THE ASTROPHYSICAL JOURNAL, 830:5 (9pp), 2016 October 10 doi:10.3847/0004-637X/830/1/5
© 2016. The American Astronomical Society. All rights reserved.

SPIN–ORBIT MISALIGNMENT AS A DRIVER OF THE *KEPLER* DICHOTOMY

CHRISTOPHER SPALDING AND KONSTANTIN BATYGIN
Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena, CA 91125, USA
Received 2016 June 6; revised 2016 July 12; accepted 2016 July 14; published 2016 September 30

ABSTRACT

During its five-year mission, the *Kepler* spacecraft has uncovered a diverse population of planetary systems with orbital configurations ranging from single-transiting planets to systems of multiple planets co-transiting the parent star. By comparing the relative occurrences of multiple to single-transiting systems, recent analyses have revealed a significant over-abundance of singles. Dubbed the "*Kepler* Dichotomy," this feature has been interpreted as evidence for two separate populations of planetary systems: one where all orbits are confined to a single plane, and a second where the constituent planetary orbits possess significant mutual inclinations, allowing only a single member to be observed in transit at a given epoch. In this work, we demonstrate that stellar obliquity, excited within the disk-hosting stage, can explain this dichotomy. Young stars rotate rapidly, generating a significant quadrupole moment, which torques the planetary orbits, with inner planets influenced more strongly. Given nominal parameters, this torque is sufficiently strong to excite significant mutual inclinations between planets, enhancing the number of single-transiting planets, sometimes through a dynamical instability. Furthermore, as hot stars appear to possess systematically higher obliquities, we predict that single-transiting systems should be relatively more prevalent around more massive stars. We analyze the *Kepler* data and confirm this signal to be present.

Key words: planet–star interactions – planets and satellites: dynamical evolution and stability

1. INTRODUCTION

In our solar system, the orbits of all eight confirmed planets are confined to the same plane with an rms inclination of $\sim 1^\circ$–2°, inspiring the notion that planets arise from protoplanetary disks (Kant 1755; Laplace 1796). By inference, one would expect extrasolar planetary systems to form with a similarly coplanar architecture. However, it is unknown whether such low mutual inclinations typically persist over billion-year timescales. Planetary systems are subject to many mechanisms capable of perturbing coplanar orbits out of alignment, including secular chaos (Laskar 1996; Lithwick & Wu 2012), planet–planet scattering (Ford & Rasio 2008; Beaugé & Nesvorný 2012), and Kozai interactions (Naoz et al. 2011).

Despite numerous attempts, mutual inclinations between planets are notoriously difficult to measure directly (Winn & Fabrycky 2015). In light of this, investigations have turned to indirect methods. For example, by comparing the transit durations of co-transiting planets, Fabrycky et al. (2014) inferred generally low mutual inclinations ($\sim 1.^\circ 0$–$2.^\circ 2$) within closely packed *Kepler* systems. Additionally, within a subset of systems (e.g., 47 Uma and 55 Cnc) stability arguments have been used to limit mutual inclinations to $\lesssim 40^\circ$ (Laughlin et al. 2002; Veras & Armitage 2004; Nelson et al. 2014). On the other hand, Dawson & Chiang (2014) have presented indirect evidence of unseen, inclined companions based upon peculiar apsidal alignments within known planetary orbits. Obtaining a better handle on the distribution of planetary orbital inclinations would lend vital clues to planet formation and evolution.

Recent work has attempted to place better constraints upon planet–planet inclinations at a population level, by comparing the number of single- to multi-transiting systems within the *Kepler* data set (Johansen et al. 2012; Ballard & Johnson 2016). Owing to the nature of the transit technique, an intrinsically multiple planet system might be observed as a single if the planetary orbits are mutually inclined. An emerging picture is that although a distribution of small $\sim 5^\circ$ mutual inclinations can explain the relative numbers of double- and triple-transiting systems, a striking feature of the planetary census is a significant over-abundance of single-transiting systems. Furthermore, the singles generally possess larger radii (more with $R_{\rm p} \gtrsim 4$ Earth radii), drawing further contrast.

The problem outlined above has been dubbed the "*Kepler* Dichotomy," and is interpreted as representing at least two separate populations; one with low mutual inclinations and another with large mutual inclinations that are observed as singles. The physical origin of this dichotomy remains unresolved (Morton & Winn 2014; Becker & Adams 2016). To this end, Johansen et al. (2012) proposed the explanation that planetary systems with higher masses undergo dynamical instability, leaving a separate population of larger, mutually inclined planets, detected as single transits. While qualitatively attractive, this model has two primary shortcomings. First, it cannot explain the excess of smaller single-transiting planets. Second, unreasonably high-mass planets are needed to induce instability within the required approximately gigayear timescales. Accordingly, the dichotomy's full explanation requires a mechanism applicable to a more general planetary mass range. In this paper, we propose such a mechanism—the torque arising from the quadrupole moment of a young, inclined star.

The past decade has seen a flurry of measurements of the obliquities, or spin–orbit misalignments, of planet-hosting stars (Winn et al. 2010; Albrecht et al. 2012; Huber et al. 2013; Morton & Winn 2014; Mazeh et al. 2015; Li & Winn 2016). A trend has emerged whereby hot stars ($T_{\rm eff} \gtrsim 6200$ K) hosting hot Jupiters possess obliquities ranging from 0° to 180°, as opposed to their more modestly inclined, cooler (lower-mass) counterparts. Further investigation has revealed a similar trend among stars hosting lower-mass and multiple-transiting planets (Huber et al. 2013; Mazeh et al. 2015). Most relevant to the *Kepler* Dichotomy, Morton & Winn (2014) concluded at 95% confidence that single-transiting systems possess enhanced spin–orbit misalignment compared to multi-transiting systems.

Precisely when these spin–orbit misalignments arose in each system's evolution is still debated (Albrecht et al. 2012; Lai 2012; Storch et al. 2014; Spalding & Batygin 2016). However, the presence of stellar obliquities within currently coplanar, multi-planet systems hints at an origin during the disk-hosting stage (Huber et al. 2013; Mazeh et al. 2015). Indeed, many studies have demonstrated viable mechanisms for the production of disk–star misalignments, including turbulence within the protostellar core (Bate et al. 2010; Spalding et al. 2014; Fielding et al. 2015) and torques arising from stellar companions (Batygin 2012; Batygin & Adams 2013; Lai 2014; Spalding & Batygin 2014, 2015). Furthermore, Spalding & Batygin (2016) proposed that differences in magnetospheric topology between high and low-mass T Tauri stars (Gregory et al. 2012) may naturally account for the dependence of obliquities upon stellar (main sequence) $T_{\rm eff}$. Crucially, if the star is inclined relative to its planetary system while young, fast-rotating, and expanded (Shu et al. 1987; Bouvier 2013), its quadrupole moment can be large enough to perturb a coplanar system of planets into a mutually inclined configuration after disk dissipation.

