RETENTION OF STELLAR-MASS BLACK HOLES IN GLOBULAR CLUSTERS

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ABSTRACT

Globular clusters should be born with significant numbers of stellar-mass black holes (BHs). It has been thought for two decades that very few of these BHs could be retained through the cluster lifetime. With masses $\sim 10 \, M_\odot$, BHs are ~ 20 times more massive than an average cluster star. They segregate into the cluster core, where they may eventually decouple from the remainder of the cluster. The small-N core then evaporates on a short timescale. This is the so-called Spitzer instability. Here we present the results of a full dynamical simulation of a globular cluster containing many stellar-mass BHs with a realistic mass spectrum. Our Monte Carlo simulation code includes detailed treatments of all relevant stellar evolution and dynamical processes. Our main finding is that old globular clusters could still contain many BHs at present. In our simulation, we find no evidence for the Spitzer instability. Instead, most of the BHs remain well mixed with the rest of the cluster, with only the innermost few tens of BHs segregating significantly. Over the 12 Gyr evolution, fewer than half of the BHs are dynamically ejected through strong binary interactions in the cluster core. The presence of BHs leads to long-term heating of the cluster, ultimately producing a core radius on the high end of the distribution for Milky Way globular clusters (and those of other galaxies). A crude extrapolation from our model suggests that the BH–BH merger rate from globular clusters could be comparable to the rate in the field.

Key words: binaries: close – globular clusters: general – gravitational waves – methods: numerical – stars: kinematics and dynamics

Online-only material: color figures

1. INTRODUCTION

Typical globular clusters (GCs) should form ~ 100–1000 black holes (BHs) within ~ 3 Myr and could retain most of them initially, if their natal kicks are sufficiently low (see Wong et al. 2012 and references therein). With masses $\sim 10 \, M_\odot$, these BHs become the most massive objects in the cluster after just ~ 10 Myr, so their dynamics will be very different from that of typical stars. The presence of BHs can affect the overall structure and evolution of the parent cluster (Mackey et al. 2008). BHs accreting from a stellar companion can be visible as bright X-ray binaries (XRBs), which are in principle detectable both in our own and in other nearby galaxies (Kalogera et al. 2004). Merging BH–BH binaries are key sources of gravitational waves (GWs) which should be detectable by upcoming interferometers such as Advanced LIGO (Harry et al. 2010).

It is well known that the formation rate per unit mass of XRBs is higher in clusters by orders of magnitude compared to the field (e.g., Pooley et al. 2003). This indicates that dynamics must play an essential role in producing cluster XRBs. Prior to 2007, however, there had not been a single detection of a BH XRB inside a GC, although many had been identified in galactic fields. This appeared to agree with many theoretical studies suggesting that essentially all BHs within a star cluster should be ejected through dynamical interactions on a timescale $\sim 10^9$ yr (Kulkarni et al. 1993; Sigurdsson & Hernquist 1993; Portegies Zwart & McMillan 2000; O'Leary et al. 2006; Banerjee et al. 2010). Key to all these previous studies is the expectation that BHs will segregate rapidly through dynamical friction, on a timescale ~ 100 Myr, and will succumb to the so-called Spitzer instability (Spitzer 1969; Kulkarni et al. 1993), i.e., dynamically decouple from the cluster by forming a central subcluster consisting primarily of BHs. The relaxation time for this small-N subcluster of BHs is very short, leading to prompt core collapse and evaporation. Through dynamical interactions, some BHs will be ejected in the form of tight BH–BH binaries that merge via GW emission within a Hubble time.

This scenario was first proposed based on simple analytic estimates by Kulkarni et al. (1993) and Sigurdsson & Hernquist (1993). The first direct N-body simulations of this effect used up to $N \sim 10^4$ particles (e.g., Portegies Zwart & McMillan 2000; Merritt et al. 2004). Larger direct N-body simulations by Banerjee et al. (2010) used $N \sim 10^5$, but with a single BH mass (10 $M_\odot$) and no primordial binaries. Using simple dynamical models, O'Leary et al. (2006) and Sadowski et al. (2008) studied the evolution of populations of BHs that were either completely decoupled from or in equilibrium with the cluster, respectively (see discussion in Downing et al. 2010).

