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—Invited Paper—

Modelocking of Semiconductor Laser Diodes*

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The history of modelocking of the semiconductor laser is reviewed. The theory of modelocking as it relates to the semiconductor laser diode system is developed and discussed. Experiments on semiconductor lasers at MIT and the Bell Laboratories under both active and passive modelocking conditions are described.

§1. Historical Background and Introduction

The generation of short pulses by the use of an active element and a nonlinear element in a feedback configuration was first proposed and realized in the microwave frequency range by Cutler.[1] He used a traveling wave tube for the amplifying medium and a crystal "expander" for the role of the saturable absorber.

De Maria[2] was first to use a saturable absorber in a laser cavity to produce short optical pulses. The field has expanded enormously in the meantime and a complete listing of references would run into the hundreds.[3] The shortest pulses have been produced by passively modelocked cw dye lasers.[4,5]

Much effort has been devoted to short pulse generation using laser diodes. The techniques fall into two general categories: relaxation oscillation and modelocking, although at times they occur both in the same system. Lee and Roldan[6] and Basov *et al.*[7] used two-section diodes to produce pulses 10–100 ps long under pulsed current excitation. The repetition rate of the pulses was determined by the relaxation times of the system and was not directly connected with the roundtrip time of a pulse within the system. Paoli and Ripper[8] generated pulses of fractions of nanoseconds by socalled "second-order mode-locking" where the natural relaxation frequency lies close to the frequency difference (caused by material dispersion) of two adjacent axial-mode frequency separations.

Morozov *et al.*[9] used a two-section diode, one for gain, the other as an absorber in an external resonator. The idea was sound but ahead of its time because the diodes had poor mode quality, and had to be operated pulsed. Due to the transient excitation and heating, appreciable transient frequency pulling must have occurred preventing effective modelocking. Only ripples were observed. Harris[10] used an anti-reflection coated diode in an external resonator and modulated the drive current. He observed pulses of 1–2 ns duration. Observation of 0.3 ps substructure in the SHG trace from a pulse excited diode was reported[11] but the results suffered from poor reproducibility.

The first cw modelocking of a semiconductor laser that produced reproducible pulses of 20 ps duration[12,13] and less[14] were carried out in a relatively short (5 cm) external resonator using uncoated diodes for convenience [Fig. 1]. At the time of these experiments stripe geometry heterostructure diodes were available that

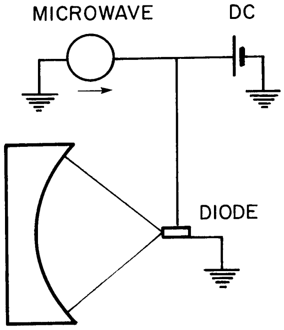

Fig. 1. Schematic of diode in external resonator as used in MIT experiments.

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could be excited cw. Their transverse mode structure was of sufficient quality to make threshold reduction via reflection from an external mirror observable. The pulses were not Fourier transform limited as evidenced by the substructure of the Second Harmonic Generation trace (see §6).

Further experiments with antireflection coated diodes (or the diode face internal to the resonator) produced 6.9 ps pulses under forced modelocking conditions.[15,16] It is likely that all modelocking experiments resulting in pulses of the order of 20 ps and less were assisted by saturable absorber effects. At least all experiments at MIT were made with diodes that exhibited self-pulsations at the roundtrip time of the external resonator (while not showing such pulsations with the external mirror blocked. These were uncoated diodes that could lase under such a condition). The shortest pulses reported to date are those obtained by Ippen *et al.*[17] with diodes antireflection coated on the face internal to the resonator and under self-pulsing conditions with no applied microwave drive [Fig. 2]. These experiments are most convincing in underscoring the importance of saturable absorption. The modelocking principle can be applied to any cw semiconductor diode of good mode quality and it is only a matter of time that diodes operating at wavelengths as long as 10 μm will be modelocked successfully.

The next step in the development of compact picosecond pulse generators is the replacement of the external resonator by a fiber or an optical waveguide. In fact, complete integration looks feasible in structures less than 1 cm long. If the index of the optical waveguide is of the order of 3.5 as in GaAs, this corresponds to a pulse repetition rate of 4.3 GHz. Recently, modelocking at 5 GHz has been reported[18] so that this higher repetition rate does not appear

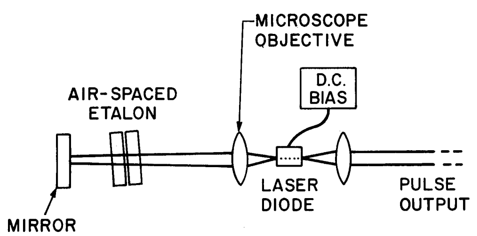


Fig. 2. Schematic of system used by Ippen *et al.*

prohibitive.

An alternate approach of obtaining short pulses from diodes is by direct pulse excitation of the diode and impressively short pulses have been produced in this way.[19] If simplicity of the structure is the dominant consideration, this modulation method will prevail. However, one may surmise that for a given modulation power the modelocking principle will be capable of producing "cleaner" pulses (with less structure) at higher repetition rates than the direct modulation process. This follows from the fact that, in a modelocking configuration, the pulse does not build up from noise but from a pulse "stored" in the external resonator from the preceding passage. Also, the various relaxation oscillations that follow a rapid turn-on of the diode do not affect the modelocked pulse shape, but do have to be combatted in direct pulse generation from a pulsed diode.

The theory of saturable absorber modelocking was pursued extensively in computer studies of the transient buildup of modelocked pulses in a *Q*-switched system.[20-27] An analytic theory of forced modelocking of a homogeneously broadened laser was developed by Siegman and Kuizenga[28] in 1970. In 1973 the author attended a conference in which A. Dienes reported the puzzle encountered in the passive modelocking of dye lasers with a dye absorber. The relaxation times of both the laser dye and absorber dye were of the order of a nanosecond, whereas the pulselengths achieved were of the order of a picosecond. Indeed the "paradox" was discovered only after the relaxation times were actually measured using the modelocked laser pulses to probe the dyes.

This puzzle intrigued the author. At the time, no analytic theory existed for saturable absorber modelocking. The dye lasers operated cw, hence the complications of transient buildup were not of concern. Indeed, a cw modelocked system is like a cw oscillator operating in a mode that bears no relation to the noise-excitation from which the mode has been excited initially.