In what follows, we analyze this process quantitatively. First, we calculate the mutual inclination induced between two planets as a function of stellar oblateness (J_2), demonstrating a proof-of-concept that stellar obliquity suffices as a mechanism for over-producing single-transiting systems. Following this, we use N-body simulations to subject the famed, six-transiting system Kepler-11 to the quadrupole moment of a tilted, oblate star. We show that not only are the planetary orbits mutually inclined, but for nominal parameters the system itself can undergo a dynamical instability, losing three to five of its planets, with larger mass planets preferentially retained. In this way, we naturally account for the slightly larger observed size of singles (Johansen et al. 2012).

2. ANALYTICAL THEORY

In order to motivate the following discussion, consider two planets orbiting in a shared plane around an inclined, oblate (high J_2) star. The effect of the stellar potential is to force a precession of each planetary orbit about the stellar spin pole, with the precession rate higher for the inner planet. If planet–planet coupling is negligible, the subsequent evolution would excite a mutual inclination between the planets of twice the stellar inclination (assuming fixed stellar orientation and negligible eccentricities). Alternatively, if planet–planet coupling is very strong, they will retain approximate coplanarity. Below, we analytically compute the system's evolution between these two extreme regimes (i.e., for general J_2).

2.1. Assumptions

We restrict our analytic calculation to small mutual inclinations between the planets and utilize Laplace–Lagrange secular theory (Murray & Dermott 1999). This framework assumes the planets to be far from mean motion resonances, allowing one to average over the orbital motion. Consequently, each planetary orbit becomes dynamically equivalent to a massive wire, a concept that is due to Gauss (Murray & Dermott 1999; Morbidelli 2002). Furthermore, we set all eccentricities to zero.[1]

The star's orientation will be held fixed. The validity of this assumption can be demonstrated by considering the ratio of stellar spin to planetary orbital angular momenta:

$$\frac{J_\star}{\Lambda_{\rm p}} \equiv \frac{I_\star M_\star R_\star^2 \Omega_\star}{m_p\sqrt{GM_\star a_p}} \tag{1}$$

where $I_\star \approx 0.21$ is the dimensionless moment of inertia appropriate for a fully convective, polytropic star (Chandrasekar 1939), and the stellar rotation rate is $\Omega_\star = 2\pi/P_\star$. Consider a young, Sun-like star, possessing a rotation period of $P_\star = 10$ days (on the slower end of observations; Bouvier 2013) and a radius of roughly $2R_\odot$ (Shu et al. 1987). A 10 Earth-mass object would need to orbit at over $\sim$100 au in order to possess the angular momentum of the star. Thus, provided we deal with compact, relatively low-mass systems, the stellar orientation can be safely fixed to zero.

A further assumption is that the dynamical influence of stellar oblateness may be approximated using only the leading order quadrupole terms, neglecting those of order $\mathcal{O}(J_2^2)$. Therefore, the disturbing part of the stellar potential (with $e = 0$) may be written as (Danby 1992)

$$\begin{aligned}\mathcal{R} &= \frac{Gm_{\rm p}M_\star}{2a_p}\left(\frac{R_\star}{a_p}\right)^2 J_2\left(\frac{3}{2}\sin^2 i_p - 1\right)\\ &\approx \frac{Gm_{\rm p}M_\star}{2a_p}J_2\left(\frac{R_\star}{a_p}\right)^2(6s_p^2 - 1),\end{aligned} \tag{2}$$

where the second step has made the assumption of small planetary inclination i_p and defined a new variable $s_p \equiv \sin(i_p/2)$ (this definition is introduced to maintain coherence with traditional notation in celestial mechanics, e.g., Murray & Dermott 1999).

Finally, it is essential to define an initial configuration for the planetary system. Both numerical and analytic modeling of planet–disk interactions suggest that embedded protoplanets have their inclinations and eccentricities damped to small values within the disk-hosting stage (Tanaka & Ward 2004; Cresswell et al. 2007; Kley & Nelson 2012). Furthermore, any warping of the disk in response to a stellar companion is expected to be small (Fragner & Nelson 2010; Batygin et al. 2011). Therefore, throughout this work, we assume that the planets emerge from the disk with circular, coplanar orbits that are inclined by some angle $\beta_\star$ relative to the star. Note that we will always fix the stellar spin direction to be aligned with the z axis, so $\beta_\star$, the stellar obliquity, constitutes the initial inclination of the planetary orbits in our chosen frame.

2.2. Two-planet System

Incorporating the above assumptions, we may now write down the Hamiltonian ($\mathcal{H}$) that governs the dynamical evolution of the planetary orbits. To second order in inclinations (and dropping constant terms), we have (Murray

[1] This approximation is simply for ease of analytics and will be lifted in the numerical analysis below.

& Dermott 1999)

$$\mathcal{H} = \underbrace{\frac{Gm_1m_2}{a_2}[(s_1^2 + s_2^2)f_3 + s_1s_2f_{14}\cos(\Omega_1 - \Omega_2)]}_{\text{Planet–planet interaction}}$$
$$\underbrace{- \frac{3Gm_1M_\star}{a_1}J_2\left(\frac{R_\star}{a_1}\right)^2 s_1^2 - \frac{3Gm_2M_\star}{a_2}J_2\left(\frac{R_\star}{a_2}\right)^2 s_2^2}_{\text{Planet–stellar quadrupole interaction}}, \quad (3)$$

where the prefactors are

$$f_3 = -\frac{1}{2}f_{14} = -\frac{1}{2}\left(\frac{a_1}{a_2}\right)b_{3/2}^{(1)}\left(\frac{a_1}{a_2}\right), \quad (4)$$

and $b_{3/2}^{(1)}$ is the Laplace coefficient

$$b_{3/2}^{(1)}(\alpha) \equiv \frac{1}{\pi}\int_0^{2\pi}\left[\frac{\cos\psi}{(1+\alpha^2 - 2\alpha\cos\psi)^{3/2}}\right]d\psi. \quad (5)$$

Because we are using Hamiltonian mechanics, the dynamics must be described in terms of canonical variables. Traditional Keplerian orbital elements do not constitute a canonical set, so we transform to Poincaré (or, modified Delauney; Murray & Dermott 1999) variables, defined as

$$Z_{\mathrm{p}} \equiv m_{\mathrm{p}}\sqrt{GM_\star a_{\mathrm{p}}}\,(1 - \cos(i_{\mathrm{p}})) \quad z_{\mathrm{p}} \equiv -\Omega_{\mathrm{p}}. \quad (6)$$

Physically, Z_{p} is the angular momentum of a circular orbit after subtracting its component in the z-direction. Notice that in the small angle limit,

$$Z_{\mathrm{p}} \approx \frac{1}{2}m_{\mathrm{p}}\sqrt{GM_\star a_{\mathrm{p}}}\,i_{\mathrm{p}}^2 \equiv \frac{1}{2}\Lambda_{\mathrm{p}}i_{\mathrm{p}}^2. \quad (7)$$