Monte Carlo (MC) methods have made it possible to model realistic GCs *self-consistently* with $N \sim 10^5$–10^6 and significant primordial binary fractions (e.g., Giersz et al. 2008; Chatterjee et al. 2010). The most realistic GC models with BHs to date are from Downing et al. (2010, 2011), who used a Hénon-type MC code to simulate clusters with $N = 5 \times 10^5$ stars, a distribution of BH masses, and primordial binaries. These works used analytic cross sections to determine the results of dynamical interactions, rather than direct integration.

Previous studies suggest that most BHs are ejected on a timescale $\sim 10^9$ yr. Hence, old GCs should have very few, if any, BHs left. However, in 2007 the first candidate BH XRB *inside* a GC was detected in NGC 4472 (Maccarone et al. 2007). Since then, several additional BH candidates have been found in clusters in other galaxies (Brassington et al. 2010; Shih et al. 2010; Barnard et al. 2011; Maccarone et al. 2011). Strader et al. (2012) discovered two stellar-mass BHs in a Milky Way (MW) GC (M22). Assuming these BHs are accreting from white dwarf companions, and using calculated formation and survival rates

from Ivanova et al. (2010), Strader et al. (2012) estimate that M22 has ∼5–100 BHs.

Furthermore, there have been a few recent theoretical suggestions that significant numbers of BHs could still remain in some old clusters (Mackey et al. 2008; Moody & Sigurdsson 2009). Mackey et al. (2008) used N-body simulations with BHs to explain the radius–age trend in the clusters in the Magellanic Clouds; with different initial retention fractions for BHs, they were able to reproduce the trend of increasing spread in core radius with age in these systems. In some models, they retained up to ≃100 BHs over a Hubble time.

Here we re-examine the BH retention question based on a realistic, large-N, fully self-consistent MC model. We find that at least some old MW clusters may indeed retain a large fraction of their primordial BHs. This dramatically different picture for the fate of BHs in GCs may have implications for both BH XRBs and the production of merging BH–BH binaries.

2. METHOD

We use a Hénon-type MC method to self-consistently model the evolution of star clusters due to the effects of two-body relaxation, strong binary scattering encounters, stellar collisions, single and binary stellar evolution, and mass loss from the Galactic tidal field. A detailed description of our code, as well as examples of its capabilities and comparisons with other methods, can be found in Joshi et al. (2000, 2001), Fregeau et al. (2003), Fregeau & Rasio (2007), and Chatterjee et al. (2010). The code has been well tested against direct N-body models whenever possible. Since the dynamical evolution of BHs in clusters is strongly dependent on interactions involving BH binaries, we perform direct calculations of all strong three-body (binary–single) and four-body (binary–binary) interactions using the small-N integrator Fewbody (Fregeau et al. 2004). These interactions are responsible for the hardening of BH–BH binaries and ejections of BHs from the cluster. Single star and binary evolution are modeled using the routines of SSE and BSE (Hurley et al. 2000, 2002). Orbital energy loss from GW emission is handled within BSE for binaries retained in the cluster. For ejected systems, which are no longer evolved with our code, we use a simplified timescale for GW inspiral in the weak field limit (Peters 1964). Neutron stars and BHs receive natal kicks with velocities drawn from a Maxwellian distribution with $\sigma = 265$ km s^{-1}. For BHs, the kick velocity is lowered according to the amount of material that falls back onto the final BH after the supernova explosion, according to Belczynski et al. (2002).

