Dynamics of Granular Matter

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Invited Review Paper

Dynamics of Granular Matter*1

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Granular matter is a typical example of a new topic in statistical (phenomenology) mechanics. Reconsidering granular matter from the physical point of view, several new aspects have been clarified, although granular matter has been studied by engineers for a long (period of) time. This review examines three topics: (1) pattern dynamics of sand ripples and dunes, (2) mathematical structure of a fluidized bed, and (3) convection and turbulence in a vibrating bed. Investigating these topics, it is found that the dynamics of granular matter exhibits many typical nonlinear phenomena, for example, formations of pattern, localized states, and turbulence.

KEYWORDS: granular matter, sand dune, sand ripple, soliton, bubble, slug, fluidized bed, convection, turbulence, vibrating bed.

1. Introduction

Granular matter is the general name that refers to sets of fine particles. How fine are they? This depends upon the situation. In volcanic activity, a kind of avalanche can sometimes occur. This avalanche is a mixture of rocks, ash and hot air. Under such circumstances, even large rocks can be regarded as 'fine' particles compared with the size of the whole avalanche. On the other hand, cigarette smoke can be regarded as a mixture of air and very 'fine' particles whose diameter is about 100 Å. Thus, we can consider many objects to be granular matter. (Even the galaxy can be treated as a set of 'fine' particles, i.e., stars!)

In this review, however, we consider only particles whose diameter ranges from 1 μm to 1 cm. Each particle itself is a macroscopic body from the conventional point of view. We can compute motion of an individual particle by standard classical mechanics. In contrast, we know very little about how a set of fine particles behaves.

Sand is a typical example of granular matter which we consider in this article. Imagine you are on a sand beach. When we try to grasp sand, it flows out through our fingers. A castle constructed by a little boy on the beach easily vanishes once a wave comes along. Beside the castle, we may find sand ripples (that wind and waves produce) on the beach. In contrast to these images, sand sometimes threatens our lives, *e.g.* when we are in a desert. A large dune can cover whole towns, or a deadly sand storm can kill off many life forms. How can we describe such a wide range of phenomena? At the moment we have no answer.

Granular matter has been studied in many fields such as chemical engineering, mechanical engineering, physics of earthquakes, and civil engineering. Granular matter can be a form of industrial materials, a method for transporting material, and the foundation of a construction. In spite of such a wide range of applications, due to lack of our knowledge, we frequently have difficulty in making use of granular matter.

The aim of engineers is to reproduce phenomena or to control phenomena, while our aim is to construct a statistical or fluid mechanics of the cooperative dynamics of powder. We also aim to determine the mathematical structure behind various kinds of powder behavior. Although these aims are not directly related to engineering questions, we hope they can be the first step toward understanding of the fundamental properties of granular matter which is needed by engineers.[1)]

In this review we will collect some typical examples of the dynamics of granular matter. We have neither enough space nor the ability to review general aspects of granular matter. The most drastic phenomenon related to granular matter is probably fluidization. This review examines three examples of fluidization, which all have different mechanisms. Fluidization can be seen in a wide range of phenomena, as shown later. Fluidization can also be the first step to understanding the dynamics of powder because we can compare fluidized granular matter with normal fluid, which also has a wide range of applications in engineering.

In §2, the dynamics of dunes and ripples are considered as the first example. Section 3 explains nonlinear waves and related topics in fluidized granular matter. Convection and turbulent flow in a vibrating bed of powder are considered in §4. Conclusions and summary will be found in §5.

2. Dynamics of Dunes and Ripples

On a windy day, a person who visits a sand beach would find a clear stripe pattern which consists of the undulation of the sand surface (Fig. 1(a)). Ripples on the sand surface emerge and disappear, depending on the strength of the wind, and generate various spatio-temporal patterns. The spatial scale of a ripple is on the order of 10 cm, and the temporal scale of its birth

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and death is about a few hours. A ripple is a dissipative structure which the external force, wind, produces on the surface of the sand and it also can be thought of as a surface wave of granular matter.

On the other hand, although we rarely have a chance to see one, a dune is another example of a dissipative structure on the sand surface (Fig. 1(b)). The wavelength of a dune is typically on the order of 10 m to several hundred meters. The shape and the dynamics of dunes have much more variety than those of ripples. For example, a barchan dune is a kind of isolated dune which has the shape of a crescent. It moves in the leeward direction with its shape and size maintained as in a soliton and damages roads, buildings and gardens in its path of motion. Starfish shaped dunes known as stardunes are generated in deserts when the wind direction temporally varies. They can reach up to a kilometer in horizontal size and up to several hundred meters in height and move very slowly. The temporal scale of dune formation is from one year to tens of thousands of years. Thus, in spite of simple appearance of the elementary dynamical process of sand, the cooperative dynamics of sand presents us with various patterns of dunes, depending upon wind strength, grain size, grain mass, and grain nonuniformity. The variety of the dynamics of dunes and ripples is no less rich than that observed in thermal convection or liquid crystal convection.[2)]

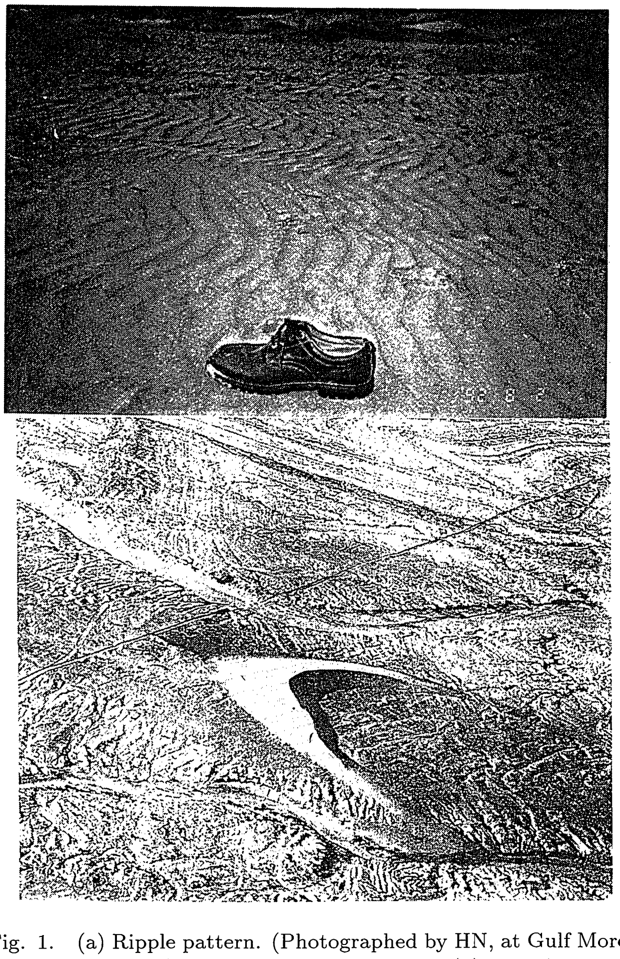


Fig. 1. (a) Ripple pattern. (Photographed by HN, at Gulf Moro, west coast, USA) Length of shoes is 25.5 cm. (b) Baruchan dune at California, USA (From Pye and Tsoar[2)]). The straight line from upper right corner is a road.

