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A Compact 2 Degree-of-Freedom Energy Harvester with Cut-Out Cantilever Beam

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In this work, a novel 2 degree-of-freedom (DOF) vibration energy harvester is proposed. The harvester comprises one main cantilever beam and one secondary cantilever beam cut out within the main beam. By varying the proof masses, the first two resonances can be tuned close to each other, while maintaining significant magnitudes, thus providing a useful wide bandwidth for energy harvesting. Unlike previous 2-DOF harvesters, the proposed harvester is compact and utilizes the beam more efficiently by generating energy from both the main and secondary cantilevers. Therefore, the proposed harvester is more adaptive and functional in practical random or frequency-variant vibrational circumstances.

Harvesting energy from ambient vibrations using piezoelectric materials is a promising alternative solution for powering small electronics like wireless sensors. A conventional piezoelectric energy harvester (PEH) usually consists of a cantilever beam with a proof mass at its free end [e.g., Fig. 1(a)]. For such a device, its high-order modes are usually neglected because they are far away from the first mode and provide much lower response level as compared to the first mode. Thus, only the first mode of the harvester is exploited for energy harvesting and the harvester is usually regarded as a single-degree-of-freedom (SDOF) system. Considering the wide bandwidth of practical vibrations in the environment, a conventional PEH using only single mode is definitely inefficient. To overcome this limitation, many researchers have attempted to develop systems with the capability of broadband energy harvesting. As summarized in a review article,[1] various approaches have been proposed for broadband vibration energy harvesting using different techniques such as multi-modal harvesting,[2–8] resonance tuning approach,[9,10] and non-linear technique.[11,12]

Various designs for multi-modal energy harvesting have been reported in the literature. Shahruz[2] and Ferrari *et al.*[3] proposed similar systems comprising an array of cantilevers with various lengths and tip masses. These cantilevers with different working frequencies can be carefully designed to cover certain range of frequency to achieve a broader bandwidth. Such design however significantly increases the volume and weight of the system, which not only sacrifices the power density but also limits its applicability. Rather than using the cantilever array configuration, some researchers have developed multiple DOF harvesters based on one single beam. Tadesse[4] presented a design of multi-modal energy harvesting beam employing both electromagnetic and piezoelectric transduction mechanisms, each of which was efficient for a specific mode. Ou *et al.*[5] presented a 2-DOF PEH by using a two-mass cantilever beam. Although two useful modes were obtained, they were quite far apart at 25 and 150 Hz, respectively. Arafa *et al.*[6] proposed a 2-DOF PEH in which a dynamic magnifier was adopted. It magnified the power output but could not achieve two close working frequencies unless an impractical huge magnifier was employed. Erturk *et al.*[7] developed a PEH with L-shaped beam configuration where the second natural frequency approximately doubled the first.

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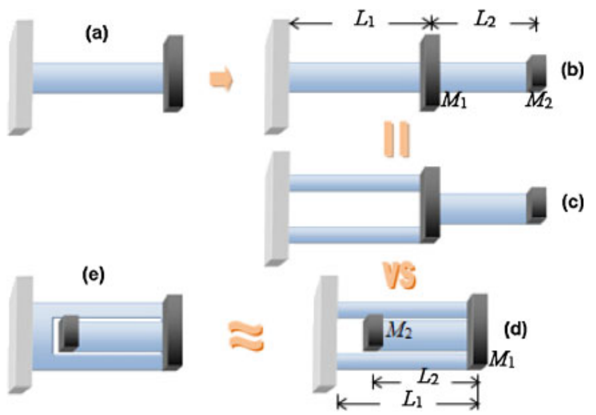

Fig. 1. (Color online) Comparison of (a) SDOF cantilever beam, (b) previous continuous cantilever beam, (c) equivalent continuous cantilever beam, (d) simplified cut-out cantilever beam, and (e) actual cut-out beam tested in experiment.

The objective of a broadband PEH is to achieve close resonances that have significant magnitudes for effective energy conversion. However, most of the above 2-DOF designs can only achieve resonances far away from each other with the second peak much smaller than the first. Kim *et al.*[8] developed a 2-DOF PEH that could achieve two close resonances, but this design required an additional vibration body to be attached to two cantilevers, which increased the volume as well as the complexity of the harvester. In this work, a novel compact 2-DOF energy harvester is developed aiming at achieving two close resonances with significant magnitudes. This harvester comprises one main cantilever and an enclosed secondary cantilever, as shown in Fig. 1(e). It can be conveniently fabricated from a conventional SDOF harvester by cutting out the inner beam and attaching an additional proof mass. This configuration is referred to as "cut-out" beam hereafter. Experiment has been performed to prove this concept.