As a first step in the development of an analytic theory of passive modelocking the author reformulated the theory of active modelocking of Siegman and Kuizenga in a simpler form[29] amenable to the treatment of passive modelocking. The analysis of passive modelocking with a fast saturable absorber followed immediately,[30] and the first attempts were made to understand a system using a slow absorber. The question remained how modelocking with a slow absorber provides for stability of the pulse against growth of perturbations along the lagging edge of pulse, which experiences net gain when the slow absorber is "bleached". New[31] pointed out the importance of gain saturation in "terminating" the gain, thus cutting off the lagging edge of the pulse. New's theory was not fully analytic and did not include the finite bandwidth of the laser medium. However, with this new insight it was easy to modify the existing analytic theory to include gain saturation and the result was the paper "Theory of modelocking with a slow saturable absorber".[32] The same fall the author started his one year of leave of absence at the Bell Laboratories which gave an excellent opportunity to test out the theory. One point of dispute was the shape of the pulse. The tails of the pulses in Q-switched modelocked systems were measured to be gaussian by Kaiser and van der Linde,[33] although more recent measurements by Auston seemed to have shown exponential tails.[34] The new analytic theory predicted unequivocally exponential tails. This is an important detail for the experimentalist, because exponential tails contain much more energy than gaussian tails. If one makes measurements of relaxation times at the limit of time resolution, the inferred times can be grossly in error, if gaussian tails are assumed for pulses that actually have exponential tails. Experiments on the very stable system of Ippen and Shank at the Laboratories showed that the pulses had exponential tails.[35] This was the major step that led to acceptance of the theory.

The development of a theory of modelocking with a slow saturable absorber was one of the motivations for renewed attempts at modelocking of a semiconductor diodes at MIT and in the sequel we touch briefly on these developments already referred to above.

While at Bell, the author looked into the progress of the Integrated Optics (IO) work. To his surprise, IO was not pursued with the same energy and sense of purpose as, for example, fiber optics. It seemed logical that integrated optics would be given a new impetus if there were available a source of picosecond pulses that could be integrated with other components so as to provide a compact source of short pulses.

We have mentioned earlier the paradox of passively modelocked dye laser, namely that they provided picosecond pulses, even though the relaxation times of the media were of the order of a nanosecond. If it was possible to achieve picosecond pulses with media that relax within nanoseconds, it ought to be possible to produce short pulses with laser diodes that also have relaxation times in the nanosecond range. One could use a laser diode above threshold as the gain medium facing a laser diode driven below threshold to provide the saturable absorber. These diodes would have to form part of a resonator in which the roundtrip time would be of the order of a nanosecond.

Back at MIT, Dr. P. -T. Ho, then a graduate student, assumed the task to design an external resonator containing two laser diodes for the aforementioned purpose. Initial attempts with lenses failed, because we did not have A. R. coated lenses of large F-number. At Lincoln Laboratory, we saw Dr. Marc Flemming's successful resonator with a curved mirror, and decided to try our luck with the new system. Fairly soon, Ho observed significant threshold current reduction when one diode was placed into the resonator. The threshold reduction was observed by comparing the powers emerging from the external endface of the diode with the mirror blocked and unblocked [see Fig. 1]. All this was done as the first step in putting two diodes into the same resonator. It was then that it occurred to us (Glasser had also joined the group by then) that it would be interesting to actively modelock the diode by applying a μ-wave current-modulation to the diode. This was soon accomplished with Glasser's help. Ho immediately observed marked changes in the μ-wave spectrum of the detector current when the drive was applied and the frequency was adjusted to coincide with the frequency of noise beats at $\Delta\omega = \pi c/nl_d$. We took this as an indication of modelocking, but were not able to get the second harmonic setup to work well enough to resolve the pulses. It was then (Fall, 1977) that Dr. Ippen joined us as a visiting professor. With his expert guidance it did not take long before the pulsewidths were measured.

The purpose of the main body of the paper is to review the salient features of the modelocking theory. Some new results will be integrated with the published formalism. We shall start with the master equation in a time domain description of the modelocking process, with bandwidth limitation by the gain and with modulation of the loss. The formalism Fourier transformed into the frequency domain provides additional insight; in particular it is in this representation that the name "modelocking" can be affixed to the process. The equations can be generalized to include bandwidth limitation by gain and modulation of gain. We look in detail at modelocking with a slow saturable absorber because it is this mechanism which, presumably, produces the short pulses observed by Ippen *et al.*

We then review in greater detail the experimental results obtained to date by our laboratory and workers at the Bell Laboratories.

§2. Master Equation of Modelocking

Consider at first a ring laser configuration with a laser medium (L), an absorber medium (A), with a traveling-wave excitation proceeding in one direction (say counter-clockwise) around the ring [see Fig. 3]. We assume that a pulse has formed so that on the m-th pass around the ring, at the reference plane, the linearly polarized E field component may be written

$$E = v_m(t)\, e^{j\omega_0 t} + \text{complex conjugate}, \tag{1}$$

where $v_m(t)$ is the envelope and ω_0 is the carrier frequency. We ignore the details of the transverse field pattern, treating E as a plane wave. If one considers the transverse field pattern, the problem may become very complicated unless irises mounted in the resonator assure the excitation of the dominant transverse mode only. Then $v_m(t)$ is to be interpreted as the amplitude of the transverse mode.

The pulse passes through the laser medium of (amplitude) gain coefficient α_L, length l_L, absorber of absorption coefficient α_A, length l_A. Finally there may be an additional phase shift ϕ due to other components in the resonator. The modification of the pulse passing the reference plane upon the m-th pass, is represented by the exponential operator

$$e^{\alpha_L l_L}\, e^{-\alpha_A l_A}\, e^{-j\phi}, \tag{2}$$

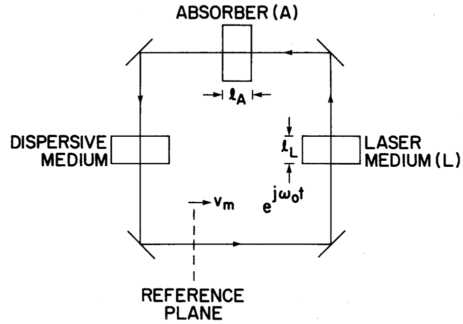


Fig. 3. Schematic of ring resonator.

operating on the amplitude v_m. If the change per pass is small, the exponential can be expanded:

$$e^{\alpha_L l}\, e^{-\alpha_A l_A}\, e^{-j\phi} \simeq (1 + \alpha_L l_L - \alpha_A l_A - j\phi) v_m = (1 + O) v_m, \tag{3}$$

where

$$O v_m \equiv (\alpha_L l_L - \alpha_A l_A - j\phi) v_m. \tag{4}$$

In this limit the action of the elements has been replaced by a sum operator O (the α's and ϕ are in general nonlinear operators) and the operations commute (whereas in general they do not). In this limit the order in which the elements are encountered is irrelevant. Also, in this limit one may incorporate in the absorber loss all other losses, such as mirror losses and the loss due to output coupling.

The pulse $v_{m'}$ operated upon by $(1+O)$ is delayed by the travel time T_R so that $(1+O)v_m(t)$, after delay, produces $v_{m+1}(t)$. Denote the delay operator by D. Then

$$v_{m+1}(t) = D(1+O) v_m(t). \tag{5}$$

In the steady state, v_{m+1} must be equal to $Dv_m(t)$ within a possible time shift δT_R. Indeed pulse shaping may produce such an additional shift by, say, shaving off the front edge of the pulse and amplifying the back edge.

$$v_{m+1}(t) = D v_m(t - \delta T_R) \simeq D\left[v_m(t) - \delta T_R \frac{d}{dt} v_m(t) \right]. \tag{6}$$

Introducing (6) into (5), operating on both sides by D^{-1} (shifting forward in time by T_R), one obtains

$$\left(O + \delta T_R \frac{d}{dt} \right) v_m(t) = 0. \tag{7}$$

This is the Master Equation of modelocking.

