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Confinement induced trajectory of a squirmer in a two dimensional channel

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Abstract

Micro-swimmers in confinement are encountered in a variety of scenarios such as locomotion of sperm cells in female reproductive tract, targeted drug delivery and biofilm formation. Using a squirmer, a surface actuating model, we simulate the trajectory of swimmers in a two-dimensional channel confinement. Exploiting the simplicity of squirmer model and performing the study in two dimensions we restrict the analysis to minimum number of parameters and isolate and analyze the confinement induced swimmer trajectories. Using exact solutions of two dimensional disk squirmers we first show that they behave qualitatively similar to three dimensional spherical squirmers near a repulsive, planar wall. In a channel, fully resolved flow and thus hydrodynamic interaction between the squirmer and the channel walls are obtained using the lattice Boltzmann method. We find that strong pullers and pushers slide along the channel walls, a behavior determined by single wall. In contrast, swimmers with weak force dipoles break the symmetry in behavior between pushers and pullers, and this behavior is determined by both walls of the channel. Weak pullers stay at the channel center and weak pushers execute an oscillatory trajectory spanning the channel width. Straight line trajectories can be solely characterized by a fixed point on a phase plane spanned by its orientation angle and the distance from the channel centerline whereas oscillatory trajectories can be solely characterized using its escape angle from the wall. The nature of the trajectories is found to be robust to the details of higher modes and the size of the confinement.

Keywords: micro-swimmer dynamics, 2D confinement, lattice Boltzmann simulation, squirmer model, numerical simulation

1. Introduction

Locomotion of microorganisms in fluids, whether it is in search of food or escape from a prey, is an essential survival mechanism in nature (Koch and Subramanian 2011). Synthetic autophoretic swimmers, the laboratory counterparts of biological swimmers, use locally available light, heat or chemical energy for propulsion (Bechinger *et al* 2016). Both biological and synthetic swimmers generally reside in a colloidal world: often they are micron-sized (thus the name micro-swimmers) and move very slowly. Correspondingly the Reynolds number (Re) defined as the ratio of inertial to viscous forces is very small, typically ranges from 10^{-4} to 10^{-2} (Brennen and Winet 1977). Thus the viscous forces dominate the fluid dynamics and the thrust generating mechanism of these micro-swimmers from the surrounding fluid differ from that in the macroscopic world of inertial swimmers (Purcell 1977). Moreover, the local environment, e.g.: physical confinement plays a decisive role in determining the velocity of the swimmer. In this work, we determine the trajectories of a model micro-swimmer confined between two parallel walls using lattice Boltzmann simulations. We then proceed to show the extent to which different factors affect the behavior of these trajectories.

Being long ranged, the hydrodynamic interaction of a micro-swimmer with its confining surface is important in several scenarios. For example, it is known that increased residence time of bacteria near a surface can facilitate biofilm formation (van Loosdrecht *et al* 1990, Kantsler *et al* 2012, Li *et al* 2017). Moreover, since interactions with the confining wall change the translational and angular velocity of a micro-swimmer, the resulting trajectories in a confinement are different from that in the bulk. Such studies are also relevant to understand the sperm locomotion in female reproductive tract (Riffell and Zimmer 2007), and in the development of artificial micro-swimmers for drug delivery and sensing applications (Fusco *et al* 2013). Another projected application is for microfluidic mixing devices since some micro-swimmers exhibit space-spanning oscillatory trajectories (Najafi *et al* 2013).

In a drop of dispersion of bull spermatozoa confined between two parallel glass walls, Rothschild (1963) found that cells accumulate near the confining walls. Later on, similar observations were found with *E. coli* as well (Berke *et al* 2008) and the origin of this accumulation has been shown to be the hydrodynamic interaction between the wall and the swimming microorganisms. On the other hand, several works show that micro-swimmers exhibit an enhanced swimming velocity (Aguilar and Yeomans 2013, Zhu *et al* 2013, Liu *et al* 2016) and an oscillatory or helical trajectory when confined in a narrow channel or tube. In experiments, it was observed that increased confinements changed the trajectory of a *Paramecium* cell from a helix to a straight line (Elgeti and Gompper 2015). Theoretical investigations using a finite swimming cylinder (Jana *et al* 2012) and numerical investigations of squirmers in a capillary tube (Zhu *et al* 2013) generated helical trajectories. While accounting for hydrodynamics of lowest order microswimmer models results in attraction/repulsion of the swimmer, higher order effects are shown to yield the onset of oscillatory motion (de Graaf *et al* 2016). However, depending on the channel geometry and the swimming dynamics, these results change too (Ao *et al* 2014, Lintuvuori *et al* 2016, Malgaretti and Stark 2017, Liot *et al* 2018).

The rheological properties of the suspending fluid will also affect the dynamics of microswimmers. In a power law fluid, the swimmers swim slower than in a Newtonian fluid and the swimming characteristics become very sensitive to the Reynolds number (Ouyang *et al* 2018). The local modification to the fluid properties due to swimming motion also modifies the fluid velocity field thus affecting the interaction between the swimmers as well as the interaction with a confining medium (Ouyang *et al* 2017, 2019). Therefore, the non-

Newtonian fluid properties can significantly alter the dynamics of swimmers compared to that in a Newtonian fluid.

Theoretical and numerical investigations mentioned above have provided insights into the origin of hydrodynamic behavior of micro-swimmers with confining walls. However, several assumptions made in such studies are re-examinable as outlined below. The simplest approach to understand the hydrodynamic collision behavior of micro-swimmer with a surface comes from the analytical calculations using the method of images (Blake and Chwang 1974, Berke *et al* 2008). This analysis models swimmers as point particles within the framework of a multipole expansion and entails the assumption that confining wall is always sufficiently far away (Spagnolie and Lauga 2012, Ider *et al* 2018). However, this is not the case in narrower channels (stronger confinements). Confinement is shown to even change the nature and motility of deformable swimmers (Männik *et al* 2009, Wu *et al* 2015). Therefore it is important to relax the point swimmer approximation and investigate the effect of the finite size of the swimmer and its proximity to the wall in determining their trajectories in a confined environment. Similarly, most of the studies restrict the details of swimming stroke to the first two modes of the multipole expansion. However, higher order modes may always be present invariably as subtle details of swimming strokes (de Graaf *et al* 2016). In an interesting case, flow fields measured around starfish larvae show the dominance of (five) different modes during swimming and feeding phase (Gilpin *et al* 2016). Presently it is not clear whether these higher modes can substantially alter the trajectories of channel confined swimmers.