After substituting, we arrive at the governing Hamiltonian

$$\mathcal{H} = -\mathcal{E}\left[\frac{Z_1}{\Lambda_1} + \frac{Z_2}{\Lambda_2} - 2\sqrt{\frac{Z_1Z_2}{\Lambda_1\Lambda_2}}\cos(z_1 - z_2)\right] - \frac{3}{2}n_1J_2\left(\frac{R_\star}{a_1}\right)^2 Z_1 - \frac{3}{2}n_2J_2\left(\frac{R_\star}{a_2}\right)^2 Z_2 \quad (8)$$

where, for compactness, we define

$$\mathcal{E} \equiv \frac{Gm_1m_2}{4a_2}\left(\frac{a_1}{a_2}\right)b_{3/2}^{(1)}\left(\frac{a_1}{a_2}\right). \quad (9)$$

In order to complete the calculation, we define a complex variable that represents the inclination of each planet

$$\eta_{\mathrm{p}} \equiv \sqrt{\frac{Z_{\mathrm{p}}}{\Lambda_{\mathrm{p}}}}\,(\cos(z_{\mathrm{p}}) + \imath\sin(z_{\mathrm{p}})) \approx \frac{1}{\sqrt{2}}i_{\mathrm{p}}(\cos(\Omega_{\mathrm{p}}) - \imath\sin(\Omega_{\mathrm{p}})), \quad (10)$$

where $\imath = \sqrt{-1}$. The purpose is to cast Hamilton's equations into an eigenvector/eigenvalue problem. Specifically, in terms of these new variables, we must solve

$$\dot{\eta}_{\mathrm{p}} = \imath\frac{\partial\mathcal{H}}{\partial\eta_{\mathrm{p}}^*}\frac{1}{\Lambda_{\mathrm{p}}}, \quad (11)$$

in which "*" denotes complex conjugation, yielding the matrix equation

$$\frac{d}{dt}\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix} = -\imath\begin{pmatrix}B_1 + \nu_1 & -B_1\\ -B_2 & B_2 + \nu_2\end{pmatrix}\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix},$$

where we have defined four frequencies as

$$B_1 \equiv \frac{1}{4}n_1\left(\frac{a_1}{a_2}\right)^2 b_{3/2}^{(1)}\left(\frac{a_1}{a_2}\right)\frac{m_2}{M_\star}$$
$$B_2 \equiv \frac{1}{4}n_2\left(\frac{a_1}{a_2}\right)b_{3/2}^{(1)}\left(\frac{a_1}{a_2}\right)\frac{m_1}{M_\star}$$
$$\nu_{\mathrm{p}} = \frac{3}{2}n_{\mathrm{p}}J_2\left(\frac{R_\star}{a_{\mathrm{p}}}\right)^2. \quad (12)$$

The equation above may be solved using standard methods, whereby the solution is written as a sum of eigenmodes

$$\eta_{\mathrm{p}} = \sum_{j=1}^{2}\eta_{\mathrm{p},j}\exp(\imath\lambda_j t). \quad (13)$$

Indeed, the problem may be easily extended to N planets, though writing down all eigenvectors $\eta_{\mathrm{p},j}$ and eigenmodes λ_j rapidly becomes cumbersome.

2.2.1. Initial Conditions and Solution

A choice must be made for the initial conditions of the problem. As already mentioned above, here we choose the condition that both orbits are initially coplanar, having recently emerged from their natal disk, with the star inclined by some angle $\beta_\star$ relative to them. Accordingly, all four boundary conditions may be satisfied by requiring that

$$\eta_{\mathrm{p}}|_{t=0} = \frac{\beta_\star}{\sqrt{2}}. \quad (14)$$

What we seek is the mutual planet–planet inclination (denoted β_{rel}). In the small angle approximation, we can compute this quantity using the relation

$$(1 - \cos(\beta_{\mathrm{rel}})) \approx \eta_1\eta_1^* + \eta_2\eta_2^* - (\eta_1\eta_2^* + \eta_2\eta_1^*) \approx \frac{1}{2}\beta_{\mathrm{rel}}^2. \quad (15)$$

After solving for eigenvalues, eigenvectors, and matching the initial conditions, we arrive at the solution for the mutual inclination of the two planets as a function of time, which takes the rather compact form

$$\boxed{\beta_{\mathrm{rel}}(t) = 2\beta_\star\mathcal{G}(J_2)\sin(\omega_0 t/2)}, \quad (16)$$

where we define the (semi-)amplitude of the oscillations between planets

$$\mathcal{G} = \mathcal{L}\left[1 + \mathcal{L}^2 + 2\left(\frac{\Lambda_2 - \Lambda_1}{\Lambda_2 + \Lambda_1}\right)\mathcal{L}\right]^{-1/2} \quad (17)$$

in terms of the ratio of frequencies

$$\mathcal{L} \equiv \frac{\nu_1 - \nu_2}{B_1 + B_2} = 6J_2\left(\frac{R_\star}{a_1}\right)^2\left(\frac{a_2}{a_1}\right)^2\frac{1}{b_{3/2}^{(1)}(\alpha)}\frac{M_\star}{m_2}\frac{1 - \alpha^{7/2}}{1 + \Lambda_1/\Lambda_2}. \quad (18)$$

A convenient consequence of the aligned initial conditions is that the oscillations are purely sinusoidal, evolving with half the frequency

$$\omega_0 \equiv [(B_1 + B_2)^2 + (\nu_1 - \nu_2) \times (\nu_1 - \nu_2 + 2(B_1 - B_2))]^{1/2}. \quad (19)$$

One aspect to notice is that the amplitude is maximized when the equality

$$\nu_1 + \mathcal{K}^2 B_1 = \nu_2 + \mathcal{K}^2 B_2 \quad (20)$$

is satisfied, where $\mathcal{K} \equiv (\Lambda_1 + \Lambda_2)/(\Lambda_1 - \Lambda_2)$. The maximum amplitude, scaled by $\beta_\star$, can then be written as

$$2\mathcal{G}_{\max} = \frac{\Lambda_1 + \Lambda_2}{\sqrt{\Lambda_1 \Lambda_2}}. \quad (21)$$

The significance of this result is best seen upon considering the outer planet to be a test particle, such that $B_1 = 0$ and $\Lambda_2 \to 0$. In such a scenario, the maximum of the amplitude $\mathcal{G}_{\max} \to \infty$. Such an unphysical result occurs as a consequence of a secular resonance (Murray & Dermott 1999; Morbidelli 2002; Spalding & Batygin 2014; Batygin et al. 2016), whereby the inner and outer bodies precess at similar rates. In reality, our earlier approximation that mutual inclinations are small breaks down in this regime and the inclusion of higher order terms is required.

