_e^2 (\omega^2 - \omega_{\text{Ae}}^2)^3} \frac{1}{5R^5} (\sqrt{\pi}) \\ &\begin{cases} \frac{1}{4} \sqrt{(\omega^2 \cos^2 k_z z + \omega_{\text{Ae}}^2 \sin^2 k_z z)(5(\omega^2 \cos^2 k_z z + \omega_{\text{Ae}}^2 \sin^2 k_z z)^2 - 16\omega^2 \omega_{\text{Ae}}^2 \cos^2 k_z z \sin^2 k_z z)} & (\text{st.}) \\ \sqrt{\frac{5}{16}} |(\omega \mp \omega_{\text{Ae}})(\omega \pm \omega_{\text{Ae}})^2| & (\text{pr.}) \end{cases} \end{aligned} \quad (13)$$

$$\begin{aligned} &= V^3 \frac{\sqrt{\pi} R}{20} \frac{\rho_e}{\omega^3} \\ &\begin{cases} \sqrt{(\omega^2 \cos^2 k_z z + \omega_{\text{Ae}}^2 \sin^2 k_z z)(5(\omega^2 \cos^2 k_z z + \omega_{\text{Ae}}^2 \sin^2 k_z z)^2 - 16\omega^2 \omega_{\text{Ae}}^2 \cos^2 k_z z \sin^2 k_z z)} & (\text{st.}) \\ \sqrt{5} |\omega^2 - \omega_{\text{Ae}}^2| |\omega \pm \omega_{\text{Ae}}| & (\text{pr.}) \end{cases} \end{aligned} \quad (14)$$

Curiously enough, the expression for the standing wave does not depend on the sign of the Elsässer variable! This probably means that the damping for the z^- is as strong as for z^+ , because they are both part of the same standing wave.

3. Nonlinear Damping of Kink Waves

In order to calculate the energy density and energy cascade rate of the standing kink waves, we extend expression (9) to also average over the wavelength:

$$\langle\langle \epsilon \rangle\rangle = \int_0^\infty r dr \left(\int_0^{2\pi} d\varphi \frac{k_z}{2\pi} \int_{-\pi/k_z}^{\pi/k_z} dz \frac{\omega}{2\pi} \int_0^{2\pi/\omega} dt \epsilon^2 \right)^{1/2}. \quad (15)$$

We sum the contributions of both Elsässer components in both the interior and exterior regions:

$$\langle\langle w_{i/e} \rangle\rangle = \langle\langle w_{i/e}^+ + w_{i/e}^- \rangle\rangle \quad (16)$$

and we obtain a wave energy density of

$$\langle\langle w \rangle\rangle = \langle\langle w_i \rangle\rangle + \langle\langle w_e \rangle\rangle = \pi R^2 \frac{\rho_i + \rho_e}{4} V^2, \quad (17)$$

which is half the energy of a propagating wave (Equation (59) in Van Doorselaere et al. 2020a) with the same amplitude. This can be understood intuitively, since a standing wave of the same amplitude as the propagating wave is the superposition of two propagating waves with half the amplitude. For the energy, it implies $2 \times (1/2)^2$, resulting in the factor of 1/2.

To compute the energy cascade rate, we start from the intermediate result in Equation (12). It is mathematically arbitrary to first sum $\epsilon^\pm$ before averaging it (this operation is noncommutative). However, physically it is preferable to first compute the total energy cascade rate before averaging it over space and time. The reason for this is that the standing wave is operating as a whole: the kink wave contains both the $z^\pm$ components simultaneously, and their combined damping is responsible for attenuating the wave. Thus, both quantities must be summed first before averaging over space and time. Even though we think it is incorrect, we have included the alternative result (with the averaging first, before the summing) in the [Appendix](#).

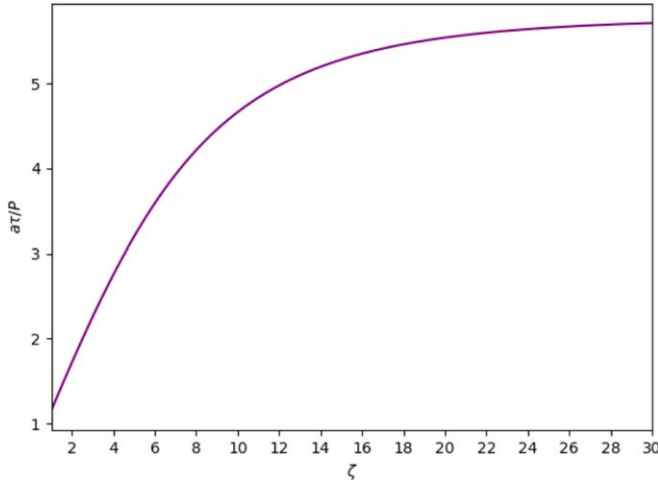


Figure 1. The damping time τ/P due to nonlinearity for a standing kink wave as a function of the density contrast ζ , using Equation (25).