The fundamental difference between our proposed harvester and previous 2-DOF harvesters is that the secondary beam is enclosed within the main beam rather than extended outwards from the main beam. This geometric discrepancy results in difference in the stiffness matrix of the proposed and previous 2-DOF harvesters. To illustrate this point, we compare our simplified cut-out cantilever beam model [Fig. 1(d)] with the previous continuous cantilever beam model [Fig. 1(b) or 1(c)].

The main cantilever beam (length L_1) in the cut-out configuration [Fig. 1(d)] is assumed to have the same elastic modulus, thickness and overall width as the secondary beam (length L_2). Thus, the flexural rigidity EI is uniform throughout. This assumption applies to the continuous configuration in Fig. 1(c). Although there is a slight difference between the simplified model [Fig. 1(d)] and the actual model [Fig. 1(e)] tested in experiment due to the different width at the root of main beam, it can be neglected as we only use the simplified model to illustrate the difference of natural frequencies between the cut-out beam and continuous beam. Both configurations can be modeled as the lumped parameter system by neglecting the distributed mass of the cantilever beam. The mass matrixes are the same for both

$$[M] = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} = M_1 \begin{bmatrix} 1 & 0 \\ 0 & \alpha \end{bmatrix}. \tag{1}$$

The stiffness matrix of the cut-out configuration is

$$[K]_a = \frac{6EI}{(3\beta^2 + 4\beta^3)L_1^3} \begin{bmatrix} 2 - 6\beta + 6\beta^2 + 2\beta^3 & 3\beta - 2 \\ 3\beta - 2 & 2 \end{bmatrix}. \tag{2}$$

The stiffness matrix of the continuous configuration is

$$[K]_b = \frac{6EI}{(3\beta^2 + 4\beta^3)L_1^3} \begin{bmatrix} 2(1+\beta)^3 & -3\beta - 2 \\ -3\beta - 2 & 2 \end{bmatrix}. \tag{3}$$

The non-dimensional parameter α denotes the proof mass ratio M_2/M_1, while β denotes the ratio L_2/L_1. Solving the eigenvalue problem of the two configurations, we can obtain two roots of ω^2 where ω is the natural frequency. The non-dimensional difference of the two roots can be written as

$$\frac{\Delta(\omega^2)^a_{1,2}}{\omega_s^2} = \frac{4\sqrt{[\alpha(1 - 3\beta + 3\beta^2 + \beta^3) + 1]^2 - \alpha(3\beta^2 + 4\beta^3)}}{\alpha(3\beta^2 + 4\beta^3)}, \tag{4}$$

$$\frac{\Delta(\omega^2)^b_{1,2}}{\omega_s^2} = \frac{4\sqrt{[\alpha(1 + 3\beta + 3\beta^2 + \beta^3) + 1]^2 - \alpha(3\beta^2 + 4\beta^3)}}{\alpha(3\beta^2 + 4\beta^3)}. \tag{5}$$

Here $\omega_s = \sqrt{3EI/L_1^3 M_1}$ denotes the natural frequency of the SDOF cantilever beam with length L_1 and proof mass M_1. Note that the only difference in the above equations is that the term 3β has opposite sign. For both configurations, when α approaches zero, two resonance frequencies approach each other. This means that the mass for the secondary beam decreases to zero, thus degrading the system to a SDOF system, which is of no interest to us. Other than that, the cut-out cantilever beam can also achieve two equal resonant frequencies when $\beta \to 2/3$ and $\alpha \to 27/17$ by taking derivative of eq. (4). However, for the continuous beam, it is not possible to obtain two close resonances from eq. (5) with non-zero α. This unique property of cut-out cantilever configuration provides a practical parametric option to implement a 2-DOF energy harvester with two close resonances.