Simplifications noted above are frequently justified by stating that they may only slightly change the instantaneous dynamics of the micro-swimmer from actual values. However, firstly, it may be noted that the deviations can accumulate over time to make large differences. One such example is the trajectory of a channel confined micro-swimmer. Secondly, trajectory is the consequence of interaction (including hydrodynamic) of the micro-swimmer with the confining walls and small effects can affect the dynamic behavior of the micro-swimmer near a wall. Therefore, in this work, we calculate the swimming trajectories of confined micro-swimmers, characterize them for various types of micro-swimmers and analyze the robustness of their trajectories with respect to the common assumptions mentioned above.

The primary aim of the paper is to calculate the trajectories of microswimmers in a long channel. Hydrodynamic interactions with the walls cause deviations from a straight line trajectory which will be quantified. For this purpose, we make several simplifications. We use squirmer as a model micro-swimmer to perform the trajectory analysis. Compared to models taking detailed geometry of microswimmers into account (Liu *et al* 2014, Wu *et al* 2015), squirmer is not a very sophisticated model. However it is simple and easy to implement in numerical studies. Moreover we restrict our study into two dimensions. It may be noted that trajectories of a single spherical squirmer in a circular tube has been studied numerically in Zhu *et al* (2013) and it was observed that neutral swimmers followed a helical trajectory while the swimmer with a force dipole showed variety of trajectories such as motion near the center of the tube, near the wall or crashing on the walls. However the interaction of a swimmer with a curved surface itself gives rise to a variety of collision behavior (Kuron *et al* 2019). Therefore we restrict our analysis to flat surfaces and in two dimensions since it allows analysis with reduced number of variables. Thus, (i) exploiting the simplicity of squirmer model and (ii) performing analysis in two dimensions we isolate and analyze the confinement induced swimmer trajectories.

Most of the earlier studies have concentrated on analyzing the hydrodynamic collision behavior of a squirmer with a single wall. The collision may result in the swimmer trapped

near the wall if the force dipole contribution dominates the fluid flow around the squirmer (Ishimoto and Gaffney 2013, Shen *et al* 2018). Otherwise the swimmer may bounce off from the wall or exhibit an oscillatory trajectory near the single wall. However, the behavior of these squirmers, especially a trajectory analysis between two parallel walls is not studied in the literature yet. Studies which have done squirmer between parallel walls have either concentrated on the hydrodynamic collision behavior with a single wall (Li and Ardekani 2014, Lintuvuori *et al* 2016, Shen *et al* 2018) or in a curved wall (Zhu *et al* 2013) or the behavior of multiple swimmers (Delfau *et al* 2016). As we show here, hydrodynamic collisions with the walls also result in confinement induced trajectories of microswimmers. These trajectories are also diverse as the collision behavior, and is of importance in describing the long term behavior of confined microswimmers.

Section 2 provides the description of the squirmer model. Lattice Boltzmann method (LBM) is used to solve for the fluid flow and the coupling between fluid flows and swimmer resulted in the dynamics of the swimmer. These details of numerical method are explained in section 3. Then we begin the results in section 4 by describing the hydrodynamic collision behavior of a single squirmer near a single wall. The results are then expanded for a squirmer in a channel confinement and described in a phase space spanning the orientation direction and the location of the swimmer. The behavior of fixed points and limit cycles obtained in this phase portrait are then analyzed by including higher modes of swimming and changing the confinement to squirmer size ratio.

2. Modeling a confined micro-swimmer

2.1. Squirmer model

We use squirmers, an envelope model developed by Lighthill (1952) and Blake (1971a) as a model micro-swimmer. Though, this model has been developed for ciliated organisms (Pedley 2016), it is used in varied contexts (Wang and Ardekani 2013, Chisholm *et al* 2016). Following Blake (1971b), we define squirmers as circular rigid particles having a tangential surface slip velocity,

$$u_\phi^s(\phi) = \sum_n B_n \sin(n\phi), \quad (1)$$

where B_n specifies the magnitude of n th mode. A schematic of the system under consideration is shown in figure 1. As shown let the orientation vector of the squirmer be $\mathbf{e}$, $|\mathbf{e}| = 1$ and the position vector on the surface of the squirmer be $\mathbf{x}_s$. Then ϕ is defined as the angle between these two vectors, $\phi = \cos^{-1}\left(\frac{\mathbf{e} \cdot \mathbf{x}_s}{|\mathbf{e}| |\mathbf{x}_s|}\right)$.

In an unconfined fluid, this model swimmer develops a flow field (Blake 1971b), in lab frame of reference,

$$\begin{aligned} u_r(r, \phi) &= \frac{B_1}{2} \left(\frac{a^2}{r^2} \right) \cos \phi + \sum_{n=2}^{\infty} \frac{n}{2} B_n \cos n\phi \left[\frac{a^{n+1}}{r^{n+1}} - \frac{a^{n-1}}{r^{n-1}} \right] \\ u_\phi(r, \phi) &= \frac{B_1}{2} \left(\frac{a^2}{r^2} \right) \sin \phi + \sum_{n=2}^{\infty} \frac{1}{2} B_n \sin n\phi \left[n \frac{a^{n+1}}{r^{n+1}} - (n-2) \frac{a^{n-1}}{r^{n-1}} \right], \end{aligned} \quad (2)$$

where u_r and u_ϕ represent the radial and angular components of the velocity field. Thus, in an unbounded fluid, a squirmer translates with a translational velocity $\mathbf{U}_0 = \frac{1}{2} B_1 \mathbf{e}$, determined only by the B_1 mode. The relative magnitudes of subsequent modes, which capture the details of near-field fluid motion but do not contribute to swimming, are characterized by defining

