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Scaling and Universality in the Integer Quantum Hall Effect.

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Abstract. – For a model of noninteracting electrons in a disorder potential under quantum Hall conditions the critical behavior near the centers of the two lowest Landau levels and its dependence on the correlation length of the disorder potential are studied. The localization length is calculated numerically for quasi-one-dimensional systems. Finite-size scaling is used to obtain the critical exponent. For the lowest Landau level universal one-parameter scaling independent of the correlation length is found. Universality in the higher Landau levels is established by an argument based on the shape of the potential matrix elements in the limit of large correlation length. In the second lowest Landau-level universality is explicitly demonstrated by comparison of numerical data obtained for a correlation length equal to the magnetic length with those obtained for the lowest Landau level. However, no scaling behavior could be found for short correlation lengths in the first Landau level.

The question of the nature of the critical behavior at the metal-insulator transition in the center of disorder broadened Landau levels in a strong magnetic field has received widespread attention due to the discovery of the quantized Hall effect. Experimentally, it was observed that the width of the peaks in the longitudinal resistance in the integer quantum Hall effect as well as the slope of the Hall resistance between consecutive quantized plateaus show power law scaling as a function of temperature [1]. The corresponding exponent κ was found in first experiments to have a universal value of 0.42 ± 0.04 even for the transition between fractional Hall plateaus [2]. It is related to the exponents ν of the localization length and $p/2$ of the phase coherence length by $\kappa = p/(2\nu)$. However, in other experiments using different samples it was found that the measured exponents κ depended both on the Landau-level index as well as on the particular doping of the sample [3, 4].

In a recent experiment Koch *et al.* were able to study the scaling of the peak width of ρ_{xx} as a function of system size directly, thus achieving an independent determination of ν and κ [5]. The measured value of $\nu = 2.3 \pm 0.1$ agrees well with the results of theoretical calculations for the lowest Landau level [6-8], while the values for p are not universal and range from 2.7 ± 0.3 to 3.4 ± 0.4.

Previously we have shown that in the lowest Landau level the normalized localization length of finite systems exhibits a single-parameter scaling relation both in the case of a

δ-correlated disorder potential and for a Gaussian correlated potential of range l_c, where l_c is the magnetic length $\hbar/eB$ [6,9]. The random Landau matrix model employed in these papers as well as in the present work assumes that the effects of Landau-level mixing can be neglected and that the scaling behavior is independent of higher-order correlation functions of the disorder potential. It was also shown that the network model of Chalker and Coddington that is related to the case of very long correlation length belongs to the same universality class [10]. However, preliminary results for δ-correlated disorder potentials projected onto the second lowest Landau level showed no sign of scaling behavior [11].

In this paper the question of universality of the scaling behavior in higher Landau levels will be discussed. More extensive numerical calculations for the second lowest Landau level $n = 1$ both for zero and finite correlation lengths will be presented. The main results are that there exists a correlation length above which the scaling behavior is universal for every Landau level. For $n = 1$ this length is numerically found to be $\lesssim l_c$. For correlation lengths small compared to the magnetic length no scaling behavior is observed. Possible explanations for this behavior are discussed.

In order to study the scaling behavior in higher Landau levels the random Landau matrix model as described in ref. [6,12] is generalized for higher Landau levels. Given a Gaussian correlation function of the disorder potential

$$\overline{V(\boldsymbol{r})\,V(\boldsymbol{r}')} = \frac{V_0^2}{2\pi\sigma^2}\exp\left[\frac{|\boldsymbol{r}-\boldsymbol{r}'|^2}{2\sigma^2}\right], \tag{1}$$

the correlation function of the Hamiltonian projected onto the N-th Landau level becomes

$$\overline{\langle Nk_1|V|Nk_2\rangle\langle Nk_3|V|Nk_4\rangle} = \frac{V_0^2}{2\sqrt{2\pi}\sigma L_y}\delta_{k_1-k_2,\,k_4-k_3}\exp\left[-\frac{(k_1-k_2)^2\sigma^2}{2}\right]\cdot$$
$$\cdot\int \mathrm{d}x\int \mathrm{d}x'\exp\left[-\frac{(x-x')^2}{2\sigma^2}\right]\varphi_{N,k_1}(x)\,\varphi_{N,k_2}(x)\,\varphi_{N,k_3}(x')\,\varphi_{N,k_4}(x'), \tag{2}$$