In principle, one may also set $\Lambda_1 \to 0$ and conclude that the above resonance persists when the inner planet is a test particle. However, the resonant criterion in terms of stellar oblateness reads

$$J_2|_{\rm res} \approx \frac{1}{6}\left(\frac{a_1}{R_\star}\right)^2 \left(\frac{a_1}{a_2}\right)^2 \left(\frac{m_2}{M_\star}\right) \times \left(\frac{b_{3/2}^{(1)}(\alpha)}{\Lambda_2}\right) \frac{(\Lambda_1 + \Lambda_2)^2}{(\Lambda_1 - \Lambda_2)(1 - \alpha^{7/2})}, \quad (22)$$

which is negative when $\Lambda_1 < \Lambda_2$. Accordingly, the condition for secular resonance can only be satisfied when $\Lambda_1 > \Lambda_2$, i.e., when the inner planet possess more orbital angular momentum than the outer planet.

As an illustration, we plot the semi-amplitude $\mathcal{G}$ in Figure 1 appropriate for a configuration where the two planetary orbits are situated at 0.05 and 0.1 au, both orbiting a solar-mass star with radius 0.01 au (about twice the Sun's radius). Three cases are shown: the red line depicts a 1 Earth-mass planet interior to a 10 Earth-mass planet while the blue line has the planets interchanged. The former configuration possesses a positive $J_2|_{\rm res}$, appearing as a maximum in the amplitude. The third situation (the cyan line) represents a 100 Earth-mass planet interior to a 1 Earth-mass body, illustrating that higher-mass inner planets may more easily misalign their outer companion (Equation (21)), but $J_2|_{\rm res}$ is correspondingly higher.

Figure 1 demonstrates that misalignments of the order twice the stellar obliquity can be readily excited for reasonable values of J_2. By geometric arguments, the potential for such misalignments to take one of the planets out of transit depends upon the ratio $R_\star/a$. However, for the cases considered above, only $\sim 4°$ of stellar obliquity are required to remove the two planets from a co-transiting configuration (less than the $\sim 7°$ present in the solar system; Lissauer et al. 2011). Conversely, planets may remain co-transiting if the innermost planet is sufficiently distant, the planets are very massive and/or tightly packed, or the stellar quadrupole moment is particularly low.

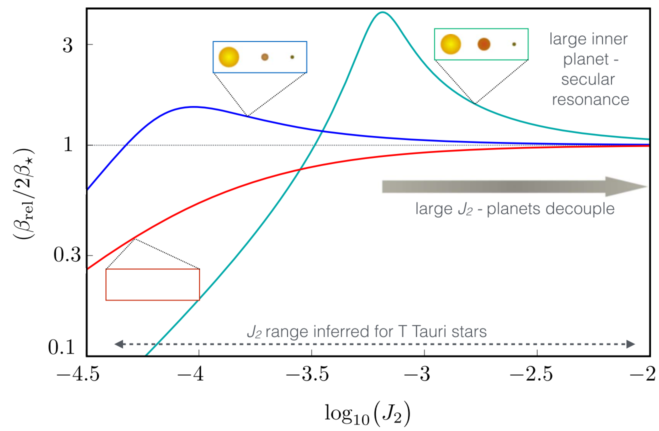

Figure 1. Amplitude of oscillations in mutual planet–planet inclinations excited between two initially coplanar, circular planetary orbits $\beta_{\rm rel}$, scaled by twice the stellar obliquity $\beta_\star$. The planets are situated at 0.05 and 0.1 au for three different mass configurations: the red line has a 10 Earth-mass planet outside a 1 Earth-mass planet, where blue has the planets switched. The cyan line augments the inner planet to 100 Earth masses. Notice that any time the inner planet has more angular momentum, there exists a peak in the misalignments, representing resonance. In the limit of large J_2, the planets entirely decouple and reach mutual inclinations equal to twice the stellar obliquity.

3. NUMERICAL ANALYSIS

Several crucial aspects of real systems were neglected in order to obtain the analytic solution (16) above. Principally, we included only two planets whose orbits were assumed to be circular and only slightly mutually inclined. Additionally, in averaging over short-term motion, our adopted secular approach is unable to describe the full dynamics. A more subtle aspect was that we considered a constant J_2, when in reality, stars are expected to spin down and shrink over time until J_2 is essentially negligible (Irwin et al. 2008; Bouvier 2013; McQuillan et al. 2013).

In order to test our hypothesis within a more general framework, we now turn to N-body simulations. To carry out the calculations, we employed the well-tested Mercury6 symplectic integration software package (Chambers 1999).[2] In addition to standard planet–planet interactions, we modified the code to include the gravitational potential arising from a tilted star of given J_2, along with a term to produce general relativistic precession (following Nobili & Roxburgh 1986).

For the sake of definiteness, the parameters of our modeled system were based off of Kepler-11, a star around which six transiting planets have been discovered (Lissauer et al. 2011). Detailed follow-up studies, using Transit Timing Variations, have constrained the masses of the innermost five planets and placed upper limits upon the mass of Kepler-11g, the outermost planet,[3] making this system ideal for dynamical investigation. Though choosing one system is not exhaustive, our goal is to demonstrate the influence of a tilted star upon a general coplanar system of planets. We follow Lissauer et al. (2013) and use their best-fit mass of 8 Earth

[2] http://www.arm.ac.uk/~jec/home.html

[3] The mass of Kepler-11g, is only loosely constrained; however, for the purposes of this work, it is not particularly imperative to choose the "real" mass.

Table 1
The Parameters of the Kepler-11 System

	Kepler-11					
Property	b	c	d	e	f	g
Mass (Earth masses)	1.9	2.9	7.3	8.0	2.0	8.0
Radius (Earth radii)	1.80	2.87	3.12	4.19	2.49	3.33
a (au)	0.091	0.107	0.155	0.195	0.250	0.466
Period (days)	10.3	13.0	22.7	32.0	46.7	118.4

Note. The mass of Kepler-11g only has upper limits set upon it, but we follow Lissauer et al. (2013) and choose a best-fit mass of 8 Earth masses here.

Masses for Kepler-11g, with the stellar mass given by $0.961 M_\odot$ (see Table 1).

