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Unraveling the dynamics of wealth inequality and the impact on social mobility and health disparities

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Abstract

Inequalities in wealth, income, access to food and healthcare have been rising worldwide in the past decades, approaching levels seen in the early 20th century. Here we study the relationships between wealth inequality and mobility for different segments of the population, comparing longitudinal surveys conducted in the USA and in Italy. The larger wealth inequality observed in the USA is reflected by poorer health conditions than in Italy. We also find that in both countries wealth mobility becomes slower at the two extremes of the wealth distribution. Households trapped in a state of persistent lack of wealth are generally experiencing greater food insecurity and poorer health than the general population. We interpret the observed association between inequality and immobility using a simple agent based model of wealth condensation driven by random returns and exchanges. The model describes well survey data on a qualitative level, but the mobility is generally overestimated by the model. We trace back this discrepancy to the way income is generated for low-wealth households which is not correctly accounted by the model. On the other hand, the model is excellent in describing the wealth dynamics within a restricted class of ultra-wealthy, as we demonstrate by analyzing billionaires lists. Our results suggest that different forms of inequality are intertwined and should therefore be addressed together.

1. Introduction

The last decades have witnessed a constant rise in wealth and income inequality worldwide [1] with an extensive literature describing the underlying economic, social, and political aspects [2, 3]. Wealth inequality has increased due to a variety of factors including tax policies and financial regulations favouring wealth concentration, increasingly unequal income distributions resulting in disparities in wealth accumulation over time, and progressive deterioration of social welfare programs that provide support to low-income households [4, 5]. Wealth inequality and income inequality are closely related. Wealth, as the total value of assets minus liabilities held by individuals or households at a given point in time, can significantly influence the flow of income over time, providing a buffer against economic shocks and uncertainty, while also generating returns that can supplement and sustain income over time [6]. The concentration of wealth among a few individuals or households can therefore have significant implications for income distribution, perpetuating patterns of income inequality across generations [4, 5].

Wealth and income inequality are often associated with high levels of social immobility, which is on rise in developed economies [7, 8]. Countries with high levels of income inequality tend to have lower levels of intergenerational social mobility [9], an observation often referred to as the ‘Great Gatsby’ effect [10]. In other words, individuals from lower-income families in more unequal societies are less likely to move up the

social ladder compared to their counterparts in more equal societies. This relationship could be explained by the fact that low income and wealth can limit access to resources and opportunities, such as quality education, healthcare, and stable employment, that are critical for upward mobility [6]. On the other hand, wealth concentration can reinforce the advantages of those who are already at an advantage [11]. Policies aimed at reducing income and wealth inequality can then help also promote intergenerational social mobility by improving access to those resources and opportunities for individuals from lower-income backgrounds [12].

Economic inequality is often associated with disparities in health and food security. Health inequality refers to the disparities in health outcomes and access to healthcare that exist among different populations or social groups. It is a pressing issue that highlights the unjust distribution of health resources and opportunities, leading to unequal health outcomes and experiences. Factors such as socioeconomic status, race, gender, and geographical location can significantly influence an individual's access to quality healthcare services, preventive measures, and health education [13]. This inequality often results in marginalized communities facing higher rates of chronic diseases, limited access to healthcare facilities, and reduced life expectancy [14]. Addressing health inequality requires a multifaceted approach, including improved healthcare infrastructure, equitable distribution of resources, targeted interventions, and policies that address the social determinants of health [15]. By striving for health equity, societies can work towards ensuring that everyone has a fair opportunity to attain optimal health and well-being.

It has been estimated that two billion people worldwide have moderate or severe food security, due to the lack of regular access to food [16]. Many papers demonstrated the connection between food security and physical health outcomes [17], including diabetes [18], hypertension, as well as depression and anxiety [19]. A recent paper pointed out the important impact of food insecurity in child development [20], showing that the impact of food insecurity extend beyond physical health, impacting mental health, child development, and overall quality of life. Health inequalities and food insecurity are therefore intertwined challenges that significantly impact the well-being of individuals and communities.

In this paper, we explore the relationship between wealth inequality and mobility and their impact on health status and food security by comparing data for USA and Italy. We selected these countries because while they are both developed economies, they have markedly different socio-economic organization. In general, the United States stands as a global economic powerhouse, renowned for its vast wealth and high standard of living. The nation boasts a robust economy, abundant opportunities, and a strong entrepreneurial culture, resulting in a relatively high average income for its citizens. In contrast, Italy, while still possessing a considerable economy, grapples with a more moderate level of wealth and income. The Italian economy is characterized by a greater emphasis on traditional industries, and its citizens generally experience a lower average income compared to their American counterparts. On the other hand, Italy has a more extensive social welfare system than the United States, providing universal healthcare, a comprehensive pension system, and a range of other social programs. The United States, in fact, has a more limited social welfare system, with healthcare and retirement benefits largely provided through employment-based private insurance and retirement plans. The two countries also have different labor market structures. Italy has a higher level of labor market regulation and protection, while the United States, on the other hand, has a more flexible labor market with fewer employment protections and weaker labor unions.

Studies of the relations of socio/economic inequality and mobility often rely on cross-correlations between indicators [10], such as the Gini coefficient [21], the intergenerational income elasticity [22] or the Shorrocks mobility index [23]. Economic indices are convenient to use because they reduce complex multidimensional information to a single parameter but are difficult to interpret and might hide relevant features [24], so that multi-parameter models should be preferred over single indices [25]. To overcome these problems, in the present paper we analyze data from longitudinal surveys and measure wealth inequality and mobility over 20 years in the two countries using standard tools used in the physics of non-equilibrium complex systems, such as persistence [26], mean-square displacements [27] and conditional expectation values in logarithmic space [28, 29]. In this way, we are able to more effectively visualize patterns of wealth inequality and mobility across different strata of the population and correlate them with health indicators and food insecurity.

Moreover, we interpret our empirical results on wealth persistence and mobility by simulating a simple agent-based model (ABM) that has been introduced to study wealth condensation [30] and the effect of wealth taxation on economic growth [31]. This is one example of a broad class of models whose main ingredients are a combination wealth exchanges and multiplicative noise [32–34]. There is a rich literature studying income and wealth inequality by combining statistical data analysis with ABM [18]. ABMs have been widely used in sociology [32], economics [35] and econophysics [36–40], to study a wide range of socio-economic phenomena, including the emergence of income and wealth inequality [30–34, 41–43] as well as its relation with social mobility [44, 45]. ABMs are based on the principle that individual agents,

whether they are consumers, firms, or financial institutions, have their own goals, strategies, and decision-making processes that influence their behavior and interactions with other agents. These agents can interact with one another and be subject to intrinsic and extrinsic noise sources, leading to emergent phenomena at the macro-level that are not always predictable from the micro-level behavior of individual agents. One of the key advantages of ABMs is that they can incorporate heterogeneity and complexity into economic models. ABMs, therefore, can capture the diversity of agents and the range of possible interactions between them and generate new insights into the behavior of complex socio-economic systems that are difficult to capture using traditional models. Through a comparative analysis, in the present paper we discuss successes and limitations of this class of models in describing the relations of wealth inequality and mobility, highlighting differences in the dynamics of wealth within the studied population.

2. Methods

2.1. Data

To study changes in wealth and income in the USA through longitudinal data, we downloaded data from the Panel Study for Income Dynamics (PSID) [46], last accessed on 24 March 2023 from <https://psidonline.isr.umich.edu/>. We restricted our analysis to the years for which wealth information was available (i.e. years in the period 1984–2019). For these years, we also considered data related to health and food security. We complemented the study with cross-sectional data for income and wealth available from the Survey of Consumer Finance (SCF) of the Federal Reserve, considering the most recent survey (2019) last accessed on 10 March 2023 from www.federalreserve.gov/econres/scfindex.htm. Longitudinal wealth and income data for Italy was obtained from the Bank of Italy (BoI) for the years 1991–2020 ('Indagine sui bilanci delle famiglie italiane') last accessed on 22 March 2023 from www.bancaditalia.it/statistiche/basi-dati/rdc/index.html. Aggregated statistical series for food insecurity and health expenditure were obtained from the World Bank (WB) database, last accessed on 20 April 2023 at <https://data.worldbank.org/>. Aggregated statistical series for excess mortality were obtained from the Organization of Economic Cooperation and Development (OECD) database, last accessed on 27 April 2023 at <https://stats.oecd.org/>. Finally, we also consider data on the wealth of billionaires for the years 2002–2022 collected by Forbes and formatted by Gapminder www.gapminder.org/data/documentation/billionaires-dataset/. Data has been downloaded on 18 January 2023 from <https://github.com/open-numbers/ddf-gapminder-billionaires>.

2.2. Statistical analysis

2.2.1. Wealth distribution

To estimate wealth distributions, we employ logarithmic bins. Given the wealth $W_i(t)$ of household i at time t , we first normalize the variable by its weighted mean as $w_i = W_i / \langle W_i \rangle$, where $\langle W_i \rangle = \sum_i p_i W_i$. The normalized weights $p_i = M_i / \sum_i M_i$ take into account the statistical weights M_i associated to each household. We select the positive values $W_i > 0$ and log-transform the normalized variable $X_i = \log_{10}(W_i / \langle W_i \rangle)$. We then construct the histogram of X_i by defining a set of linearly-spaced intervals summing the statistical weights of the data points falling in each bin. The resulting counts in each interval are then divided by the length of the interval. Zero and negative values are analyzed separately. The same strategy is used for other variables such as income.

2.2.2. Wealth quantiles

In our analysis, we classify each household according to the quantile interval to which it belongs within the wealth distribution. Given a variable x whose cumulative distribution is $F(x)$, we define Q -quantiles x_q as $F(x_q) = q/Q$ for $q = 0, \dots, Q-1$. The quantile intervals are then defined as $[x_0, x_1], \dots, [x_q, x_{q+1}], [x_{Q-1}, \infty]$. For simplicity, we denote the quantile interval $[x_q, x_{q+1}]$ by q and consider only deciles ($Q = 10$). In practice, we sort households according to their wealth and divide them in $Q = 10$ intervals of equal weight. Thus, to each data point x_i we associate a number $0 \leq q_i \leq Q-1$, where $q = 0$ represent the bottom 10% of the wealth distribution and $q = 9$ the top 10%. We refer to this number as the wealth decile q_i of household i . Income quantile intervals are defined in the same way.

2.2.3. Persistence

We study the dynamics $q_i(t)$ of the wealth quantiles associated to each household by following its time evolution for the time span available in the longitudinal surveys. For the sake of comparison, we restrict most of the analysis to a time span of 20 years. From the quantile trajectories $q_i(t)$, we estimate transition probability matrices $P_Q(q', t|q, t_0) = P(q_i(t) = q' | q_i(t_0) = q)$ describing the probability that $q_i = q'$ at time t when $q_i = q$ at time t_0 .