We have recently added to our code a prescription for three-body binary formation, which is important for the evolution of BHs (Kulkarni et al. 1993; Sigurdsson & Hernquist 1993; Portegies Zwart & McMillan 2000; O'Leary et al. 2006; Banerjee et al. 2010). We follow a similar procedure to Ivanova et al. (2005), Ivanova et al. (2010), and O'Leary et al. (2006) to obtain an expression for the binary formation rate as a function of hardness ratio

$$\eta = \frac{G m_1 m_2}{r_p \langle m \rangle \sigma^2}. \tag{1}$$

We keep both the geometric and gravitational focusing contributions to the cross section (in contrast to Ivanova et al. 2010, where the geometric part of the cross section for the third star to interact with stars 1 and 2 is dropped). For local number density n and average relative velocity at infinity v_∞ the rate at which two stars (m_1 and m_2) form a binary with hardness $\eta \geqslant \eta_{\min}$
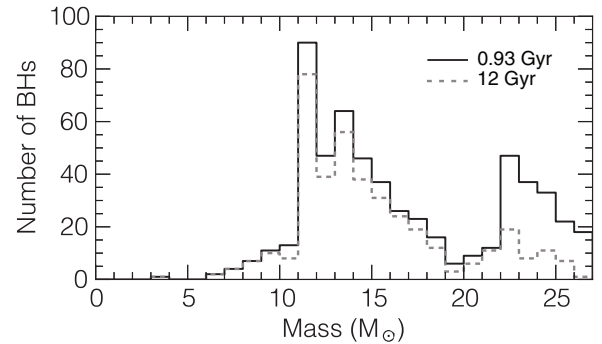
Figure 1. Mass functions for BHs present *in the cluster* at 0.93 Gyr (solid black line) and at 12 Gyr (dotted gray line). The population at 0.93 Gyr is representative of the original population of *initially retained* BHs, as some (about 100) are ejected from the cluster at formation from natal kicks. Between 0.93 and 12 Gyr, the most massive ($m_{\rm bh} \geqslant 20 \, M_\odot$) are ejected preferentially over the lower mass BHs because they have higher interaction rates.

through an interaction with a third star (m_3) is given by

$$\Gamma(\eta \geqslant \eta_{\min}) = \sqrt{2}\pi^2 n^2 v_\infty^{-9}$$
$$\times (m_1 + m_2)^5 \eta_{\min}^{-5.5}(1 + 2\eta_{\min})$$
$$\times \left[1 + 2\eta_{\min}\left(\frac{m_1 + m_2 + m_3}{m_1 + m_2} \right) \right]. \tag{2}$$

As we expect only dynamically hard binaries to survive (Heggie 1975), we only consider the formation of hard binaries with $\eta \geqslant 5 = \eta_{\min}$. We allow three-body binary formation only for BHs. When forming a three-body binary, we choose a value for η from a distribution according to the differential rate, $d\Gamma/d\eta$, with lower limit $\eta_{\min}$. The rest of the properties of the system are calculated from conservation of momentum and energy.

We have checked that our MC prescription produces binaries at a rate that is in agreement with the analytic rate from Equation (2). We have also done a set of tests using a direct N-body code (Farr & Bertschinger 2007) to check that our prescription produces hard binaries at the correct rate. Using one of our cluster snapshots, we integrated our system of BHs for a short period of time with direct N-body, and compared our binary formation probability prediction to the actual binary formation probability in the direct integration. We find good agreement with the direct N-body trials, which gives us confidence that we are actually producing hard binaries at the correct rate.