How can we deal with the variety that dunes and ripples show? In order to describe its complexity, is it necessary for us to know all the detailed information of the system, *e.g.*, local velocity field and boundary conditions, as is required for solving problems in fluid mechanics? Can we use a phenomenological equation as is frequently the case in statistical physics? Can we understand the essence of the system without treating seriously the discreteness of the system? So far great efforts have been made to explain the formation dynamics of dunes and ripples. Among them the most outstanding is the comprehensive research carried out by Bagnold based on extensive observations and experiments.[3)] However, his argument concerning the pattern formation process of ripples and dunes is rather qualitative. Kawamura took an analytical approach[4)] to explaining the ripple formation, using a stability analysis. Recently some simulations have been performed to reproduce the ripple and dune formation.[5-8)] Among them, the simulations made by Anderson et al. using the cellular automata rule show some good agreement with experimental facts. Our approach shown below is the simplest one which connects the previous theoretical approaches and more recent numerical method.[9,10)] Its main purpose is to quantitatively clarify the the ripple and dune pattern formation scenario.

Before going into the details of our model the basic dynamics of wind-blown sand will be introduced. Bagnold[3)] categorized the dynamics of the sand surface into three types (1) creep: sand particle moves with rolling, (2) saltation: sand particle jumps out from the surface for some reason and falls back onto the surface downwind, (3) suspension: strong wind makes sand fly over a long period. Among the above, (3) is ignorable when we consider dunes and ripples because the spatial scale of the motion of a suspension is too large. Here, (1) and (2) constitute the main processes in producing dunes and ripples. When wind exceeds the critical velocity, creep starts. Particles creep, collide with obstacles, and are launched (i.e., saltation). A sand particle to which wind has given additional moment along the wind direction during saltation will, when it lands on the sand surface, hit other particles and eject them (or itself) into the air. Meanwhile, sand grains continue to move along the surface by the creep process. Thus the chain process of both saltation and creep generates ripples and dunes.

Next let us model the dynamics of the system. One of the conventional ways, in which the system is treated as a multiphase flow, requires us to construct a set of equations of conserved variables. However, with this approach, not only analytical calculation, but also numerical simulation is considered to be difficult. This is because the system has a rapidly changing boundary condition, the dynamics of which is decided by the dynamics of the system.

For such a case, for simplicity, we ignore the detailed information of the system, such as the turbulent nature of the wind near the surface, and begin by constructing the simplest phenomenological model. At the same time, we include the discrete nature of the dynamics, that is, we realize the dynamics where neighboring particles may separate from each other after a saltation step. Actual modeling is based on a two-dimensional coupled map lattice (CML) model.[9,10] In the CML model each lattice point represents a large area compared with individual particles. The state variable at each lattice point, the deviation of the surface height from the whole spatial average, is the local average in the territory of each lattice point.

First we assume that flight distance L during a saltation step depends upon one of two physical variables:

I) the deviation of surface height from the spatial average $h_n(x,y)$, or,

II) the spatial derivative of it $\nabla h_n(x,y)$,

where (x,y) is the take-off position and n is the time step. I) is the case of ripple formation and II) is the dune formation case. For both cases L is independent of the topography of the landing point, so that the model is regarded as a kind of mean field approximation. The two forms of $L_n(x,y)$ used are:

model I (ripple formation): $L_n(x,y) = L_0 + bh_n(x,y)$,
model II (dune formation):

$$L_n(x,y) = L_0 - b'(\tanh(\nabla_x h_n(x,y)) + 1 + \varepsilon).$$

Here the control parameter, L_0, is the average distance of flight, which is a monotonic increasing function of wind strength. b and b' are constants and $1 \gg \varepsilon > 0$. In model I, flight distance increases as the height of the take-off point increases. In model II, flight distance depends upon whether the take-off point is on the windward side of the dune or not. This is because the scale of the dune is much larger than that of saltation; when the starting point is on the windward side of the dune, the particle collides with the slope on the same side, independent of the height of the take-off position. On the other hand, for the particles on the lee side, the probability of take-off is small because the wind force is weak and the probability of being ejected by other jumping grains are also small. Thus effective flight length is shorter than that in the case of the windward grains. Model I assumes that the flight distance is compatible with the characteristic length of the geometry (i.e., ripple wavelength), and model II assumes that the flight distance is smaller than the geometrical scale. To model the creep, anisotropy caused by the wind is ignored and only the effect of gravity is taken into account. In other words the creep is replaced by the simple diffusion process. These are all our models. Here saltation and creep are the only elementary dynamical processes introduced.

Starting from the initial conditions of an almost flat surface with small roughness, the system evolves with alternating saltation and creep steps. This extremely simplified model can reproduce various phenomena. Figure 2 shows ripples reproduced by model I, and the pattern generated by model II is shown in Fig. 3 (for details, see refs. 11 and 12). For both figures, the wind direction is from left to right. The darkness of the gray indicates height. In Fig. 2, dislocations seen in the stripes of the ripples move in several ways, for example drift and oscillation. Moreover, the whole structure drifts along the wind direction, i.e., from left to right. The stripe pattern can be observed only when L_0 exceeds threshold value L_c when the diffusion constant of creep takes a constant value. This means that wind has to be strong enough for the appearance of the ripple pattern.

On the other hand, in Fig. 3, we can observe dunes shaped similar to baruchan. Dunes move along the wind direction, retaining these shapes. Many other patterns can be observed in model II by changing the parameters. Moreover, we[11,12] performed linear stability analysis of models I and II, and found answers to the two fundamental questions:

1) mechanism of appearance of ripples: why ripples appear only when the wind blows strongly enough.

2) variety of dune shapes: why a simple elementary process can results in many kinds of shapes.

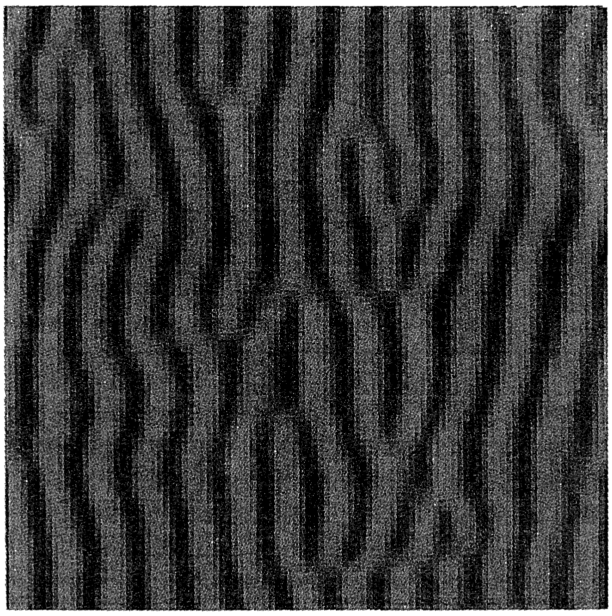


Fig. 2. Ripple patterns reproduced by model I. The brightness is proportional to height.