The time-dependent persistence of quantile q is then defined as the probability to remain in the initial quantile

$$\Phi_Q(t|q, t_0) = P_Q(q, t|q, t_0). \quad (1)$$

The Shorrocks mobility index [23], often used in the economic literature to quantify social mobility, is given by

$$S_Q(t, t_0) = \frac{Q - \sum_{q=0}^{Q-1} \Phi_Q(q, t, t_0)}{Q - 1}. \quad (2)$$

The index ranges in $0 \leq S_Q \leq 1$ and is equal to $S_Q = 0$ when the position in the wealth distribution is unchanged while complete mixing leads to $S_Q \rightarrow 1$.

2.2.4. Mean-square displacements

Persistence is an estimate of the immobility within the wealth distribution but does not inform about the patterns of mobility. To this end, we define time-dependent mean-square displacements of the quantiles as

$$M(t|t_0, q_i(t_0)) = \sqrt{\langle (q_i(t) - q_i(t_0))^2 \rangle} \quad (3)$$

where the average is obtained by summing over i . If there were no restrictions to wealth mobility, in the large time limit $t \gg t_0$ the position of each household within the wealth distribution is expected to be random. In this conditions, the mean-square displacement for $q_i(t_0) = q$ should approach the limit

$$M_\infty(q) = \sqrt{\sum_{q'=0}^{Q-1} (q' - q)^2 / Q} = \sqrt{(q^2 - q(Q-1) + Q^2/3 - Q/2 + 1/6)}. \quad (4)$$

We denote this limit as ‘perfect mix’.

2.2.5. Conditional expectation values

We estimate conditional expectation values $\langle X(Y) \rangle$ of a first variable X (e.g. income) with respect to a second variable Y (e.g. wealth), defined as the mean of X when the the second variable is equal to Y . This is a well established method used to describe relations among variables in non-equilibrium systems with power law scaling [28, 29]. As for the case of distributions, we use logarithmically spaced bins. We consider positive values of Y_i , compute the logarithm of the variable $Z = \log_{10} Y$, define a set of linearly spaced intervals and calculate the weighted averages of X for all the points for which Z falls into the interval considered. In an analogous way, We also compute the conditional variance $\sigma(X|Y)$ of the first variable X when the second variable is equal to Y .

2.3. Model

We simulate the AGM of wealth inequality proposed by Bouchaud and Mézard [30]. The model describes the evolution of the wealth W_i of the agent i , within a population of N agents, according to the following stochastic differential equations

$$\frac{dW_i}{dt} = \eta_i(t) W_i + \sum_j (J_{ij} W_j - J_{ji} W_i) \quad (5)$$

where J_{ij} controls wealth redistribution between agent i and agent j and $\eta_i(t)$ is a Gaussian random noise with mean m and variance σ . The multiplicative noise term $\eta_i(t) W_i$ is interpreted in the Stratonovic sense. We consider a redistribution interaction embedded in regular random network with degree $c = 4$ with $J_{ij} = J_0 / c A_{ij}^0$, where A_{ij}^0 is the adjacency matrix of the network ($A_{ij}^0 = 1$ if the node i is connected to j and zero otherwise) [42, 43].

To integrate numerically the equations, it is convenient to express them in terms of an N -dimensional system of equations for the vector $X = (W_0, \dots, W_N)$ which obeys

$$dX_i = \sum_j A_{ij} X_j dt + B_i X_i d\psi_i \quad (6)$$

where $A_{ij} = A_{ij}^0 J_0 / c + \delta_{ij} (m - J_0 / c \sum_k A_{ki}^0 - \phi)$, $B_i = \sigma$ and $d\psi_i$ are independent Wiener processes (with variance 1 and zero mean).

We exploit the scaling invariance of the equations [30] and study the model as a function of the reduced parameter J_0/σ^2 , measuring time in units of $1/\sigma^2$. Since we are only interested in relative inequality, we rescale the wealth W_i by the total wealth $W_T = \sum_i^N W_i$ defining the wealth fraction $\tilde{w}_i = W_i/W_T = Nw_i$ belonging to agent i [30]. In this way, the results are independent on the parameter m .

3. Results

3.1. Wealth inequality in USA and Italy

Wealth inequality is conventionally measured considering the wealth distribution or more often by defining simplex indices, such as the Gini index [21]. The upper tail of the wealth probability density has received wide attention in the literature and is well described by the Pareto distribution $P(W) \sim 1/W^{\alpha+1}$, where the Pareto exponent α ranges between 1 and 2 for most countries [37]. A lower Pareto exponent correspond to higher wealth inequality.

Here we use national survey data (PSID for USA and BoI for Italy, as discussed in the Methods section) to estimate the wealth distribution for households in USA and Italy in different years. Since the distribution spans several order of magnitudes in wealth, we analyze it using logarithmically-spaced bins, which is appropriate for a power laws. As discussed in the methods section, we consider normalized wealth since we are interested in inequalities rather than in the absolute values of wealth. The estimated wealth probability density functions, for positive wealth only, are for different years are reported in figures 1(a) and (b) for Italy and USA, respectively. As expected, we observe a Pareto tail for high wealth, with exponents $\alpha \simeq 1.45$ for USA and $\alpha \simeq 1.95$. The lower part of the distribution reveals another power law exponent $W^{-\beta}$, with $\beta \simeq 0.5$ which is more apparent in US data, but also observable in Italian data. We also notice that the general shape of the distribution remains the same across the years with only small deviations, so that distributions for different years are superimposed.

We have confirmed the double power law behavior observed in PSID using an independent cross-sectional survey, the SCF by the Federal Reserve. The results shown in figure 2 confirms this result. In particular, we fit the data with the interpolating function

$$f(w) = C_1 \frac{w^{-\beta}}{1 + (w/w_c)^{(1+\alpha-\beta)}}, \quad (7)$$

obtaining $\alpha = 1.46$ and $\beta = 0.47$ as best fit parameters. The crossover value is estimated to be at $w_c = 0.87$ which corresponds to 730 000 USD.

We now turn our attention to the part of the wealth distribution that is not included in figures 1(a) and (b): the region of zero or negative net wealth. In figures 1(c) and (d), we report the fraction of households with zero or negative wealth as a function of time for USA and Italy, respectively. The data show a much higher fraction in USA with respect to Italy and in both cases the fraction has increased in the past 30 years, reaching up to 20% in the USA. Hence, while the relative distribution of positive wealth remains roughly constant, the increase in the fraction of households that have no wealth is consistent with the general growth of wealth inequality reported in the literature.

3.2. Time-dependent persistence and mobility patterns of wealth

Having established the general form of the wealth distribution, we now concentrate on the mobility of individual households within the distribution. To this end, we label households according to their wealth decile and follow their time evolution for a time span of 20 years using longitudinal national surveys for USA (PSID) and Italy (BoI). This is illustrated in figures 3(a) and (b) for USA and Italy, respectively, using a heatmap showing the time-dependent transition probability $P_Q(q', t|q, t_0)$ that an household falling into a decile q (here $q = 0, 3, 5, 7, 9$) at $t = t_0$ ($t_0 = 1999$ for USA and $t_0 = 2000$ for Italy) is found in decile q' at time t . The figure show some degree of diffusion across the deciles which is, however, minimal for households starting from the upper decile.

A summary of the mobility pattern can be obtained by computing the Shorrocks mobility index [23] S_{10} , as discussed in the methods section. The result reported in figure 3(c) reveal little differences between USA and Italy and display a slow time dependence. The Shorrocks index, however, averages together the mobility of all the wealth quantiles into a single number and therefore does not allow to appreciate the difference in mobility of households of different wealth. To this end, we consider the time-dependent persistence into the top and bottom decile for households in the USA and in Italy as shown in figures 3(d) and (e). The results show in both cases, a marked difference in persistence between the top and bottom decile with households in the top decile being the most persistent. All the curves are well fit with a stretched exponential function

$$\Phi_Q(t|q, t_0) = (1 - \Phi_Q^\infty) \exp\left(- (t/\tau(q))^{\gamma(q)}\right) + \Phi_Q^\infty, \quad (8)$$

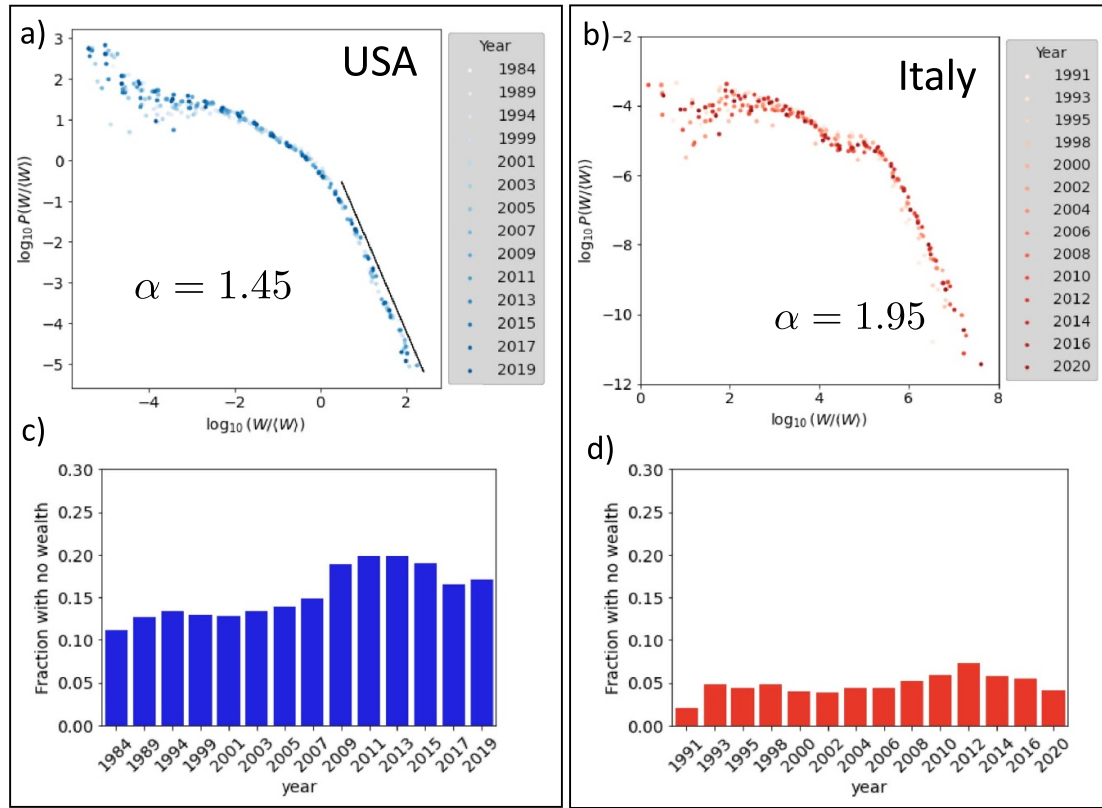


Figure 1. Differences in wealth inequality between USA and Italy. Panels (a) and (b) report the distribution household wealth shares plotted in double logarithmic scales for (a) USA (PSID data) and (b) Italy (data from the Bank of Italy). Both distribution displays a Pareto tail for large wealth with exponents $\alpha \simeq 1.45$ and $\alpha \simeq 1.9$ for USA and Italy, respectively. Panels (c) and (d) report the fraction of households with no wealth (either zero or negative) for USA and Italy, respectively. Sources: PSID (c) and BoI (d).