## 3. THE EVOLUTION OF A CLUSTER WITH STELLAR-MASS BLACK HOLES

We present the results of a cluster model starting with $N = 3 \times 10^5$ stars following a King profile with $W_0 = 5$, half-mass radius $r_h = 2.44$ pc, metallicity of $Z = 0.001$, and initial binary fraction $f_b = 0.1$. We choose our stellar masses from the Kroupa (2001) initial mass function ranging from 0.1 to 100 $M_\odot$. The cluster has initial total mass $M_{\rm tot} = 2.03 \times 10^5 \, M_\odot$ and half-mass relaxation time $T_{\rm rh} \approx 6.5 \times 10^8$ yr. The central escape speed is[1] 31 km s^{-1}. Only about 12% of the BHs formed in the cluster received natal kick speeds above this value. We choose our remnant masses according to Belczynski et al. (2002), which produces BH masses in the range ∼5–30 $M_\odot$ for $Z = 0.001$. We form about 700 BHs in total, of which about 600 are retained

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[1] Our cluster is well below the velocity dispersion limit of ∼40 km s^{-1} found in Miller & Davies (2012), above which growth of a massive BH through successive mergers might occur.
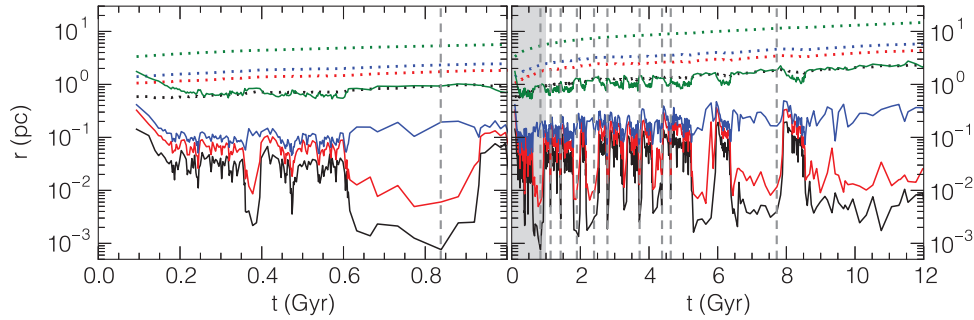
Figure 2. Evolution of the Lagrange radii of the BHs and all *other* objects. Plots show the radii enclosing 1%, 5%, 10%, and 50% of the mass, from bottom to top, for each category of object: BHs (solid curves) and non-BH (dotted curves). The left panel shows the evolution over the shorter interval corresponding to the gray band in the right panel. The BHs segregate from the rest of the cluster on a timescale of a few hundred Myr. The vertical gray dashed lines indicate the times when three-body binaries form. Interactions involving these hard binaries cause most of the oscillations of the innermost Lagrange radii. However, interactions involving hard binaries *not* created by three-body formation can also cause the same effect.

(A color version of this figure is available in the online journal.)
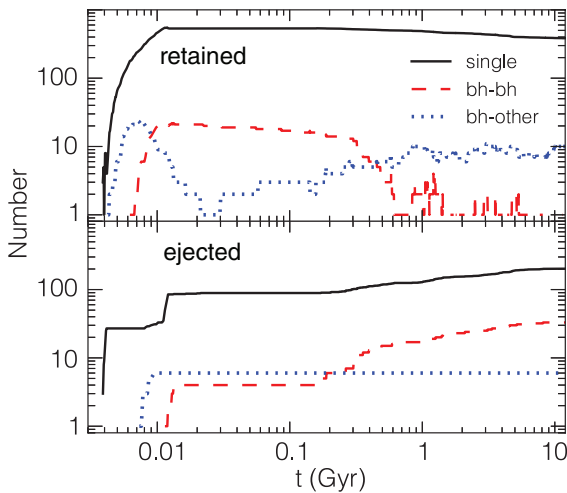
Figure 3. Evolution of BH population retained in (top) and ejected from (bottom) the cluster. Each plot shows the number of single BHs (solid black line), BH–BH binaries (dashed red line), and BH binaries with a *non-BH* companion (dotted blue line), either retained or ejected. The number of BH–BH binaries within the cluster tends to *decrease* over time until there are just a few. Both single BHs and BH–BH binaries are ejected efficiently from about 300 Myr until about 6 Gyr, at which point the ejection rate slows down significantly. This occurs in conjunction with a drop in the overall binary interaction rate, which is caused by the ongoing cluster expansion due to heating by the BHs. BH–other binaries tend to increase in number very slowly over time. At 12 Gyr, the cluster still has 395 BHs, more than half of the cluster's initial population. The majority of the BHs are *single* at all times.