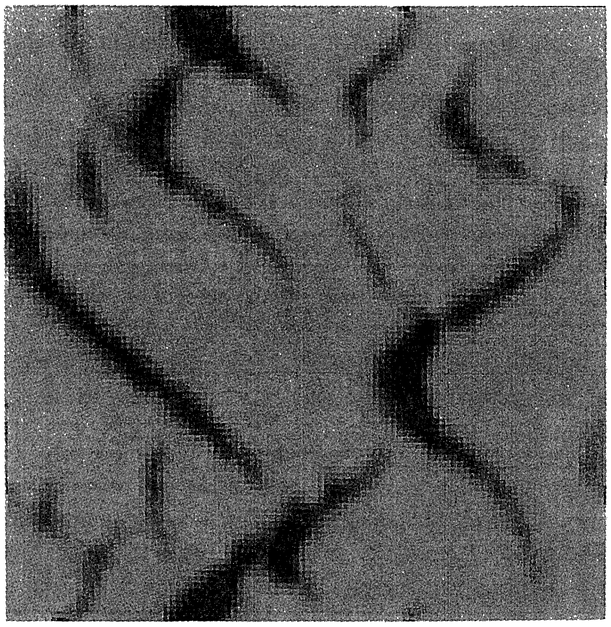


Fig. 3. Baruchan dune reproduced by model II. The darkness is proportional to height.

Linear stability analysis, however, can explain only small parts of the rich variety of spatiotemporal patterns. The most interesting phenomena are sure to be beyond simple linear stability analysis.

Among interesting problems beyond linear stability analysis, solitary motion of barchan dunes and motion of dislocations in ripples are important. These can be treated using the weakly nonlinear analysis explained in the next section. Other than this method, considering a mapping of $x_{n+1} = x_n + L(h_n(x,y), \nabla h_n(x,y))$ that represents particle motion due to saltation, we notice that the mapping induces folding of the sand surface because the mapping $x_n \to x_{n+1}$ changes as a function of time. The folding results in violent mixture which seems to be incompatible with coherent structures like ripples and dunes. It is interesting to study the relationship between the coherent structure of the whole system and the chaotic dynamics of each element.

When we further take discreteness of the particles into account (see §4), we can recognize its importance. The discreteness allows surface particles to move independently of the particles under the surface. Saltation can easily cause neighboring particles to separate from each other. Thus individual surface particles move closer or far them away from one another from time to time. In contrast to the surface particles, the particles in the lower layers retain their positions and behave like solids. This is because particles in the lower layer are trapped in many local minimum points caused by static friction. However, one should remember that a particle in the lower layer can move freely once it has eventually reached the surface.

If we try to explain the behavior of the system simply by using the fluid dynamical description, we will not succeed, nor we can explain the coherent motion of numerous sand grains only by using the particle description.

Hopefully, we can find a new method to describe these complicated systems.

3. Mathematical Structure of Fluidized Bed

In this section, we briefly summarize recent developments in the understanding of fluidization of granular particles mainly from the theoretical side. The most attractive aspect of fluidization is flow-induced phase change in which the collection of particles, a kind of solid material without flow, changes its properties completely to a kind of mixture consisting of flow and particles due to an imposed flow. We can regard patterns in sand ripples and dunes as a problem of surface waves in solid-gas mixtures as in the previous section. On the other hand, the bulk flow in a mixture[13] of solid-gas also exhibits a fascinating behavior.[14,15] In experiments, we prepare a vessel containing granular particles and impose gas flow from the bottom of it. In this simple experiment, the following phase changes take place. First, we observe uniformly fluidized bed, a uniform and homogeneous state of the solid-gas mixture, when the imposed velocity of gas is fairly low. Second, bubbles appear (see Fig. 4) after the uniformly fluidized bed becomes unstable when the imposed velocity exceeds a critical value. As the imposed velocity increases, bubbles become as large as the

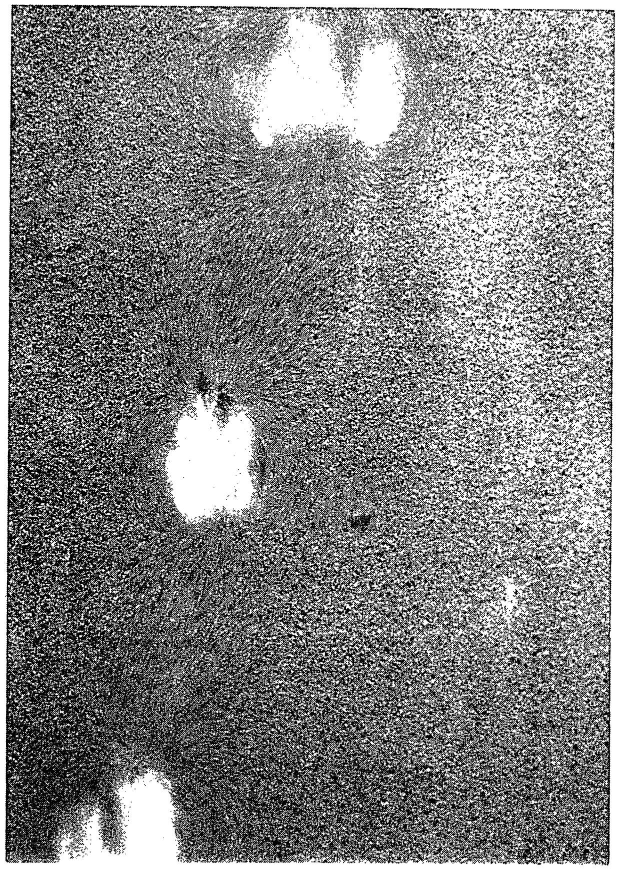


Fig. 4. Bubbles in an experiment on a fluidized bed. The diameter of bubbles is a few centimeters. The depth and the width of the container are 1 cm and 20 cm, respectively. The height of granular collection is about 50 cm without flow, the mean diameter of particles is about 100 μm, the injected speed of gas is a few cm/s. (This photograph is from Profs. Mori and Yamazaki, Nagoya Univ.).

width of the vessel (slugging), and moreover the phase changes to a turbulent state. Finally we obtain a suspension state in which the concentration of granular particles is small. In dilute suspensions, the distribution of particles is, again, uniform and homogeneous. We can note a similarity between bubbling in this problem and the boiling of water, and a similarity between nonuniform suspension in this system and combustion processes.[16,17] The collection of granular particles, however, does not have any equilibrium state and the above phase changes are typical features of nonequibrium dynamics of phase transitions in open systems.

We note that the problem of fluidized beds has an essentially different feature when we compare it with other problems such as the convections of water and liquid crystals. This means, horizontal motions of convection roll arrays are investigated in the convections of water and liquid crystals, on the other hand, vertical motions of bubbles are considered in the fluidized bed. Of course, as will be shown, the problem of fluidized bed can also be reduced to a horizontal problem in some special cases. However, interesting features in fluidization can be understood in the space including the vertical coordinate. Therefore, many theorists have analyzed one-dimensional models to capture the essence of fluidization.