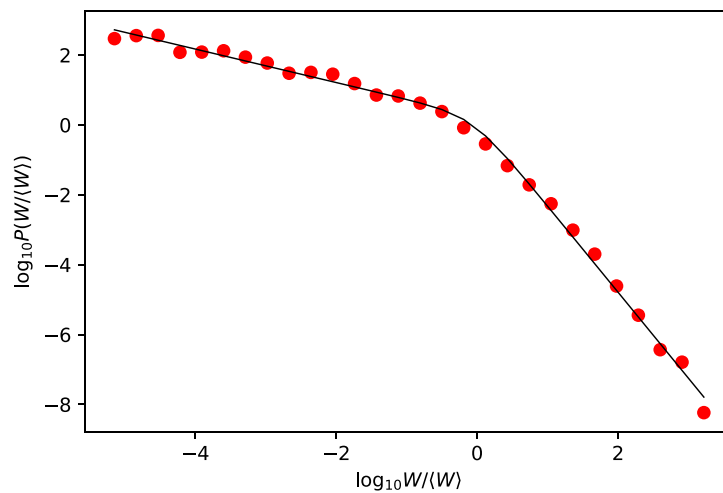


Figure 2. Crossover-scaling in the US wealth distribution. The US wealth distribution estimated from SCF data (2019) fitted with a crossover function between two power laws, with exponents $\beta = 0.47$ and $\alpha = 1.46$.

where Φ_Q^∞ is the value expected assuming that after an infinite time the wealth is uncorrelated with its initial value (i.e. $\Phi_Q^\infty = 1/Q = 1/10$). The best fits yield characteristic times $\tau(0) = 1.7$ and $\tau(9) = 10^5$ years with $\gamma(0) = 0.37$ and $\gamma(9) = 0.07$ for the USA and $\tau(0) = 3$ and $\tau(9) = 415$ years with $\gamma(0) = 0.27$ and $\gamma(9) = 0.13$ for Italy. Figure 3(f) reports residual persistence, defined as the value of the persistence observed after 20 years for households in different deciles minus the expected asymptotic value (i.e. $\Phi_Q(t=20) - \Phi_Q^\infty$). These results suggest that persistence depend on the initial decile and is highest for top

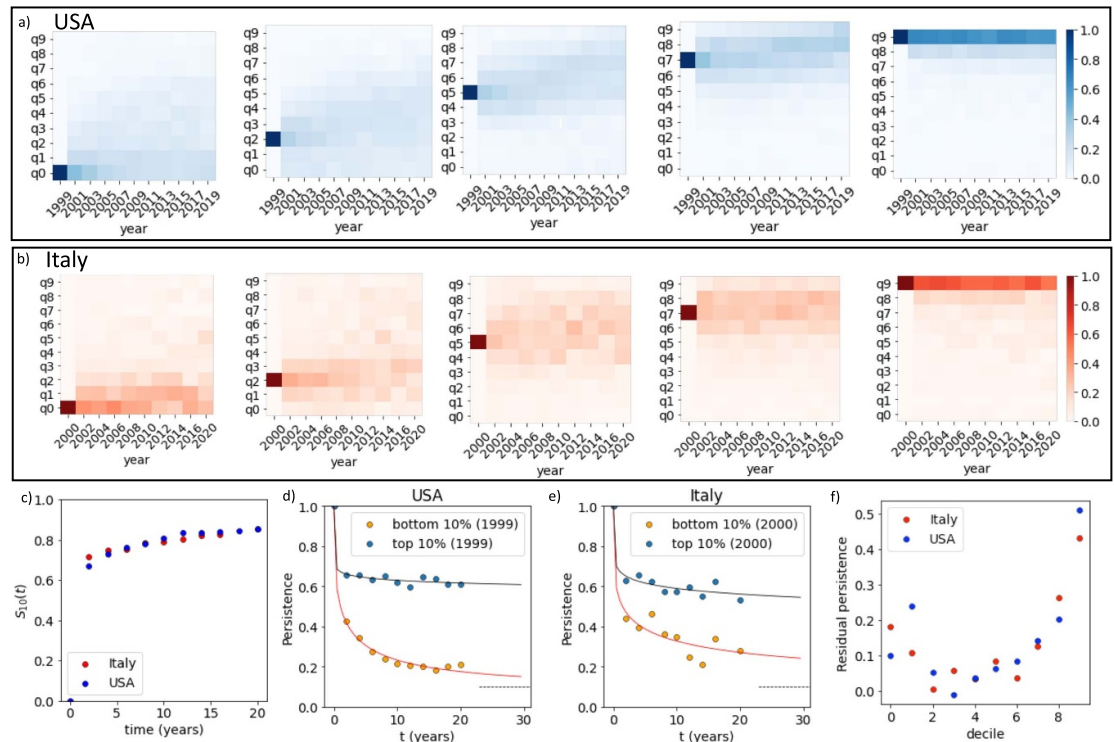


Figure 3. Wealth mobility and persistence. Wealth mobility patterns are illustrated for (a) USA and (b) Italy as heatmaps representing the probability that the wealth falls into each decile of the distribution at time t for households that at time t_0 (1999 for USA and 2000 for Italy) belonged to a given wealth decile (here $q(t_0) = 0, 3, 5, 7, 9$). (c) The Shorrocks wealth mobility index as a function of time. Time-dependent persistence in the upper and lower decile of the wealth distribution for (d) USA and (e) Italy. (f) Residual persistence estimated for USA Italy. Sources: PSID for USA and Bank of Italy for Italy.

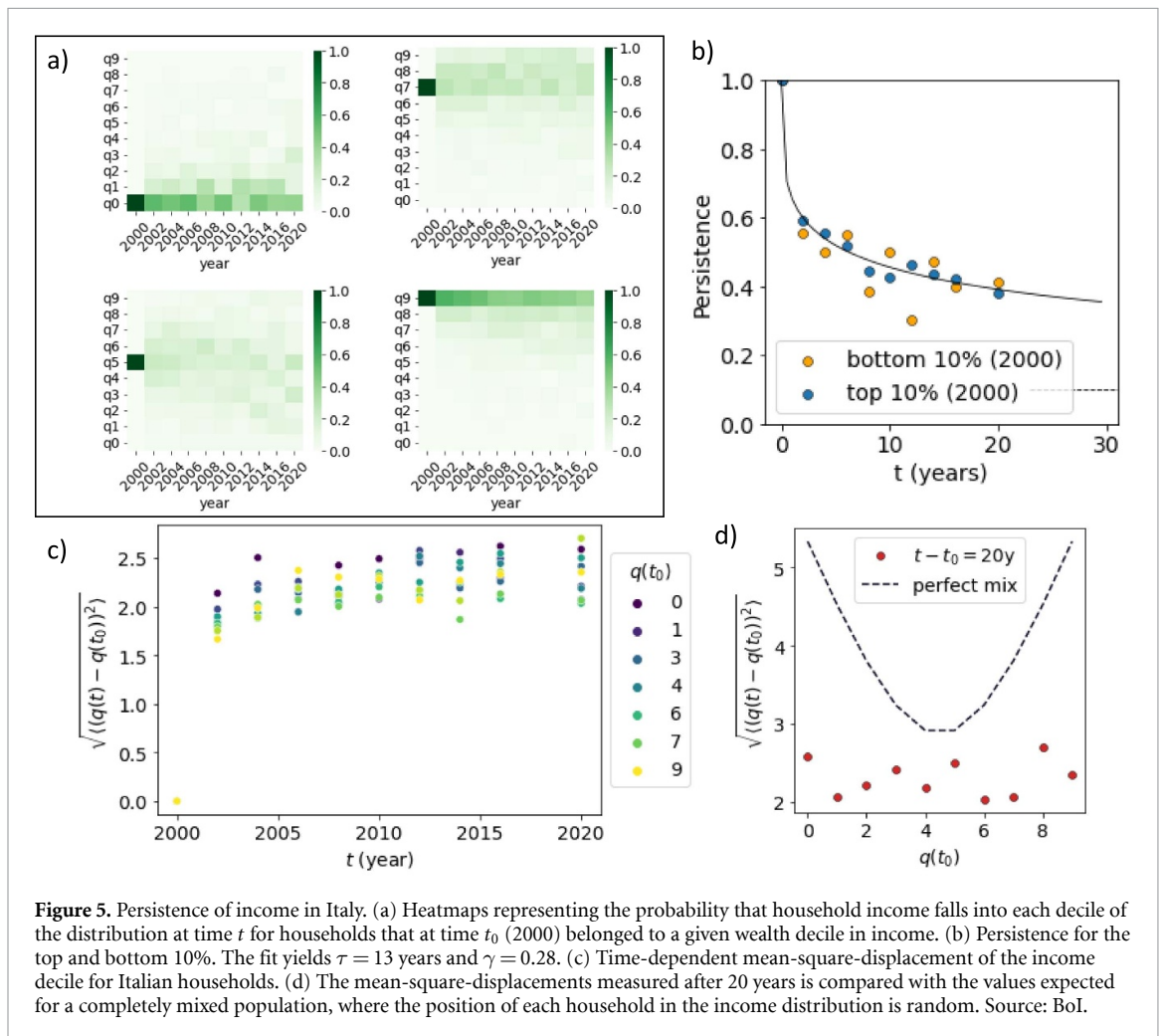
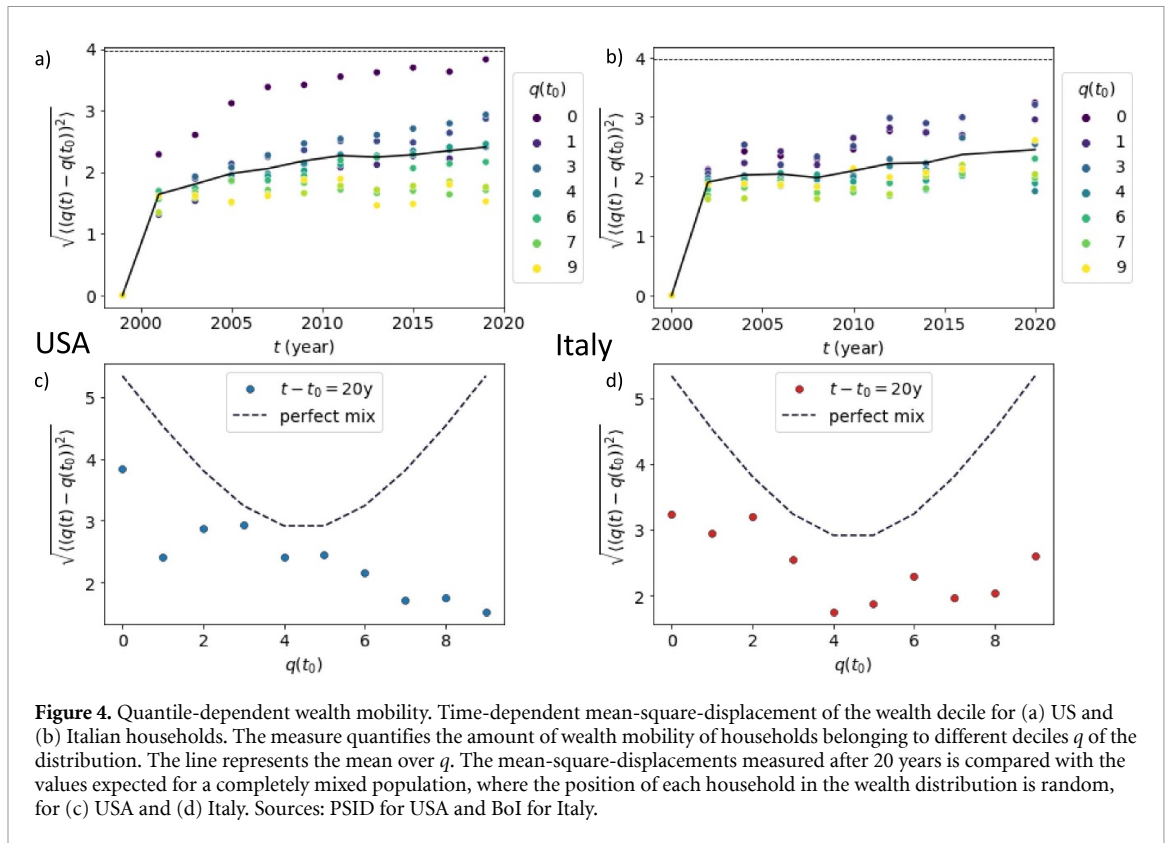
wealth owners that are for a large part immobile. Residual persistence is instead lowest for households in intermediate wealth deciles.