(A color version of this figure is available in the online journal.)

initially (the remainder are ejected by natal kicks). The BH mass distribution at an early time is shown in Figure 1.

Within a few Myr, the BHs begin to segregate, leading to a central collapse by about 400 Myr. Formation of three-body binaries and their subsequent interactions lead to repeated core oscillations (see Figure 2). After about 300 Myr, strong binary interactions involving the mass-segregated BH population start to become dynamically important, and the rate of ejection of BHs (both single and binary) increases abruptly. Ejections continue through the end of the simulation, but the rate slows down over time. The evolution of the numbers of single and binary BHs retained in and ejected from the cluster is shown in Figure 3. For the entire simulation, most of the BHs are *single*; in fact, beyond about 300 Myr, there are typically no more than about 10 BH binaries in the cluster.

Statistics of the BHs at different times are shown in Table 1. By 12 Gyr, the cluster has ejected 202 single BHs,

33 BH–BH binaries, and 6 BH binaries with non-BH companions. Throughout the simulation, 13 BH–BH binaries merge due to GW emission; 6 of these mergers occur within the cluster, while the rest occur post-ejection. Most of the BH ejections and BH–BH mergers occur within about the first 6 Gyr of evolution. At 12 Gyr, our model still has nearly 400 BHs, more than half of the initially retained population.

In Figure 4 we show the fractions of single BHs and all single stars in radial bins at several times. The BH fraction in the central bin, which always contains 20 BHs, grows to unity within about 600 Myr (left three panels), meaning that the innermost 20 objects are *all* BHs. Just outside the central bin, the BH fraction is typically less than 0.4, and it decreases to negligible fractions beyond about 1 pc. This indicates that, while the BHs do indeed segregate to some extent, most of the BHs *do not* dynamically decouple from the cluster (i.e., they do not become Spitzer unstable). All but the innermost 20 or so most massive BHs remain well mixed with the cluster at all times.

The most massive BHs tend to be preferentially ejected from the cluster (see Figure 1 and Table 1). Nearly 75% of the ejected BHs have masses $\gtrsim 20\,M_\odot$, despite the fact that these more massive BHs are much less common than lower mass BHs. Since the most massive BHs sink the deepest, they tend to have the highest rates of strong interactions, which provide the energy needed to eject them from the cluster.

We end at 12 Gyr, a typical age for MW GCs, with the cluster having $N = 2.47 \times 10^5$ stars, $M_{\rm tot} = 1.05 \times 10^5\,M_\odot$, $r_h = 12.7$ pc, and binary fraction $f_b = 0.098$. The final mass of our cluster is just slightly larger than the median value for MW GCs ($M_{\rm med} \approx 8 \times 10^4$; Heggie & Hut 2003). For our model, we find an observational core radius $r_c \simeq 5$–7 pc, which falls within the high end of the core radius distribution of the MW GC system, and is also consistent with the range of core radii associated with old ($\sim$10 Gyr) GCs in the Magellanic Clouds (see Figures 1 and 2 in Mackey et al. 2008).

4. DISCUSSION AND CONCLUSIONS

The evolution and survivability of BHs in clusters, as well the effect that BHs have on their host cluster, will depend strongly on the degree to which the BHs are able to decouple from the cluster. Our MC method allows us to include realistic initial conditions as well as all the relevant physics for studying these types of systems in detail.