Based on the cut-out cantilever concept, we devised a 2-DOF harvester as well as a conventional SDOF harvester. Experiment has been performed to compare the two harvesters and to show the advantage of this novel design. Figure 2 demonstrates the fabricated prototype installed on a vertical seismic shaker. As the actual experiment prototype is slightly different from the simplified model used in

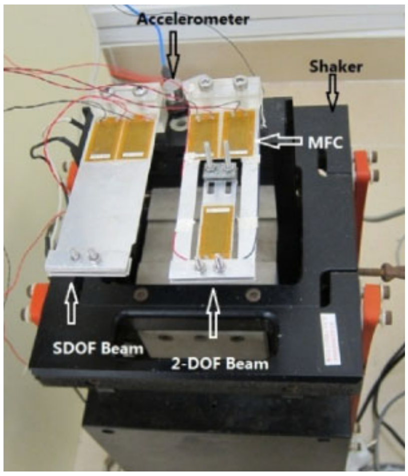

Fig. 2. (Color online) Conventional SDOF and proposed 2-DOF cut-out harvesters installed on seismic shaker.

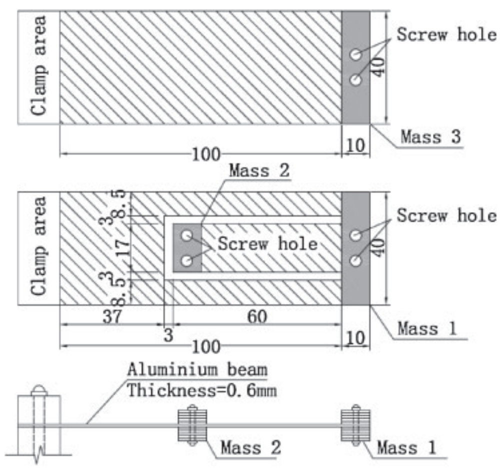

Fig. 3. Geometry of conventional SDOF and proposed 2-DOF harvesters (all dimensions in mm).

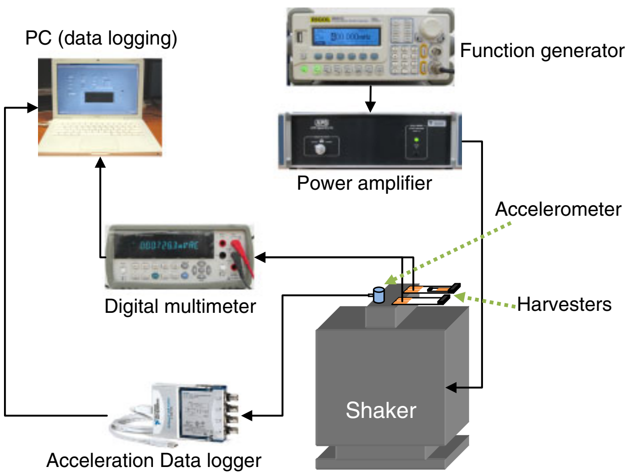

Fig. 4. (Color online) Schematic of experiment setup.

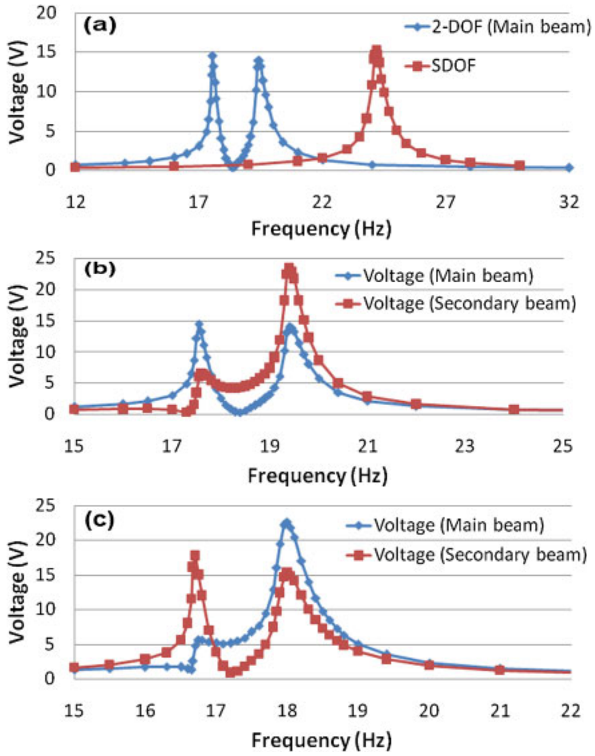

Fig. 5. (Color online) Open circuit voltage response (a) Comparison between 2-DOF configuration (from main beam, $M_1 = 7.2g$, $M_2 = 11.2g$) and SDOF configuration ($M_3 = 7.2g$). (b) Voltage output from main and secondary beams of 2-DOF configuration ($M_1 = 7.2g$, $M_2 = 11.2g$). (c) Voltage output from main and secondary beams of 2-DOF configuration ($M_1 = 7.2g$, $M_2 = 14.2g$).

