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To cite this article: M. Konopik *et al* 2020 *EPL* **131** 60004

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Nonequilibrium information erasure below $kT \ln 2$

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received 16 June 2020; accepted in final form 7 September 2020

published online 2 November 2020

PACS 05.70.Ln – Nonequilibrium and irreversible thermodynamics

PACS 42.50.Wk – Mechanical effects of light on material media, microstructures and particles

Abstract – Landauer’s principle states that information erasure requires heat dissipation. While Landauer’s original result focused on equilibrium memories, we here investigate the reset of information stored in a nonequilibrium state of a symmetric two-state memory. We derive a nonequilibrium generalization of the erasure principle and demonstrate that the corresponding bounds on heat and work may be reduced to zero. We further introduce reset protocols that harness energy and entropy of the initial preparation and so allow to reach these nonequilibrium bounds. We finally provide numerical simulations with realistic parameters of an optically levitated nanosphere memory that support these findings. Our results indicate that local dissipation-free information reset is possible away from equilibrium.

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Introduction. – According to the standard (equilibrium) formulation of Landauer’s principle, the erasure of one bit of information generates at least $kT \ln 2$ of heat and consumes the same amount of work [1]. Here T is the temperature of the environment to which the memory device is coupled, k the Boltzmann constant and $\ln 2$ the information content of one bit. Information erasure is thus unavoidably dissipative [2,3]. Landauer’s principle is a central result of the thermodynamics of information that applies to all logically irreversible transformations [4,5]. It additionally imposes a fundamental physical limit to the downsizing of binary switches, such as field effect transistors [6–8]. The existence of the Landauer bound has been established in a number of experiments in which two-state memories have been realized with an optical tweezer [9], an electrical circuit [10], a feedback trap [11] and nanomagnets [12,13]. Meanwhile, growing energy consumption and dissipation in modern integrated electronics has become a major technological challenge that threatens future progress [6–8]. It has recently been shown that the work required for erasure may be reduced to zero in nonequilibrium asymmetric memories in the overdamped regime, thus allowing to reduce energy consumption [14–16]. However, these findings do not address the more pressing issue of control and suppression of heat dissipation.

The study of nonequilibrium memories is not purely academic. Two different types of electronic storage devices

are usually distinguished [17]. Read-only memories (ROM) (and their variants EPROM and EEPROM¹) are non-volatile memories that retain the information stored in them in the absence of a power source. Information is here encoded in an equilibrium state, as considered in Landauer’s original principle [1]. By contrast, random-access memories (RAM), the most common memories in modern computers, are volatile and the stored data is lost when power is switched off. Information is in this case encoded in a nonequilibrium state, whose preparation requires a given amount of energy and entropy. At the same time, novel switching devices, beyond the standard FET technology, are currently being explored in order to decrease power dissipation [18]. Promising examples include tunable nanomechanical oscillators that operate in the weakly damped regime [19–22].

Motivated by these observations, we here perform a detailed investigation of nonequilibrium information erasure. We first derive a generalization of Landauer’s principle that holds for information initially stored in a nonequilibrium state. We show that both heat and work associated with the reset process may be theoretically reduced below the equilibrium Landauer bounds of $kT \ln 2$, provided the preparation energy and entropy are properly harnessed.

¹EPROM is an erasable programmable read-only memory and an EEPROM is an electrically erasable programmable read-only memory [17].

Both quantities may even change sign, indicating that work may be produced and heat absorbed with the help of the prepared initial state. We stress that these findings do not violate the second law, but directly follow from it when applied to the considered nonequilibrium situation. We further introduce novel erasure protocols that allow us to reach these nonequilibrium bounds in a generic, symmetric double-well potential. We finally discuss a possible experimental verification of the nonequilibrium erasure principle in the underdamped regime using an optically levitated nanosphere [23] and provide extensive numerical simulations of the process with realistic parameters.