Measures of persistence, including the Shorrocks index, provide information about the extent of wealth immobility of different strata in the population but do not inform about wealth mobility patterns encoded in the transition probability matrix. To this end, we compute mean-square quantile displacements $M(t|t_0, q)$ as defined in the Methods section. Figures 4(a) and (b) shows that $M(t|t_0, q)$ grows slowly in a q dependent manner, for the USA and Italy. In both cases, the slowest growth is observed for the highest quantiles the fastest for the lower one, particularly in the USA. As in the case of persistence, we can compare the observation with the result expected asymptotically if we assume that the final wealth decile is independent on the initial one (perfect mix). In this case, $M(t|t_0, q) \rightarrow M_\infty(q)$ can be computed analytically (see methods section). A comparison between the observed residual mean-square quantile displacements after 20 years and $M_\infty(q)$ is reported in figures 4(c) and (d) for the USA and Italy. The figure show that for all value of q the observed mobility patterns are far from perfect mix since mobility is rather localized. The lowest level of mobility is observed for highest wealth deciles. Furthermore, the typical changes in wealth decile over a span of 20 years is between 2 and 3.

For completeness, we repeated our analysis of persistence and mobility using income instead as wealth. A summary of the results are reported in figure 5 for Italy and figure 6 for the USA. The results are qualitatively similar to the analysis based on wealth, except for the fact that there is less disparity in persistence and mobility between the top and bottom decile. For all cases, however, residual persistence is considerable and mobility is restricted.

3.3. Health inequality

Our analysis shows that a fraction of the population in USA and Italy has either zero or negative wealth, lying at the bottom of the wealth distribution (figure 1). Moreover, studying the mobility across the wealth distribution, we observed persistence within each wealth decile that is largest at the extremes (figure 4). We then examine the possible consequences for health of persistent lack of wealth. Since PSID contains longitudinal information on how health-related variables associated with wealth for US households but we could not access comparable data for Italy, we decided to compare the two countries using aggregated statistical data.



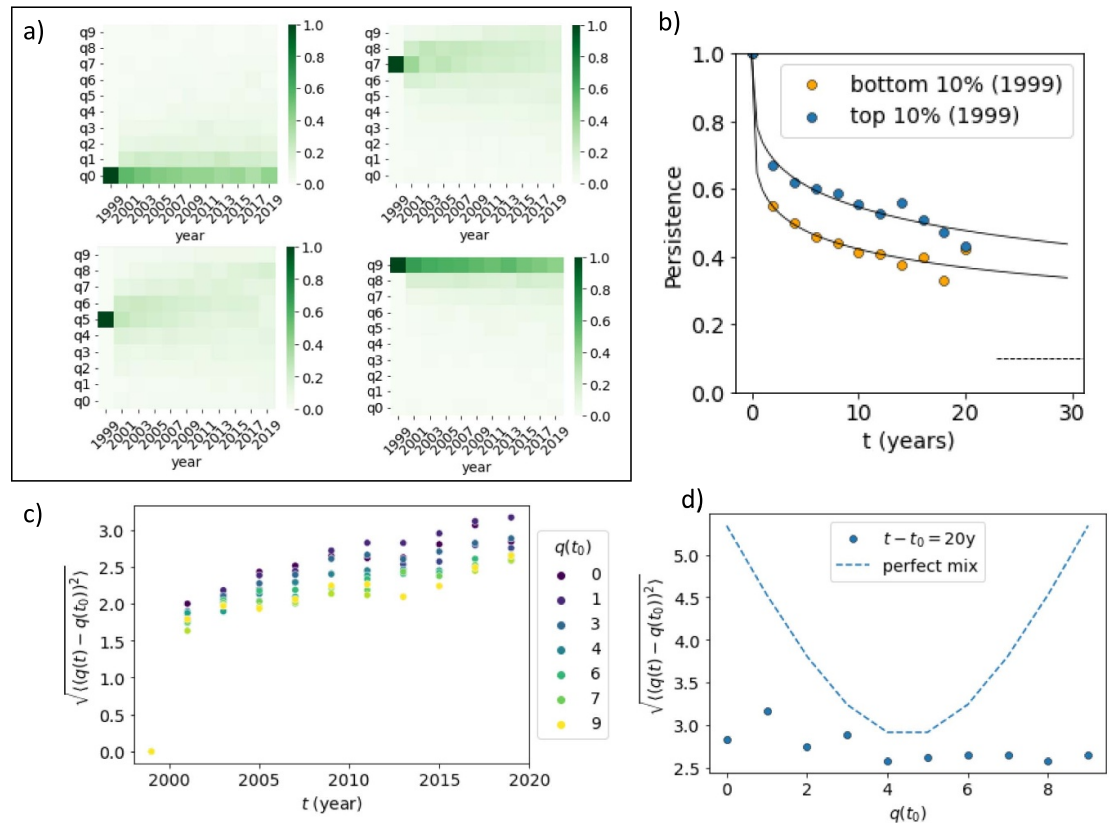


Figure 6. Persistence of income in the USA. (a) Heatmaps representing the probability that household income falls into each decile of the distribution at time t for households that at time t_0 (1999) belonged to a given wealth decile in income. (b) Persistence for the top and bottom 10%. The fits yields $\tau = 9$ years and $\gamma = 0.24$ for the bottom 10% and $\tau = 31$ years and $\gamma = 0.31$ for the top 10%. (c) Time-dependent mean-square-displacement of the income decile for US households. (d) The mean-square-displacements measured after 20 years is compared with the values expected for a completely mixed population, where the position of each household in the income distribution is random. (d) Source: PSID.

Aggregated data show that the rate of avoidable mortality, both preventable and treatable, in the USA was almost twice as large as in Italy for the years 2010–2020 (see figure 7). It is interesting to point out that this occurs despite the fact that the healthcare expense per capita, governmental and private, is considerably larger in the USA than in Italy (figure 7).

We then studied in detail the excess mortality due to various diseases from chronic diseases to mental ones, considering the years lost per 100 000 people for the same period (figure 8). For all the diseases considered, including diseases related to metabolism and nutrition, the circulatory system and the nervous system, mortality in the USA is larger than in Italy. The only exception is found in the case of cancer (figure 9). In fact, for colon cancer mortality in USA is higher than in Italy, but for bladder, skin and stomach cancer Italy is higher than USA. Finally, for lung, leukemia and pancreatic cancer the mortality is similar between the two countries. The complex etiology of this diseases and the possible link with prevention/therapeutic strategies that are different between the two countries as well as in general the different lifestyle (from the quality of the food to the practise of physical activities) could be involved in the different evolution of the tumors.

Considering that in general USA show higher mortality for a broad range of diseases (the only exception being cancer), we then turn our attention on the possible association between health and persistence at the bottom of the wealth distribution, analysing the longitudinal national surveys for USA (PSID). As shown in figure 10(a), approximately 40% of the households with no wealth in the year 1999 persist in this condition until 2019. We thus consider households that had zero or negative wealth in 1999 and in 2019 (persisters) and compare these households with those that had zero or negative wealth in 1999 but a positive wealth in 2019 (non persisters), and also consider the general population (all) for reference.