The type of bandwidth limiting and modulation is contained in the operator O. Note that we have assumed the existence of pulses that repeat themselves. If we obtain as a solution of (7) an isolated pulse, the implication is that the pulse is part of a pulse-train with period $T_R + \delta T_R$. The period is externally imposed, if the modelocking is active, or "internally" determined if the modelocking is passive. In the latter case, δT_R is a parameter that plays the role of an eigenvalue, except for the fact that the equation to be solved for $v_m(t)$ is a nonlinear one.

Thus far, we have studied a ring resonator. The analysis is equally applicable to a standing wave resonator, as long as the laser medium and modulator are short compared with the laser pulse and the two are positioned near the mirror. The pulse, instead of running round and round, bounces back and forth between the mirrors. With care, the analysis can be applied also to other positions in the standing wave resonator of the laser medium and modulator.

§3. Active Modelocking by Loss Modulation

A schematic description of active modelocking in the time domain is shown in Fig. 4. The loss is modulated sinusoidally in time. Net gain "windows" extend over periodically spaced time intervals. The pulse experiences net gain within the window, loss in the wings extending beyond the window. This causes a sharpening of the pulse. The limited bandwidth of the gain causes pulse spreading. In the steady state, the sharpening is kept in balance by the spreading.

For the analytic treatment, we must obtain explicit expressions for the operator O of (4). Assume the following time dependence of the modulated loss

$$\alpha_A l_A = \alpha_A^{(0)} l_A [1 + 2M(1 - \cos \omega_M t)] \simeq \alpha_A^{(0)} l_A [1 + M\omega_M^2 t^2], \tag{8}$$

where $\alpha_A^{(0)}$ is a constant and M is the modulation coefficient.

Next consider the operator $\alpha_L l_L$. In the frequency domain, for a Lorentzian gain profile,

$$\alpha_L = \frac{\alpha_L^{(0)}}{1 + j\dfrac{\omega - \omega_0}{\omega_L}}, \tag{9}$$

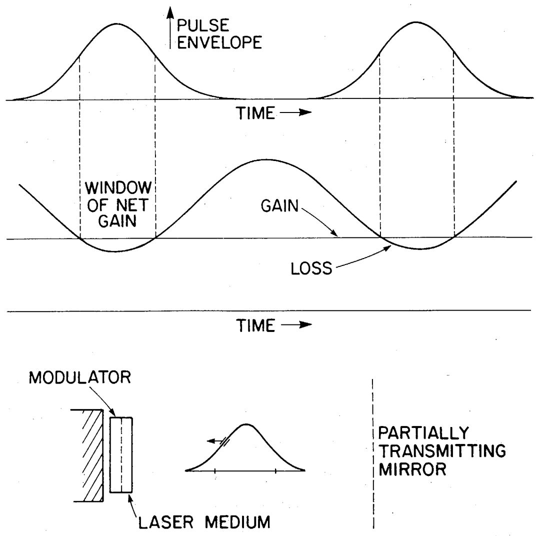


Fig. 4. Active modelocking description in time domain.

where we have picked ω_0 [the carrier frequency of (1)] to be the center frequency of the gain line. If one expands α_L in powers of $(\omega - \omega_0)/\omega_L$ one obtains

$$\alpha_L l_L = \alpha_L^{(0)} l_L \left[1 - j\left(\frac{\omega - \omega_0}{\omega_L}\right) - \left(\frac{\omega - \omega_0}{\omega_L}\right)^2 + \cdots \right]. \tag{10}$$

If we break off the expansion with the second order term we retain a parabolic dependence of α_L upon frequency. In this limit any bell-shaped gain profile is represented by the expansion (10). The envelope $v_m(t)$, Fourier transformed, is a function of $(\omega - \omega_0)$. Operation upon $v_m(\omega - \omega_0)$ by the multiplier $j(\omega - \omega_0)$ corresponds to differentiation with respect to time in the time domain:

$$\alpha_L l_L v_m(t) \to \alpha_L^{(0)} l_L \left[1 - \frac{1}{\omega_L}\frac{d}{dt} + \frac{1}{\omega_L^2}\frac{d^2}{dt^2} \right] v_m(t). \tag{11}$$

The action of the gain medium on the pulse is a change in amplitude represented by the constant multiplier, a (small) time delay $\alpha_L^{(0)} l_L / \omega_L$ represented by the first derivative, and spreading of the pulse in time represented by the second derivative—a diffusion operator. The peak gain parameter $\alpha_L^{(0)}$ is treated as time independent—no modulation of the pulse by gain variation. This assumption holds when the relaxation time of the gain medium is long compared to the period of the pulse train; the gain settles down to a level below the small signal value and the time dependence of the gain due to saturation by one single pulse is ignored. In the model of saturable absorber modelocking in §5 this assumption is lifted.

The phase ϕ can represent interesting physical mechanisms. One may expand ϕ in powers of $(\omega-\omega_0)$

$$\phi=\phi_0+\frac{\mathrm{d}^2k}{\mathrm{d}\omega^2}l(\omega-\omega_0)^2. \tag{12}$$

The first constant term is a constant phase shift; we have omitted a linear term because it corresponds to a time delay which is taken into account separately. The second derivative of k, $(\mathrm{d}^2k/\mathrm{d}\omega^2)$ expresses the group velocity dispersion in a medium of length l and propagation constant k. This may be the contribution of dispersive components in the resonator, intentional or unintentional. Combining (7) with the definition of the operation O of (4), using (11) and (12), one obtains

$$\left\{\alpha_{\mathrm{L}}^{(0)}l_{\mathrm{L}}\left[1-\frac{1}{\omega_{\mathrm{L}}}\frac{\mathrm{d}}{\mathrm{d}t}+\frac{1}{\omega_{\mathrm{L}}^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\right]-\alpha_{\mathrm{A}}l_{\mathrm{A}} -\mathrm{j}\left[\phi_0-\frac{\mathrm{d}^2k}{\mathrm{d}\omega^2}l\frac{\mathrm{d}^2}{\mathrm{d}t^2}\right]+\delta T_{\mathrm{R}}\frac{\mathrm{d}}{\mathrm{d}t}\right\}v(t)=0, \tag{13}$$

where we have omitted the subscript m on $v_m(t)$.