Continuing from the summed energy cascade rates in Equation (5), we have subsequently:

$$\begin{aligned} \langle\langle\epsilon\rangle\rangle &= V^3 \frac{\sqrt{\pi} R}{10} \frac{\rho_e}{\omega^3} \left(\frac{k_z}{2\pi} \int dz \omega^2 \cos^2 k_z z (4\omega^4 \cos^4 k_z z \right. \\ &\quad \left. + (\omega^2 \cos^2 k_z z - \omega_{Ae}^2 \sin^2 k_z z)^2) \right)^{1/2} \end{aligned} \quad (18)$$

$$\begin{aligned} &= V^3 \frac{\sqrt{\pi} R}{10} \frac{\rho_e}{\omega^3} \left(\frac{k_z}{2\pi} \int dz (5\omega^6 \cos^6 k_z z \right. \\ &\quad \left. - 2\omega^4 \omega_{Ae}^2 \cos^4 k_z z \sin^2 k_z z + \omega^2 \omega_{Ae}^4 \cos^2 k_z z \sin^4 k_z z) \right)^{1/2} \end{aligned} \quad (19)$$

$$= V^3 \frac{\sqrt{\pi} R}{10} \frac{\rho_e}{\omega^3} \frac{1}{4} (25\omega^6 - 2\omega^4 \omega_{Ae}^2 + \omega^2 \omega_{Ae}^4)^{1/2} \quad (20)$$

$$= V^3 \frac{\sqrt{\pi} R}{10} \frac{\rho_e}{4} \left(25 - 2 \left(\frac{\omega_{Ae}}{\omega} \right)^2 + \left(\frac{\omega_{Ae}}{\omega} \right)^4 \right)^{1/2}. \quad (21)$$

We use the approximate expression for the kink frequency

$$\omega^2 = \frac{\rho_i \omega_{Ai}^2 + \rho_e \omega_{Ae}^2}{\rho_i + \rho_e} = \omega_{Ae}^2 \frac{2}{1 + \zeta}, \quad (22)$$

with $\zeta = \rho_i/\rho_e$. In this equation, the last equality is only valid if the magnetic field is uniform. We can then further simplify the energy cascade rate to

$$\langle\langle\epsilon\rangle\rangle = V^3 \frac{\sqrt{\pi} R}{10} \frac{\rho_e}{4} \left(25 - 2 \left(\frac{1 + \zeta}{2} \right) + \left(\frac{1 + \zeta}{2} \right)^2 \right)^{1/2} \quad (23)$$

$$= V^3 \frac{\sqrt{\pi} R}{10} \frac{\rho_e}{8} \sqrt{\zeta^2 - 2\zeta + 97}. \quad (24)$$

Somewhat surprisingly,⁵ this expression does not tend to 0 as $\zeta \rightarrow 1$, in contrast to the damping of propagating waves by uniturbulence (Equation (58) of Van Doorselaere et al. 2020a). This agrees with the findings of Howson et al. (2019) that the

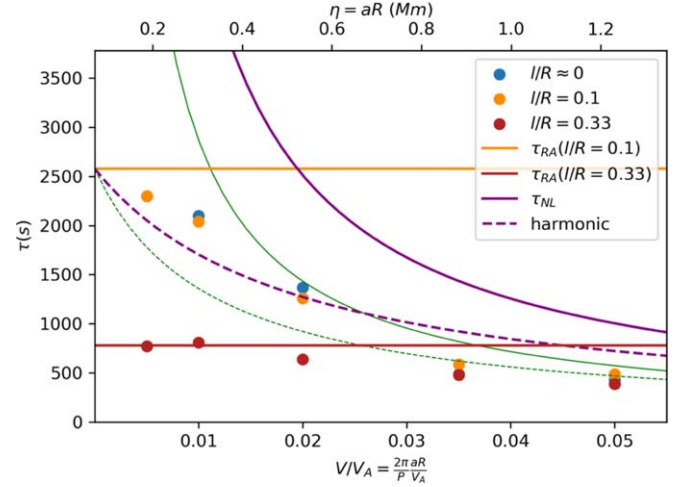


Figure 2. The damping times τ as a function of initial oscillation amplitude V/V_{Ai} (or equivalently η in the top horizontal axis). The points correspond to the results of Magyar & Van Doorselaere (2016), with the color showing different l/R as displayed in the legend. The full purple line shows the results of Equation (25). The horizontal red and orange lines are the damping times from resonant absorption (Equation (26)), with the appropriate l/R as indicated in the legend. The dashed purple line is the harmonic average of the resonant damping time for $l/R = 0.1$ and the nonlinear damping time. For completeness, the damping time with the alternative method in the Appendix (Equation (A4)) is shown with the full green line, and the harmonic average is shown with the dashed green line.

nonlinear damping by KHI also works in a coronal loop model with a uniform density but varying magnetic field.