where L_y is the width of periodic strip, the $\varphi_{N,k}(x) = \phi_N((x-kl_c^2)/l_c)$ are harmonic-oscillator functions and the Landau gauge $\boldsymbol{A} = (0, Bx)$ was used [12]. Matrix elements with this correlation function are generated from uncorrelated, complex random variables $u_0(x, k)$ by

$$\langle Nk_1|V|Nk_2\rangle = \frac{V_0}{\sqrt{2\pi L_y\sigma}}\exp\left[-\frac{(k_1-k_2)^2\sigma^2}{4}\right]\cdot$$
$$\cdot\int \mathrm{d}x\int \mathrm{d}x'\,\varphi_{N,k_1}(x)\,\varphi_{N,k_2}(x)\exp\left[-\frac{(x-x')^2}{\sigma^2}\right]u_0(x', k_1-k_2), \tag{3}$$

and

$$\overline{u_0(x,k)\,u_0(x',k')} = \delta(x-x')\,\delta_{k,-k'}. \tag{4}$$

After extracting the Gaussian factors from the oscillator functions and some algebra eq. (3) becomes

$$\langle Nk_1|V|Nk_2\rangle = \frac{V_0\beta l_c}{\sqrt{2\pi L_y\sigma}} \exp[-\Delta\kappa^2] \int d\xi\, u_0\left(\beta\xi + \frac{K}{\beta}, 2\frac{\Delta\kappa}{\beta}\right) \exp[-\xi^2]\cdot$$
$$\cdot\frac{1}{2^N N!\sqrt{\pi}} \int d\eta \exp\left[-\frac{l_c^2+\sigma^2}{\sigma^2}\eta^2\right] H_N\left(\eta + \frac{\xi}{\beta} - \frac{\Delta\kappa}{\beta}\right) H_N\left(\eta + \frac{\xi}{\beta} + \frac{\Delta\kappa}{\beta}\right), \tag{5}$$

with $\beta^2 = (\sigma^2 + l_c^2)/l_c^2$, $\Delta\kappa = (k_1 - k_2)\, l_c\beta/2$ and $K = (k_1 + k_2)\, l_c\beta/2$. $H_N(x)$ are the Hermite polynomials. $\langle Nk_1|V|Nk_2\rangle$ is in general a N-dependent function of $k_1 - k_2$ and $k_1 + k_2$. In the limit $\sigma \to \infty$, however, the matrix elements become independent of the Landau-level index, since eq. (5) reduces to

$$\langle Nk_1|V|Nk_2\rangle = \frac{V_0\beta l_c}{\sqrt{2\pi L_y\sigma}} \exp[-\Delta\kappa^2] \int d\xi\, u_0\left(\beta\xi + \frac{K}{\beta}, 2\frac{\Delta\kappa}{\beta}\right) \exp[-\xi^2]. \tag{6}$$

If we assume that higher correlation functions are irrelevant for scaling, this implies that in this limit the scaling behavior is indeed independent of the Landau-level index. Correspondingly, the critical exponent obtained in the semi-classical limit [13] coincides with the numerical value for short-ranged potentials in the lowest Landau level and the network model of Chalker and Coddington shows the same scaling behavior as the random Landau matrix model in the lowest Landau level [6, 10].

In order to get more detailed insight into the scaling behavior in higher Landau levels for shorter correlation lengths, we performed numerical calculations of the exponential decay length $\lambda_M(E)$ of the modulus of the single-particle Green's function for very long cylinders of circumference M in the second lowest $n = 1$ Landau band. In fig. 1 and 2 the ratio λ_M/M is

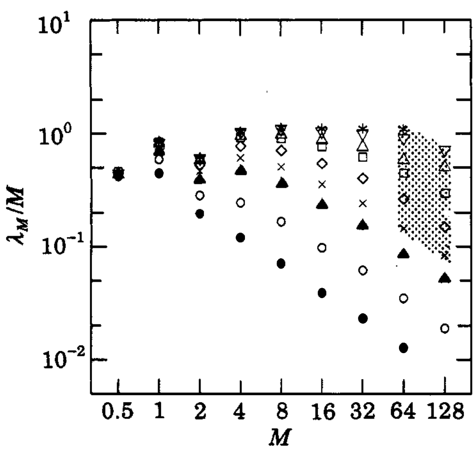

Fig. 1.