3.1. N-body Simulation

For our numerical runs, we choose 10 values of stellar obliquity and 11 of initial stellar $J_2 = J_{2,0}$ (i.e., the oblateness immediately as the disk dissipates). Once again, we fix the stellar orientation aligned with the z axis, but choose the initial planet–star misalignments:

$$\beta_\star \in \{5, 10, 20, 30, 40, 50, 60, 70, 80, 85\}. \tag{23}$$

The value of J_2 for a star deformed by its own rotation may be related to its spin rate $\omega_\star$ and Love number (twice the apsidal motion constant) k_2 as follows (Sterne 1939):

$$J_2 = \frac{1}{3}\left(\frac{\Omega}{\Omega_b}\right)^2 k_2, \tag{24}$$

where Ω_b is the break-up spin frequency at the relevant epoch. The Love number can be estimated by modeling the star as a polytope with index $\chi = 3/2$ (i.e., fully convective; Chandrasekar 1939), which yields $k_2 \approx 0.28$. Observations constrain the spin-periods of T-Tauri stars to lie within the range of ~1–10 days (Bouvier 2013), while the break-up period is given by

$$T_{\rm b} = \frac{2\pi}{\Omega_b} \approx \frac{1}{3}\left(\frac{M_\star}{M_\odot}\right)^{-1/2}\left(\frac{R_\star}{2R_\odot}\right)^{3/2} \text{days}. \tag{25}$$

In our simulations below, we use the current mass of Kepler-11 for the star, but suppose its radius to be somewhat inflated relative to its current radius ($R_\star = 2R_\odot$), reflecting the T Tauri stage (Shu et al. 1987). With these parameters, we arrive at a reasonable range of $J_{2,0}$ of

$$10^{-4} \lesssim J_{2,0} \lesssim 10^{-2}, \tag{26}$$

within which we choose 11 values uniformly separated in log-space:

$$J_{2,0} \in \{10^{-4}, 10^{-3.8} \ \ldots \ 10^{-2}\}. \tag{27}$$

In all runs, rather than allowing both $R_\star$ and J_2 to vary, we simply left $R_\star$ as a constant, letting J_2 decay exponentially over a timescale of $\tau = 1$ Myr

$$J_2(t) = J_{2,0}\, \mathrm{e}^{(-t/\tau)}. \tag{28}$$

The choice for τ is essentially arbitrary, provided J_2 decays over many precessional timescales, owing to the adiabatic nature of the dynamics (Lichtenberg & Lieberman 1992; Morbidelli 2002). Our choice of 1 Myr roughly coincides with a Kelvin–Helmholtz timescale (Batygin & Adams 2013) but is chosen also to save computational time.

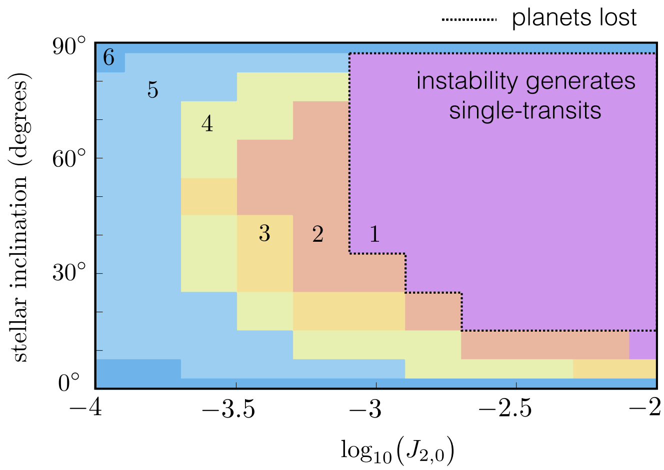

Figure 2. Maximum number of transits detectable after 22 million years of integrating Kepler-11 with a tilted, oblate star. The x-axis denotes the value of J_2 immediately after disk dispersal ($J_{2,0}$) and the y-axis represents the stellar inclination. The runs where planets were lost through instability are outlined by a dotted line, which corresponds closely to the region where only single transits can be observed (the purple region).

For each case, our integrations span 22 million years, beginning with the initial condition of a coplanar system possessing the current semi-major axes of the Kepler-11 system, but with eccentricities set to zero (Lissauer et al. 2013). In order to analyze the results, we sample the system six times between 19 and 22 Myr, and at each step calculate the maximum number of transiting planets that could be observed from a single direction. The results at all six times were then averaged.

The determination of the maximum number of transits was accomplished as follows. We begin by checking whether all possible pairs within the six planets mutually transit, where the criterion for concluding a pair of planets to be non-transiting is

$$|\sin(\beta_{\rm rel})| > \sin(\beta_{\rm crit}) \approx \frac{R_\star}{a_1} + \frac{R_\star}{a_2}, \tag{29}$$

where in the above formula we used the current radius of Kepler-11 ($R_\star = 1.065 R_\odot$, as opposed to the inflated value relevant to the T Tauri stage). If any single pair of planets was non-transiting, we proceeded to choose each possible combination of five out of the six and performed a similar pairwise test to identify potentially observable five-transiting systems. If no set of five passed the test, we chose all sets of four, etc., until potentially finding that only one planet could be seen transiting. We note that the criterion above neglects the possibility of fortunate orbital configurations allowing two mutually inclined orbits to intersect along the line of sight. This complication, however, does not affect our qualitative picture.

4. RESULTS

The numerical results are presented in Figure 2, where the x-axis depicts the initial stellar $J_2 = J_{2,0}$. The y-axis refers to the misalignment $\beta_\star$ between the stellar spin axis and the initial plane of the six planets. Each run has been given its own rectangular box, within which the color represents the maximum number of co-transiting planets that may be observed around the star (as discussed above). The numerics verify our analytic result, in that the observable multiplicity may be significantly reduced solely as a consequence of stellar obliquity.

As expected, higher values of $J_{2,0}$ result in fewer transiting planets, provided the star is tilted relative to the planetary orbits. As with our analytic results, even small stellar obliquities are sufficient to reduce the transit count, with 5° of obliquity reducing the transit number to as little as three (Figure 2). However, planet–planet mutual inclination was not the only source of the reduction in transit number. A crucial finding was that for large enough $J_{2,0}$ and $\beta_\star$, the stellar quadrupole potential caused the system to go unstable, casting three to five planets out of the system or into the central body, with planet–planet collisions existing as an additional possibility not captured in our simulations (Boley et al. 2016).

The region of instability (i.e., where at least one planet was lost) is outlined by a dotted line in Figure 2. Interestingly, the areas of instability map closely onto the regions where only a single transit remains. In other words, almost every single-transiting system coming out of the integration had lost planets through dynamical instability. Furthermore, each time instability occurred, the two lowest mass members were lost: Kepler-11b and f, with the next lowest mass body, Kepler-11c often joining them. Such a preference for retaining more massive planets is indeed reflected in the data as a slightly larger typical radius for single-transiting systems (Johansen et al. 2012). However, more testing is required to determine whether this is a generic feature of our model.

We showed that for any given two-planet system, there exists a resonant J_2 if the inner planet has more angular momentum than the outer. However, the picture becomes much more complicated in a multi-planet system, where each planet introduces two additional secular modes (one for eccentricity and one for inclination; Murray & Dermott 1999), increasing the density of resonances in Fourier space. As the stellar J_2 decays, its influence sweeps across each resonance, providing ample opportunity to excite mutual inclinations. If the two planets, Kepler-11d and f, were alone, they could become resonant at $J_2|_{\rm res} \sim 10^{-2.4}$, which coincides approximately with the onset of instability in the low-inclination runs (Figure 2) but not exactly, for the reasons mentioned above.