Now we can find the expression for the damping time by dividing the energy density (Equation (17)) by the energy cascade rate (Equation (24)), using the same method as in Van Doorselaere et al. (2020a)

$$\begin{aligned} \tau &= \frac{\langle w \rangle}{\langle\langle\epsilon\rangle\rangle} = 20\sqrt{\pi} \frac{R}{V} \frac{1 + \zeta}{\sqrt{\zeta^2 - 2\zeta + 97}} \\ &= 20\sqrt{\pi} \frac{P}{2\pi a} \frac{1 + \zeta}{\sqrt{\zeta^2 - 2\zeta + 97}}. \end{aligned} \quad (25)$$

Here $a = \eta/R$ is the relative displacement amplitude, i.e., the ratio of the displacement η compared to the loop radius R . The strongest damping occurs for $\zeta = 1$ and is equal to $\tau/P = \frac{1}{a} \frac{20}{\sqrt{96\pi}} = 1.15/a$. For an infinitely dense loop or vacuum exterior $\zeta \rightarrow \infty$, the damping saturates with a maximum of $\tau/P = 10/a\sqrt{\pi} \approx 5.64/a$. The full graph of τ is shown in Figure 1.

The damping time τ computes how fast the kink wave energy is cascaded to smaller scales. This formula considers the inertial regime of the turbulent cascade (see Figure 10 of Van Doorselaere et al. 2020b), which is an ideal MHD process. In this turbulent cascade, the energy dissipation rate does not depend on the scale of the eddies, resulting in a power law behavior (see Kolmogorov or Iroshnikov–Kraichnan scaling). Once the turbulent eddies enter the dissipative range, the energy cascade rate will take a different form (which does depend on the dissipation coefficients) and also have a different power law slope.

The analogy with damping by resonant absorption is clear. The damping in resonant absorption is occurring also in ideal MHD. The details of viscous/resistive damping of the resonant

⁵ Perhaps it is even more surprising that the largest prime number below 100 would occur in the mathematical description of a physical phenomenon!

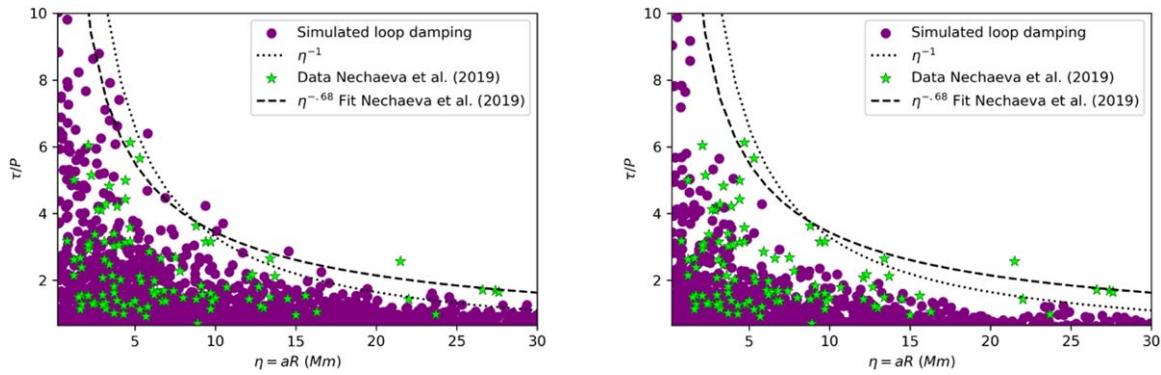


Figure 3. Scatter plot of the quality factor τ/P vs. the amplitude η for the data of Nechaeva et al. (2019) in green stars. Monte Carlo simulation of damping of kink waves in our current model with purple dots (left panel: only nonlinear damping, right panel: harmonic average of nonlinear damping and resonant absorption). The dotted line shows the possible outer envelope with η^{-1} we obtained in this work and the dashed line shows the outer envelope $\eta^{-.68}$ as determined by Nechaeva et al. (2019).

Alfvén modes is different, and not needed to find the resonant damping rate (see Equation (26)). In a sense, resonant absorption is a cascade to smaller radial wavenumbers (albeit independent of the amplitude), reinforcing the analogy with the nonlinear damping (Equation (25)), which represents a cascade in the radial and azimuthal wavenumber.

4. Comparison to Simulations

Here we compare the results for the damping time (Equation (25)) to those of Magyar & Van Doorselaere (2016). In that work, 3D simulations were performed of standing kink waves, of which the amplitude and scale of the inhomogeneous layer l/R was varied in a parameter study. The authors found that the damping is determined by resonant absorption for small amplitudes. In this regime, the damping is independent of the wave amplitude (Goossens et al. 1992). However, in the high amplitude regime, they found that the damping was caused by the nonlinear evolution of the kink mode, namely the formation of the KHI (Terradas et al. 2008). They considered a loop with an Alfvén speed of $V_{Ai} = 0.6 \text{ Mm s}^{-1}$, radius of $R = 1.5 \text{ Mm}$, and density contrast of $\zeta = 5$. They considered for the amplitude parameter of the initial velocity perturbation the values $V = \{0.005, 0.01, 0.02, 0.035, 0.05\} V_{Ai}$, given in terms of the internal Alfvén speed. Their results for the damping times as shown in their Figure 6 are displayed as dots in Figure 2, for three different values of the thickness of the inhomogeneous layer between the interior and exterior regions $l/R = \{0, 0.1, 0.33\}$.