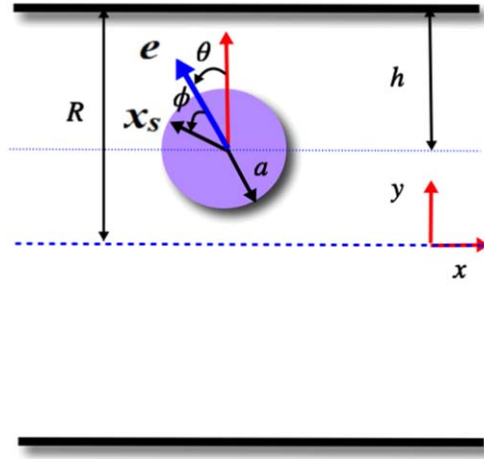


Figure 1. A squirmer of radius a , oriented along $\mathbf{e}$ is placed between two plates separated by a distance $2R$.

$$\beta_n = B_n/B_1, n = 2, 3, 4 \dots \quad (3)$$

The leading order singularities associated with first two modes (B_1, B_2) are a source dipole and a force dipole (Pak and Lauga 2014) respectively. These are the two modes often used in the literature (Götze and Gompper 2010, Li *et al* 2014, Chisholm *et al* 2016) and they categorize the micro-swimmers into following three types. Microorganisms such as *Chlamydomonas*, categorized as pullers ($\beta_2 > 0$), generate thrust in front of their body by drawing fluid from the front and the back and ejecting the fluid sideways. On the other hand, pushers, $\beta_2 < 0$, (e.g.: bacteria) generate thrust behind their body by sucking fluid from the sides and pushing itself through the fluid. $\beta_2 = 0$ corresponds to neutral swimmers, e.g.: Volvox that has a source dipole flow field around their body. A neutral swimmer is different from a passive particle, as the former has a slip velocity due to which it swims in a fluid, but the latter can be driven only by an external force.

2.2. Confined squirmer

The domain under consideration is shown in figure 1. A two-dimensional disk squirmer of radius a is placed between two parallel plates separated by a distance $2R$. Incompressible Navier–Stokes equations describe the fluid dynamics around the squirmer,

$$\nabla \cdot \mathbf{u} = 0 \quad (4)$$

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \nabla^2 \mathbf{u}, \quad (5)$$

where $\mathbf{u}$ is the velocity field, p is the pressure field, ρ is the density and μ is the viscosity of the surrounding fluid. Section 3 describes the implementation details.

3. Numerical method

Fluid motion is solved using the method of lattice Boltzmann and by appropriately coupling to the squirmer dynamics as described below.

3.1. Lattice Boltzmann method (LBM)

LBM is used to solve Navier–Stokes equations numerically (Succi 2001). In this approach, Boltzmann transport equation describing the non-equilibrium evolution of probability distribution functions, $g(\mathbf{x}, \mathbf{v}, t)$, is discretized in physical and velocity space and then evolved temporally according to

$$g_k(\mathbf{x} + \mathbf{v}_k \Delta t, t + \Delta t) = g_k^*(\mathbf{x}, t) \quad (6)$$

$$g_k^*(\mathbf{x}, t) = g_k(\mathbf{x}, t) - \frac{\Delta t}{\tau} [g_k(\mathbf{x}, t) - g_k^{eq}(\mathbf{x}, t)]. \quad (7)$$

Here $g_k(x, t)$ represents the k th population in the direction of discrete lattice velocity $\mathbf{v}_k$ at position $\mathbf{x}$ and time t . The zeroth and first moments of distribution functions give the fluid density $\rho = \sum_k g_k$ and the momentum $\rho \mathbf{u} = \sum_k g_k \mathbf{v}_k$ where $\mathbf{u}$ is the fluid velocity. The distribution functions undergo successive streaming and collision. According to equation (6), during the streaming process, the populations are propagated from one lattice node to another. According to equation (7), during the subsequent collision operation, the populations are relaxed to the equilibrium distribution function

$$g_k^{eq} = \rho w_k \left[1 + \frac{3(\mathbf{u} \cdot \mathbf{v}_k)}{c_s^2} - \frac{3\mathbf{u}^2}{2c_s^2} + \frac{3(\mathbf{u} \cdot \mathbf{v}_k)^2}{2c_s^4} \right]. \quad (8)$$

Both mass and momentum are conserved during the collision. Collision process is controlled by the relaxation time scale τ which relates to viscosity of the fluid as $\frac{\mu}{\rho} = c_s^2(\tau - 0.5\Delta t)$. Also, c_s is the sound speed and w_k is the weight factor corresponding to each discrete velocity direction k . We will use a $D2Q9$ model of LBM in our simulations which solves fluid motion in 2 dimensions with 9 discrete velocity directions.

No-slip boundary condition is imposed on the two channel walls and is achieved using a midpoint bounce back scheme (Aidun and Clausen 2010). Boundary conditions on the squirmer is described in the next subsection.

3.2. Squirmer dynamics

Consider a squirmer located instantaneously at (x, y) and oriented at an angle θ as shown in figure 1. If the squirmer has a translational velocity $\mathbf{U}$ and rotational velocity $\mathbf{\Omega}$, then its surface velocity is given by,

$$\mathbf{u}^s(\mathbf{x}_s) = \sum_n (B_n \sin(n\phi)) \mathbf{e}_\phi + \mathbf{U} + \mathbf{\Omega} \times \mathbf{x}_s. \quad (9)$$

$\mathbf{u}^s(\mathbf{x}_s)$ drives the fluid flow around the squirmer and this flow field is calculated via LBM as explained above.