Further, we introduce the symbols used in previous publications.[29,32] The quality factor Q of the resonator representing the time independent part of the loss

$$\alpha_{\mathrm{A}}^{(0)}l_{\mathrm{A}}=\frac{\omega_0T_{\mathrm{R}}}{2Q}. \tag{14}$$

The peak gain g normalized to the time independent part of the loss

$$\alpha_{\mathrm{L}}^{(0)}l_{\mathrm{L}}=\frac{\omega_0T_{\mathrm{R}}}{2Q}g, \tag{15}$$

and the detuning parameter

$$\delta\equiv-\omega_{\mathrm{L}}\delta T_{\mathrm{R}}\frac{2Q}{\omega_0T_{\mathrm{R}}}. \tag{16}$$

One obtains from (13)

$$\left\{g\left[1-\frac{1}{\omega_{\mathrm{L}}}\frac{\mathrm{d}}{\mathrm{d}t}+\frac{1}{\omega_{\mathrm{L}}^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\right] -1-\frac{2Q}{\omega_0T_{\mathrm{R}}}\mathrm{j}\left[\phi_0-\frac{\mathrm{d}^2k}{\mathrm{d}\omega^2}l\frac{\mathrm{d}^2}{\mathrm{d}t^2}\right] -\frac{\delta}{\omega_{\mathrm{L}}}\frac{\mathrm{d}}{\mathrm{d}t}-M\omega_{\mathrm{M}}^2t^2\right\}v=0. \tag{17}$$

Synchronous modulation is achieved when the transit time, as modified by the gain medium, is equal to the period of the modulation $2\pi/\omega_{\mathrm{M}}$. Then, δT_{R} is equal to the change from the free space round trip time caused by the laser medium.

$$\frac{\omega_0T_{\mathrm{R}}}{2Q}\frac{g+\delta}{\omega_{\mathrm{L}}}=\frac{\omega_0T_{\mathrm{R}}}{2Q}\frac{g}{\omega_{\mathrm{L}}}-\delta T_{\mathrm{R}}=0, \tag{18}$$

The modelocking equation reduces to a Schroedinger equation in a parabolic "potential" well, except that some of the coefficients are complex, if dispersion is included, $(\mathrm{d}^2k/\mathrm{d}\omega^2)l\neq0$. Then

$$\left[\left(g-1-\mathrm{j}\frac{2Q}{\omega_0T_{\mathrm{R}}}\phi_0\right)+\frac{1}{\omega_{\mathrm{D}}^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}-M\omega_{\mathrm{M}}^2t^2\right]v=0, \tag{19}$$

where

$$\frac{1}{\omega_{\mathrm{D}}^2}\equiv\frac{g}{\omega_{\mathrm{L}}^2}+\mathrm{j}\frac{2Q}{\omega_0T_{\mathrm{R}}}\frac{\mathrm{d}^2k}{\mathrm{d}\omega^2}l. \tag{20}$$

The solutions are Hermite Gaussians

$$v(t)=H_\nu(\omega_{\mathrm{p}}t)\exp(-\omega_{\mathrm{p}}^2t^2/2), \tag{21}$$

with

$$\omega_{\mathrm{p}}=\sqrt[4]{M}\sqrt{\omega_{\mathrm{M}}\omega_{\mathrm{D}}}, \tag{22}$$

and

$$g-1-\mathrm{j}\frac{2Q}{\omega_0T_{\mathrm{R}}}\phi_0=\frac{\omega_{\mathrm{p}}^2}{\omega_{\mathrm{D}}^2}(2\nu+1). \tag{23}$$

The real part of this equation determines the difference between the peak gain and the loss:

$$g-1=\mathrm{Re}\,[\omega_{\mathrm{p}}^2/\omega_{\mathrm{D}}^2](2\nu+1). \tag{24}$$

The imaginary part of $\omega^2/\omega_{\mathrm{D}}(2\nu+1)$ dictates the phase angle ϕ_0. We shall see in the next section that this corresponds to a uniform detuning of the axial modes from their natural resonance frequencies.

Equation (24) contains g implicitly in the parameter $\mathrm{Re}\,[1/\omega_{\mathrm{D}}^2]=g/\omega_{\mathrm{L}}^2$, (20). Generally $|\omega_{\mathrm{D}}^2|\gg|\omega_{\mathrm{p}}^2|$ and the gain will not exceed unity by much. Thus g can be replaced by unity in the definition (20), and (24) is then explicitly an expression for the excess gain $g-1$. When the fundamental solution is excited with its required gain g, then the higher order solutions have insufficient gain to be operating. They are unexcited in the absence of noise excitation. Conversely, if one of the higher order solutions were set up, then the lowest order solution would experience an excess gain and its amplitude would grow until the gain is depressed to the required steady state value (remember, the gain is a function of power P).

$$g=\frac{g_0}{1+\dfrac{P}{P_L}}, \tag{25}$$

where P_L is the saturation power of the laser, and g_0 is the small signal gain. When detuning is present, the "Ansatz"

$$v(t)=H_\nu(\omega_p t)\cdot\exp(-\omega_p^2 t^2/2)\,e^{\alpha t}, \tag{26}$$

reduces the differential equation (17) to the Schroedinger equation (19); α corresponds to a time shift of $v(t)$ with respect to the minimum of the modulated loss at $t=0$. Equating terms with the same time dependence,

$$\frac{2\alpha}{\omega_D^2}=\frac{g+\delta}{\omega_L}, \tag{27}$$

and

$$g-1-j\frac{2Q}{\omega_0 T_R}\phi_0=\left(\frac{g+\delta}{2}\right)^2\frac{\omega_D^2}{\omega_L^2}+\frac{\omega_p^2}{\omega_D^2}(2\nu+1). \tag{28}$$

The excess gain increases with increased detuning, increasing $|g+\delta|$. When $g+\delta>0(g+\delta<0)$, the period of the applied modelocking signal is longer (shorter) than the round trip time (as modified by the gain medium); the pulse occurs before (after) the loss minimum. The loss decreases (increases) on the average in the duration of the pulse so that the pulse gets pushed back (forward) from its natural return time.

The above solutions allow the following conclusions.

(1) The pulse is gaussian.

(2) The higher order modes are not excited in the noise-free steady state.

(3) The pulsewidth is inversely proportional to the fourth root of the modulation depth M, and the square root of the effective bandwidth Re $\sqrt{\omega_D}$.

(4) The dispersion $d^2k/d\omega^2$ causes "a chirp"—of the pulse frequency.

(5) Detuning of the modulation frequency causes shifts of the pulse peak with respect to the loss minima. The required excess gain increases.

§4. The Modelocking Equation in the Frequency Domain

For certain applications, it is more appropriate to represent the modelocking equation in the frequency domain. One of these applications is the case of a periodic gain- or loss-profile as a function of frequency as it occurs in a composite Fabry-Perot resonator.[36,37] Also, in the frequency domain the modelocking equation lends itself to an interpretation in terms of the locking of axial modes—the origin of the term "modelocking".