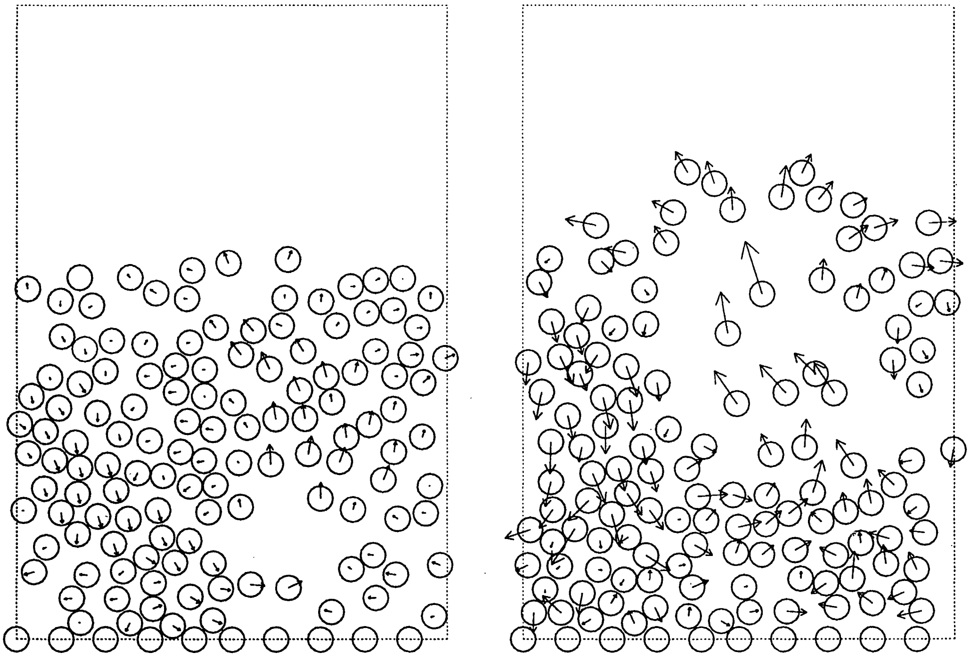


Fig. 5. Bubbling obtained from the three-dimensional simulation by Ichiki and Hayakawa.[19] The fluid is air (its shear viscosity is $\mu = 1.7\times10^{-4}$ poisse and the mass density is $\rho_f = 1.1\times10^{-3}$ g/cm^3), and the particle is assumed to be a hard-core particle which are constrained in two-dimensional space (the density $\rho_p = 2.5$ g/cm^3, the radius $a = 10^{-3}$ cm) affected by the gravitational force (the gravitational acceleration is $g = 9.8\times10^2$ cm/s^2). We assume the periodic boundary condition for all directions. The injected gas speed is $0.3U_0$ where $U_0 = 2a^2(\rho_p - \rho_f)g/(9\mu) \simeq 3.28$ cm/s is the equilibrium falling velocity of one particle. The time interval between these configurations is 5.0×10^{-3} s.

How do we describe fluidization mathematically? At present, we do not have any definite answer to this important question. We shall introduce some typical approaches to this problem and summarize characteristics of them. These approaches start from various space-time scales, microscopic models and macroscopic phenomenological models. We note that a microscopic approach is not always superior to a macroscopic approach.

At the most microscopic level, we need to solve the motion of granular particles, taking into account the effects of the fluid field at each numerical step. This moving boundary problem is difficult to solve using a computer and successful in limited cases. Ichiki and Hayakawa[19] have introduced a dynamical model in which flow is assumed to be described by the Stokes flow. This model is regarded as an extension of the Stokesian dynamics for colloid particles.[20] The Stokes approximation for fluid is valid when we discuss fairly small particles (its radius is on the order of 10^{-3} cm).[18] From their simulation, they have reproduced bubbling (Fig. 5) and slugging (Fig. 6) as observed in experiments. We observe that convection is necessary to produce bubbles in which particles move upward through the center of bubbles. On the other hand, convection does not occur in slugs in which particles sedimentate. Their simulation also suggests the importance of the boundary condition in producing bubbles. Although we observe realistic bubbles when we introduce fixed particles at the bottom of the container, we do not observe any stable convection, nor as a result, any stable bubbles when we impose the periodic boundary condition. It is clear that this problem is closely related to the sedimentation problem, which is

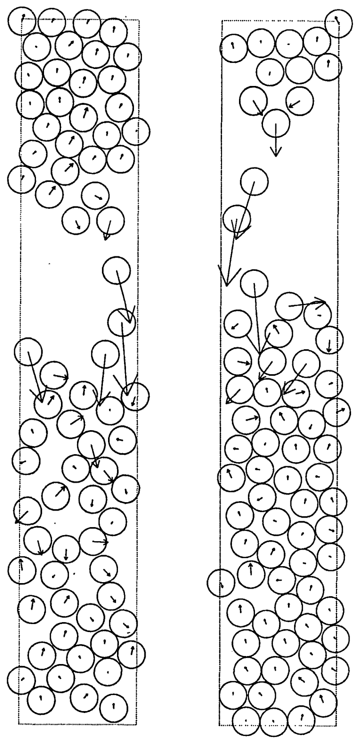


Fig. 6. Slugging obtained from the three-dimensional simulation by Ichiki and Hayakawa.[19] The data for fluid and particles are the same as those in Fig. 5. We assume the periodic boundary condition for all directions. The volume fraction of particles is about 0.35. The time interval between these configurations is 2.0×10^{-2} s.

a fundamental many-body problem interacting through long-range hydrodynamic force.[21-24] Another interesting discovery resulting from their simulation is the universality of the concept of powder turbulence proposed by Taguchi,[25] originally for vibrating beds. See ref. 19 for the details of their analysis.

A modern and popular approach is the simulation by the distinct element method (DEM).[26-28] The DEM is also a powerful tool to describe vibrating beds, as in the next section. In this approach, the particles are replaced by a mechanical model which consists of springs, dashpots and sliders, and their magnitudes are determined by empirical methods. In their simulation, fluid motion and the mutual friction between fluid and particles obey a phenomenological model. They have succeeded in simulating hundreds of thousands of particles and reproducing realistic motion of particles. It is, however, difficult to understand the physical structure of powder turbulence, such as the onset of turbulence and the physical insight of fluidization, from the simulation based on the DEM method.