Nonequilibrium erasure principle. – We begin by analyzing an erasure cycle that consists of a preparation and a reset phase. To that end, we consider a general memory device weakly coupled to a heat bath at temperature T . The total entropy change for system and bath during the reset phase is $\Delta S_{\text{res}} = \Delta S_{\text{mem}} + \Delta S_{\text{bat}} \geq 0$ [24]. Owing to its large size, the bath always remains in equilibrium and thus $\Delta S_{\text{bat}} = Q/T$, where Q is the heat dissipated into the environment. The system is assumed to be initially in a nonequilibrium state with phase-space distribution $\rho(x, p, 0)$ where x denotes the position and p the momentum. After reset of duration τ , the memory is in state $\rho(x, p, \tau)$. The work done on the system during reset is $W = \Delta F_{\text{eq}} + T\Delta I + T\Delta S_{\text{res}}$ [25–27], where ΔF_{eq} is the equilibrium free energy difference and $I(t) = S(\rho(t) || \rho_{\text{eq}}(t)) = -k \int dx dp \rho(t) \ln[\rho(t)/\rho_{\text{eq}}(t)]$ the relative entropy between the nonequilibrium state $\rho(t)$ and the corresponding equilibrium state $\rho_{\text{eq}}(t)$. The entropic distance I may be interpreted as the amount of information needed to prepare $\rho(t)$ from $\rho_{\text{eq}}(t)$ [28]. When the potential of the memory device is the same before and after reset, initial and final equilibrium states are equal, $\rho_{\text{eq}}(0) = \rho_{\text{eq}}(\tau)$. The free energy difference therefore vanishes, $\Delta F_{\text{eq}} = 0$. Using the first law, the dissipated heat may next be written as $Q = W - \Delta U_{\text{res}} = T\Delta I + T\Delta S_{\text{res}} - \Delta U_{\text{res}}$, where ΔU_{res} is the variation of internal energy during reset. For a complete erasure cycle consisting of preparation and reset, we have $\Delta U = \Delta U_{\text{pre}} + \Delta U_{\text{res}} = 0$. According to the second law, the total entropy production is nonnegative, $\Delta S_{\text{res}} \geq 0$ [24]. As a result, we obtain the following two inequalities for heat and work:

$$Q \geq Q_L = T[I(\tau) - I(0)] - \Delta U_{\text{res}}, \quad (1)$$

$$W \geq W_L = T[I(\tau) - I(0)]. \quad (2)$$

Equations (1), (2) are nonequilibrium generalizations of Landauer’s erasure principle to which they reduce for initial and final equilibrium states that correspond to $I(\tau) - I(0) = k \ln 2$ and $U(0) = U(\tau)$. They may be compactly written as $Q \geq \Delta \mathcal{F} - \Delta U_{\text{res}}$ and $W \geq \Delta \mathcal{F}$, where $\mathcal{F} = F + TI$ is the information free energy [5]. We observe that the nonequilibrium Landauer bounds for heat and work, Q_L and W_L , can in principle be controlled through the initial entropic distance to equilibrium $I(0)$ and the preparation energy, $\Delta U_{\text{pre}} = -\Delta U_{\text{res}}$. This opens

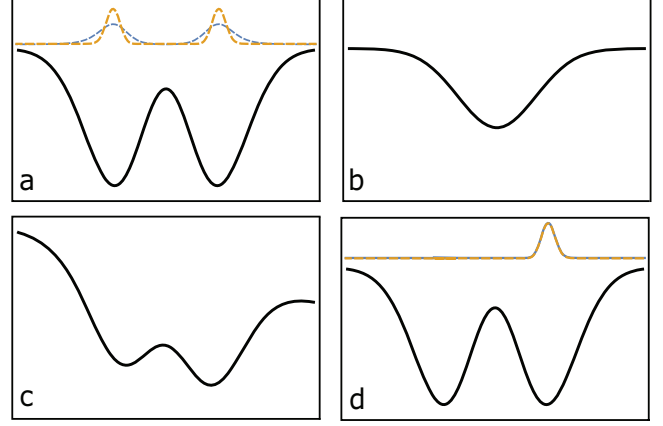


Fig. 1: Two-state memory. Double-well potential (4) used as a generic symmetric two-state memory. (a) Initially, 1 bit of information is stored in a configuration where the two potential wells are occupied with equal probability $1/2$. Blue (yellow) dashed lines represent equilibrium (nonequilibrium) distributions used for storage. (b)–(d) Information is reset by bringing the particle with probability 1 to the right well in a metastable state by cyclically lowering the barrier and applying a tilt.

the fascinating possibility to reduce both the amounts of heat and work required for reset below the equilibrium value of $kT \ln 2$. The two essential questions that we here address are a) whether the nonequilibrium Landauer bounds (1), (2) can be actually reached in practice and b) if yes, how?