For households in the persisters group the fraction of family heads with poor health status is larger than that of family heads of non persister households which on their turn is larger that of the general population (figure 10(b)). We then look at the incidence of several conditions including cancer (figure 10(c)), diabetes (figure 10(d)), hearth disease (figure 10(e)), hypertension (figure 10(f)) and psychological problems

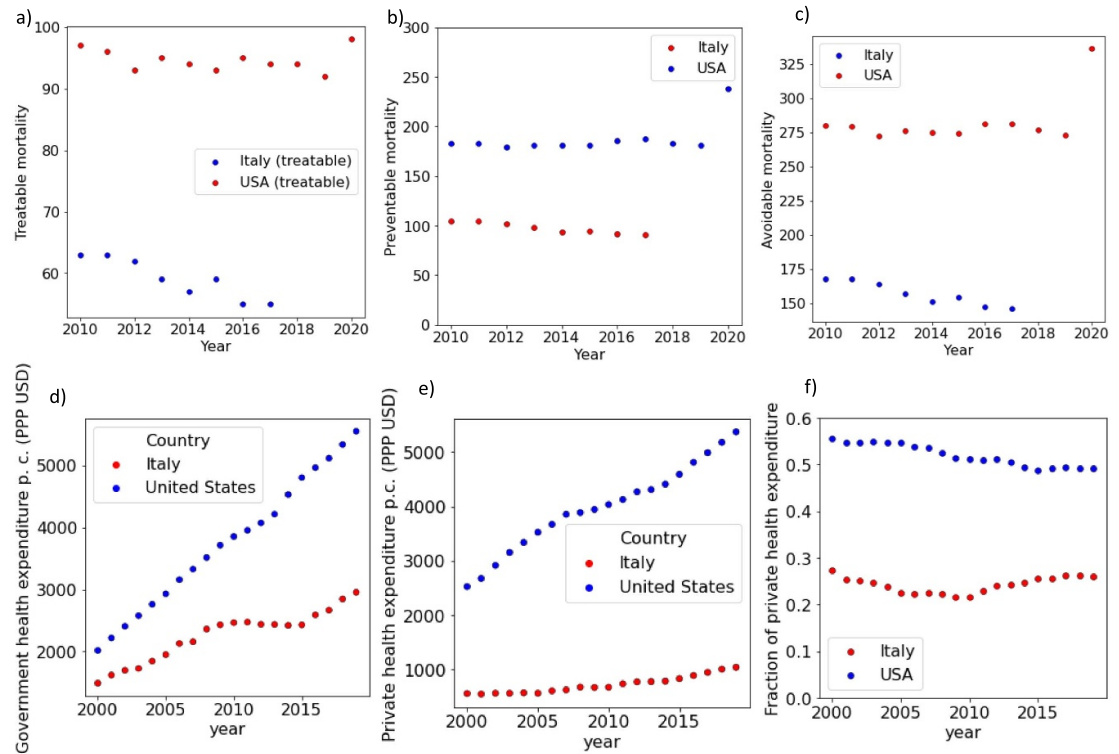


Figure 7. Comparison of avoidable mortality and health expenditure for USA and Italy. (a) Treatable mortality, (b) preventable mortality and (c) avoidable mortality per 100 000 population for USA and Italy. (d) Per capita government health expenditure and (e) private health expenditure for USA and Italy (in purchasing power parity USD) for USA and Italy. (f) Fraction of private health expenditure over total expenditure for USA and Italy. Source: WB.

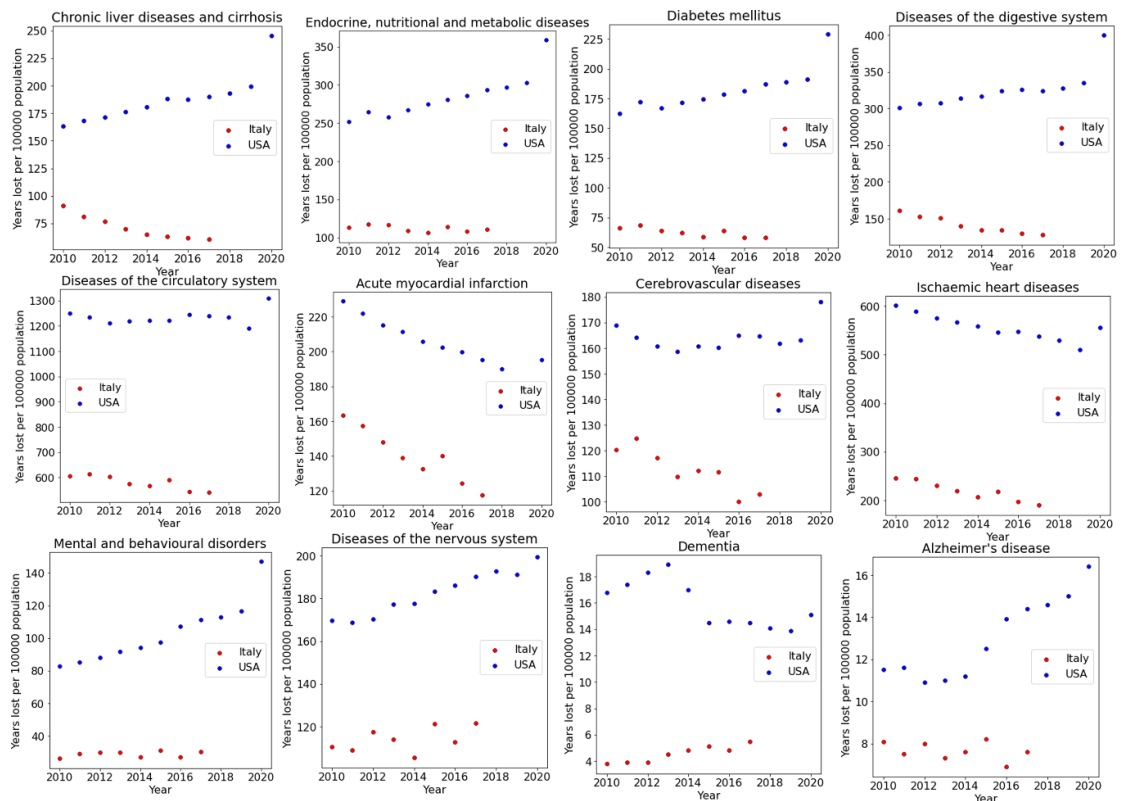
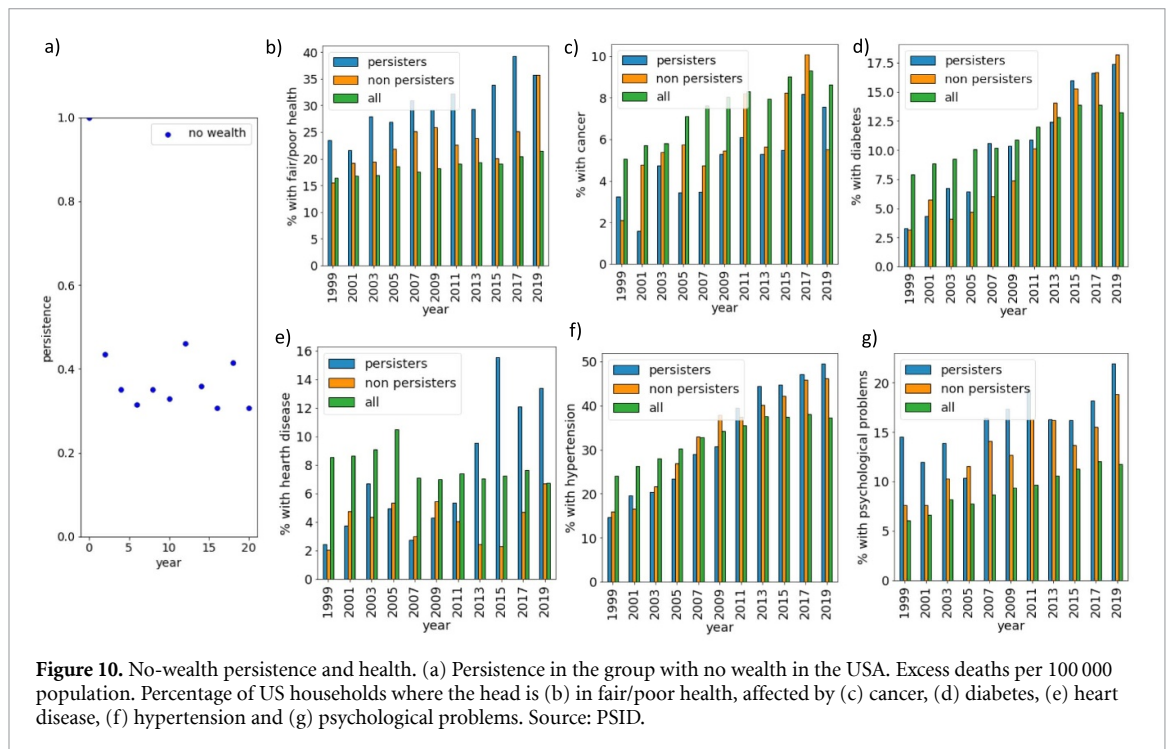
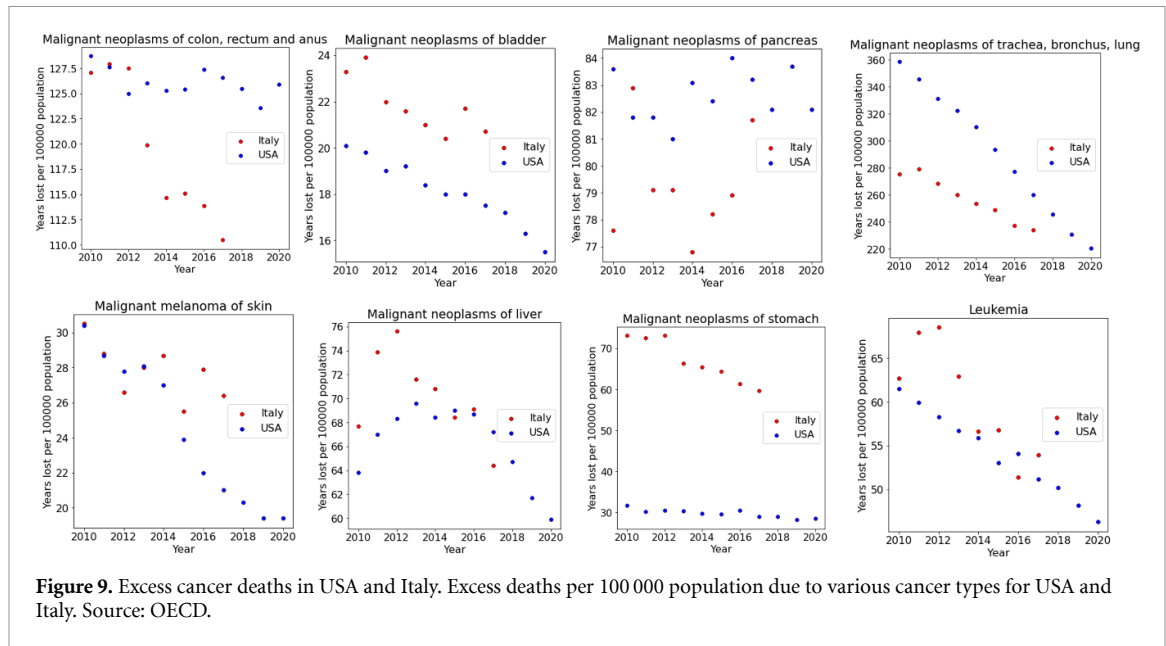


Figure 8. Excess deaths in USA and Italy. Excess deaths per 100 000 population due to various diseases as reported in the panels for USA and Italy. Source: OECD.