In the most optimistic model of Mackey et al. (2008) (run 4), about 50% of their BHs ($\approx$100) are retained over $\sim$10 Gyr. This
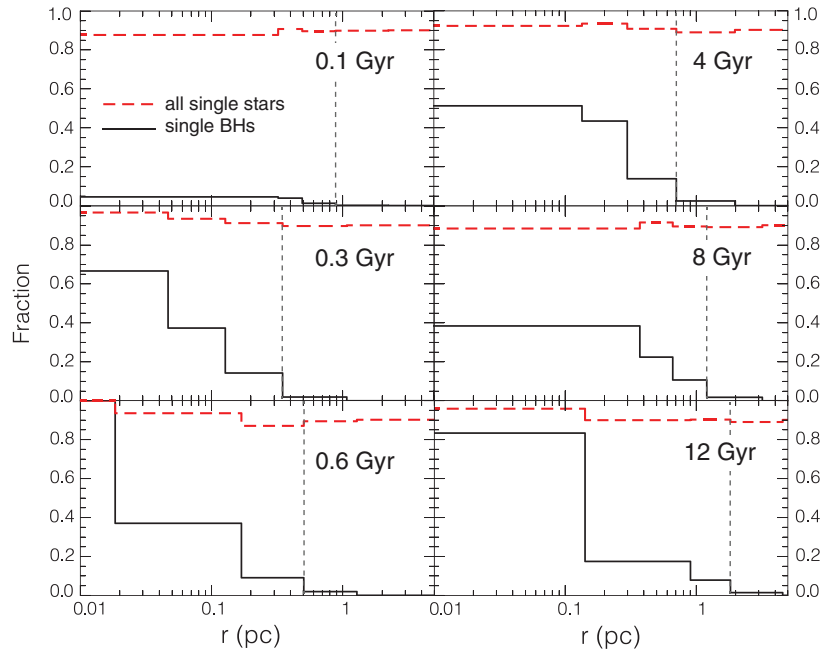
Figure 4. The fraction of *single BHs* (solid black line) and *all single stars* (dashed red line) at several times; the remainder of the objects are binaries. The innermost bin contains 20 BHs, and the number of BHs inside each subsequent bin doubles (40, 80, etc.). The BH fraction in the central bin (containing 20 BHs) reaches unity by about 600 Myr (left panels), and then fluctuates between 0.4 and 1 for the rest of the simulation (right panels). Beyond the first bin, the BH fraction decreases, reaching negligible fractions beyond about 1 pc. The vertical gray dashed line shows the extent of the innermost 140 BHs ($20 + 40 + 80 = 140$ contained within the first three bins), which is at all times less than half of the retained BH population.

(A color version of this figure is available in the online journal.)

Table 1
Properties of BH Population at Different Evolutionary Stages: 0.93 Gyr, 3.25 Gyr, 6.5 Gyr, 9.77 Gyr, and 12 Gyr

Type	$T = 0.93$ Gyr $= 1.4\, T_{rh}$		$T = 3.25$ Gyr $= 5\, T_{rh}$		$T = 6.50$ Gyr $= 10\, T_{rh}$		$T = 9.77$ Gyr $= 15\, T_{rh}$		$T = 12$ Gyr $= 18.5\, T_{rh}$	
	ret	ej	ret	ej	ret	ej	ret	ej	ret	ej
N_{sBH}	534	89	434	166	395	197	387	201	385	202
N_{BH-BH}	17	4	1	26	0	32	0	33	0	33
$N_{BH-other}$	3	6	10	6	8	6	9	6	10	6
$m_{ave,BH}\,(M_\odot)$	16.7	13.3	15.6	20.6	15.1	21.8	14.9	22.0	14.9	22.0
$N_{BH}(m \geqslant 20\,M_\odot)$	178	27	101	110	69	143	63	148	63	149
N_{mgr}	4	0	5	1	6	6	6	7	6	7

Notes. The table shows the number of single BHs (N_{sBH}), number of BH–BH binaries (N_{BH-BH}), number of BH–other (non-compact) binaries ($N_{BH-other}$), number of BHs with masses above $20\,M_\odot$ ($N_{BH}(m \geqslant 20\,M_\odot)$), and average individual BH mass ($m_{ave,BH}$) that are retained in/ejected from the cluster, at the different times. We also show the number of BH–BH mergers (N_{mgr}) that have occurred up to the time given, either inside the cluster or post-ejection.