To answer these questions, we investigate a Brownian particle in a symmetric double-well potential. Such a two-state system may be regarded as a generic model for an elementary memory and has been employed in the experiments [9,11–13]. We describe the dynamics of the particle with the underdamped Langevin equation [29],

$$m\ddot{x} + \gamma\dot{x} + V'(x, t) - Af(t) = F(t), \quad (3)$$

where m is the mass of the particle, γ the friction coefficient, $f(t)$ a tilting force with amplitude A , and $F(t)$ a centered white noise force with variance $\langle F(t)F(t') \rangle = 2m\gamma kT \delta(t - t')$. For concreteness, the symmetric double-well potential $V(x, t)$ is taken to be of the form (fig. 1)

$$V(x, t) = - \left[h(t)a + g(t)\frac{b}{2}x^2 \right] \exp \left(\frac{-cx^2}{2} \right), \quad (4)$$

with a tunable barrier height via $g(t)$ and a tunable barrier width via the function $h(t)$. Such a potential appears naturally in the optomechanical setup discussed below. We stress that our findings do not depend on the specific shape of the double-well potential nor on the level of damping.

Equilibrium erasure protocol. – To put our nonequilibrium results into proper perspective, we first investigate commonly used equilibrium erasure protocols [9,11–13]. The particle is initially prepared to occupy either of the two wells with equal probability $1/2$

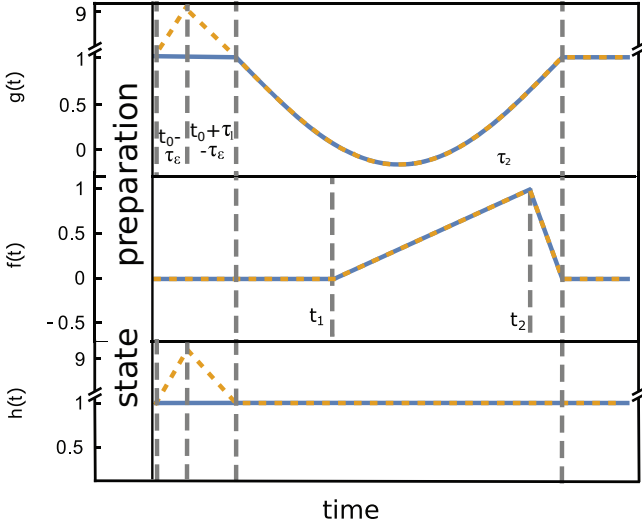


Fig. 2: Reset protocols. The sawtooth function $f(t)$, eq. (5), applies the tilt toward the right well, while the function $g(t)$, eq. (6), cyclically modulates the barrier height. The barrier width is controlled by $h(t)$. The blue (solid) lines show the equilibrium protocol (5), (6) and the yellow (dashed) lines the nonequilibrium protocol (9), (10) designed to harness the nonequilibrium preparation energy and entropy.

(fig. 1). In this configuration $S_{\text{mem}}(0) = k \ln 2$, $I(0) = 0$ and the memory stores one bit of information. That information is erased by modulating the shape of the confining potential during time τ such that the particle ends up with probability 1 in one of the wells, $S_{\text{mem}}(\tau) = 0$, $I(\tau) = k \ln 2$, irrespective of its initial location [30]. The reset operation is implemented by decreasing the height of the barrier via $g(t)$ and applying the tilt $f(t)$ in a cyclic manner, $h(t) = 1$ throughout this process (fig. 2, blue solid lines) [31]:

$$f(t) = \begin{cases} (t - t_1)/(t_2 - t_1), & t_1 < t \leq t_2, \\ 1 - (t - t_2)/(\tau - t_2), & t_2 < t \leq \tau, \\ 0, & \text{otherwise,} \end{cases} \quad (5)$$

$$g(t) = \begin{cases} 1 - B \sin \left[\frac{\pi(t - t_0)}{\tau - t_0} \right], & t_0 < t \leq \tau, \\ 1, & \text{otherwise.} \end{cases} \quad (6)$$

The parameter B controls the amplitude of the barrier lowering. Erasure protocols of this type have been implemented in the recent experiments [9,11–13], where information was encoded in an initial equilibrium state. We therefore call them equilibrium erasure protocols.