Multiplication of $v(t)$ by a time-function $\alpha_A l_A(t)$ is a convolution in the frequency domain. A convolution is a complicated operation and it is not convenient to treat the case of passive modelocking in the frequency domain, because then $\alpha_A l_A(t)$ becomes a nonlinear kernel in the convolution. On the other hand, for a sinusoidal active modulation, $\alpha_A l_A$ given by (8), the solution $v(t)$ of (13) is a periodic function of the same frequency. Multiplication of the Fourier component $V_n\, e^{j\omega_0 t}\, e^{jn\omega_M t}$ of $v(t)$ by the function $\alpha_A l_A(t)$ produces two sidebands $MV_n\, e^{j\omega_0 t}\, e^{j(n\pm1)\omega_M t}$. These act as injection signals for the Fourier components $V_{n\pm1}$. Indeed, the discrete Fourier transform of (13) using (8) for $\alpha_A l_A$, is

$$\begin{aligned}\Bigg\{\alpha_L^{(0)} l_L\left[1-j\frac{n\omega_M}{\omega_L}-\left(\frac{n\omega_M}{\omega_L}\right)^2\right]-\alpha_A^{(0)} l_A\\ -j\left[\phi_0-n\omega_M\delta T_R+(n\omega_M)^2\frac{d^2k}{d\omega^2}l\right]\Bigg\}V_n\\ =\alpha_A^{(0)} l_A M[V_{n+1}-2V_n+V_{n-1}].\end{aligned} \tag{29}$$

This form of the equation can be cast into an equivalent-circuit representation.[29]

The laser medium presents the admittance to the n-th mode:

$$Y_L\equiv-\alpha_L^{(0)} l_L\left[1-j\frac{n\omega_M}{\omega_L}-\left(\frac{n\omega_M}{\omega_L}\right)^2\right]. \tag{30}$$

The resonator (cavity) admittance of the n-th mode is:

$$\begin{aligned}Y\tau\equiv\alpha_A^{(0)} l_A+j\Bigg[\phi_0+\frac{n\omega_M\delta}{\omega_L}\frac{\omega_0 T_R}{2Q}\\ +(n\omega_M)^2\frac{d^2k}{d\omega^2}l\Bigg].\end{aligned} \tag{31}$$

The resonator admittance exhibits, in addition to the frequency independent loss $\alpha_A^0 l_A$, a detuning term consisting of a part common to all modes, and a part that increases with the mode number n, counting the mode with frequency ω_0 as $n=0$. Finally there is a dispersive part causing uneven mode spacing so that Y_C depends quadratically upon n. We may now ascertain that the constant phase ϕ_0 represents oscillation of all modes at a constant frequency offset that is equal to the offset of the $n=0$ mode from its center frequency.

On the right hand side appears an "injection current" produced by the sidebands of the adjacent modes V_{n+1} and V_{n-1}. For convenience, the self-term proportional to V_n is also included on the right hand side so as to produce a second order difference. The equivalent circuit is shown in Fig. 5. The dimensions of the quantities can be rendered into amps, mho's etc. by normalization to an arbitrary admittance.

The equivalent circuit of Fig. 5 shows clearly the "modelocking" aspect of the process. The n-th axial mode of the resonator is locked to the $(n-1)$st and $(n+1)$st mode through their modulation sidebands.

If one treats the spectrum as a continuum, introducing the continuous frequency variable ω for $\omega_n-\omega_0$, one replaces the second order difference by a second order derivative in (29). A Schroedinger equation with a parabolic well results:

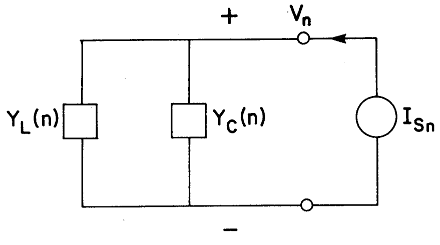
$$\left\{\left[g-\mathrm{j}g\frac{\omega}{\omega_L}-(\omega/\omega_D)^2\right]-1-\mathrm{j}\frac{2Q}{\omega_0 T_R}\phi_0 -\mathrm{j}\delta\frac{\omega}{\omega_L}-M\omega_M^2\frac{d^2}{d\omega^2}\right\}V(\omega)=0, \quad (32)$$

where we have used the definitions (14), (15), (16), and (20). This form renders evident the invariance under Fourier transformation of the equations (19) and (32), and their Hermite Gaussian solutions.

Figure 6 shows schematically the features associated with the modelocked Gaussian solution for the synchronous case with no dispersion.

$$g+\delta=0, \quad \frac{1}{\omega_D^2}=\frac{g}{\omega_L^2}. \quad (33)$$

The abscissae of the vertical lines in Fig. 6a give the frequencies of the axial modes and their heights are equal to the normalized loss. In the absence of any power in the resonator,



$$I_{Sn}=-a_A^{(0)}\ell_A M[V_{n+1}-2V_n+V_{n-1}]$$

Fig. 5. Equivalent circuit for one resonator mode.

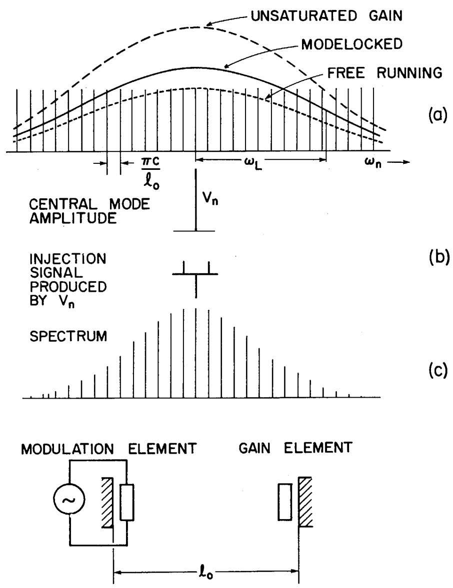


Fig. 6. Active modelocking description in frequency domain. The gain profile is shown as a Lorentzian, not a parabola.

the gain is at the unsaturated level. The power in the resonator in the modelocked state causes the gain to decrease to the required level. In the center there is excess gain, in the wings there is a deficiency of gain. The discrepancy between gain and loss is made up by the injection signal of (29) as shown in Fig. 6b. Figure 6c shows the modelocked gaussian spectral shape.

The gaussian spectrum $\exp(-\omega^2/2\omega_p^2)$ has a spectral width determined by

$$\omega_p=\sqrt[4]{M/g}\sqrt{\omega_L\omega_M}\simeq\sqrt[4]{M}\sqrt{\omega_L\omega_M}; \quad \text{with } g\simeq 1. \quad (34)$$

Note that the spectral width is proportional to the fourth root of the modulation coefficient of the loss, M, and the fourth root of the inverse of the second derivative of the gain versus frequency curve, g/ω_L^2. The gain versus frequency curve plays the role of the "potential" in the Schroedinger equation (32) with $1/\omega_D^2 \to g/\omega_L^2$ when there is no dispersion.

If the modulation is not synchronous, the modes of larger $|n|$ are progressively more detuned. If dispersion is included, we know from before that $\phi_0 \neq 0$. Then the modes are not oscillating at the resonance frequencies. Further, they experience a detuning proportional to n^2. The form of (29) is particularly suited for adaptation to new problems of interest that cannot be handled conveniently in the time domain description.