5. DISCUSSION

In recent years, the *Kepler* data set has grown sufficiently comprehensive to facilitate statistically robust investigations at a population level. Out of this data has emerged a so-called "*Kepler* Dichotomy"; the notion that single-transiting systems are too common to be explained as resulting from a simple distribution of mutual inclinations within systems of higher multiplicity (Johansen et al. 2012; Ballard & Johnson 2016).

In separate studies, significant misalignments have been detected between the stellar spin axes and the planetary orbits they host, particularly around stars with main sequence $T_{\rm eff} \gtrsim 6200$ K (Winn et al. 2010; Albrecht et al. 2012; Mazeh et al. 2015; Li & Winn 2016). This trend initially became apparent within the hot-Jupiter data set and was consequently often interpreted as evidence for a post-disk, high-eccentricity migration pathway for hot-Jupiter formation.[4] However, a similar trend has now emerged within *Kepler* systems, including the multi-transiting sub-population (Huber et al. 2013; Mazeh et al. 2015), with little evidence supporting a tidal origin (Li & Winn 2016). These observations cumulatively suggest that many of the misalignments originated from directly tilting the protoplanetary disk, thereby inclining all planets in the system at once (Batygin 2012; Lai 2014; Spalding & Batygin 2014, 2015; Fielding et al. 2015).

A consequence of primordially generated spin–orbit misalignments is that stellar obliquity would be present at the end of the natal disk's life, leaving the planetary orbital architecture subject to the quadrupole moment of their young, rapidly spinning, and expanded host star. This paper has demonstrated that such a configuration naturally misaligns close-in systems and, furthermore, provides a mechanism for dynamical instability that by-passes the problem encountered in earlier work that unreasonably large planets were required to induce instability (Johansen et al. 2012).

The observable multiplicity of transiting systems can be reduced either by inclining planetary orbits relative to each other, or by intrinsically reducing the number of planets. Here, we have shown that both can be at play, with modest J_2 and stellar obliquity causing misalignments, whereas sufficiently large values thereof lead to dynamical instability, shedding planets. The origin of the instability is likely secular in nature, and significant planet–planet inclinations have been shown to reduce the inherent stability of planetary systems in numerous previous works (Laughlin et al. 2002; Veras & Armitage 2004; Nelson et al. 2014). In support of this interpretation, our simulations resulted in planetary instability at much smaller J_2 when obliquity was high $\gtrsim 40^\circ$. Accordingly, we would expect multiplicity (both transiting and intrinsic) to be lower around hot stars, which tend to possess higher obliquities (Winn et al. 2010; see below).

5.1. Predictions

Imposing stellar obliquity as a source of the *Kepler* Dichotomy leads to several predictions. Naturally, stars leaving the disk-hosting stage with larger J_2 and obliquity are more likely to end up observed as exhibiting single transits, either as a result of dynamical instability or the excitation of mutual planet–planet inclinations. As mentioned above, there is an observed trend whereby stars with $T_{\rm eff} \gtrsim 6200$ K exhibit higher obliquity (Winn et al. 2010; Albrecht et al. 2012; Mazeh et al. 2015) and so, on the face of it, one would expect a higher relative incidence of singles around higher-mass stars. The picture is, however, complicated by the universal feature of stellar evolution models that more massive stars contract along their Hayashi tracks faster (Siess et al. 2000). Accordingly, the influence of J_2 in more massive stars may have decayed to a greater extent than in lower-mass stars by the time their natal disk dissipates, partly offsetting the impact of their larger typical obliquities.

[4] The dependence on host star temperature (and therefore mass) was attributed to tidal dissipation within the convective regions of lower-mass stars (Winn et al. 2010; Lai 2012).

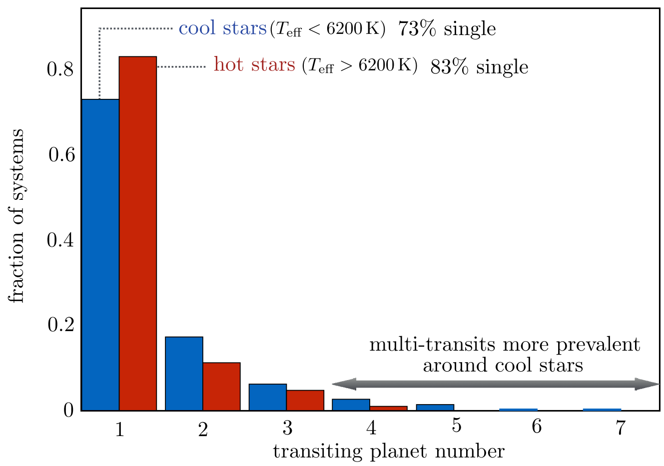

Figure 3. Fraction of systems exhibiting each number of transiting planets from one to seven within the hot ($T_{\rm eff} > 6200$ K, red bars) and cool ($T_{\rm eff} < 6200$ K, blue bars) sub-samples of planet-hosting *Kepler* stars. There were 132 hot stars and 1504 cool stars in the data used, of which 83% and 73%, respectively, exhibited single transits. Accordingly, transiting systems around hot stars show a stronger tendency toward being single, in agreement with the predictions of our presented model (see the text).

The above complications notwithstanding, both our analytical and numerical analyses suggest a greater sensitivity of the degree of misalignment to stellar obliquity than to stellar J_2. Consequently, we make the prediction that hot stars possess more abundant single-transiting systems relative to cool stars.

Previous work has already suggested the existence of our predicted trend. Specifically, both hotter stars and, independently, single-transiting systems appear to exhibit higher obliquities (Morton & Winn 2014; Mazeh et al. 2015). The overlapping of these two findings implies at least a weak trend toward more singles around hotter stars. In order to further test this prediction, we carried out a simple statistical analysis of confirmed *Kepler* planets, as we now describe.

5.2. Kepler Data

To obtain data on confirmed, *Kepler* systems, we downloaded the data from the "Confirmed Planets" list (as of 2016 July) on the NASA Exoplanet Archive website.[5] The systems were filtered to include only those in the *Kepler* field, though the conclusions that follow do not change qualitatively if non-*Kepler* detections are included.

In Figure 3, we split the data into "hot" stars with $T_{\rm eff} > 6200$ K (132 in total) and"cool" stars, with $T_{\rm eff} < 6200$ K (1504 in total). For each sub-population, we illustrate the fraction of systems as a function of the number of planets observed in transit. The hot stars clearly demonstrate a larger fraction of singles and a smaller fraction of multiples for each value of multiplicity, in agreement with the predictions of our model. In order to quantify the significance of this agreement, we carry out a statistical test that quantitatively compares the two populations.