However, both $\mathbf{U}$ and $\mathbf{\Omega}$ are not known *a priori* in this calculation. Therefore we impose the force-free, torque-free conditions

$$\int \mathbf{f}(\mathbf{x}_s) dS = 0 \quad (10)$$

$$\int \mathbf{x}_s \times \mathbf{f}(\mathbf{x}_s) dS = 0 \quad (11)$$

on the squirmer. Here, $\mathbf{f}(\mathbf{x}_s)$ is the force density on the surface of the swimming particle and it is related to the first moment of the lattice Boltzmann distribution function (Aidun and Clausen 2010).

$$\mathbf{f}(\mathbf{x}_s) = \sum_k -2\mathbf{v}_k \left[g_{\bar{k}}(\mathbf{x}, t^+) + \frac{\rho w_k}{c_s^2} (\mathbf{u}^s \cdot \mathbf{v}_k) \right], \quad (12)$$

where t^+ refers to a time post-collision but pre-propagation and $\bar{k}$ represents the direction opposite to k .

Thus, this procedure ensures the dynamic coupling between the fluid and the micro-swimmer: equation (9) acts as the boundary condition driving the fluid flow, but the resulting stress distribution on the surface of the swimmer has to satisfy equations (10) and (11). Thus, $\mathbf{U}$ and Ω in equation (9) are modified iteratively to satisfy force-free, torque-free conditions of the squirmer. In order to simultaneously solve equations (9)–(11), Broyden's iterative algorithm (Burden and Faires 2011) is used. This is a numerical technique developed to solve nonlinear equations in which an identity matrix is taken as an approximation for the Jacobian matrix and then proceeded by updating it in each iteration till convergence. During this iterative procedure, the squirmer is not allowed to move in the computational domain. In other words, we have taken a 'quasi-stationary' approach to determine the instantaneous values of the translational and rotational velocities of the squirmer.

The trajectory of the squirmer in two dimensions is then constructed by integrating the kinematic equations of motion

$$\dot{x} = U_x, \quad \dot{y} = U_y, \quad \text{and} \quad \dot{\theta} = \Omega \quad (13)$$

from any initial position and orientation using Euler's method.

3.3. Exploiting linearity

At $Re = 0$, Navier–Stokes equations reduce to Stokes equations which are linear. The linearity of these governing equations can be exploited in two different ways in computing squirmer dynamics. This approach is useful since it reduces the amount of computations dramatically. Also, the problem under consideration is translationally invariant in the x -direction. Therefore, we need to investigate squirmer dynamics only in y – θ space. The two different ways in which linearity is exploited in this work are explained below.

- (i) *To calculate the velocities due to individual modes:* Consider the computation of $\mathbf{U}^{B_n}$, the instantaneous translational swimming velocity of an individual squirmer solely due to n th mode of swimming (see equation (1)) at a given y location and an arbitrary orientation θ . It can be done by superposition of two known solutions, $\mathbf{U}_I^{B_n}$ and $\mathbf{U}_{II}^{B_n}$ which are the translational swimming velocities of the squirmer for two different orientations $\theta = \theta_I$ and $\theta = \theta_{II}$ respectively, but at the same y location. These 'basis' orientations are chosen to be as $\theta_I = 0$ and $\theta_{II} = \frac{\pi}{2n}$. Mathematically,

$$\mathbf{U}^{B_n}(y, \theta) = \mathbf{U}_I^{B_n}(y, \theta_I = 0) \cos n\theta + \mathbf{U}_{II}^{B_n}\left(y, \theta_{II} = \frac{\pi}{2n}\right) \sin n\theta. \quad (14)$$

For example, for B_1 mode the two basis orientations are squirmer oriented perpendicular to the wall ($\theta_I = 0$) and squirmer oriented parallel to the wall ($\theta_{II} = \frac{\pi}{2}$). Calculations done as explained in section 3.2 give $\mathbf{U}_I^{B_1}$ and $\mathbf{U}_{II}^{B_1}$ respectively. Using equation (14), the total translational velocity due to B_1 mode at any orientation can be calculated. The magnitude of each mode (B_n) is always chosen as unity to simplify the next step.

- (ii) *To calculate the total velocity of the squirmer:* The total translational swimming velocity of the squirmer is calculated as a superposition of velocities from individual modes,

$$\mathbf{U}(y, \theta) = \sum_n B_n \mathbf{U}^{B_n}(y, \theta) \quad (15)$$

for any desired β_n .

A similar superposition principle is followed to calculate the angular velocity corresponding to each mode and at any y location and orientation.

This procedure of using equations (14) and (15) reduced the number of computations substantially.

3.4. Non-dimensional variables

There are two relevant non-dimensional numbers in the problem arising from the geometry.

- (i) A non-dimensional variable $\alpha = y/(R - a)$ is defined to indicate the transverse location of the squirmer in the channel, $\alpha \in [-1, 1]$. $\alpha = 0$ represents the center of the channel and $\alpha = \pm 1$ represent the two walls.
- (ii) a/R is another non-dimensional number that indicates the squirmer size with respect to the channel width.

If we choose squirmer radius a as the characteristic length and translational swimming speed of an unconfined squirmer $U_0 = B_1/2$ as the characteristic velocity then the Reynolds number of the squirmer is calculated as $Re = \frac{(B_1/2)a}{(\mu/\rho)}$. A small Re is maintained in the simulations in order to analyze the results in creeping flow regime, however variations in Re can bring in inertial effects to the fluid dynamics (Chisholm *et al* 2016, Ouyang *et al* 2018). In the simulations $Re = 0.015 \ll 1$ is used and hence Re is not a parameter in this study. In the results below, translational and rotational velocities of the squirmer are reported after normalising with U_0 and U_0/a respectively. Also, it may be noted that positive Ω shows anticlockwise rotation of the squirmer.