is slightly less than our final retention fraction (about 65%), but with N three times that of Mackey et al. (2008), this amounts to more than a factor of three difference in the actual *number* of BHs that we retain. In contrast with Mackey et al. (2008) who found no BH–BH mergers within a Hubble time, we produce 13 mergers. In clusters with low central escape velocities, recoil kicks from strong dynamical encounters may tend to eject BH–BH binaries before they are tight enough to merge within a Hubble time. Although some of the Mackey et al. (2008) models do indeed have significantly lower escape velocities than the model we present, their run 4 actually has a comparable escape velocity, so this cannot reconcile the difference in merger rate. Instead, the discrepancy may be explained by the larger number of BHs, as well as the inclusion of primordial binaries, resulting in a higher interaction rate in our simulation, which is consistent with the larger number of ejected BHs (but see discussion in Downing et al. 2010 about the competing effects of *hardening*

and *destruction* of BH–BH binaries that go along with high BH interaction rates). We also compare to Downing et al. (2010), who track BH–BH mergers in a set of MC simulations. Their model 10low75 is most similar to ours, with the same binary fraction and metallicity, and $N = 5 \times 10^5$, $r_h = 2$ pc, and $T_{rh} = 5.25 \times 10^8$ yr. They produced 6 ± 3 mergers (averaged over 10 simulations) within T_H, about half as many as we produce in our simulation. Agreement to within a factor of two is reasonable, considering their use of cross sections for predicting the outcomes of strong binary interactions (rather than direct integration), which may overestimate the disruption rate for tight BH–BH binaries (Downing et al. 2010).

A crude extrapolation from our model can be used to estimate the rate of BH–BH mergers in a Milky Way equivalent galaxy (MWEG). In our model, the total merger rate is ~ 1 per Gyr. Our Galaxy may have had ~ 300 GCs (about half of which have since dissolved). We therefore estimate a merger rate of

~0.3 per MWEG per Myr from star clusters. This exceeds the estimated merger rate from primordial binaries in the galactic field (Abadie et al. 2010). Thus, our model indicates that it is important to include GCs in calculations of the BH–BH merger rate of the Universe.

Our results indicate that at least *some* old GCs could have *hundreds* of stellar-mass BHs at present. Since nearly all of our BHs are single, our prediction is consistent with the small number of BH XRBs detected in clusters to date. This result is timely, considering the recent discovery of *two* BH XRBs in an MW GC by Strader et al. (2012), who suggest that there may actually be ~5–100 BHs in M22 at present. Our main conclusion is different from that of many other studies in the literature. This difference is not easily reconciled, but will be the subject of future investigations.

As has been suggested by Mackey et al. (2008), the presence of BHs can indeed cause heating that can lead to significant core expansion, as we confirm with our model. The smaller cores observed in MW globulars may indicate larger BH kicks than assumed in this work; intriguingly, Repetto et al. (2012) suggested such a change to the kick distribution on the basis of a population synthesis study of Galactic BHs.