(figure 10(g)). In particular for heart disease, hypertension and psychological problems, the data indicates that lack of wealth and no-wealth persistence are associated with poorer health status.

3.4. Inequality in food security

Food insecurity is known to be connected to diseases such as hypertension, depression and anxiety [19]. Figure 11(a) reports the percentage of population affected by moderate or severe food insecurity in USA and Italy according to the World Bank, while figure 11(b) reports the percentage related to severe food insecurity for the period 2005–2020. We observe that food insecurity is generally larger in the USA than in Italy, but severe food insecurity is larger in Italy and on the rise in the past few years.

We thus consider the possible relationship between food insecurity and no-wealth persistence analysing the longitudinal national surveys for USA (PSID). We follow the same strategy as for the case of health status: we first compare aggregated data for the two countries and then analyze more closely the case of the USA. Figure 11(c) shows the fraction of households with food insecurity as a function of time subdivided into persisters, non persisters and all. We see that the percentage of households with food insecurity among

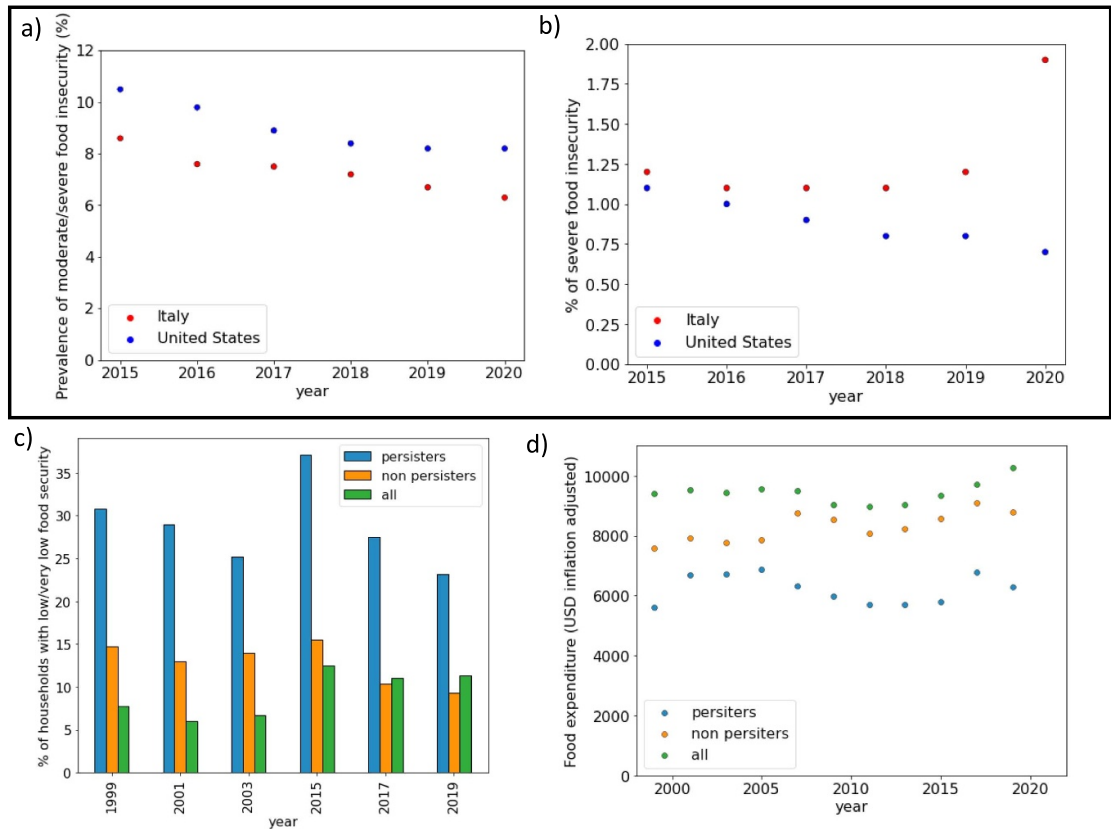


Figure 11. No-wealth persistence and food insecurity. Percentage of (a) moderate/severe and (b) severe food insecurity in the population for USA and Italy as a function of time. (c) Percentage of US households with low/very low food security as a function of time considering the entire population, those with no wealth in 1999 and in 2019 (persisters) or those who had no wealth in 1999 but had wealth in 2019 (non-persisters). (d) Food expenditure for the households belonging to the same categories (inflation adjusted USD). Source: WB (a), (b), PSID (c), (d).

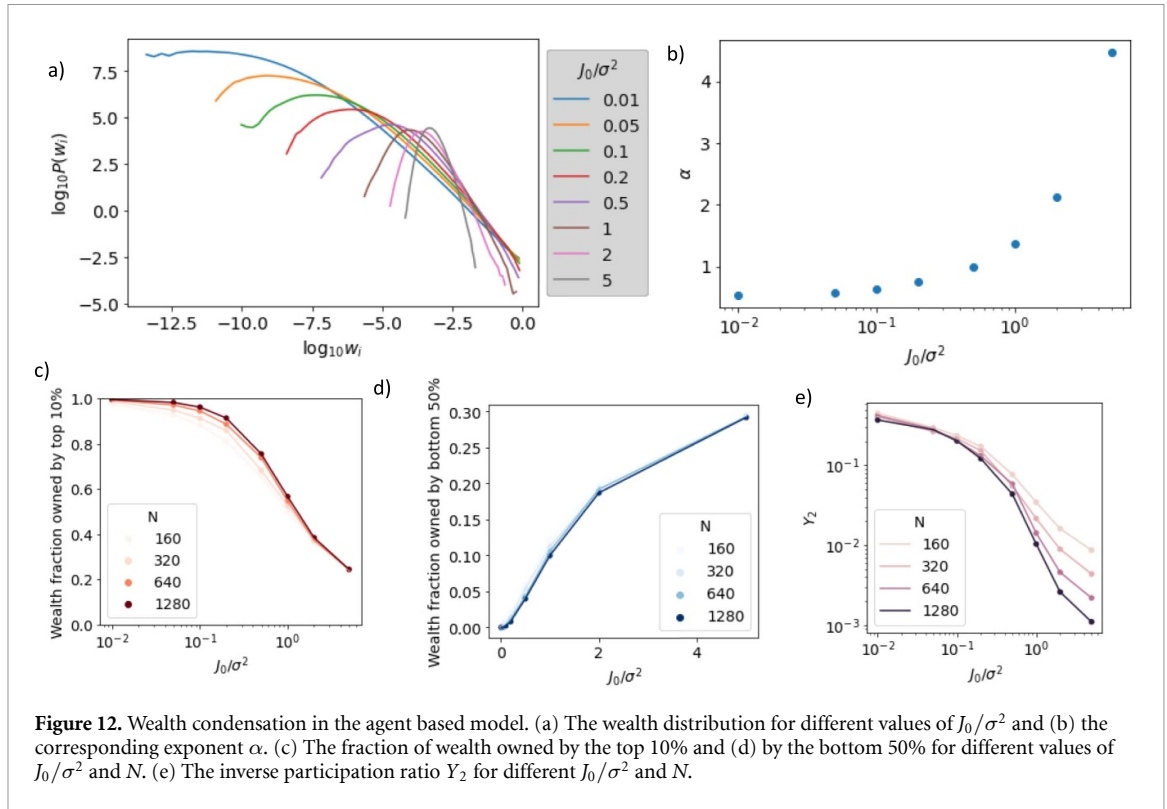
persisters is not decreasing in time, being almost twice that of non persisters and three times larger than that of the general population. On the other hand, the percentage of households with food insecurity decreases among non-persisters approaching the levels of the general population after 20 years. Similar trends are observed in the mean food expenditures (figure 11(d)): persisters spend in average less than non persisters and even less than the general population.

3.5. Wealth persistence in an agent based model

To better interpret our findings on wealth persistence and mobility within a population, we resort to an agent based model in which a series of identical agents accumulate and loose wealth subject to randomness and redistribution. In particular, we concentrate on the model of wealth condensation proposed by Bouchaud and Mezard [30] and studied in detail in a series of subsequent papers by various authors [31, 42, 43]. The steady-state properties of the model are well understood analytically [30, 31, 42, 43] and in good agreement with empirical statistics. In particular, the model is attractive because it is able to recover a distribution of wealth which displays Pareto (power law) tails resulting from the stochastic dynamics of interacting agents in a growing economy. Furthermore, the distribution of relative wealth converges to a steady state and the Pareto exponent is controlled only by the ratio of two parameters, representing the redistribution rate (J_0) and the variance of the noise distribution (σ^2).

Here we perform numerical simulations of the model focusing on its out-of-equilibrium behavior (see the Methods section for more technical details on the model). We first verify that our simulations recover known steady-state properties of the model [30], including the power law wealth distribution, the Pareto tail exponent dependence on J_0/σ^2 , the system-size dependence of the inverse participation ratio and wealth condensation for low J_0/σ^2 (see figure 12).

Figure 13(a) report the temporal evolution of the wealth quantiles (for $Q = 10$) of N agents as a function of the ratio J_0/σ^2 which controls the extent of wealth inequality (the lower J_0/σ^2 the higher the inequality).



The figure shows that after a sufficiently long time the quantile of each agent appears to be uncorrelated with the initial value. The time to approach to this steady-state depends on J_0/σ^2 , which is consistent with analytical results for the correlation time obtained in the mean-field version of the model [30]. To be more quantitative, we compute the persistence in the top and bottom decile for different values of J_0/σ^2 (see figures 13(b) and (c)) in analogy with the analysis of longitudinal surveys reported in figure 3. We thus fit the simulated persistence curves with equation (8) and obtain a convincing match for all values of J_0/σ^2 . We find that the stretched-exponential exponent $\gamma(q) \simeq 0.5$ for $q = 0$ and $\gamma(q) \simeq 0.6$ for $q = 9$, with little dependence on J_0/σ^2 .