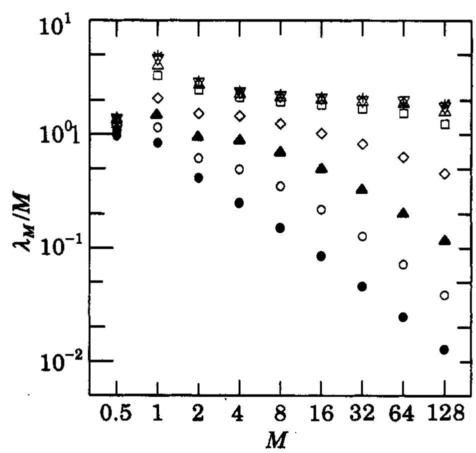

Fig. 2.

Fig. 1. – The normalized exponential decay length λ_M/M as a function of system width M in the first Landau band and $\beta^2 = 2$ for energies 0.01 ($*$), 0.03 ($\triangledown$), 0.05 ($\triangle$), 0.07 ($\square$), 0.1 ($\diamond$), 0.14 ($\times$), 0.18 ($\blacktriangle$), 0.3 ($\circ$) and 0.5 ($\bullet$).

Fig. 2. – The normalized exponential decay length λ_M/M as a function of system width M in the first Landau band and $\beta^2 = 1$ for energies 0.01 ($*$), 0.1 ($\triangledown$), 0.18 ($\triangle$), 0.3 ($\square$), 0.5 ($\diamond$), 0.65 ($\blacktriangle$), 0.8 ($\circ$) and 1.0 ($\bullet$).

plotted as a function of M for different energies. In fig. 1 the correlation length is one magnetic length, while in fig. 2 the disorder potential is δ-correlated. The difference between these two cases are striking. The results in fig. 1 look very similar to those obtained for the lowest Landau band [9]. In fact, the data from the dotted region of fig. 1 scale to the same scaling function. Figure 3 shows all the data, those in fig. 1 and those obtained previously [6, 9, 10], fitted by a single curve and the insert shows the power law divergence of the localization length $\xi(E)$ for $n = 0$, $\sigma = 0$, l_c, and $n = 1$, $\sigma = l_c$. The common slope of the curves in the insert is given by the critical exponent $\nu = 2.35 \pm 0.03$. For small values of M/ξ the scaling function can be well approximated by $\lambda_M/M = (\lambda_M/M)_c + a(M/\xi)^{1/\nu}$, with the fixed-point value $(\lambda_M/M)_c = 1.19 \pm 0.04$. It should be emphasized here that the decision whether or not data scale is made using a quantitative procedure taking into account the statistical uncertainties of the data [14]. This procedure allows for the decision that the data in the tails of the band and for smaller system sizes—outside the dotted region in fig. 1—do not scale.

Universal scaling is completely missing from the data shown in fig. 2. This was insured by the same quantitative procedure as that used to prove the scaling of the data in the dotted region of fig. 1. Scaling behavior with a different scaling function and critical exponent cannot completely be ruled out. The data for energies between 0.3 and 0.8 and system sizes between 32 and 128 can be fitted by a scaling function with an exponent of about 6.4. Due to the small number of data that can be fitted, the statistical significance of the fit is very poor. Since, furthermore, the energy range of the data that can be fitted is very narrow and far away from the center of the band, it seems unlikely that this fit reflects the actual critical behavior as was suggested by Ando and Aoki [8] and Mieck [15]. Still the localization length seems to diverge in the center of the band for infinite system size and becomes constant in the tails of the bands. However, due to the absence of any observable scaling a reasonable extrapolation

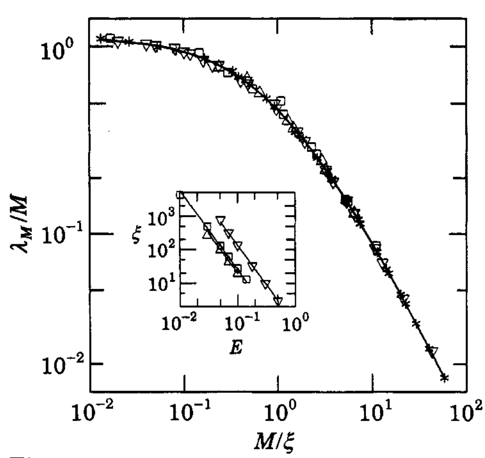

Fig. 3.