We choose the initial nonequilibrium distribution of the symmetric memory device to be given by (fig. 1)

$$\rho(x, p, 0) = \frac{1}{Z'_{b+\varepsilon}} \exp \left[-\beta \left(\frac{p^2}{2m} + V'_{b+\varepsilon}(x) \right) \right], \quad (7)$$

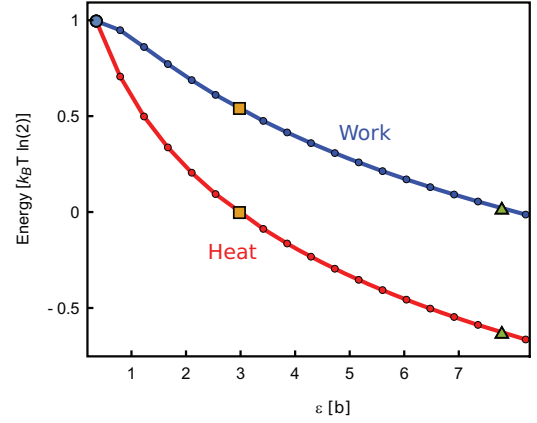


Fig. 3: Nonequilibrium Landauer bounds. Heat Q_L (red), eq. (1), and work W_L (blue), eq. (2), as a function of the parameter ε that quantifies the departure from equilibrium of the initial state, eq. (7). Both nonequilibrium bounds decrease and become negative with increasing values of ε .

where $Z'_{b+\varepsilon}$ is the normalization constant and $\beta = 1/(kT)$ the inverse temperature. The modified potential reads

$$V'_{b+\varepsilon}(x) = - \left[a'(b + \varepsilon) + \frac{b + \varepsilon}{2} x^2 \right] \exp \left(-\frac{cx^2}{2} \right), \quad (8)$$

with $a'(b') = b'/(2c)(2 - c\bar{x}^2)$ and $\bar{x} = \sqrt{2/(bc)(b - ca)}$ the fixed position of the potential minima (fig. 1). Equations (7) and (8) are chosen in order to decrease energy and entropy of the initial nonequilibrium state as compared to the equilibrium state. In particular, the nonequilibrium state is narrower than the corresponding equilibrium state (fig. 1). The parameter ε controls the departure from equilibrium and eq. (7) reduces to the equilibrium distribution $\rho_{\text{eq}}(x, p, 0)$ for $\varepsilon = 0$. The initial nonequilibrium state (7) may be prepared by letting the system equilibrate in the modified potential (8) and then instantaneously, that is, much faster than the relaxation time of the system, switching to the unmodified potential (4). The energetic cost of this preparation is ΔU_{pre} . Additional energy may be dissipated if the initial state preparation is done in finite time.

In order to study the approach to the nonequilibrium Landauer bounds (1), (2), we simulate the reset process by numerically solving the Langevin equation (3) for the protocols (5) and (6) with experimentally realistic parameters, using a 4th-order Runge-Kutta method (see the Supplementary Material [Supplementarymaterial.pdf](#) (SM)). The starting points of the simulations are randomly generated according to $\rho(x, p, 0)$, eq. (7). The final distribution $\rho(x, p, \tau)$ is determined from the end points of the simulated trajectories. We evaluate the stochastic work as $\tilde{W} = \int dt \partial V(x, t)/\partial t$ and the stochastic heat from the first law, $\tilde{Q} = \Delta \tilde{U} - \tilde{W}$ [31,32]. We calculate their respective mean values, W and Q , by averaging of many ($\sim 10^5$) trajectories. Figure 3 shows the Landauer bounds Q_L and W_L , eqs. (1), (2), as a function of the parameter ε , for