The relaxation time of the persistence $\tau(q)$ is found to depend on J_0/σ^2 so that a more unequal wealth (figure 13(d)) distribution is reflected by a longer relaxation time. Furthermore, the relaxation of the bottom decile is faster than that of the top decile. Both observations are in qualitative agreement with the data from US and Italian surveys: wealth inequality is larger in the USA than in Italy and so are the relaxation times and the relaxation time for $q = 0$ is larger than the one for $q = 9$ in both countries. We next consider the Shorrocks mobility index and see that it converges to S_{10} with in a time that increases as J_0/σ^2 decreases (figure 13(e)). Finally, we measure mean-square quantile displacements which are illustrated in figure 13(f) for $J_0/\sigma^2 = 1$. All the curves approach a q dependent steady-state limit corresponding to the perfect mix limit $M_\infty(q)$ (equation (4)).

3.6. Limitations of the agent based model

The simulation results discussed above is in qualitative agreement with the time evolution of wealth observed in panel surveys from USA and Italy. In particular, persistence and mean-square quantile displacements follow similar time-dependent functions, including a stretched exponential relaxation. On a quantitative level, however, there are fundamental differences between model predictions and empirical data. First of all, the large differences in persistence between households in the top and bottom decile is not explained by the model. Data show that persistence in the top decile decays extremely slowly with a large fraction of households that are effectively immobile. Furthermore, the exponent of the stretched exponential $\gamma(q)$ is smaller in the data than in the model, indicating again that that mobility is much slower in the data than what would be suggested by the model. This is corroborated by the analysis of mean-square quantile displacements that are more localized in the data than in the model. Finally, agents with zero or negative wealth are by construction not present in the model.

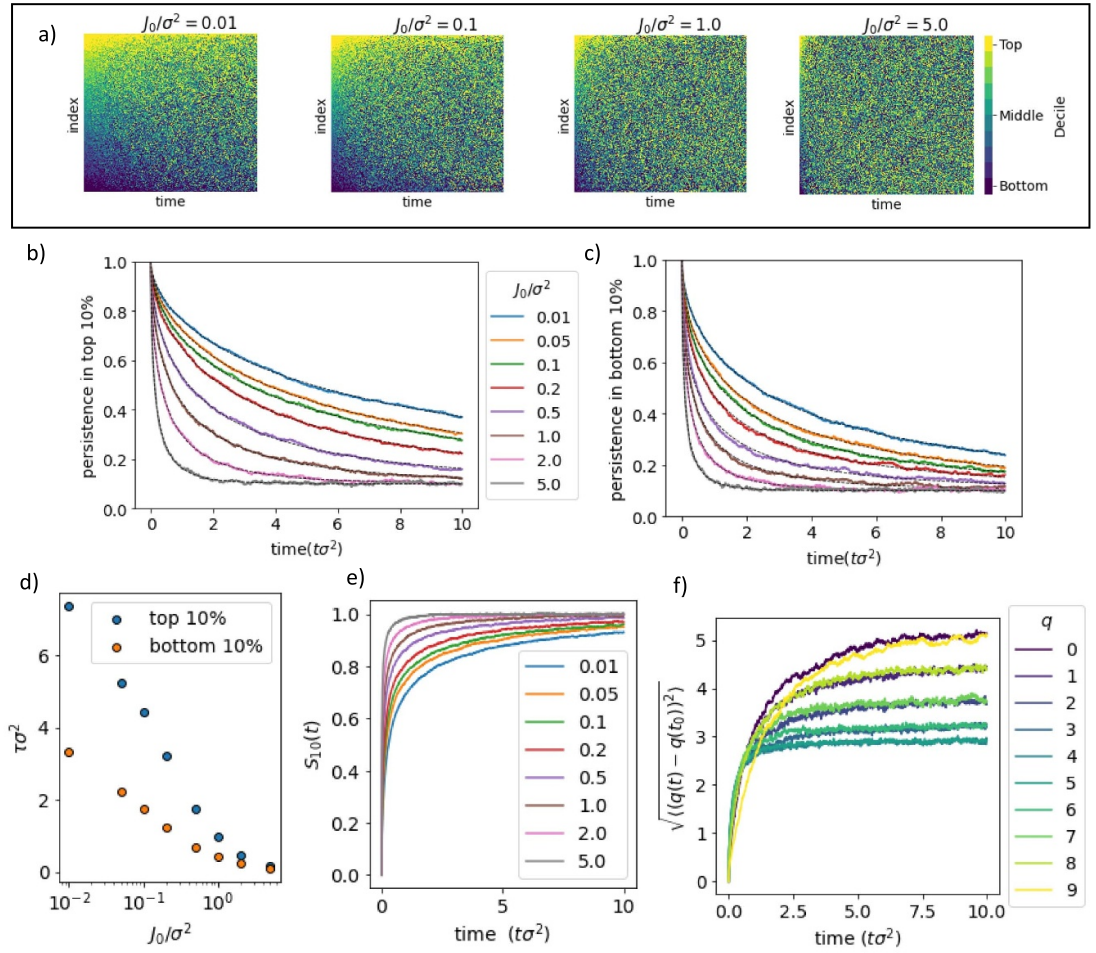


Figure 13. Wealth persistence and mobility in the agent based model. (a) Time-dependent variations of the wealth decile for a population of $N = 1280$ agents for different values of J_0/σ^2 . The shown time axis is within the $[0,10]$ interval, in units of $1/\sigma^2$. Persistence for the (b) top and bottom (c) decile for different values of J_0/σ^2 . Persistence curves are fit with a stretched exponential as discussed in the text. (d) The time constant of the stretched exponential as a function of J_0/σ^2 from the fits in panels (b) and (c). (e) Time-dependence of the Shorrock mobility index for different values of J_0/σ^2 . (f) Time-dependent mean-square-displacements of the wealth decile for different initial deciles $q(t_0)$.

To understand why the wealth mobility patterns predicted by the model are not in line with the data, we reconsider the main assumption of the model, namely that the time-dependent changes of the agents wealth are directly dependent on their current wealth. This fact is summarized in figure 14(a) showing that the standard deviation of the changes in wealth conditioned to the current wealth is proportional to the wealth itself $\sigma(dW|W) = CW$, with $C < 1$. When we measure the same quantity in empirical data for USA (figure 14(b)) and Italy (figure 14(c)), we find that proportionality is only found for large wealth, while for low wealth $\sigma(dW|W)$ becomes independent on wealth.

To corroborate this conclusion, we analyze the evolution of personal wealth using the list of ultra-wealthy individuals compiled by Forbes for the years 2002–2022. As shown in figure 14(d), the standard deviation of wealth changes for billionaires is proportional to wealth as assumed in the model. We thus analyze persistence in the group of the 300 top wealthiest individuals. The rank dynamics is illustrated in figure 14(e) and the time-dependent persistence is reported in figure 14(f). The curve is well fitted by equation (8), setting $\Phi^\infty = 0$, and obtaining $\gamma = 0.7$ and $\tau = 13$ years. Within this restricted group, rank turnover is relatively fast and well described by the agent based model.

The linear dependence of the wealth fluctuations on wealth is associated to the way personal income is generated. As shown in figure 15(a) for the case of US households, income is on average proportional to wealth for large wealth while for low wealth it is not. By analyzing the main source of income of each households, we see that for low wealth income is mainly due to wages, pension or transfers, while for higher wealth it is due to business ownership and capital gains/dividends (see figure 15(b)). The situation is similar for Italian households (figures 15(c) and (d)), although the transition is less sharp than in the USA.

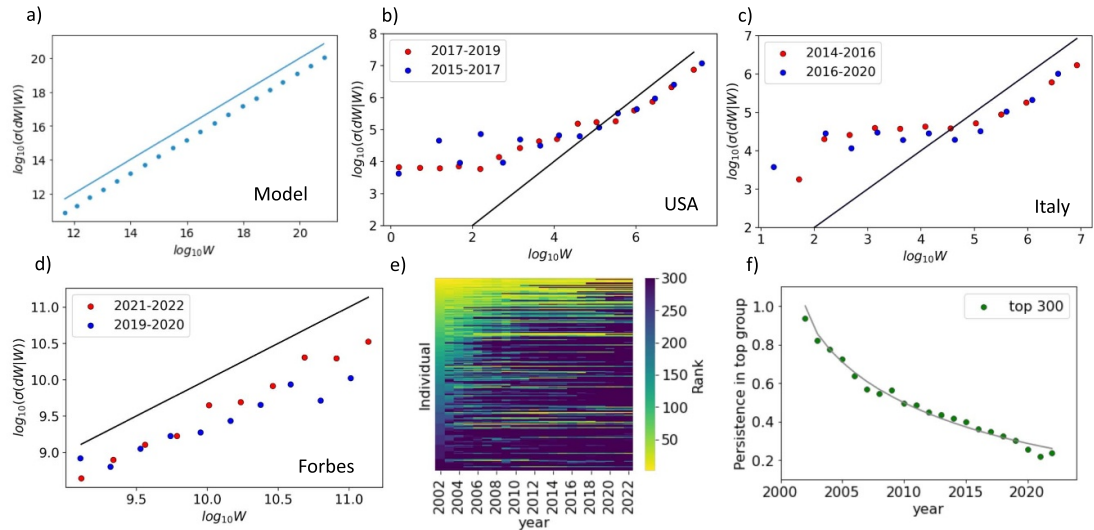


Figure 14. ABM fails to describe dynamics for low wealth. (a) Double-logarithmic plot of the standard deviation of the wealth changes dW as a function of the wealth W obtained from simulations of $N = 1258$ agents ($J_0/\sigma^2 = 1$) and from (b) US and (c) Italian survey data and from (d) Forbes billionaires data. A line with unit slope (i.e. $\sigma(dW|W) = W$) is reported for reference. (e) Time-dependent variations of the wealth rank for the top 300 billionaires of the Forbes list of 2002. (f) Time-dependent persistence in the group of the top 300 billionaires from 2002 to 2022. The curve fit follows the predictions of the model. Source: PSID (b), BoI (c), Forbes (d)–(f).