3.5. Simulation parameters

A domain size of 400×400 is chosen for the simulations, unless mentioned otherwise. Wall boundary conditions are applied at the top and bottom of the domain and periodic boundary conditions are applied on the other two sides. As usual in LBM, spatial and temporal resolutions are chosen as unity. Density and viscosity of the fluid are taken as 1 and $2/3$ lattice units respectively. A single squirmer of radius 20 lattice units is placed at a specified y and at a specified orientation θ . Iterative procedure to calculate the squirmer velocities using equations (10) and (11) is done on the steady state flow fields obtained using LBM. The iterative procedure is continued as long as the tolerance limit for the force and torque exceeds 10^{-6} . For an unconfined swimmer, we find the swimming velocity U_0 to be within an error of 1% of the theoretical value $B_1/2$.

4. Results and discussion

4.1. Instantaneous velocities of a squirmer near a wall

The instantaneous translational and angular velocities (U_x , U_y and Ω) of a squirmer located near a wall are shown in figure 2 as a function of squirmer orientation θ . (a)–(c) Correspond to squirmer with only B_1 mode and (d)–(f) with only B_2 mode of swimming. The case of a wall bound squirmer is much studied in the literature Li and Ardekani (2014), Lintuvuori *et al* (2016), Rühle *et al* (2017), Shen *et al* (2018), however we discuss these results first as it (i)

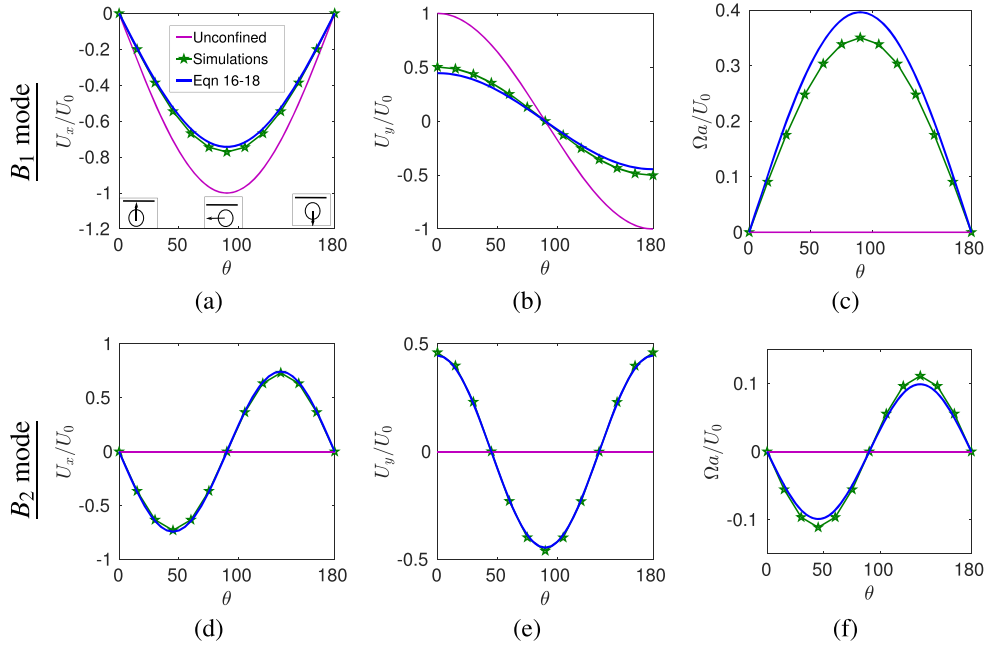


Figure 2. Normalised translational (U_x/U_0 , U_y/U_0) and angular ($\Omega a/U_0$) velocities of squirmer located at a distance $h = 1.25a$ from a wall, as a function of orientation of the squirmer (θ). (a)–(c) B_1 squirmer (d)–(f) B_2 squirmer. The insets of (a) schematically show three orientations of the squirmer $\theta = 0^\circ$, 90° and 180° . Legends for all graphs are same as that given in (a).

naturally leads to explaining the behavior of channel confined squirmers and (ii) serves as a validation of our numerics.

Figure 2 shows the translational velocities of an unconfined swimmer, $U_{o,x}$ and $U_{o,y}$, as well. On comparison between the unconfined swimmer and the wall bound swimmer it may be seen that presence of a wall reduces the velocity of the B_1 squirmer in both directions—velocity parallel to the wall and velocity perpendicular to the wall. The reduction in U_y may be interpreted as a wall induced repulsion of a B_1 squirmer due to hydrodynamic interaction with the wall. While there is no qualitative change in U_x and U_y compared to $U_{o,x}$ and $U_{o,y}$, this is not the case with Ω . The angular velocity of an unconfined squirmer in figure 2(c) is zero for all θ since the squirmer does not rotate by itself in the bulk fluid. But a B_1 squirmer rotates near a wall. The direction of this wall induced rotation is such that the B_1 squirmer gets oriented away from the wall always. Therefore, the hydrodynamic interactions of a squirmer with the wall may lead to attraction or repulsion and reorientation resulting in a *hydrodynamic collision* with the wall.

Unlike a B_1 squirmer which exhibits only quantitative changes in the translational velocities, a B_2 squirmer shows changes qualitatively too. The nature of wall-induced effects on a B_2 squirmer as a function of squirmer orientation is shown in figures 2(d)–(f). An unconfined squirmer with B_2 mode alone does not swim or rotate. Figures 2(d) and (e) show that a wall induces a translational velocity, along as well as perpendicular to the wall. The direction of the induced velocity depends upon the orientation of the squirmer.

On analyzing figure 2(e) further, it may be observed that a B_2 squirmer oriented close to and parallel to the wall ($\theta = 90^\circ$) has a negative U_y indicating a repulsion from the wall. On

the other hand, a B_2 squirmer oriented normal to the wall, whether towards ($\theta = 0^\circ$) or away ($\theta = 180^\circ$) is attracted towards the wall. We will see later that this attraction can cause trapping of squirmers near channel walls in certain cases. The angular velocity of B_2 squirmer also depends upon the orientation of the squirmer: if it is oriented towards the wall, it rotates further towards the wall; if it is oriented away from the wall, it gets rotated further away from the wall (figure 2(f)).