Conventionally, we often use a two-fluid model,[14,15] the fluid of granular material and the real fluid, to describe compound systems of granular particles and fluids. This model has an advantage in elucidating the macroscopic pattern formation by using bifurcation analysis and stability analysis of hydrodynamics. On the other hand, it is not easy to decide which model is suitable for this problem, the role of granular particles is not clear in this model, and the model itself contains some empirical parameters. We may partially resolve an important question concerning the ambiguity of relevant models for a fluidized bed since Batchelor[29] have ascertained what the relevant terms are. We recognize that a suitable two-fluid model for slow motion relative to its sound velocity should consist of two parts. One is mass conservation and another is the momentum conservation. Of course, we can assume the incompressibility of fluids. In the equations of momentum conservation, important terms are the mutual friction, the effective pressure for each phase and viscous terms in particle phase except for the gravitational effects. We may neglect the viscous effects from real fluid and assume that the fluid is Newtonian. We also note that this two-fluid model actually corresponds to the DEM model[26-28] in which the solid phase pressure and the viscous terms in the two-fluid model correspond to the elastic collision due to springs and the inelastic collision from dashpots and sliders in DEM. The direct simulation of this kind of two-fluid models can also produce realistic motion of granular flow[15,30] (see Fig. 7[30]) supplemented with empirical laws.

It is easy to discuss the linear stability of a uniformly fluidized bed. The equation for the two-fluid model has a trivial solution in which the velocity of the particle phase is zero, the velocity of fluid phase is constant in the vertical component, and the volume fraction of particles is uniform and constant. When we linearize the two-fluid model around this trivial solution, the growth rate of the plane wave becomes positive when the destabilized effects from the mutual friction exceed the stabilized effects from the elastic collision among particles. Thus a

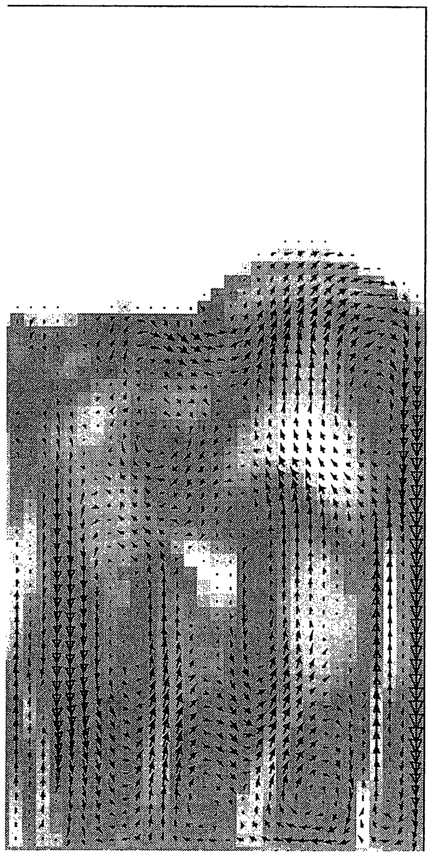


Fig. 7. Bubbling obtained from two-dimensional simulation based on a two-fluid model by Komatsu.[30] The arrows represent the velocity field of granular particles. The dense region of granular particles is dark-shaded in this figure.

uniformly fluidized bed becomes unstable, resulting in the appearance of the negative diffusion constant in the vertical direction. This unstable mode propagates from the bottom to the top of the container with the increase of its amplitude.

We need nonlinear analysis to understand interesting behaviors of two-phase flow. In general, the nonlinear analysis is complicated and difficult. It is possible, however, to capture the universal feature of the fluidization when we restrict ourselves to weakly nonlinear regions. In fact, several authors[31-35] have demonstrated that the soliton described by the Korteweg-de Vries (KdV) equation plays an important role, at least in one-dimensional models, near the onset of the instability of the uniformly fluidized bed. This picture has been confirmed from the fact that the reduced equation can reproduce the numerical solutions of two-field model quantitatively.[31-34] A direct simulation of the two-fluid model shows the formation of pseudosolitons which are similar to the solitons of the KdV equation (see Fig. 8). Note that pseudosolitons of this kind do not describe strong phase separation between the granular phase and the fluid phase. Even in one-dimensional models, at present, we do not know how solitons change to phase-separated states.

In contrast to the one-dimensional case, we do not have any consensus in weakly nonlinear analysis in a multidimensional case. In this review, we describe a recent the-

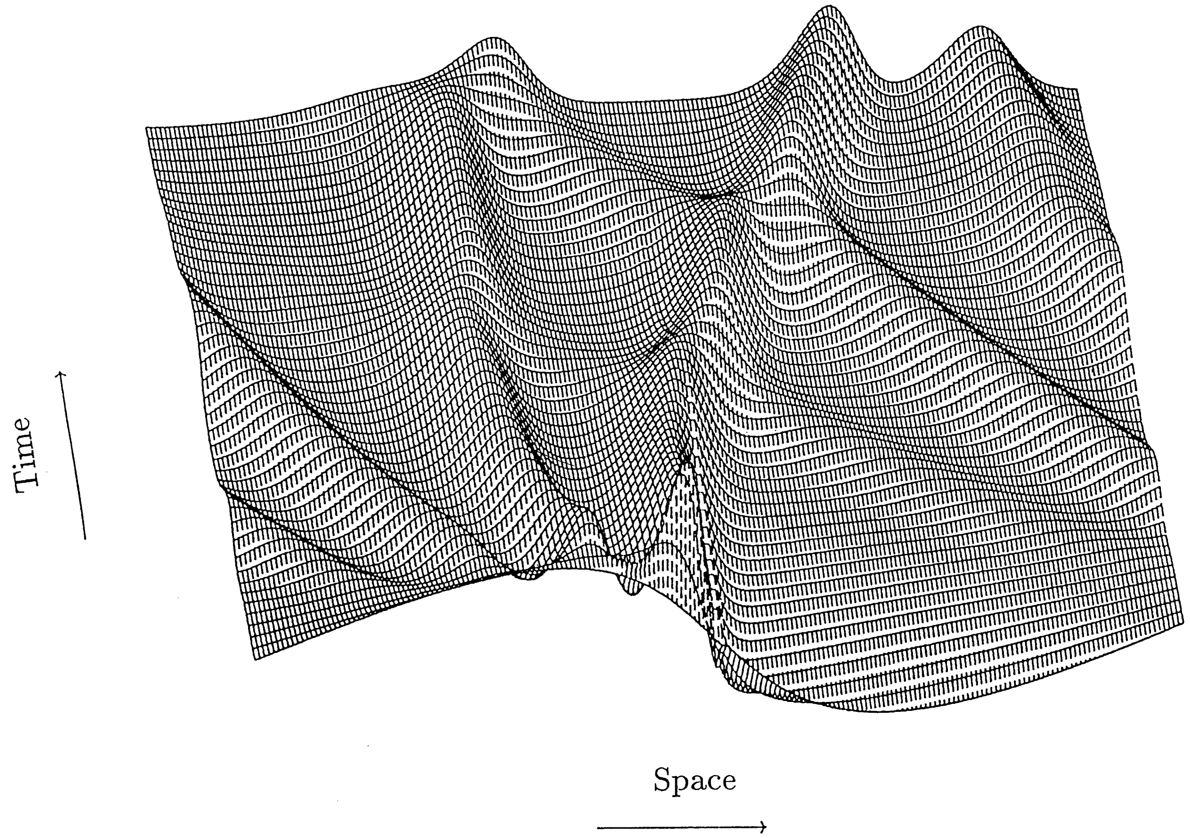


Fig. 8. The spatiotemporal pattern obtained from the one-dimensional two-fluid model. This figure is plotted in the frame of the propagating velocity of fluctuation obtained from the linear stability analysis. We impose the periodic boundary condition in space. The initial condition is the sinusoidal wave whose period is the same as the space size. As in the KdV equation, the wave is steepened by the nonlinear term, and the wave is separated into solitons by the dispersive effects. The "mountain part" represents the region in which the gas volume fraction is large.