On their data points, we overplot a few lines. The purple line corresponds to the damping time computed with the formula Equation (25), which was derived for $l/R = 0$. The orange and red horizontal lines are the expected damping times from the thin-tube, thin-boundary limit in resonant absorption (Ruderman & Roberts 2002) using a sinusoidal density profile in the inhomogeneous layer:

$$\frac{\tau_{RA}}{P} = \frac{2 R \zeta + 1}{\pi l \zeta - 1}, \quad (26)$$

for the parameters used in the Magyar & Van Doorselaere (2016) simulations.

From this graph, it can be seen that the purple line captures the behavior of the damping quite well in the region of high amplitude, apart from a vertical offset. The latter offset could be explained by the shortcomings of the present analytical

model that was derived under the assumption that $l/R = 0$. Another transition region from the interior to the exterior of the loop could easily alter the constant prefactor ($20\sqrt{\pi}$) of the nonlinear damping, as it also does for resonant absorption (Soler et al. 2013).

Another shortcoming is that the nonlinear damping does not capture well the apparent saturation that occurs for small amplitudes. To address this, the dashed purple line shows the harmonic average of the nonlinear damping and the damping by resonant absorption. In the dashed line, we have taken $1/\tau = 1/\tau_{RA} + 1/\tau_{NL}$, where we take τ_{NL} from Equation (25). A better correspondence with the numerical simulation points is indeed found. However, we still cannot match the observed inflection point in the simulation data. Consider the damping time's dependence on the wave amplitude a . For resonant absorption, we know $\tau_{RA} \sim C$, while for the nonlinear damping, we know $\tau_{NL} \sim D/a$. Then, $\tau \sim CD/(aC + D)$. This is a rational function. Since both C and D are positive, it will have a pole at $a < 0$. Beyond this, it is a monotonically decreasing section of the hyperbola. Thus, with the harmonic average it is always impossible to obtain an inflection point in this case.

In taking the harmonic average, we have assumed that the nonlinear damping and resonant absorption operate independently from each other. However, it may be possible that the interaction between these damping mechanisms is more complicated (e.g., TWIKH rolls lengthen the resonant layer), and then a better fit could be obtained.

For completeness, the result obtained in the Appendix (in particular Equation (A4)) is also shown in Figure 2 in green, and its harmonic average is shown by dashed green lines.

5. Matching with Observations

In this section, we investigate how the theory fits with data of damping of standing loop oscillations. We use the catalog mentioned in Nechaeva et al. (2019), and displayed as green stars in Figure 3. To generate the data points from our analytical theory, we follow the approach of Verwichte et al. (2013). They performed a Monte Carlo simulation for the resonant damping τ_{RA} and period P in which the loop parameters are drawn randomly from given distributions. Here we use the same procedure in order to study how τ/P depends on the amplitude η .

We take 5000 realizations of oscillating coronal loops. We take the following as random variables for each loop:

1. the density contrast ζ is drawn from a uniform distribution $[1, 9.5]$, where the latter value for $\zeta_{\max}$ is taken from Verwichte et al. (2013),
2. the thickness of the inhomogeneous layer l/R is drawn from a uniform distribution $[0, 2]$,
3. the amplitude η is drawn from a uniform distribution between $[0.2, 30]$ Mm, as suggested from the data, and
4. the radius R is uniformly drawn from a distribution between $[0.5, 5]$ Mm.

All of these distributions are very crude. In principle, we could take the more advanced Bayesian inferences of Pascoe et al. (2018), but at this stage we have chosen to keep the results as simple as possible. Moreover, we have not taken into account projection effects on the oscillation amplitude, which would push down the horizontal scale by the cosine of the viewing angle.

For each randomly generated loop, we have plotted the nonlinear damping time τ/P in the left panel of Figure 3, while the right panel shows the harmonic average of the nonlinear damping time and resonant absorption, following the results of Section 4.

After drawing the random numbers and computing the associated damping times with the nonlinear damping time or the harmonic average, we have added 50% noise to the damping rate τ/P to mimic observational constraints. With this, we mean that we multiply the theoretically obtained damping rate for the i th loop $(\tau/P)_i$ with $1 + 0.5n_i$, where the noise n_i for the i th loop is drawn from the standard normal distribution $N(0, 1)$. The specific shape of the noise (relative or absolute) does not have a significant effect on our results.

The results of this Monte Carlo process are shown in Figure 3. There seems to be a good match between the green stars and purple dots in a statistical sense. The left panel shows an overpopulation toward long damping times, but this is solved in the right panel where the resonant absorption is taken into account. The lower part of the graph is densely populated with simulated data points, and this seems to lack in the observations. However, these points correspond to damping times $\tau/P < 1$, which are hard to observe, and would rather be classified as nonoscillatory loops. Therefore, the observational points are probably biased toward the higher values of τ/P . Consequently, the discrepancy between the purple points and the green points for small τ/P is a result of the observational bias.