Ishimoto and Crowdy (2017) have derived exact expressions for describing the dynamics of a two dimensional squirmer near a no-slip wall,

$$U_x = \frac{1}{2}(1 - \rho^2)[-B_1 \sin \theta + 2\rho B_2 \sin 2\theta] \quad (16)$$

$$U_y = \frac{(1 - \rho^2)^2}{2(1 + \rho^2)}[B_1 \cos \theta + 2\rho B_2 \sin 2\theta] \quad (17)$$

$$\Omega = \frac{\rho^2}{a(1 + \rho^2)}[-2\rho B_1 \sin \theta + (1 - 3\rho^2)B_2 \cos 2\theta], \quad (18)$$

where, $\rho = \frac{h}{a} - \left[\frac{h^2}{a^2} - 1\right]^{1/2}$, h is the distance from the wall. The instantaneous translational and rotational velocities obtained from these equations are also plotted in figure 2. Indeed the simulation results match well with the exact solutions thus serving (also) as a validation of our numerical results.

4.2. Instantaneous velocities of a squirmer in a channel

Figure 3 shows the instantaneous translational and angular velocities (U_x , U_y and Ω) of channel confined squirmers as a function of squirmer orientation θ . (a)–(c) Correspond to squirmer with only B_1 mode and (d)–(f) correspond to squirmer with only B_2 mode of swimming. Different plots in each graph correspond to different α i.e. different y locations in the channel, from location near the wall to the channel centerline. The results are symmetric with respect to the centerline of the channel and therefore only the results for the upper half of the channel ($\alpha \geq 0$) are shown.

The qualitative behavior in the instantaneous velocities of a channel confined squirmer is similar to that of a wall bound squirmer. For example, we have seen that the translational velocity of a B_1 squirmer decreases while it acquires an angular velocity near a single wall. Similarly, both the translational velocity and the magnitude of acquired angular velocity increase as the squirmer approaches the channel walls. The acquired velocities of B_2 squirmer also increase as it approaches the walls of the channel.

Before we proceed, it is worth thinking about the consequence of these plots on the trajectory of a channel confined squirmer. While reduced translational velocities indicate that squirmer will spend more time near the wall than the case without hydrodynamic interactions, the presence of a wall induced angular velocity shows that the squirmer may just reorient and escape from the wall. Thus the competition between these two effects determines the actual effect of hydrodynamic interactions on the trajectory of a squirmer even for a B_1 or B_2 squirmer.

Thus, the consequence of the diverse behavior shown in figure 3 is that a squirmer having B_1 and B_2 modes of surface actuation will show a complex hydrodynamic collision process with channel boundaries. An example of this process is shown schematically in figure 4. Consider a puller located near the upper wall but oriented towards it (figure 4, case I). It experiences an axial velocity in the negative x -direction due to hydrodynamic interactions of

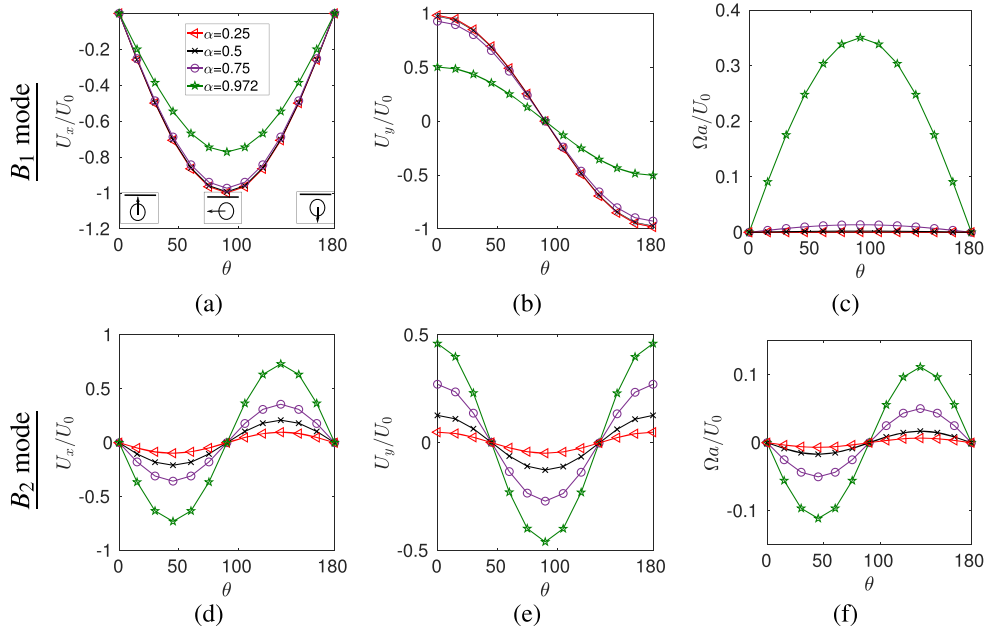


Figure 3. Normalised translational (U_x/U_0 , U_y/U_0) and angular ($\Omega a/U_0$) velocities of the squirmer in a channel as a function of orientation of the squirmer (θ) at various α (y-locations). (a)–(c) B_1 squirmer (d)–(f) B_2 squirmer. The insets of (a) schematically show three orientations of the squirmer $\theta = 0^\circ$, 90° and 180° close to the upper wall of the channel. Legends for all graphs are same as that given in (a).