ory by Hayakawa.[36] Hayakawa derived a scalar equation in the weakly nonlinear region in multidimensional cases which reduces to the one-dimensional KdV equation at the lowest order and contains the small contribution from the dissipation in the vertical direction, the horizontal diffusion, and the nonlocal term in the horizontal coordinate. Using this reduced equation, when we discuss the linear stability of one soliton in the vertical space, we obtain a simple horizontal amplitude equation. However, the scalar equation is not sufficient for discussing the bubble formation, because this equation is irrotational.

The appearance of the amplitude equation in the horizontal plane is analogous to that in the Rayleigh Beard problem.[37] In the latter case, the sine wave in the vertical direction is important in the linear stability of the uniform state. The amplitude equation in horizontal space can be derived from the direct perturbation with the solvability condition. As is well known, analysis of this kind of amplitude equation is a hot subject in the physics of pattern formation.[37] In our case, solitons play an essential role in the vicinity of the critical point of uniformly fluidized beds. Therefore, we expect that essential progress in this problem will be made by comparing it with convection problem.

This kind of weakly nonlinear approach, of course, has a severe limitation. As mentioned above, our simulation[19,30] have suggested that the convection is important in the bubbly phase. In addition, Komatsu[30] has demonstrated that there is no correlation between volume fraction and the solid-phase velocity in the bubbly phase in contrast to the result of the weakly nonlinear analysis, where a strong correlation between them exists. This may suggest that we need an alternative approach to understanding bubbling in fluidized beds.

In this section, we have summarized the present state of understanding of fluidized beds while introducing the work carried out by the present authors. Therefore, we note that this review is far from being a complete and fair summary of the present state of the investigation of fluidization. We merely hope that the readers will now understand how fascinating fluidization is. At the end of this section we stress the following. One of the most remarkable characteristics in fluidization is the coexistence of various space-time scales in fluidized beds. We have introduced the most microscopic approach by Ichiki and Hayakawa,[19] DEM approach, two-fluid model, psudosoliton equation, and the amplitude equation in a weakly nonlinear region, where the space-time scales are coarsened according to the order of the approaches. At present, there are some missing links between different scales. In particular, an important problem is that of connecting the microscopic model and two-fluid model. In addition, the bubbling and slugging cannot be understood by weakly nonlinear analysis at present. Therefore we need some conceptual changes to understand this problem.

4. Convection and Turbulence in the Vibrating Bed of Powder

In this section, we consider, pure granular systems which consist of only granular particles. Problems we dealt with in previous two sections correspond to difficult ones in fluid mechanics, i.e., instability of surface and instability of multiphase flow. Both of these problems have been investigated and are recent topics in fluid mechanics. Thus, it is natural that surface and multiphase flow in granular matter are difficult to analyze because granular matter does not have any established basic equation. Can we expect that the pure granular system, which consists of only granular particles, is easier than the former two examples? The answer is 'No'. Contrary to our expectation, the problem of the pure granular system is even more difficult. The above two cases have phenomenological descriptions although one of them is numerical and the other is analytical. In contrast to them, the pure granular system has no phenomenological description except in some special cases. The only possible method that has been applied to the pure granular system is a molecular dynamics method. In spite of such limitations on the approach for the pure granular system, we can obtain many interesting results using molecular dynamics calculation.

In this review, we study the dynamics of granular matter. It is easy to imagine the meaning of the term 'dynamics' in fluid mechanics because fluid usually has momentum and flows. (*e.g.*, the complicated flow pattern of cigarette smoke).

Granular matter like salt and sand, however, usually does not move without external forces. In §2 and §3, the gas flow coexists with the powder and causes granular flow. For a pure granular system, we need other methods to induce granular flow, *e.g.*, gravitational force and shear force. In this section, we employ only vibrational force to produce a collection of fluidized powders because a vibrating bed of powder is easily treated both numerically and experimentally.

Figure 9 shows a schematic of an experiment with a vibrating bed of powder. A vessel of horizontal size about 10 cm × 10 cm and depth of a few centimeters is filled with granular matter, typically glass beads whose diameter is less than 1 mm. A speaker shakes the vessel vertically. Such a simple setup can exhibit many interesting features as seen below.[38-40] The experiment for a vibrating bed is so simple that Faraday had already studied it more than one hundred and sixty years ago[41] and found a dynamical phase transition in which the bed is fluidized as the strength of the vibration increases. In modern experiments with a vibrating bed, the control parameter turns out to be an acceleration amplitude Γ, which is, *e.g.*, $\Gamma = b\omega_0^2$ when vertical vibration has the time dependence $b\cos\omega_0 t$. When Γ exceeds the gravity acceleration g, convection starts in the bed (Fig. 10). Convection usually consists of two convecti on cells, and it flows downwards along the side wall and upwards in the center. Furthermore, the upward flow results in heaping on the surface of the bed. Thus the simple vertical vibration causes convection and heaping. Faraday carried out an experiment in three dimensions, but recent experiments can be carried out in two dimensions.[42]

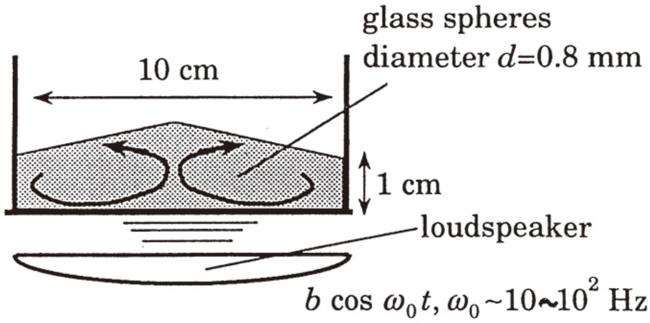
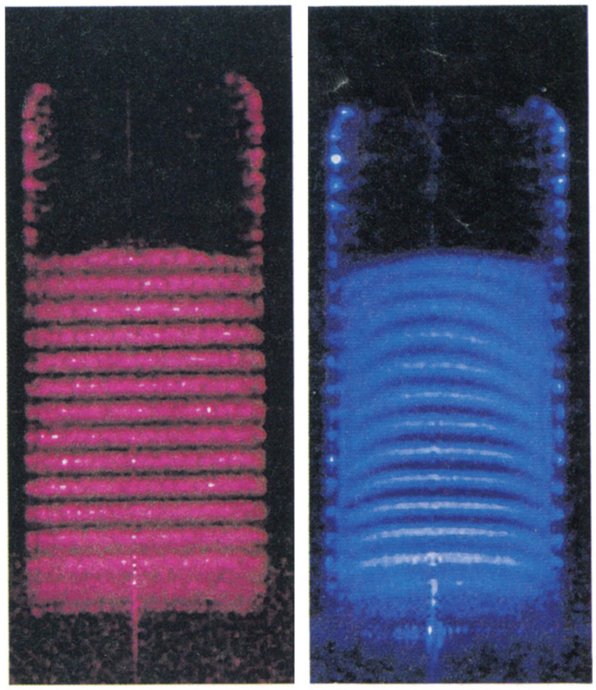


Fig. 9. Schematics of experiment on vibrating bed of powder.