Using Bayes' theorem, with a uniform prior, we generated a binomial distribution for hot stars and cool stars separately that illustrates the probability of the data, given an assumption about what fraction of systems are single. Such an argument is similar to determining the fairness of a coin flip, where heads equates to a single system and tails a multi system. Specifically, we plot

$$\mathcal{P}(\{\text{data}\}|S) = AS^{N_s}(1 - S)^{N_t - N_s}, \tag{30}$$

where N_s is the number of single systems within a population of N_t total stars and A is a normalization coefficient (Sivia 1996). The variable S is the single bias weighting; the probabilistic tendency for a population of planet-hosting stars to display single transits as opposed to multiples. The quantity $\mathcal{P}(\{\text{data}\}|S)$ gives the probability of reproducing the data if the underlying tendency is S. In the hot population, $N_t = 132$ and $N_s = 110$, whereas the cool population had $N_t = 1504$ and $N_s = 1098$.

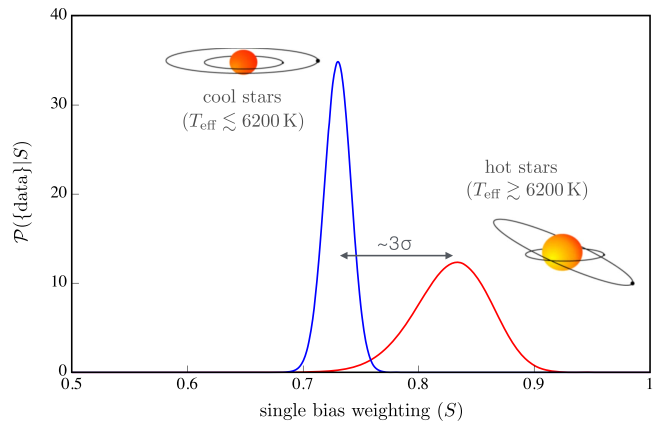

Figure 4. Probability of the data, given an intrinsic fraction S of singles out of systems with hot (red line) and cool (blue line) stars. The separation of the peaks is roughly 2.9σ (as defined in the text). Therefore, to a very high confidence, hot stars possess relatively more singles, as our hypothesis predicts.

As can be seen from Figure 4, the two distributions are visually distinct, with hotter stars possessing a stronger bias toward singles, with a significance of 2.9σ.[6] More data are needed to tease out this relationship further and to isolate the influence of a tilted star versus other confounding factors, such as the dependence upon stellar mass of the occurrence rate of giant planets. For now, we conclude that the data supports our general prediction, that hotter, more oblique stars possess a relatively greater abundance of single-transiting planets.

A separate prediction relates to the distance of the planets from the host star. Specifically, the quadrupole moment falls off as $R_\star^2/a^2$, whereas the coplanarity required for transit grows as $a/R_\star$, and so the overall magnitude of our proposed mechanism should become negligible within more distant systems. In consequence, we predict that the "*Kepler* Dichotomy" signal will weaken for systems at larger orbital distances. As future missions, such as *TESS* collect more data, this unique aspect of our model will become amenable to observational tests.

A caveat to the above analysis is that the dichotomy appears to persist within the population of planets around M-dwarfs (Ballard & Johnson 2016). This is problematic as these stars are expected to exhibit lower inclinations, being cooler on the main sequence. We interpret this as stellar oblateness being effective across all stellar masses, but being relatively more important within the hotter, more inclined population. This is supported

5 http://exoplanetarchive.ipac.caltech.edu

6 Where σ^2 here is defined as the sum of the squares of the standard deviations of each individual distribution.

by our numerical simulations, where even small obliquities reduced the transit number if oblateness was large enough (Figure 2).

5.3. The Origin of Spin–Orbit Misalignments

Our work here essentially relies upon the assumption that stellar obliquity is excited early on, in the disk-hosting stage. This is not the only potential origin for spin–orbit misalignments, with alternative pathways including secular chaos (Lithkwick & Wu 2012), planet–planet scattering (Ford & Rasio 2008), and Kozai interactions (Naoz et al. 2011; Petrovich 2015). These mechanisms are traditionally inseparable from the idea that hot Jupiters migrate through a post-disk, high-eccentricity pathway (Wu & Murray 2003; Petrovich 2015). Whereas it is likely that some hot Jupiters formed in this way, it is unlikely to constitute the dominant pathway (Dawson et al. 2014) and, furthermore, cannot explain the spin–orbit misalignment distribution in *Kepler* systems (Mazeh et al. 2015; Li & Winn 2016). Rather, disk-driven migration constitutes a favorable mechanism that may retain multiple planet systems within the same plane, and can account for the observed spin–orbit misalignments if the disk itself becomes misaligned with respect to the host star.

To that end, multiple studies have shown that a stellar companion is dynamically capable of exciting star–disk misalignments across the entire observed range of spin–orbit misalignments (Batygin 2012; Lai 2014; Spalding & Batygin 2014). Specifically, the tidal potential of a companion star, or even that of the star cluster itself, induces a precession of the disk orientation, leading to significant star–disk misalignments, usually by way of a secular resonance (Spalding & Batygin 2014). Although observations of disk orientation in young binary systems are elusive, there exists at least one known example of a binary where each star has a disk with its plane misaligned to that of the binary (Jensen & Akeson 2014), just as in the aforementioned theoretical picture. Furthermore, stellar multiplicity appears to be a nearly universal outcome of star formation (Duchêne & Kraus 2013; Beuther et al. 2014).

Within the framework of primordial excitation of spin–orbit misalignments, the dependence upon stellar mass (or $T_{\rm eff}$) has been linked to the observed multipolar field topology of higher-mass T Tauri stars compared to the more dipolar configuration seen in lower-mass T Tauri stars (Gregory et al. 2012). The weaker dipoles of higher-mass stars increase their magnetic realignment timescales above that of their lower-mass counterparts, naturally explaining the observed trend in spin–orbit misalignments with stellar $T_{\rm eff}$, and therefore mass (Spalding & Batygin 2016). Our work here has demonstrated an additional consistency between observations and primordially excited spin–orbit misalignments, namely that the *Kepler* Dichotomy naturally arises from the dynamical response of multi-planet systems to the potential of an oblate, tilted star.

6. CONCLUSIONS

This paper investigates the origin of the "*Kepler* Dichotomy," within the context of primordially generated spin–orbit misalignments. We have shown that the quadrupole moment of such misaligned, young, fast-rotating stars is typically capable of exciting significant mutual inclinations between the hosted planetary orbits. In turn, the number of planets available for observation through transit around such a star is reduced, either through dynamical instability or directly as a result of the mutual inclinations, leaving behind an abundance of single-transiting systems (Johansen et al. 2012). The outcome is an apparent reduction in the multiplicity of tilted, hot stars, with their observed singles being slightly larger, as a consequence of many having undergone dynamical instabilities, in accordance with observations.

Through the conclusions of this work, the origins of hot Jupiters, of compact *Kepler* systems, the *Kepler* Dichotomy, and spin–orbit misalignments, are all placed within a common context.