Nechaeva et al. (2019) provided a fit for the outer bound of the data cloud, and found that it was given by $\eta^{-0.68}$ (shown as a dashed black line in Figure 3). In contrast, we have overplotted the outer envelope with an η^{-1} shape, as was found in our Equation (25). Indeed, it seems that the η^{-1} matches better with the purple simulated points in the left panel. However, moving to the right panel where resonant absorption is also taken into account, it is hard to say which power law fits better. Intuitively speaking, we could say that $\eta^{-0.68}$ is an observational power law that results from averaging the η^{-1} power law of nonlinear damping and the η^0 power law of resonant absorption.

In any case, the data does not seem to contradict the current theory of nonlinear damping of standing kink oscillations. However, Figure 5 for the alternative theory also provides a reasonable match of the data to the theoretical points. Perhaps a

Bayesian model comparison approach (Montes-Solís & Arregui 2017) could confirm that the data fits better with our model (based on physical reasoning) than the model in the Appendix, and this should be investigated in future work.

6. Conclusions

In this paper, we have started from our previous results on the description of kink waves through Elsässer variables (Van Doorsselaere et al. 2020a). We have used those results to compute the energy density and nonlinear energy cascade rate for standing kink waves. By taking the ratio of those, we have computed the predicted damping time of standing kink waves by the nonlinear development of KHI. We found that the damping time is proportional to the period and has a complicated dependence on the density contrast, but that the damping does not vanish for density contrast equal to 1 (density of the loop is the same as the exterior). More importantly, we found that the damping time is inversely proportional to the amplitude of the kink oscillation.

It was also found previously in simulations (Magyar & Van Doorsselaere 2016) and observations (Goddard et al. 2016; Nechaeva et al. 2019) that the damping of kink waves gets stronger for increasing amplitude. In this paper, we have confronted our derived damping time with both the simulation and observational results.

From the comparison with the simulations of Magyar & Van Doorsselaere (2016), we found that the nonlinear damping dominates for high amplitude oscillations, but that resonant absorption plays the dominant role for low amplitudes. Our analytical results capture the behavior of the numerical results reasonably well, maybe aside from a vertical offset, which could be caused by a difference of the assumed transition layer. We established that taking the harmonic average of the resonant absorption damping time and the nonlinear damping time gives a good match with the overall behavior of the simulated loops.

We also compared our analytical formula to the results of Nechaeva et al. (2019). We have generated random loops in a Monte Carlo process, by varying the loop radius, loop density contrast, loop inhomogeneity, and oscillation amplitude. For these random loops, we have computed the expected nonlinear damping time. From the comparison of the simulated loops with the observed loops, we see that there is a good match between the two data clouds. This includes the upper bound of the data cloud that is modeled in our case with η^{-1} , where η is the amplitude of the oscillation.

Continuing on the Monte Carlo process for recreating the observed data points, our method offers perspective on a Bayesian inference of loop parameters (see, e.g., Arregui et al. 2013a; Verwichte et al. 2013; Pascoe et al. 2020) by finding the best statistical match between the two data clouds. This has the potential to seismologically determine the radii of coronal loops, the substructure of the loop, their density contrast distribution, and the oscillation amplitude distribution. We will investigate the possibilities in a future work.

Further future work is in the numerical verification of the predicted nonlinear damping rate of kink waves due to the formation of TWIKH rolls (Equation (25)). In the simulations of Magyar & Van Doorsselaere (2016), the effects of resonant damping and nonlinear damping are intermixed, as was shown in Section 4. However, it is possible to run specialized

simulations with $I/R \simeq 0$ to eliminate the influence of resonant absorption. We plan to run those simulations in the near future.

T.V.D. was supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation program (grant agreement No. 724326) and the C1 grant TRACEspace of Internal Funds KU Leuven. The research benefited greatly from discussions at ISSI-BJ.

Appendix

Nonlinear Damping of Standing Kink Waves: Alternative Calculations

In Section 3, we have explained that there is no mathematical preference in the averaging and summing of the two Elsässer components. We have taken the physically correct approach to first add the contributions before averaging them and these are the results we presented in the main text. This is because the kink wave contains both Elsässer components, which are damped by both $\epsilon^\pm$ simultaneously. For completeness, we also list the alternative computations even though we think that these are incorrect.

Mathematically, the problem is the noncommutativity of the averaging and summing. We can write down that

$$\langle\langle\epsilon\rangle\rangle = \langle\langle\epsilon^+ + \epsilon^-\rangle\rangle \neq \langle\langle\epsilon^+\rangle\rangle + \langle\langle\epsilon^-\rangle\rangle. \quad (\text{A1})$$

In the main text of this paper, we have taken the left-hand side of the inequality sign. Here we consider the right-hand side.