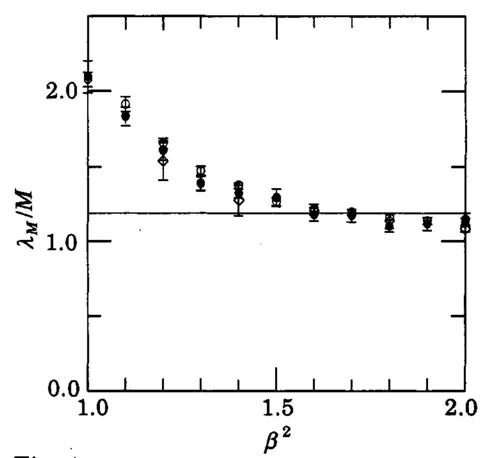

Fig. 4.

Fig. 3. – The normalized exponential decay length λ_M/M as a function of the scaling variable $M/\xi(E)$. The data plotted are taken from the dotted area of fig. 1 and ref. [6, 9, 10]. The corresponding parameters and symbols are: $n = 0$, $\sigma = 0$ ($\triangledown$), $n = 0$, $\sigma = l_c$ ($\triangle$), $n = 1$, $\sigma = l_c$ ($\square$), and the network model ($*$). The solid curve approximates the scaling function. The insert shows the localization length $\xi(E)$.

Fig. 4. – The normalized exponential decay length λ_M/M for energy $E = 0.01$ as a function of β^2 for system sizes $M = 16$ ($\circ$), $M = 32$ ($\bullet$) and $M = 64$ ($\diamond$). The horizontal line is the fixed-point value of the scaling function in fig. 3. Error bars are $\pm$ 2 standard deviations.

to infinite system size is not possible. For energies close to the band center within about 30% of the bandwidth the data show very little energy and system size dependence. A further remarkable feature of fig. 2 is that λ_M/M in the band center does not significantly decrease with system size, and is even for the largest system sizes about a factor of two larger than the fixed-point value in fig. 3. If the absence of scaling were just a finite-size effect, then this value would have to reach its fixed-point value for sufficiently large system sizes above which scaling would be observed. The data suggest that, without any abrupt change in the system size dependence, this critical length scale would have to be exponentially large compared to that in the lowest Landau level and for finite correlation length in the first Landau level.

Figure 4 shows the dependence of the value of λ_M/M close to the center of the band as a function of the correlation length for three different system sizes. Changing the system width by a factor of 4 does not significantly change λ_M/M. While for $\beta^2 \geqslant 1.6$ the data are consistent with the fixed-point value, the larger value for smaller correlation lengths shows the absence of scaling behavior in this regime. The lack of system size dependence in this plot again does not suggest that going to moderately larger system sizes would restore universal scaling.

The numerical data of fig. 2 are compatible with the existence of a band of critical states in the center of the first Landau level with a width of the order of one third of the bandwidth. With «critical states» we denote states for which $\lambda_M \propto M$. There is certainly no indication for any extended states since the exponential decay length λ_M grows at most as fast as M. Another explanation for the absence of scaling for the presented system sizes would be the existence of a critical length scale several orders of magnitudes larger than the magnetic length for the δ-correlated potential in the first Landau level. For $\beta^2 = 1$ this length scale is about 64 (in units of $\sqrt{2\pi}l_c$). From fig. 4 one would expect this length scale to be of that order for β^2 down to about 1.5. Below this value the length scale would have to rise very rapidly, since even near $\beta^2 = 1.5$ no size dependence is observed. However, there is no obvious reason for such a behavior of the critical length.

In conclusion, we have presented evidence for the universality of the scaling behavior of the localization length in the quantum Hall regime for sufficiently large correlation length of the disorder potential. Universal scaling is absent in numerical calculations for δ-correlated potentials projected onto the first Landau level.

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