Fig. 1(d) because of the non-uniform stiffness and distributed mass, finite element simulation was conducted to determine the proper dimensions and other parameters needed for the experiment. The detailed dimensions of the two harvesters are shown in Fig. 3. The SDOF cantilever beam and the 2-DOF cut-out cantilever beam were both made from pieces of aluminum plates with the same size ($110 \times 40 \times 0.6\,\text{mm}^3$). Specially, the cut-out 2-DOF cantilever beam was fabricated by cutting out the inside of a SDOF beam. Pieces of small steel plates were screwed at the free end of the beams such that the weight of the proof masses can be adjusted conveniently. Macro-fiber composites (MFC) sheets with d_{31} piezoelectric effect (M-2814-P2) were used for vibration-to-electricity transduction. Two pieces of MFC were bonded at the root of the main beam, while another one piece bonded at the root of the secondary beam. For comparison, the conventional SDOF harvester also has two pieces of MFC at its root.

The schematic of experiment setup is shown in Fig. 4. A harmonic excitation signal was generated by a function generator, adjusted by the power amplifier and finally fed to the seismic shaker. In the experiment, the excitation frequency was swept from 10 to 30 Hz. During this sweeping procedure, the excitation acceleration was monitored by an acceleration data logger as feedback loop and controlled at $0.102g$ ($= 1\,\text{m/s}^2$). The open circuit voltage output generated by the MFC was logged by the digital multimeter.

The open circuit voltage responses are shown in Fig. 5. Figure 5(a) compares the response from the two MFCs at the root of the cut-out 2-DOF beam with that of the conventional SDOF beam. In this case, the weights of tip masses are $M_1 = M_3 = 7.2g$ and $M_2 = 11.2g$. The first two resonant frequencies obtained from the experiment are 17.4 and 19.5 Hz respectively, which are close to those obtained from the simulation (17.8 and 19.8 Hz). However the theoretical results obtained by using the simplified model are 14.6 and 17.8 Hz, respectively, due to the difference between the simplified model and the actual experimental prototype. It is obvious that the response of the cut-out configuration has two close peaks with the same level of magnitude as the SDOF configuration (around 15 V). For 2-DOF harvester, the harvested power at two resonances are 0.52 and 0.48 mW, respectively. For the SDOF harvester, the harvested power at resonance is 0.61 mW. But the 2-DOF harvester has significantly wider bandwidth than the conventional SDOF device. As shown in Fig. 5(a), the bandwidth in the open circuit voltage spectrum at voltage level of 3 V for the cut-out 2-DOF harvester is about 3.0 Hz (by adding up the two segments near the two resonances), which is more advantageous over 2.1 Hz of the SDOF harvester.

Other than the broader response bandwidth, another advantage of the proposed cut-out design is that it can fully utilize the cantilever beam by attaching one more transducer at the root of the secondary beam. Conventionally this area is not used or used inefficiently because of the low voltage output (due to low strain level) in the SDOF configuration. In the cut-out configuration, by adjusting the tip masses, the response level at the secondary beam can be tuned to be comparable to that of the main beam, as shown in Figs. 5(b) and 5(c). In Fig. 5(b), when $M_2 = 11.2g$, the main beam retained two equal peaks, and the secondary beam also generated significant output. When M_2 was further increased to $14.2g$, two equal peaks appeared from the secondary beam. Although the peaks from the main beam are no longer equal, they are still effective for energy conversion, especially the second peak, as shown in Fig. 5(c). The above results indicate that the cut-out configuration can be further optimized to achieve maximum overall output. Thus, the proposed harvester exerts the full capacity of the cantilever beam for energy harvesting.

In summary, this novel design of 2-DOF energy harvester provides larger bandwidth as compared to that of the conventional SDOF energy harvesters. It is more compact than the previous 2-DOF harvesters. Moreover, it efficiently utilizes the material of the cantilever beam by generating significant voltage output from both the main and secondary beams.

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