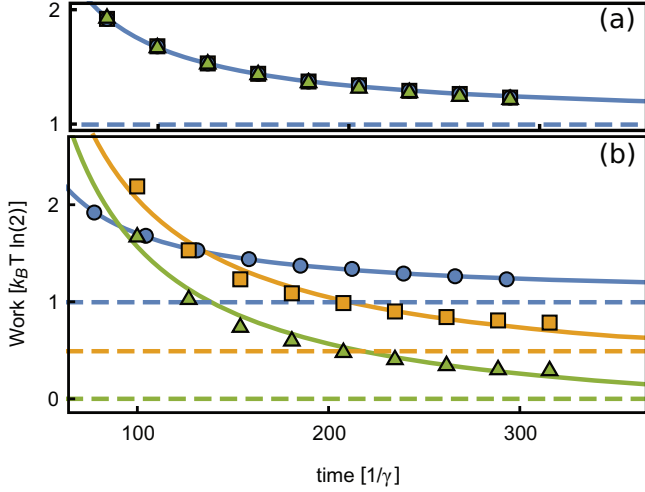


Fig. 4: Work consumed during reset. Work W as a function of the reset time τ for three values of the nonequilibrium parameter ε : $\varepsilon = 0$ (blue circles), $\varepsilon = 3b$ (yellow squares), $\varepsilon = 8.5b$ (green triangles). (a) is obtained with the equilibrium protocols (5), (6); (b) is obtained with the nonequilibrium protocols (9), (10). The two nonequilibrium curves are shifted in time by the duration of the nonequilibrium process, τ_ε (~ 20 ms), compared to the equilibrium one. The solid lines display a fit with the function $\sim 1/\tau$. The dashed lines show the nonequilibrium Landauer bound W_L (2).

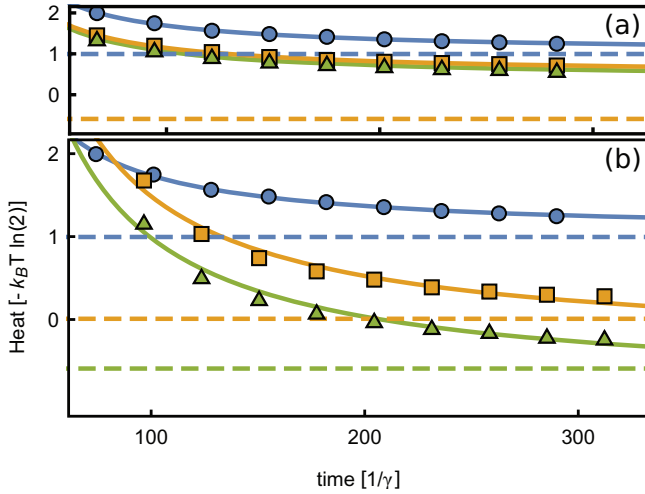


Fig. 5: Heat dissipated during reset. Heat Q as a function of the reset time τ for the same three values of the nonequilibrium parameter as in fig. 4. (a) is obtained with the equilibrium protocols (5), (6); (b) is obtained with the nonequilibrium protocols (9), (10). The solid lines display a fit with the function $\sim 1/\tau$. The dashed lines show the nonequilibrium Landauer bound Q_L (1).

an infinitely large τ , which corresponds to a final equilibrium state with $I(\tau) = 0$. As expected, both Q_L and W_L decrease with increasing ε , that is, with increasing initial departure from equilibrium $I(0)$, and eventually turn negative. The actual work and heat, W and Q , for the equilibrium reset process are plotted in figs. 4(a) and 5(a)

as a function of the duration τ for three different values of ε , indicated by the circle, square and triangle symbols in fig. 3. We observe that, for all values of the parameter ε , work W remains above the equilibrium Landauer bound of $kT \ln 2$ (horizontal dashed lines) in the limit of long times, while heat Q is only slightly reduced below that value. The nonequilibrium bounds (1) and (2) are thus not reached. By naively implementing the commonly used equilibrium protocols (5) and (6) [9,11–13], the extra energy and entropy that are required to prepare the initial nonequilibrium state $\rho(0)$ are simply dissipated at the beginning of the erasure process, and therefore lost.

Nonequilibrium erasure protocol. – In order to successfully harness the preparation energy and entropy to reduce work and heat dissipated during reset, we modify the equilibrium protocol $g(t)$, eq. (6), and $h(t)$ by adding an additional modulation of the potential at the start of the process (fig. 2, yellow dashed lines):

$$\bar{g}(t) = \begin{cases} 1 + \frac{b' - 1}{b} \frac{t'}{\tau_1}, & t_0 - \tau_\varepsilon < t \leq t_0 - \tau_\varepsilon + \tau_1, \\ 1 + \frac{b' - 1}{b} \left(\frac{\tau_\varepsilon - t'}{\tau_\varepsilon - \tau_1} \right), & t_0 - \tau_\varepsilon + \tau_1 < t \leq t_0, \\ 1 - B \sin \left[\frac{\pi(t - t_0)}{\tau - t_0} \right], & t_0 < t \leq \tau, \\ 1, & \text{otherwise,} \end{cases} \quad (9)$$