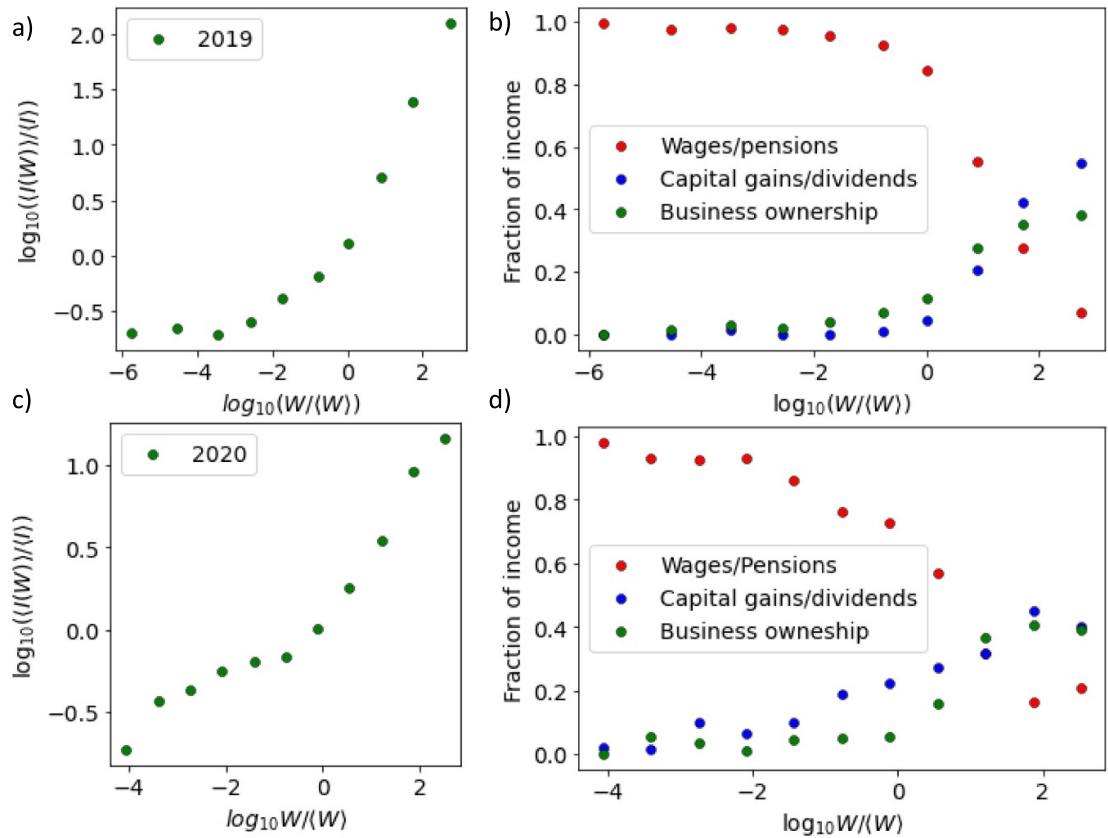


Figure 15. Income-wealth relationships. The expected value of household income given the household wealth for USA in double logarithmic scale (only positive wealth is considered). (b) Fraction of household income due to wages pensions and transfers, business ownership, and dividends and capital gains for households of given net wealth in the USA. (c), (d) Same as (a), (b) but for Italy. Source: SCF (USA) BoI (Italy).

4. Discussion

In this paper, we have studied the time-dependent persistence of wealth inequality in two developed countries, USA and Italy, exploring the relations with health status and food security. We have analyzed longitudinal panel surveys covering more than 20 years to extract wealth dynamics of individual households and quantify the time-dependent persistence within quantile intervals of wealth distribution and the mobility across them. We then investigated how the persistence of households in conditions of lack of wealth was associated with health. The goal was to better understand how personal well-being and health is related to personal wealth. The choice of USA and Italy was motivated by the large differences in healthcare systems in these two developed countries, since Italy relies on public universal healthcare while the USA mainly of private insurance. Moreover, Italy and USA exhibit contrasting characteristics when it comes to wealth and well-being in terms of wealth and income.

When comparing Italy and the United States in terms of wealth and well-being, several notable differences emerge. The United States has a significantly higher GDP and is considered one of the wealthiest countries globally. It has a dynamic economy, a robust financial sector, and is home to many multinational corporations. As a result, the average income in the United States tends to be higher than in Italy. In terms of well-being, the United States provides a high standard of living, with access to quality healthcare, advanced infrastructure, and a wide range of consumer goods and services. However, it is important to note that the distribution of wealth and well-being in the United States is not uniform, with significant income inequality and disparities in access to healthcare and education. Italy, on the other hand, has a rich cultural heritage and traditionally a strong emphasis on quality of life. Italians often prioritize family, community, and work-life balance. While Italy's overall GDP is lower compared to the United States, it still possesses a substantial economy, with a focus on industries such as fashion, automotive, and tourism. Italy's well-being indicators often excel in areas such as healthcare and education. The country has a comprehensive healthcare system that provides universal coverage, and its citizens enjoy long life expectancy. Italy also places a strong emphasis on education, with a well-regarded public education system and a long tradition of higher education institutions.

Our analysis confirms that the wealth distribution is more unequal in the USA than in Italy, with a fraction of households that have zero or negative wealth that is increasing in both countries but it is much larger in the USA. By analyzing in detail the mobility patterns of households across the wealth distribution, we found that low and high wealth households are generally the most immobile. To reach this conclusion, we used tools inspired by the non-equilibrium dynamics of complex systems in statistical physics, such as persistence and mean-square displacements, and connected them to methods employed in economics such as the calculation of transition matrices and mobility indices. In statistical physics, persistence represents a probabilistic measure of the permanence in a state as a function of time [26]. Here we use it to quantify the probability that an household remains in a given quantile interval of the wealth distribution. We find that both in USA and Italy, persistence decays as a stretched exponential with a time scale that depends on the initial wealth. Stretched exponential is well studied in condensed matter physics since it is often used as phenomenological description of the relaxation in disordered and glassy systems [47]. It has been suggested that stretched exponential relaxation may arise from an heterogeneous system with multiple relaxation times [48]. When the relaxation time distribution is very broad, the exponent γ is small indicating fast relaxation at short time scales and very slow at larger times. Our analysis shows that $\gamma \simeq 0.1\text{--}0.4$ with the lowest values found for the top 10% of the population, which also shows the highest characteristic times. Furthermore, we always observe a very fast decay of the persistence in the first year, followed by a very slow relaxation. High persistence in top and bottom wealth quantile intervals is also associated with low mobility as revealed studying mean-square displacements, that are localized within two or three intervals.

We next investigated how wealth inequality and persistence are associated with health inequality. Health inequalities can have a significant impact on inequality in general since people who experience poorer health outcomes, such as chronic illnesses or disabilities, often face barriers to education, employment, and economic opportunities. These individuals may have limited access to quality healthcare, especially in heavily privatized healthcare systems like the USA one, resulting in higher healthcare costs and decreased productivity. As a result, they may be trapped in a cycle of poverty, which exacerbates overall economic inequality. On the other hand, health disparities can affect educational attainment and employment opportunities. Poor health can lead to increased absenteeism, reduced cognitive function, and limited access to educational resources. This can result in lower academic achievement and reduced employment prospects, further widening the inequality gap. Our analysis shows that in the USA displays higher mortality than Italy when considering preventable or treatable deaths in general, but also for a variety of diseases from chronic disease to dementia and Alzheimer. This important differences between the two countries could be due to multiple factors from the important intake of junk food in people's diets which is well known to be related to

multiple diseases including a negative impact on mental and metabolic health of children [49]. Moreover, individuals with no wealth display poorer health conditions than the average population. Interestingly, this situation is exacerbated for households that persist without wealth at least for the entire period here studied. In fact, in USA where we analysed a longitudinally dataset we show that there is a persistence in the group with no wealth.

Health inequalities are closely linked to social and environmental determinants of health. Individuals from disadvantaged socioeconomic backgrounds often face greater exposure to environmental hazards, inadequate housing, food insecurity, and limited access to safe recreational spaces. These conditions contribute to poorer health outcomes, which disproportionately affect marginalized communities and perpetuate overall social and environmental inequality. Our study confirms that wealth inequality is also related to food security, since our analysis shows that food insecurity is generally higher in the USA than in Italy, in line with the differences in wealth inequality between the two countries. It should be noticed, however, that households with severe food insecurity are slightly larger in Italy in the last years analysed. This aspect might be due to possible changes in the Italian society related to the healthcare system. By looking at US longitudinal data, we found that households with no wealth, particularly those that are persistent in this state, experience more food insecurity and have also lower food expenditure.

Our study suggests complex associations between wealth inequality, social immobility, health inequality and food insecurity but establishing precise quantitative relations between all these factors is a daunting task. We thus restrict our attention to relation between wealth inequality and mobility using a simple ABM of wealth condensation in a growing economy [30] to assist our interpretation of survey data. The model contains two basic assumptions: (i) the wealth of an agent is increased or decreased in proportion to its current wealth and (ii) wealth is exchanged among agents in proportion to their respective wealth. The ratio between the amplitude of wealth exchanges and the variance of the wealth fluctuations J_0/σ^2 determines the extent of inequality and the Pareto exponent α [30]. The model is part of a large class of stochastic wealth exchange models, starting with the pioneering work by Angle [32] and followed by other models in the econophysics literature [33, 34] (for critique of these models from a classical economics perspective see [50]). We find that persistence in the model follows a stretched-exponential relaxation in analogy with panel survey data. Contrary to the data, however, the exponent γ in the model depends only weakly on the starting wealth quantile interval. Similarly, mean-square displacements follow similar qualitatively similar trends in model and data, but are more confined in the data. These results show that the model differ from reality mainly because it would predict that after a sufficiently long relaxation time the initial wealth would not matter. This is in contrast with survey data suggesting instead persistent lack of mobility at the top and to a less extent at the bottom of the wealth distribution.

By comparing the assumptions of the model with survey data, we show that assumption (i) is realized only for large enough wealth, both for the USA and Italy. This reflects the fact that for a large part of the population income is not dependent on wealth, either because it is absent or because it is too small. Hence, the model is able to describe well the wealth dynamics of high-wealth individuals, as confirmed by the analysis of wealth persistence in the Forbes list of billionaires, it overestimates the mobility within the general population. A large part of the population (87% in the USA of according to SCF data) derives the majority of their income from wages or pensions and thus in a way that is independently on wealth. This observation suggests that to understand quantitatively wealth inequality and its time evolution the model needs to be modified. One could for instance consider additive noise terms whose variance does not depend on the agents wealth or a distinction between the income coming from wages and that resulting from returns over investments.

In conclusions, our paper provides a quantitative assessment of inequality in wealth, social mobility, health and food security by comparing the USA and Italy. All these forms of inequality are intertwined and should therefore be addressed in a coherent manner.

Data availability statement

PSID data are available from <https://psidonline.isr.umich.edu/>. SCV data www.federalreserve.gov/econres/scfindex.htm. BoI data are available at www.bancaditalia.it/statistiche/basi-dati/rdc/index.html. WB data are available at <https://data.worldbank.org/>. OECD data are available at <https://stats.oecd.org/>. Forbes data are available at <https://github.com/open-numbers/ddf--gapminder--billionaires>.

Code availability

Codes to replicate this study are deposited in Github at https://github.com/ComplexityBiosystems/wealth_condensation.

Authors contributions

C A M L P and S Z designed the study, analyzed data and wrote the paper. S Z performed numerical simulations.

Conflict of interest

The authors declare no competing interests.

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