Thus, for example, the gain as a function of frequency appears in the first term in (29), an expansion of the Lorentzian profile. One may treat the profile exactly, by not going through the expansion. In (31), the parabolic well of the Schroedinger equation is replaced by a Lorentzian well. Note, however, that the "potential" is complex and some properties of the eigenfunction of Hermitean operators of quantum mechanics do not pertain to the solutions of this problem.

Of particular interest to the active modelocking experiment of an uncoated semiconductor diode in an external resonator is the case of a parabolic gain profile with periodic "wiggles". In this case, the parabolic well of (31) must be replaced by a parabolic well with periodic wiggles. The period of the wiggles is

$$\Delta\omega = \pi c/nl_d, \tag{35}$$

where n is the index of the diode material and l_d is its length. The eigenvalue equation leads to solutions with spectral peaks centered around the gain maxima. The solutions with increased phase shifts from peak to peak correspond to increased eigenvalues, i.e. increased required excess gains. Noise can lead to excitation of the whole "spectrum" of eigenfunctions of these phase shifted peaks. This is the "modelocking in clusters model"[36-38] which results, after Fourier transformation, in an envelope with statistical substructure that has the diode round-trip time $c/2nl_d$ as its period. As before in (35), the second derivative of the potential at the gain maxima (potential well minima) determines the spectral widths of the mode clusters; Fig. 7. ω_L of (34) is to be replaced by an effective band-

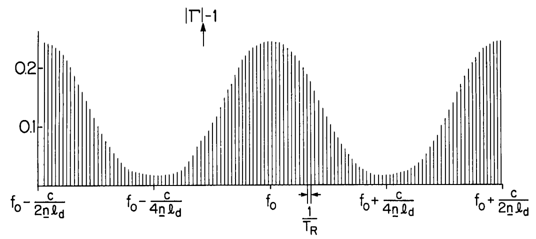


Fig. 7. The periodic wiggles of gain, $|\Gamma|-1$, in a composite Fabry-Perot formed of cw uncoated laser diode in an external resonator. Assumed parameters: $2nl_d/c = 5.2$ ps, $n=3.5$, $r=0.8$, $T_R=0.33$ ns.

width ω_{Leff},[37]

$$\omega_{\text{Leff}} = \frac{\Delta\omega}{2\pi}\sqrt{\frac{2\left(A^2+2\dfrac{A}{r}+1\right)}{A\left(\dfrac{1}{r}-r\right)}}, \tag{36}$$

where $r^2 = R$ is the effective reflectivity of the mirror, $\Delta\omega$ is the period of the wiggles and

$$A \equiv \frac{n-1}{n+1}, \tag{37}$$

with n, the index of the laser diode.

The full width at half maximum of the pulse intensity is according to (34) and (21), for $\nu = 0$,

$$\text{FWHM} = \frac{2\sqrt{\ln 2}}{\sqrt[4]{M}\sqrt{\omega_M \omega_{\text{Leff}}}}. \tag{38}$$

With the values $2nl_d/c = 5.2$ ps, $M=0.25$, $n=3.5$, $r=0.8$, one obtains 18 ps for the FWHM. This is in reasonable agreement with experiments as presented later. Note that the pulsewidth increases with the square root of the diode length.

Experiments show, as mentioned earlier and to be discussed in greater detail below, that saturable absorption plays an important role in the obtainment of short pulses by modelocking.[17] For this reason we want to discuss in some detail the theory of saturable absorber modelocking with an absorber that has a time constant slow compared with the achieved pulsewidth, the case pertinent to semiconductor diodes in which relaxation times are of the order of one nanosecond.

§5. Passive Modelocking with a Slow Saturable Absorber

No solutions have been obtained for the modelocking equation for a slow saturable absorber in the case of dispersion. We shall

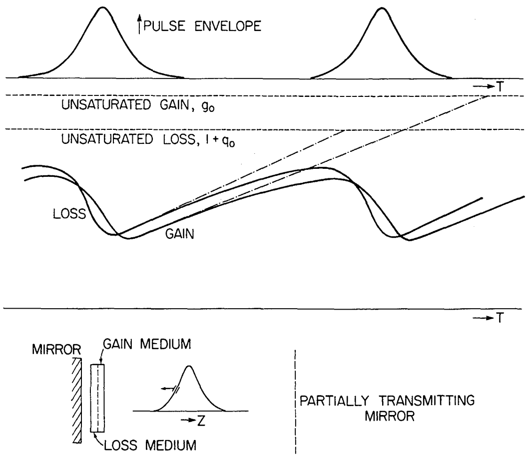


Fig. 8. Passive modelocking description, slow absorber.

neglect dispersion, and make some further approximations necessitated by the nature of the problem. Figure 8 shows schematically how the system operates in the steady state. A pulse traveling back and forth within the resonator depletes the loss first so that the peak of the pulse experiences net gain. The gain saturates towards the end of the pulse, so that net loss is experienced by the trailing edge of the pulse. This is a necessary condition for the stability of the pulse. In the dye laser system, the saturation of the absorber is made to precede that of the gain medium by proper focusing of the mode into the absorber. In the semiconductor system the absorption occurs near the endfaces of the diode because the carrier density is presumably less at the electrode ends. The average intensity is larger near the end face(s) of the diode. An absorption section further enhances the intensity relative to the intensity in the gain section, thus, saturation of the loss may be made to precede that of gain. As far as the diode system is concerned the ensuing analysis assumes antireflection coating of the diode endface internal to the resonator.

Next consider the recovery of loss and gain. Figure 8 is drawn for the case when the roundtrip time in the resonator is shorter than the recovery times of the loss and gain. This is the case in the semiconductor system with pulse repetition frequencies of 3 GHz or higher. Note that the small signal level of normalized gain, g_0, is above the small signal level of normalized loss, $1+q_0$. This must be the case, if the system is to be self-starting. The gain and loss recover between pulses, but do not reach the small signal level . Because the loss must exceed the gain just before arrival of the pulse, so as to prevent the growth of noise in the time slot(s) preceding the pulse, the recovery time of loss must be shorter than that of the gain. This is shown in the figure by the asymptotes. In the semiconductor laser a faster relaxation time for the saturable loss may be due to the enhancement of diffurion in the *short* lossy section by diffusion currents along the laser axis. We denote the initial values of gain and loss just before the pulse arrival by $g^{(i)}$ and $q^{(i)}$.

As a function of time

$$g=g^{(i)}\exp\left\{-\int^{t}|v|^2\,\mathrm{d}t/E_{\mathrm{L}}\right\}, \tag{39}$$

where E_{L} is the saturation energy of the laser medium. The normalized loss is now denoted by

$$1+q^{(i)}\exp\left\{-\frac{\int|v|^2\,\mathrm{d}t}{E_{\mathrm{A}}}\right\}, \tag{40}$$

where $q^{(i)}$ is the initial value of the saturable loss normalized to the linear loss, which in turn is normalized to unity.