$$\bar{h}(t) = \begin{cases} 1 + \frac{a' - 1}{a} \frac{t'}{\tau_1}, & t_0 - \tau_\varepsilon < t \leq t_0 - \tau_\varepsilon + \tau_1, \\ 1 + \frac{a' - 1}{a} \left(\frac{\tau_\varepsilon - t'}{\tau_\varepsilon - \tau_1} \right), & t_0 - \tau_\varepsilon + \tau_1 < t \leq t_0, \\ 1, & \text{otherwise,} \end{cases} \quad (10)$$

with $b' = b + \varepsilon$ and $t' = t - t_0 + \tau_\varepsilon$. The constants τ_ε (~ 20 ms) and τ_1 (~ 0.2 ms) depend in general on the parameter ε . They are chosen in order to effectively extract the preparation energy and entropy of the initial nonequilibrium state. Figures 4(b) and 5(b) show the erasure work W and the dissipated heat Q computed with the nonequilibrium erasure protocol (9), (10) for the same three values of ε as before (the tilting function $f(t)$, eq. (5), is kept unmodified). We see that both quantities now asymptotically approach the nonequilibrium Landauer bounds (1), (2) for higher values of time. In particular, heat vanishes for $\varepsilon = 3b$ and work vanishes for $\varepsilon = 8.5b$. In these cases, the memory has been reset without work and heat dissipation.

Experimental setup. – To experimentally verify the nonequilibrium Landauer principle (1) and (2), we propose to use an optically levitated nanoparticle. The two main advantages of this setup are that the optical confining potential can be flexibly tuned and that the underdamped

regime, described by the Langevin equation (3), is easily accessible. While first experiments on optical levitation have already been realized in the early 1970s [33], more recently an excellent experimental control in ultra-high vacuum has been demonstrated in optical tweezers [34]. In order to implement a double-well potential with a controlled barrier height $g(t)$, we suggest to form an optical trap by the combination of a TEM_{00} and a TEM_{01} mode inside a Fabry-Perot cavity, leading to a potential of the form (4) (alternative options to create complex potential landscapes may be found in ref. [35]). This configuration ensures particularly low intensity fluctuations of the optical trap and the additional cavity power enhancement allows to use a wide trap (here approximately 80 micrometers) with low driving powers of only a few milliwatts. Stable optical trapping directly inside an optical cavity has been lately successfully demonstrated in refs. [36–38].

The height function $g(t)$, eqs. (6) and (9), may be changed by varying the power of the cavity modes, while the tilt $f(t)$, eq. (5), may be implemented using the radiation pressure from a cavity-independent light source. The feasibility of this approach is underlined by a similar strategy that has been recently followed to achieve radial feedback cooling of levitated microparticles [39]. The experimental parameters chosen for the numerical simulations (table 1 in the SM) are taken from ref. [36]. Additional information on experimental details and position readout may be found in the SM.

Discussions. – We have studied the erasure of information encoded in a nonequilibrium state of a symmetric memory. We have concretely derived a nonequilibrium extension of Landauer’s principle and shown that the corresponding bounds for heat and work, eqs. (1) and (2), may both be reduced to zero. Using a generic model based on an underdamped Brownian particle in a double-well potential, we have demonstrated that these nonequilibrium limits may be reached by properly tuning the reset protocol to harness the initial preparation energy and entropy. We have further performed detailed numerical simulations of the nonequilibrium Landauer principle with realistic parameters using an optically levitated nanosphere to support these findings. In contrast to the standard equilibrium situation, the complete nonequilibrium erasure cycle here consists of distinct preparation and reset stages (the preparation stage being absent for equilibrium erasure). This offers new and powerful means to control the thermodynamics of logically irreversible operations. By, for instance, considering a computer architecture where preparation and processing zones are spatially separated [40], logically irreversible transformations, such as the reset-to-one operation, could be performed locally at no energetic cost and without dissipating any heat. The thermodynamic cost associated with the generation of the initial nonequilibrium state would be restricted to the remote preparation zone before the bit is transferred to the

processing area. Remarkably, while information erasure can never be performed for free, going away from equilibrium permits local dissipation-free logically irreversible reset.

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