As before, we start from the result in Equation (12). We have subsequently:

$$\begin{aligned} \langle\langle\epsilon^\mp\rangle\rangle &= V^3 \frac{\sqrt{\pi} R}{20} \frac{\rho_e}{\omega^3} \left(\frac{k_z}{2\pi} \int dz (\omega^2 \cos^2 k_z z \right. \\ &\quad \left. + \omega_{\text{Ae}}^2 \sin^2 k_z z) \right. \\ &\quad \times (5(\omega^2 \cos^2 k_z z + \omega_{\text{Ae}}^2 \sin^2 k_z z)^2 \\ &\quad \left. - 16\omega^2 \omega_{\text{Ae}}^2 \cos^2 k_z z \sin^2 k_z z) \right)^{1/2} \end{aligned} \quad (\text{A2})$$

$$= V^3 \frac{\sqrt{\pi} R}{20} \frac{\rho_e}{32} \sqrt{25\zeta^3 + 73\zeta^2 + 67\zeta + 219}. \quad (\text{A3})$$

As in the main text, the damping does not go to 0 as $\zeta \rightarrow 1$, also confirming in this case the results of Howson et al. (2019) that the nonlinear damping by KHI also works in a uniform density.

Now we can find the expression for the damping time by dividing the energy by the energy cascade rate, using the same approach as before:

$$\begin{aligned} \tau &= \frac{\langle w \rangle}{\langle\langle\epsilon\rangle\rangle} = 80\sqrt{\pi} \frac{R}{V} \frac{1 + \zeta}{\sqrt{25\zeta^3 + 73\zeta^2 + 67\zeta + 219}} \\ &= 80\sqrt{\pi} \frac{P}{2\pi a} \frac{1 + \zeta}{\sqrt{25\zeta^3 + 73\zeta^2 + 67\zeta + 219}}. \end{aligned} \quad (\text{A4})$$

Remarkably, the dependence on the density contrast is entirely different from before. It is shown graphically in Figure 4.

The comparison with the simulated data is shown with the thin green lines in Figure 2, showing a somewhat better match with the data. For completeness, we also show in Figure 5 the simulated data points with the Monte Carlo method of Section 5. The fit of the simulated data with the observed data points is somewhat less than in Figure 3, but not enough to distinguish the two theories, leaving it to physical grounds

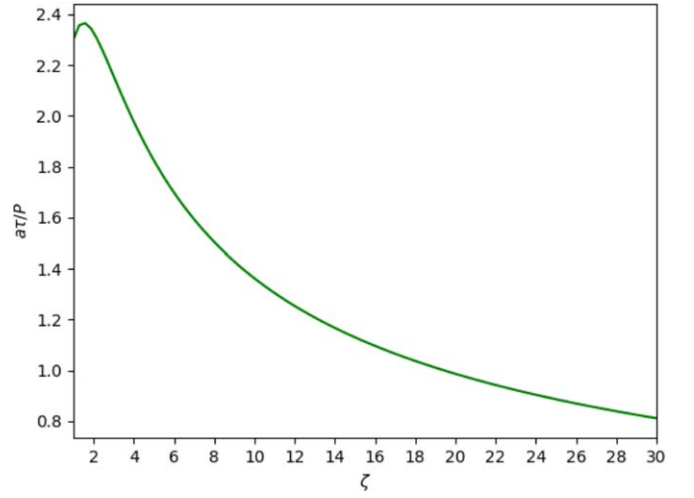


Figure 4. $a\tau/P$ for a standing kink wave as a function of the density contrast ζ , following formula Equation (A4).

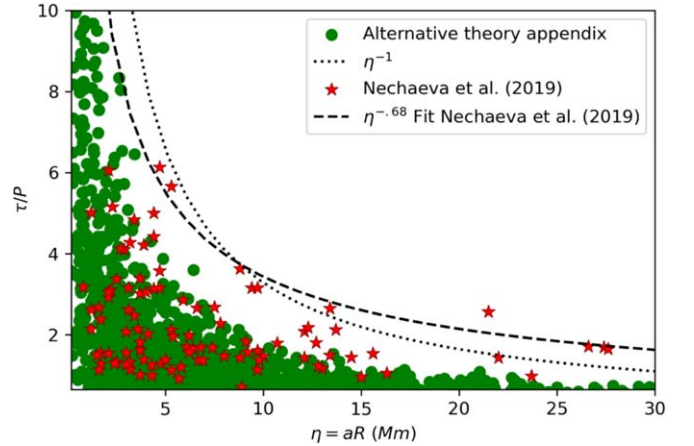


Figure 5. Scatter plot of the quality factor τ/P vs. the amplitude η for the data of Nechaeva et al. (2019) in red stars. Monte Carlo simulation of damping of kink waves in the appendix with green dots. The dashed and dotted lines are the same as in Figure 3.

alone. Still, we believe this alternative theory in the appendix to be incorrect.