This research is based in part upon work supported by NSF grant AST 1517936 and the NESSF Graduate Fellowship in Earth and Planetary Sciences (C.S). We would like to thank Erik Petigura, Henry Ngo, and Peter Gao for helpful discussions, along with Fred Adams and Joe O'Rourke for useful suggestions. We are grateful to the anonymous reviewer for helpful comments that significantly improved the manuscript.

REFERENCES

Albrecht, S., Winn, J. N., Johnson, J. A., et al. 2012, ApJ, 757, 18
Ballard, S., & Johnson, J. J. 2016, ApJ, 816, 66
Bate, M. R., Lodato, G., & Pringle, J. E. 2010, MNRAS, 401, 1505
Batygin, K. 2012, Natur, 491, 418
Batygin, K., & Adams, F. C. 2013, ApJ, 778, 169
Batygin, K., Bodenheimer, P. H., & Laughlin, G. P. 2016, arXiv:1511.09157
Batygin, K., Morbidelli, A., & Tsiganis, K. 2011, A&A, 533, 7
Beaugé, C., & Nesvorný, D. 2012, ApJ, 751, 119
Becker, J. C., & Adams, F. C. 2016, MNRAS, 455, 2980
Beuther, H., Klessen, R. S., Dullemond, C. P., & Henning, T. K. (ed.) 2014, Protostars and Planets VI (Tucson, AZ: Univ. Arizona Press)
Boley, A. C., Contreras, A. G., & Gladman, B. 2016, ApJL, 817, L17
Bouvier, J. 2013, in EAS Publications Ser. 62, Angular Momentum Transport During Star Formation and Evolution, ed. P. Hennebelle & C. Charbonnel, (Les Ulis: EDP Sciences), 143
Chambers, J. E. 1999, ApJ, 304, 793
Chandrasekar, S. 1939, An Introduction to the Study of Stellar Structure (Chicago: Univ. Chicago Press)
Cresswell, P., Dirksen, G., Kley, W., & Nelson, R. P. 2007, A&A, 473, 329
Danby, J. 1992, Fundamentals of Celestial Mechanics (2nd ed.; Richmond: Willman-Bell), 1
Dawson, R. I., & Chiang, E. 2014, Sci, 346, 212
Dawson, R. I., Murray-Clay, R. A., & Johnson, J. A. 2014, ApJ, 798, 66
Duchêne, G., & Kraus, A. 2013, ARA&A, 51, 269
Fabrycky, D. C., Lissauer, J. J., Ragozzine, D., et al. 2014, ApJ, 790, 146
Fielding, D. B., McKee, C. F., Socrates, A., Cunningham, A. J., & Klein, R. I. 2015, MNRAS, 450, 3306
Ford, E. B., & Rasio, F. A. 2008, ApJ, 686, 621
Fragner, M. M., & Nelson, R. P. 2010, A&A, 511, A77
Gregory, S. G., Donati, J. F., Morin, J., et al. 2012, ApJ, 755, 97
Huber, D., Carter, J. A., Barbieri, M., et al. 2013, Sci, 342, 331
Irwin, J., Hodgkin, S., Aigrain, S., et al. 2008, MNRAS, 383, 1588
Jensen, E. L., & Akeson, R. 2014, Natur, 511, 567
Johansen, A., Davies, M. B., Church, R. P., & Holmelin, V. 2012, ApJ, 758, 39
Kant, I. 1755, General History of Nature and Theory of the Heavens (Königsberg: Petersen)
Kley, W., & Nelson, R. P. 2012, ARA&A, 50, 211
Lai, D. 2012, MNRAS, 423, 486
Lai, D. 2014, MNRAS, 440, 3532
Laplace, P. S. 1796, Exposition dy Système du Monde (Paris: Cerie-Social)
Laskar, J. 1996, CeMDA, 64, 115
Laughlin, G., Chambers, J., & Fischer, D. 2002, ApJ, 579, 455
Li, G., & Winn, J. N. 2016, ApJ, 818, 5
Lichtenberg, A. J., & Lieberman, M. A. 1992, Regular and Chaotic Dynamics (2nd ed.; Berlin: Springer)
Lissauer, J. J., Fabrycky, D. C., Ford, E. B., et al. 2011, Natur, 470, 53
Lissauer, J. J., Jontof-Hutter, D., Rowe, J. F., et al. 2013, ApJ, 770, 131
Lissauer, J. J., Ragozzine, D., Fabrycky, D. C., et al. 2011, ApJSS, 197, 8
Lithkwick, Y., & Wu, Y. 2012, ApJL, 756, L11

Mazeh, T., Perets, H. B., McQuillan, A., & Goldstein, E. S. 2015, ApJ, 801, 3
McQuillan, A., Mazeh, T., & Aigrain, S. 2013, ApJL, 775, L11
Morbidelli, A. 2002, Modern Celestial Mechanics: Aspects of Solar System Dynamics, Vol. 1 (Boca Raton, FL: CRC Press)
Morton, T. D., & Winn, J. N. 2014, ApJ, 796, 47
Murray, C. D., & Dermott, S. F. 1999, Solar System Dynamics (Cambridge: Cambridge Univ. Press)
Naoz, S., Farr, W. M., Lithwick, Y., Rasio, F. A., & Teyssandier, J. 2011, Natur, 473, 187
Nelson, B. E., Ford, E. B., Wright, J. T., et al. 2014, MNRAS, 441, 442
Nobili, A., & Roxburgh, I. W. 1986, Relativity in Celestial Mechanics and Astrometry: High Precision Dynamical Theories and Observational Verifications, Vol. 114 (Dordrecht: Springer)
Petrovich, C. 2015, ApJ, 799, 27
Petrovich, C. 2015, ApJ, 805, 75
Shu, F. C., Adams, F. C., & Lizano, S. 1987, ARA&A, 25, 23
Siess, L., Dufour, E., & Forestini, M. 2000, A&A, 358, 593
Sivia, D. S. 1996, Data analysis: a Bayesian tutorial (Oxford: Oxford Univ. Press)
Spalding, C., & Batygin, K. 2014, ApJ, 790, 42
Spalding, C., & Batygin, K. 2015, ApJ, 811, 82
Spalding, C., Batygin, K., & Adams, F. C. 2014, ApJL, 797, L29
Sterne, T. E. 1939, MNRAS, 99, 451
Storch, N. I., Anderson, K. R., & Lai, D. 2014, Sci, 345, 1317
Tanaka, H., & Ward, W. R. 2004, ApJ, 602, 388
Veras, D., & Armitage, P. J. 2004, Icar, 172, 349
Winn, J., & Fabrycky, D. C. 2015, ARA&A, 53, 409
Winn, J. N., Fabrycky, D., Albrecht, S., & Johnson, J. A. 2010, ApJL, 718, L145
Wu, Y., & Murray, N. 2003, ApJ, 589, 605