REFERENCES

Abadie, J., Abbott, B. P., Abbott, R., et al. 2010, CQGra, 27, 173001
Banerjee, S., Baumgardt, H., & Kroupa, P. 2010, MNRAS, 402, 371
Barnard, R., Garcia, M., Li, Z., Primini, F., & Murray, S. S. 2011, ApJ, 734, 79
Belczynski, K., Kalogera, V., & Bulik, T. 2002, ApJ, 572, 407
Brassington, N. J., Fabbiano, G., Blake, S., et al. 2010, ApJ, 725, 1805
Chatterjee, S., Fregeau, J. M., Umbreit, S., & Rasio, F. A. 2010, ApJ, 719, 915
Downing, J. M. B., Benacquista, M. J., Giersz, M., & Spurzem, R. 2010, MNRAS, 407, 1946
Downing, J. M. B., Benacquista, M. J., Giersz, M., & Spurzem, R. 2011, MNRAS, 416, 133
Farr, W. M., & Bertschinger, E. 2007, ApJ, 663, 1420
Fregeau, J. M., Cheung, P., Portegies Zwart, S. F., & Rasio, F. A. 2004, MNRAS, 352, 1
Fregeau, J. M., Gürkan, M. A., Joshi, K. J., & Rasio, F. A. 2003, ApJ, 593, 772
Fregeau, J. M., & Rasio, F. A. 2007, ApJ, 658, 1047
Giersz, M., Heggie, D. C., & Hurley, J. R. 2008, MNRAS, 388, 429
Harry, G. M., & LIGO Scientific Collaboration, 2010, CQGra, 27, 084006
Heggie, D., & Hut, P. 2003, The Gravitational Million-Body Problem: A Multidisciplinary Approach to Star Cluster Dynamics (Cambridge: Cambridge Univ. Press)
Heggie, D. C. 1975, MNRAS, 173, 729
Hurley, J. R., Pols, O. R., & Tout, C. A. 2000, MNRAS, 315, 543
Hurley, J. R., Tout, C. A., & Pols, O. R. 2002, MNRAS, 329, 897
Ivanova, N., Belczynski, K., Fregeau, J. M., & Rasio, F. A. 2005, MNRAS, 358, 572
Ivanova, N., Chaichenets, S., Fregeau, J., et al. 2010, ApJ, 717, 948
Joshi, K. J., Nave, C. P., & Rasio, F. A. 2001, ApJ, 550, 691
Joshi, K. J., Rasio, F. A., & Portegies Zwart, S. 2000, ApJ, 540, 969
Kalogera, V., King, A. R., & Rasio, F. A. 2004, ApJL, 601, 171
Kroupa, P. 2001, MNRAS, 322, 231
Kulkarni, S. R., Hut, P., & McMillan, S. 1993, Natur, 364, 421
Maccarone, T. J., Kundu, A., Zepf, S. E., & Rhode, K. L. 2007, Natur, 445, 183
Maccarone, T. J., Kundu, A., Zepf, S. E., & Rhode, K. L. 2011, MNRAS, 410, 1655
Mackey, A. D., Wilkinson, M. I., Davies, M. B., & Gilmore, G. F. 2008, MNRAS, 386, 65
Merritt, D., Piatek, S., Portegies Zwart, S., & Hemsendorf, M. 2004, ApJL, 608, 25
Miller, M. C., & Davies, M. B. 2012, ApJ, 755, 81
Moody, K., & Sigurdsson, S. 2009, ApJ, 690, 1370
O'Leary, R. M., Rasio, F. A., Fregeau, J. M., Ivanova, N., & O'shaughnessy, R. 2006, ApJ, 637, 937
Peters, P. C. 1964, PhRv, 136, 1224
Pooley, D., Lewin, W. H. G., Anderson, S. F., et al. 2003, ApJL, 591, 131
Portegies Zwart, S. F., & McMillan, S. L. W. 2000, ApJL, 528, 17
Repetto, S., Davies, M. B., & Sigurdsson, S. 2012, MNRAS, 425, 2799
Sadowski, A., Belczynski, K., Bulik, T., et al. 2008, ApJ, 676, 1162
Shih, I. C., Kundu, A., Maccarone, T. J., Zepf, S. E., & Joseph, T. D. 2010, ApJ, 721, 323
Sigurdsson, S., & Hernquist, L. 1993, Natur, 364, 423
Spitzer, L., Jr. 1969, ApJL, 158, 139
Strader, J., Chomiuk, L., Maccarone, T. J., Miller-Jones, J. C. A., & Seth, A. C. 2012, Natur, 490, 71
Wong, T. W., Valsecchi, F., Fragos, T., & Kalogera, V. 2012, ApJ, 747, 111