Because the gain changes with time it is not possible to assign the bandwidth limitation to the gain medium and still hope for an analytic solution. However, in dye laser systems it is always necessary to introduce bandwidth limiting by a linear filter in order to set the frequency of the peak of the dye-gain at the peak of the absorption of the saturable absorber. Similar bandwidth limiting has been used in self locked semiconductor diode systems.[17)] We shall, therefore, represent the dispersion by the term $(1/\omega_c^2)(\mathrm{d}^2/\mathrm{d}t^2)$ where ω_c expresses the bandwidth of this filter. The modelocking equation results from (7) with proper identification of the terms:

$$\left[g^{(i)}\exp\left\{-\frac{\int|v|^2\,\mathrm{d}t}{E_{\mathrm{L}}}\right\}-1-q^{(i)}\times\exp\left\{-\frac{\int|v|^2\,\mathrm{d}t}{E_{\mathrm{A}}}\right\}-\frac{\delta}{\omega_{\mathrm{L}}}\frac{\mathrm{d}}{\mathrm{d}t}+\frac{1}{\omega_c^2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\right]v=0, \tag{41}$$

where we have used definition (16).

If one expands the exponentials up to second order in their arguments, and neglects higher order terms, the solution of the resulting equation is

$$v = \frac{V_0}{\cosh \dfrac{t}{\tau_p}}. \tag{42}$$

Define the symbol E_p for 1/2 of the pulse energy

$$E_p \equiv V_0^2 \tau_p. \tag{43}$$

Then, equating terms of equal powers of $\tanh(t/\tau_p)$, one obtains

$$g^{(i)}\left[1 - \frac{E_p}{E_L} + \frac{1}{2}\left(\frac{E_p}{E_L}\right)^2\right] - 1 - q^{(i)}\left[1 - \frac{E_p}{E_A} + \frac{1}{2}\left(\frac{E_p}{E_A}\right)^2\right] - \frac{1}{\omega_c^2 \tau_p^2} = 0, \tag{44}$$

$$g^{(i)}\left[\frac{E_p}{E_L} - \left(\frac{E_p}{E_L}\right)^2\right] - q^{(i)}\left[\frac{E_p}{E_A} - \left(\frac{E_p}{E_A}\right)^2\right] - \frac{\delta}{\omega_L \tau_p} = 0, \tag{45}$$

$$\frac{1}{2} g^{(i)}\left(\frac{E_p}{E_L}\right)^2 - \frac{1}{2} q^{(i)}\left(\frac{E_p}{E_A}\right)^2 + \frac{2}{\omega_c^2 \tau_p^2} = 0. \tag{46}$$

We note from (46) that the condition must be met:

$$g^{(i)}\left(\frac{E_p}{E_L}\right)^2 < q^{(i)}\left(\frac{E_p}{E_A}\right)^2, \tag{47}$$

if the pulsewidth is to be real. This means that the saturable absorber must be more easily "bleached" than the laser medium. In general, the deviation $\delta T_R = -\delta\omega_0 T_R/(2Q\omega_L)$ from the free-space roundtrip time is not zero. The simplest algebra results, when

$$g^{(i)}\left[\frac{E_p}{E_L} - \left(\frac{E_p}{E_L}\right)^2\right] = q^{(i)}\left[\frac{E_p}{E_A} - \left(\frac{E_p}{E_A}\right)^2\right], \tag{48}$$

and, correspondingly $\delta T_R = \delta = 0$. From (44)

$$g^{(i)} - 1 - q^{(i)} + \frac{1}{\omega_c^2 \tau_p^2} = 0. \tag{49}$$

It follows that

$$g^{(i)} < 1 + q^{(i)}, \tag{50}$$

the initial gain (before the arrival of the pulse) is less than the initial loss. This is a necessary condition for stable pulse solutions.

After passage of the pulse, the net gain is, in this special case within our approximation

$$g^{(i)}\left[1 - \frac{2E_p}{E_L} + \frac{1}{2}\left(\frac{2E_p}{E_L}\right)^2\right] - 1 - q^{(i)}\left[1 - \frac{2E_p}{E_A} + \frac{1}{2}\left(\frac{2E_p}{E_A}\right)^2\right] = -\frac{1}{\omega_c^2 \tau_p^2}, \tag{51}$$

where we have used (46). The net gain is negative, as required for stability of the pulse against amplification of perturbations following the pulse. It is equal to the net gain before passage of the pulse, and equal in magnitude and opposite in sign to the net gain at the peak of the pulse.

The algebra is a bit more involved when $\delta T_R \neq 0$ and we shall not pursue it here.[29] One may draw the following conclusions.

(1) The wings of the pulse are exponential.
(2) The laser medium must be harder to saturate than the absorber medium.
(3) The net gain is negative before arrival of the pulse.
(4) The net gain is also negative after passage of the pulse.
(5) The pulsewidth is not limited by the relaxation times of the media.
(6) The saturable absorber has to recover between pulses—or at least recover more than the laser medium.
(7) To obtain large pulse energies, and hence strong pulse shaping through saturation, the cavity roundtrip time has to be comparable to the recovery time of the laser medium.

§6. Experimental Observations of Modelocked Semiconductor Lasers

In Fig. 1 we show a schematic of the laser diode in the curved mirror resonator as used in the first experiments of active modelocking. The mirror radius was 5 cm, the diode was mounted very near the center of the curvature. The mirror was moved with micrometer screws and fine-adjusted with piezoelectric positioners. Success of alignment was judged from the change of the power-current characteristic with the mirror blocked or unblocked.

The schematic of the setup used for the intensity correlation measurement with InGaAsP diodes operating at 1.2 μm and 1.3 μm is shown in Fig. 9. The red blocking filter was used to remove all second harmonic background emanating directly from the laser. The emerging pulse-train was separated in an interferometer with one moving arm and recombined and focused on the $LiIO_3$ nonlinear crystal. The second harmonic was detected in a photomultiplier and amplified in a lock-in amplifier. During the experiment, the μ-wave spectrum of the current in a fast detector (100 ps rise time) illuminated by the pulse-train was displayed on a μ-wave spectrum analyzer.

It can be shown[3] that the second-harmonic generation (SHG) trace is proportional to

$$\overline{I^2(t)}+2\overline{I(t)I(t+\tau)},$$

if the light of intensity $I(t)$ is split evenly in the two arms of the interferometer. Figure 10 shows a typical SHG trace. The trace shows a background, a broad pedestal and a superstructure of evenly spaced narrow spikes. The pedestal is indicative of the pulsewidth; the superstructure is due to the fact that the laser pulse is formed of several axial modes which interfere with each other and give rise to the spikes. The width of the spikes is inversely proportional to the number of participating axial modes. The heights of pedestal and background are predicted equal if the above model applies. It is of interest to note that Ho observed broadening of the axial mode spectrum whenever the modelocking drive was turned on [Fig. 11].