(a) (b)

Fig. 10. Convection due to vibration. (MRI, provided by Prof. H. M. Jaeger). (a) Initial stage (b) Later stage. Downward flow along the side wall can be seen.

Although this phenomenon has been known since long ago, it is impossible to study theoretically due to lack of an appropriate mathematical model. Recent experiments, however, have attracted the attention of physicists, and some of them have succeeded in reproducing convection with soft-core potential molecular dynamics study.[43,44] This scheme has been named the 'distinct element method' (DEM, see also §3) by powder engineers. Recently convection has been reproduced using hard-core molecular dynamics[45,46] and hard-core Monte-Carlo simulation.[47]

From the numerical point of view, convection is easily reproducible. For example, using DEM, the number of particles necessary to reproduce convection is about one hundred. Since the collision of hard spheres is known to be a chaotic process, we can expect that the convection is caused by cooperative dynamics of a simple chaotic process. This situation is similar to the ripple and dune model, where coherent structure appears due to collaboration of chaotic microscopic elements (see §2). There may be some universal framework which can explain the dynamics of granular matter.

In spite of the ease of reproduction, the physics of convection has not yet been clarified. For example, we do not have any clear explanation of the physical origin of the convection. As mentioned, the instability starts only when Γ exceeds g. At that time, the motion of powder can be essentially a free fall, and the vessel loses contact with the granular particle. This will be an origin of instability. However, we cannot explain why instability causes convection. It is clear that the friction between side wall and particle is an important cause of convection. When the friction between wall and particles is less than the friction among particles, DEM produces inverse convection: i.e., upwards along the wall and downwards in the center. This behavior is also observed in hard-core molecular dynamics simulation.[45)] Without the side walls, convection disappears in DEM.[48)] In experiments, no convection occurs in a horizontally periodic cell.[42, 49)]

This situation differs from thermal convection in normal fluid, where the driving force is buoyancy. Although we call the motion in a vibrating bed a convection, the similarity between powder convection and fluid convection is limited to their appearance. In the vibrating bed of powder, convection is caused by internal horizontal stress. Horizontal flow induced by the horizontal stress must go upwards somewhere because the horizontal size of the cell is finite. When the internal friction among particles is less than the friction between wall and particles, the horizontal flow goes upwards at the center. Otherwise, the horizontal flow goes upwards along the side wall. In the thermal convection of fluid, buoyancy induces the convection, and the existence of the side wall is not essential. Actually, it is unclear how many properties the vibrating bed of powder shares with the fluid. In particular, in the vibrating bed without side wall we found turbulence[25, 50-52)] similar to that observed in fluidized beds.

Figure 11 shows how fluidization proceeds as Γ increases. When Γ slightly exceeds the critical value Γ_c, the fluidized phase appears near the surface. A solid region still remains under the fluidized region. As Γ increases, the depth of the fluidized region increases, and finally the whole bed is fluidized. In this fluidized region, we observe convection if there are side walls. Without side walls, the fluidized region resembles turbulence.

In order to compare the fluidized vibrating beds with the fluid turbulence, we first review some of the characteristics of the fluid turbulence. First, we consider sheared turbulence and convection turbulence. Sheared turbulence is generated by external shear force. Experimentally, we observe it when fluid passes through obstacles like grids, plates, cylinders, and spheres. Examples in nature include the tides and wind. From the statistical point of view, two important characteristics of the sheared turbulence are:

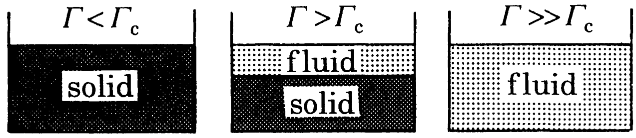


Fig. 11. Process of fluidization.

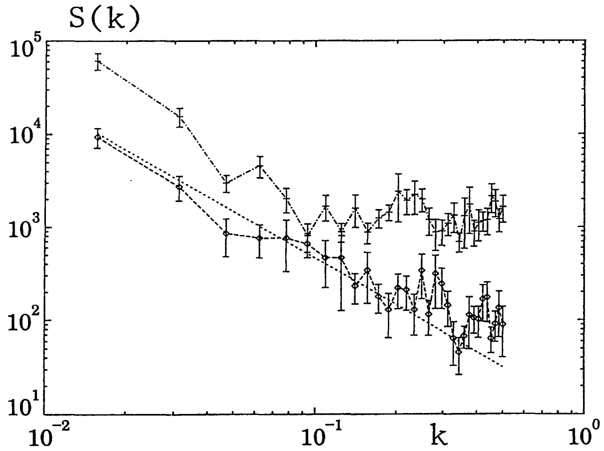


Fig. 12. Power spectrum for fluid region. Lower curve is for lower layer, and upper curve is for layer near surface. Straight line indicates $k^{-5/3}$ dependence.

(1) Spatial energy spectrum depends upon wave number k as $k^{-5/3}$ (Kolmogorov law[53)]).

(2) Spatial derivatives of velocity have a probability distribution function (PDF) which deviates from Gaussian.

Next let us consider convective turbulence which can be observed in thermal convection when the temperature difference between the hot heat bath and cool heat bath is large enough. PDFs of both velocity and temperature are Gaussian for a relatively small temperature difference even after turbulence occurs, but they deviate from Gaussian when the temperature difference increases further.[54)] This is called the soft-to-hard turbulence transition.

We observe similar behavior in the fluid phase in the vibrating bed of powder mentioned above. First, the spatial power spectrum of displacement vectors exhibits $k^{-5/3}$.[25)] As shown in Fig. 12 the power spectrum flattens for the larger wave number region. This flattening of the Kolmogorov spectrum near the surface is also observed in the fluid.[55)]

One might think that observing the Kolmogorov spectrum in granular matter is strange since granular matter is not fluid. Kolmogorov, however, did not explicitly use properties of fluid to derive his theory. We have already confirmed that granular matter satisfies the basic requirement assumed by Kolmogorov.[50)] This Kolmogorov scaling was also observed by Ichiki and Hayakawa[19)] in a fluidized bed.