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Erratum: “Nonlinear Damping of Standing Kink Waves Computed With Elsässer Variables” (2021, ApJ, 910, 58)

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1. Introduction

In Van Doorselaere et al. (2020, 2021) we computed the damping of kink waves due to the development of the Kelvin–Helmholtz instability (for standing waves) and uniturbulence (for kink waves). For this, we started from the MHD equations:

$$\frac{\partial \mathbf{z}^{\pm}}{\partial t} \mp V_A \frac{\partial \mathbf{z}^{\pm}}{\partial z} = -\mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm}, \quad (1)$$

written in terms of the perturbed Elsässer variables:

$$\mathbf{z}^{\pm} = \mathbf{v} \pm \left(\frac{\mathbf{b}}{\sqrt{\mu} \rho_0} - \mathbf{e}_z V_A \frac{\rho'}{2\rho_0} \right), \quad (2)$$

where $V_A = \frac{B_0}{\sqrt{\mu} \rho_0}$ is the usual Alfvén speed, with background density ρ_0 , background uniform magnetic field B_0 in the z -direction, velocity perturbation $\mathbf{v}$, magnetic field perturbation $\mathbf{b}$ and density perturbation ρ' . In the published articles, we proceeded to calculate the energy density w and turbulent cascade rate ϵ of kink waves, using expressions for the wave perturbations following Edwin & Roberts (1983):

$$w^{\pm} = \frac{\rho_0 (\mathbf{z}^{\pm})^2}{4}, \quad (3)$$

$$\epsilon^{\mp} = \frac{\rho_0}{2} \mathbf{z}^{\mp} \cdot (\mathbf{z}^{\pm} \cdot \nabla \mathbf{z}^{\mp}) = \mathbf{z}^{\pm} \cdot \nabla w^{\mp}. \quad (4)$$

We averaged these expressions over the wavelength, period, and loop cross section to obtain the average energy density $\langle w \rangle$ and average energy cascade rate $\langle \epsilon \rangle$. By taking the ratio of those quantities, we obtained the damping time τ :

$$\tau = \frac{\langle w \rangle}{\langle \epsilon \rangle}. \quad (5)$$

This equation (and the analytical results of Van Doorselaere et al. 2020, 2021) is mathematically correct, but the notation is confusing at best.

2. Clarification

To justify Equation (5), it is possible to reconsider Equation (1) and take the scalar product with $\rho \mathbf{z}^{\pm}/2$. The motivation for this is to obtain $\epsilon^{\pm}$ in the right-hand side. We obtain

$$\frac{dw^{\pm}}{dt} = \frac{\partial w^{\pm}}{\partial t} \mp V_A \frac{\partial w^{\pm}}{\partial z} = -\frac{\rho_0}{2} \mathbf{z}^{\pm} \cdot (\mathbf{z}^{\mp} \cdot \nabla \mathbf{z}^{\pm}) = -\epsilon^{\pm}, \quad (6)$$

where we have used the notation $\frac{d}{dt}$ for the derivative copropagating in the frame of reference with the Alfvén speed. Dividing by $w^{\pm}$, we obtain

$$\frac{1}{w^{\pm}} \frac{dw^{\pm}}{dt} = -\frac{\epsilon^{\pm}}{w^{\pm}}. \quad (7)$$

If we then say that $w^{\pm} \sim \exp(-t/\tau)$, then we recover the equation in Van Doorselaere et al. (2020):

$$\tau = \frac{w^{\pm}}{\epsilon^{\pm}}. \quad (8)$$

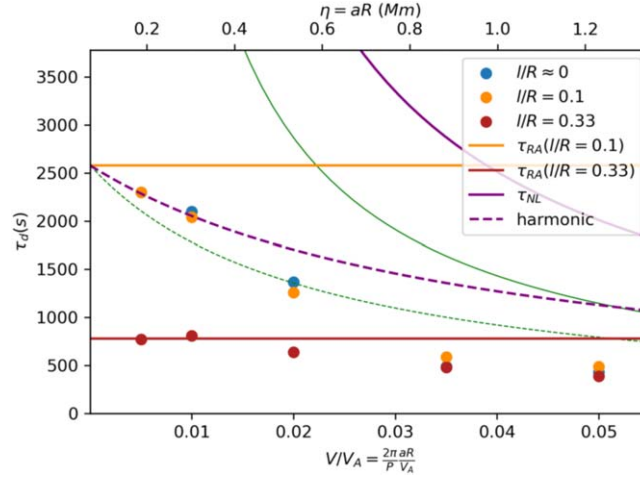


Figure 2. Replacement figure for Figure 2 of Van Doorselaere et al. (2021). The different data points come from the simulations and the full lines correspond to damping-time predictions from the theory.

However, normally in the field, it is assumed that one of the linear quantities (such as displacement ξ) is proportional to an exponentially damped profile:

$$\xi \sim \exp(-t/\tau_d). \quad (9)$$

If this dependence is considered, we find

$$w^\pm \sim \exp(-2t/\tau_d), \quad (10)$$

because w is quadratic in ξ . We would then have

$$\tau_d = 2 \frac{w^\pm}{\epsilon^\pm}, \quad (11)$$

which has a factor of 2 compared to the results presented in Van Doorselaere et al. (2020, 2021). In reality, it is of course more complicated, because we know that ξ is not a purely damped exponential, but rather a damped sine wave. Let us say

$$\xi \sim \exp(-t/\tau_d) \sin(k_z z - \omega t) \quad (12)$$

for a propagating wave with wavenumber k_z and frequency ω . Now we come back to Equation (6) and we find

$$w^\pm \sim \exp(-2t/\tau_d) \sin^2(k_z z - \omega t), \quad (13)$$

$$\epsilon^\pm = -\frac{\partial w^\pm}{\partial t} \pm V_A \frac{\partial w^\pm}{\partial z} \sim \frac{2}{\tau_d} w^\pm + 2 \frac{\cos(k_z z - \omega t)}{\sin(k_z z - \omega t)} (\omega \pm \omega_A) w^\pm. \quad (14)$$