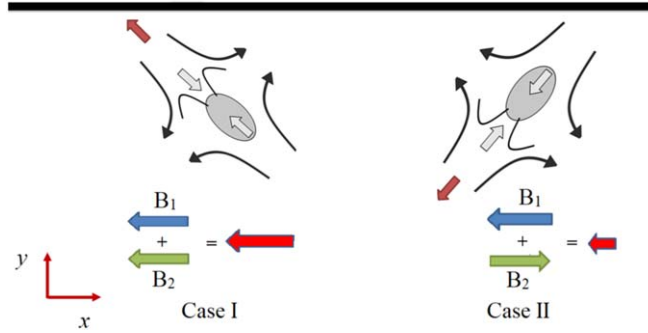


Figure 4. Schematic representation of diversity in the swimming velocity along the channel direction (U_x) of a microswimmer (puller) near the upper channel wall. Green and blue arrows respectively represent the induced velocity by B_1 and B_2 modes and the red arrow gives the resultant velocity in channel direction.

both B_1 and B_2 modes. The two arrows below the swimmer represent the direction of induced velocities. Consequently, the puller experiences a net velocity in the negative x -direction. On the other hand, a puller oriented away from the wall (case II) experiences a lower x -directional velocity since the induced velocity due to hydrodynamic interactions of both these modes are in opposite directions. The direction of net induced velocity depends upon the magnitudes of B_1 and B_2 . The instantaneous behavior of the pusher is exactly opposite—it has a higher

velocity along the channel when it is oriented away from the wall compared to when it is oriented towards the wall.

4.3. Role of wall repulsion in hydrodynamic collision with a wall

In this section we study the dynamic behavior of a squirmer, again near a single wall but when the wall is exerting a repulsive potential. Since analytical results are available for a single squirmer near a wall, we use these expressions to analyze the role of wall potential.

Wall potentials are common, for example with charged surfaces or polymer coated surfaces and it has already been shown that a repulsive potential can affect the behavior of swimmers near a no-slip wall (Lintuvuori *et al* 2016). We implement a hard wall potential in our calculations. There is only one parameter to be tuned to change the hard wall potential, namely the cut-off distance from the wall, δ_c within which the infinite repulsive force acts. We have varied both δ_c and β_2 and observe a variety of hydrodynamic collision behaviors. It turns out that, in presence of wall repulsion both 2D and 3D squirmers behave in a qualitatively similar fashion as discussed below.

Figure 5(a) shows the collision behaviors observed for various values of β_2 and $\delta_c = 0.25a$. A strong pusher (large, negative β_2) gets attracted to the wall and remains trapped and slides near the wall. A weak pusher (intermediate β_2) may show oscillations while trapped near a wall. If $|\beta_2|$ is very small then pushers, pullers and neutral swimmers bounce back from the wall. As β_2 increases, a weak puller shows oscillatory behavior and a strong puller gets trapped and slides near a wall.

This diverse collision behavior with a wall is illustrated in figure 5(b) in a phase space of δ_c and β_2 . As seen, the collision behavior is strongly dependent upon the range of repulsion chosen. A neutral squirmer always exhibits bounce back irrespective of the range of wall potential, whereas pusher and puller dynamics depend on the range of repulsion potential. The five types of collision behavior mentioned above has also been seen for a 3D squirmer (Shen *et al* 2018), except that (i) the range of β_2 in which each behavior observed is different and (ii) the absence of decaying oscillatory behavior before being trapped. The probable reason for the latter is the different form of repulsive potential chosen here. However, the order in which the different collision behavior occurs is similar for both 2D and 3D squirmers.

In order to reveal the importance of wall potential and hence to put things into perspective, we compare our results with previous analysis available in the literature both in two and three dimensions. Analysis using lubrication approximation of spherical squirmers shows that pullers have a stable trajectory near the wall (Ishimoto and Gaffney 2013, Lintuvuori *et al* 2016) while pushers have no stable or stable oscillatory trajectories. On the other hand disk-squirmers, whether pushers or pullers have no stable trajectories due to the underlying Hamiltonian structure involved (Ishimoto and Crowdy 2017). Hence, 2D and 3D hydrodynamics of squirmers near the wall are different in the absence of wall potential. The long range nature of two dimensional hydrodynamics is well known and thus it is not surprising to see qualitatively different results in two and three dimensions. However, as shown above, our analysis shows that, in presence of a wall repulsion force, which is most usual the case, both disk and spherical squirmers behave in a similar fashion near a wall.

Wall repulsion playing a crucial role in determining the collision behavior has been discussed in literature before. Ishimoto and Gaffney (2013) showed that fixed points can emerge when wall repulsion is considered for disk squirmers. However a systematic analysis was not performed in their work. The investigations on the effects of wall repulsion by Lintuvuori *et al* (2016) were also not complete since analytical calculations were restricted to an approximate solution. The approximate solution was obtained from a match of far field and