Furthermore, it is rather surprising that the $k^{-5/3}$ spectrum can appear in a solid phase where the particles do not flow. Figure 13 shows the powder spectrum

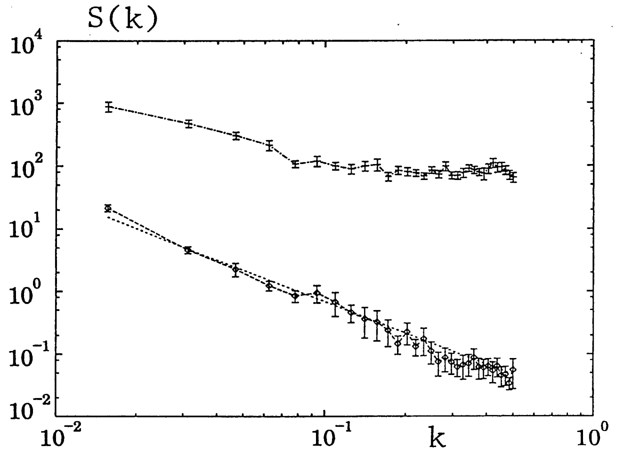


Fig. 13. Power spectrum for solid region. Lower curve is for lower layer, and upper curve is for layer near surface. Straight line indicates $k^{-5/3}$ dependence.

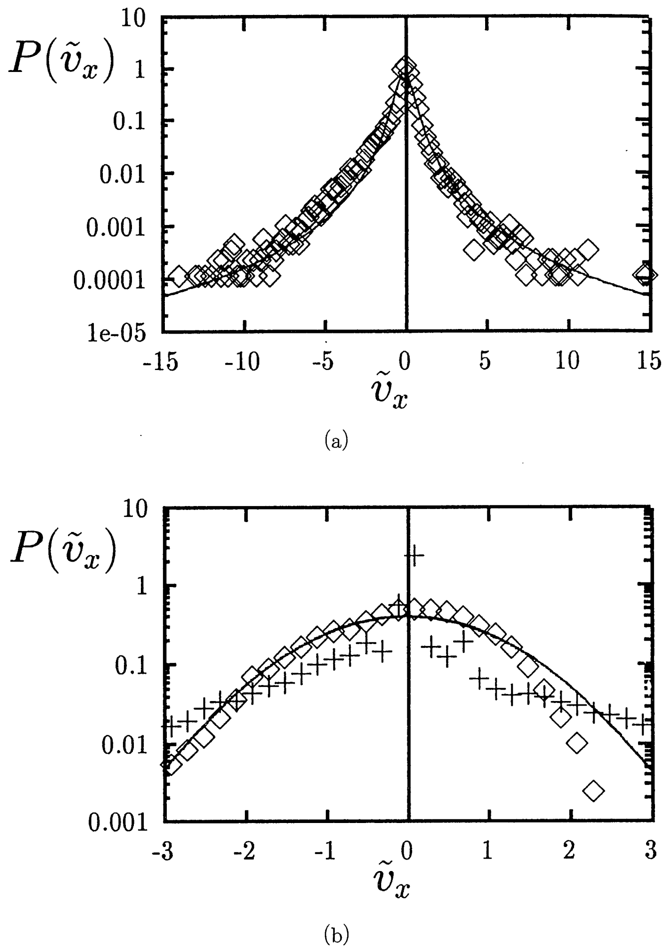


Fig. 14. (a) PDF of displacement vector (time averaged velocity) in fluid region. Solid line indicates power distribution. (b) PDF in coexistence phase (solid and fluid region). + represents fluid region and ◇ represents solid region.

for the solid region. No qualitative changes are observed. Thus, we can conclude that Kolmogorov theory is more universal than previously believed and is valid for a system without flow.

If both the solid region and fluid region have the same power spectrum, what is the difference between the solid and the fluid region? A difference appears in PDFs.[52,56] Solid phase and fluid phase have different PDFs of displacement vectors: Gaussian in solid phase and non-Gaussian (power distribution) in fluid phase. This is coincident with the soft-to-hard turbulence transition observed in thermal convection turbulence when we increase Rayleigh number. That is, solid region corresponds to soft turbulence and the fluid region corresponds to the hard turbulence. Figure 14(a) shows the PDF in the fluid region. It is a power distribution. Figure 14(b) shows the PDF when solid and fluid regions coexist. The from of the PDF is independent of whether a solid region exists below the fluid region. Deviation of PDF the from Gaussian has been observed recently experimentally.[57] In a vibrating bed with very high Γ, the velocity distribution differs from Gaussian. These results suggest that granular matter in a fluidized vibrating bed behaves like fluid, but does not obey the Navier-Stokes equation at all.

In addition to the above progress in numerical treatment, magnetic resonance imaging (MRI) technology has recently been applied to vibrating beds of powder.[58,59] Most difficulty in experimentation comes from the fact that we cannot observe the inside of a three dimensional bed. When we employ a two-dimensional setup, we can observe granular flow, but the effect of the front and rear plate may not be ignored. MRI allows us to see the inside of a three-dimensional bed dynamically. Jaeger[58] succeeded in observing convection in vibrating bed in a three-dimensional setup, and confirmed that convection inside resembles outside. Development along this direction can be expected.

In concluding this section, we can say that even a pure granular system exhibits complicated features. When vibrating, a granular bed shows several nontrivial phenomena such as convection and turbulence. In order to understand the basic mechanism of a pure granular system, numerical results should be compared with experiments in detail, although their comparison has remained at qualitative levels. Quantitative comparison will make clear the basic mechanism of a vibrating bed of powder.

Another direction of development is to construct a phenomenological treatment similar to those for fluidized bed and sand dune/ripple. For this purpose, we have to understand the connection between microscopic chaotic elements and macroscopic coherent structure.

5. Summary and Conclusion

In this article, we have reviewed three topics of fluidization of granular matter: dynamics of sand ripples and dunes, mathematical structure of a fluidized bed, and convection and turbulence in a vibrating bed. Although we have no universal framework for dealing with all of them, we find some common aspects among them in spite of apparent differences.

First, from the phenomenological point of view, statistical mechanical methods in the weakly nonlinear region seem to be valid for both dynamics of dune/ripple and fluidized bed. In particular, we are of the impression that a solitary localized mode is important for the weak nonlinear region of granular matter. This is a new field of physics because a conventional system of fluid has its basis in the sinusoidal spatial pattern in the weak nonlinear region.

Second, turbulent nature universally appears in both a fluidized bed and vibrating bed. In both cases, we find two typical turbulent properties: Kolmogorov spectrum and non-Gaussian PDFs. These findings facilitate not only understanding of granular matter but also that of fluid turbulence.

Third, we can conclude in general that an essential aspect of the dynamics of granular matter is the cooperative dynamics of microscopic chaotic elements. In the sand dune/ripple problem, the chaotic element is the lattice point of the map, and individual particles are chaotic elements in both the fluidized bed and vibrating bed. Since our understanding of cooperative dynamics of chaotic elements is still at the point of developing topics in statistical physics, we are far from fully understanding the dynamics of powder.

Thus, at the moment, we cannot say that we can make a contribution to powder engineering. However, we have been stimulated by the work of powder engineers, and look forward to meeting the readers on some occasions.

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