When we average the latter expression over time, the last term will result in 0, because it is an antisymmetric, periodic function. We are then left with an equivalent equation to Equation (11), namely

$$\tau_d = 2 \frac{\langle w \rangle}{\langle \epsilon \rangle} = 2\tau. \quad (15)$$

So the damping time in observations of the displacement is twice the damping-time formula of the propagating waves in Van Doorselaere et al. (2020). For the standing waves, the same formula holds, but then an average over both period and wavelength needs to be taken.⁵ This has repercussions for the results of Van Doorselaere et al. (2021), where the damping-time formula was directly compared to observations, without the factor 2.

3. Corrections

In Van Doorselaere et al. (2020), no changes are necessary. However, one needs to be careful in interpreting the calculated expressions for τ .

For Van Doorselaere et al. (2021), the figures do change, taking into account the extra factor 2. The vertical axis is now showing the τ_d , which corresponds to the observations and what has been previously the standard in the field.

In the replacement figure for the comparison with the simulations (Figure 2), the top panel now shows a weaker correspondence between the analytical model of Van Doorselaere et al. (2021) and the numerical model of Magyar & Van Doorselaere (2016). The

⁵ Note that we do not take the rms average as for the energy cascade rate ϵ , as we have done in Van Doorselaere et al. (2020, 2021). It is not needed here, because the regular average is different from 0 here.

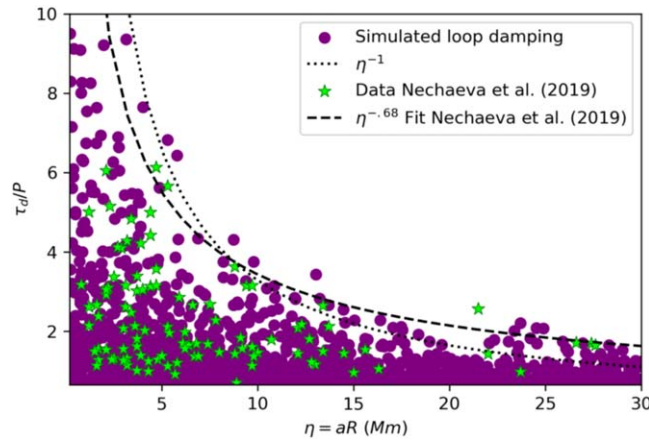


Figure 3. Replacement figure for Figure 3 (right panel) in Van Doorselaere et al. (2021). These simulated data points represent damping times for a random selection of loops. These are generated with the same random seed as the figure in the published article.

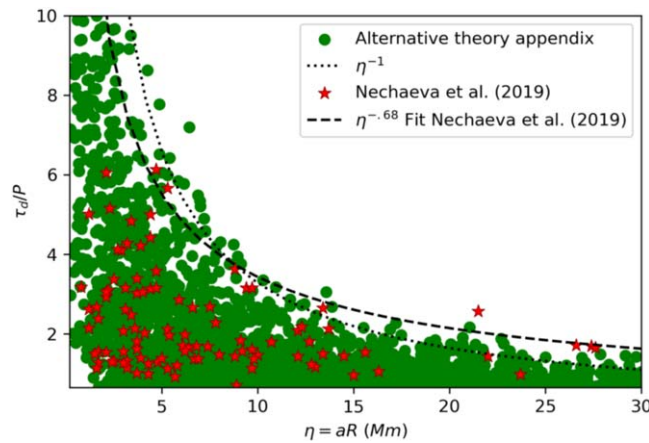


Figure 5. Replacement figure for Figure 5 in Van Doorselaere et al. (2021). These simulated data points show the randomly generated damping times with the damping-time formula in the Appendix of Van Doorselaere et al. (2021).

reason for this is unclear, and could be related to the differences between the numerical parameters and the used parameters for the analytical calculations. Moreover, the numerical model of Magyar & Van Doorselaere (2016) has a nonuniform layer at the edge of the loop, and this was not incorporated in the analytical model.

For the revised figure of the Monte Carlo simulation (Figure 3), the simulated data points have been generated using the same random numbers as in Van Doorselaere et al. (2021), and using the same distributions. The match between the data points and the simulated data points seems even better with the newer corrected damping times.

For completeness, we have also remade Figure 5 of Van Doorselaere et al. (2021) and this is shown in Figure 5, which was made for the damping-time formula in the Appendix, which we believe to be wrong.

This correction presented in this paper also has a potential impact on the results of Arregui (2021).

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