Table I gives the results of different diodes achieved to date. We have also entered recent results achieved by van der Ziel[15,16] and Ippen *et al.*[17] in an external resonator consisting of an A.R. coated diode (on the internal endface) a lens (A.R. coated) and a plane end-mirror. Ippen *et al.* used a bandwidth limiting etalon in the resonator section between the lens and the planar mirror. The pulses were Fourier transform limited. They could be maintained without an externally applied μ-wave drive, they were the result of self-locking. The diodes that showed good pulse performance invariably showed dark (absorption) regions near the external end-face of the diode. Because the external resonator was formed with a lens and plane mirror, the spacing of the mirror could be varied. For pulse-repetition rates between 1–3 GHz no appreciable change in pulse width

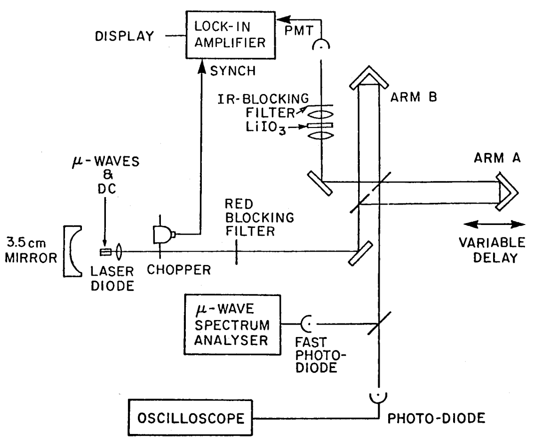


Fig. 9. Schematic of diagnostic set-up.

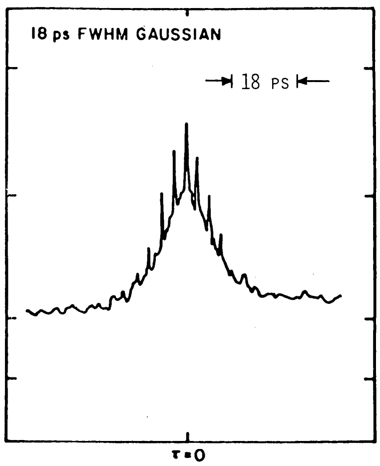


Fig. 10. Typical SHG trace taken from ref. 14.

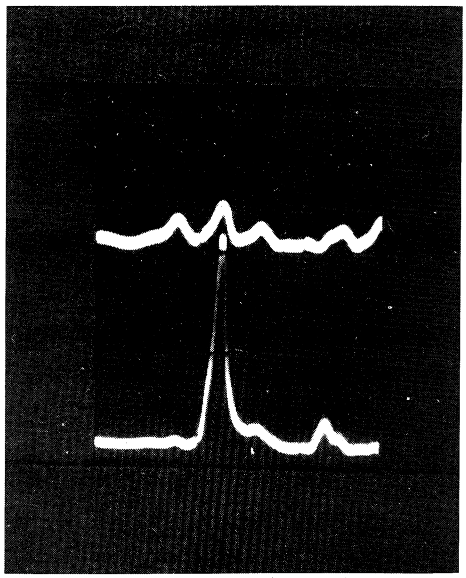


Fig. 11. The spectrum with and without modelocking drive. The peaks are separated by the axial-mode frequency separation of the diode $f=c/2nl_d$. [ref. 13]

Table I.

Material	Wavelength nm	Repetition rate (GHz)	Pulse length (ps)	Coating	Reference	Model
GaAlAs	819	1.5 & 3.0	23	None	12, 13	Active
GaInAsP	1210	1.5 & 2.1	16	A.R.	14	Active
GaAlAs	810	1–3.0	5.1	A.R.	17	Passive
GaAlAs	880	1.9	35	None	15	Active
GaAlAs	880	1.9	6.9	A.R.	16	Active

was observed. The pulses were Fourier transform limited.

Van der Ziel shows SHG traces of pulses from an A.R. coated diode in an external resonator that suggest exponential tails of the pulses.[16] The pulses are inferred to be 6.9 ps FWHM. The presence of exponential tails suggests saturable absorber action, even though the system was actively modelocked.

§7. Conclusions

We have reviewed past work on the modelocking of semiconductor lasers. We have derived the equations of modelocking in a unified way. Common to all modelocking schemes is modulation of gain, or loss, in such a manner that a "window" of net gain is presented to the pulse. The wings of the pulse experience loss, the center of the pulse experiences gain and this leads to pulse sharpening. In the steady state, the sharpening is countered by the pulse spreading due to the finite gain-bandwidth and/or group velocity dispersion.

The modelocking equations in the frequency domain were adapted to treat modelocking of a system with a gain profile exhibiting periodic variation such as occur with an uncoated laser diode in an external resonator. An expression for the achievable pulsewidth was presented.

We have described the development that led to the first successful modelocking of a laser diode in an external resonator. The ease of modulation permits relatively high frequencies of modulation, correspondingly short external resonators and compact systems. We have also mentioned that all diodes that have been modelocked successfully, self-pulsed in the external resonator, even though they ran cw when the external resonator was removed, or blocked. Such behavior cannot be explained unless one postulates the presence of some saturable loss in the system, and many candidates for such loss mechanisms have been proposed in the past.[39,40]

The modelocking of a semiconductor laser diode in an external resonator is the first step in the development of a source of picosecond pulses that is compatible with integrated optics. Indeed, the external resonator can be replaced by a fiber, or an optical waveguide. If 3 GHz modulation is envisaged, an "external" wave-

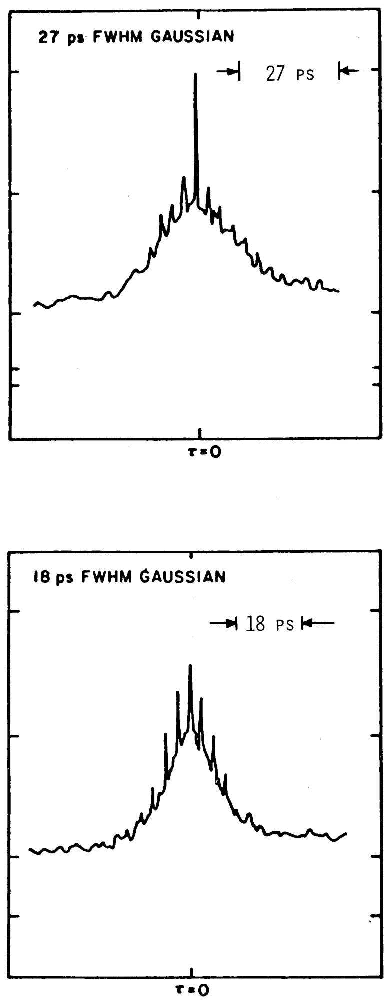


Fig. 12. Two SHG traces. Top trace without modelocking drive, bottom with modelocking drive.

guide resonator with an optical index of 3.5 would be only 1.43 cm long. Higher modulation frequencies would correspond to shorter waveguides.

Ippen's experiments have established the fact that saturable absorption (self-pulsing) plays an important role in the formation of very short pulses. This is a consideration in attempts to build an integrated optics version of the modelocked semiconductor laser diode. It would be desirable to control the amount of saturable absorption in the system by designing an absorption section into the system, via split electrodes that are provided with different biases. The next step in integrating the device will not be an easy one, but several laboratories, including our own are engaged in the pursuit of this worthwhile goal.

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