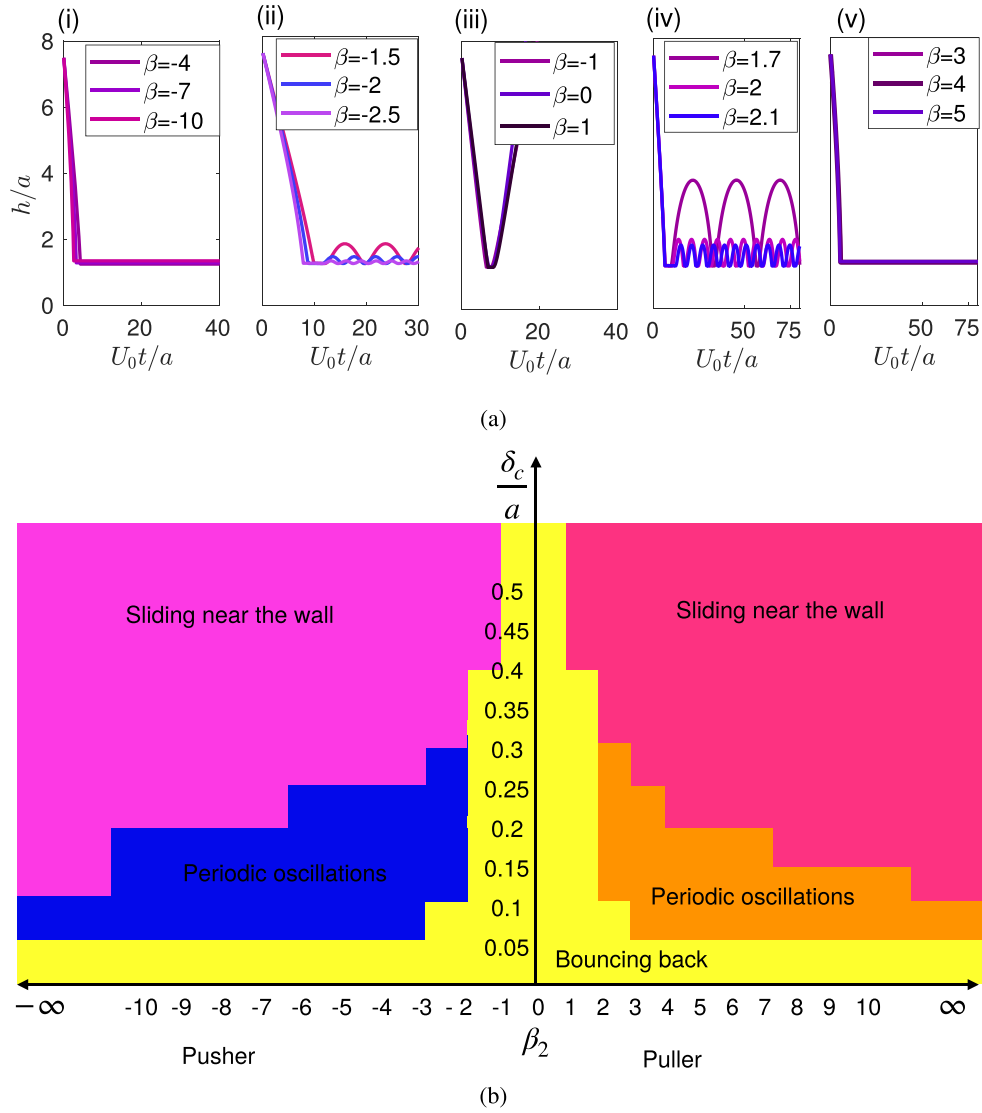


Figure 5. (a) Distance between the wall and the squirmer as a function of time for various β_2 during a hydrodynamic collision process. At $t = 0$, squirmer is located at $h = 7.5a$ and $\theta = -120^\circ$. As the swimming mode is changed from a strong pusher (large but negative β_2) to a strong puller, we find five different types of behaviors in the following order (i) trapped and sliding near the wall (ii) periodic oscillations (iii) bouncing back from the wall (iv) periodic oscillations and (v) sliding along the wall. (b) Phase diagram in β_2 - δ_c plane showing different hydrodynamic collision behavior.

lubrication solutions. Moreover this comparison was only for the rotational motion of the squirmer as the solution to the leading order translational problems in lubrication approximation were unavailable. On the other hand, here, since exact solutions are available in two dimensions we have been able to clearly demonstrate the effect of wall repulsion in the hydrodynamic collision behavior of squirmers.

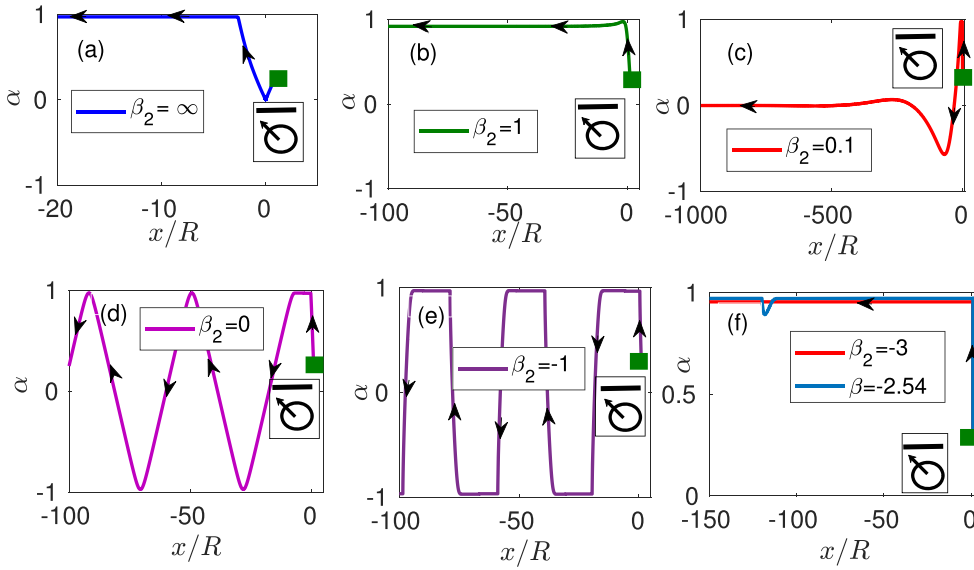


Figure 6. Trajectories of different squirmers in a channel. Initially (marked with green rectangle) they are located slightly displaced from the center at $\alpha = 0.27$ and oriented at $\theta = 60^\circ$ (as shown in the schematic in the inset). Weak puller ($\beta_2 = 0.1$) takes much longer distance to stabilize its trajectory compared to other squirmers.

Therefore, if the effects of hydrodynamics and wall repulsion are considered together, the generic behavior of a squirmer near a wall may be described as follows. Starting from most negative β_2 , squirmers get trapped and slide near a wall and then, as β_2 increases squirmers exhibit oscillations, finally resulting in scattering from the wall as $\beta_2 \rightarrow 0$. Periodic oscillations may re-emerge as β_2 increases further, and finally strong pullers with large β_2 remain trapped and slide near the wall.

With the above analysis we proceed further with 2D simulations as they will give insights into the qualitative behavior of swimmers in channels. In our simulations described below we have chosen $\delta_c = 0.25a$.

4.4. Trajectory of a channel confined squirmer

The variety in the response of B_1 and B_2 squirmers during their hydrodynamic collision with confining walls and the role of wall potential show that it is difficult to gauge their long time behavior in a channel from an instantaneous picture (as in figure 3). Therefore, we proceed to construct the trajectory of different classes of swimmers.

Figure 6 shows the trajectories of squirmers for six different values of β_2 , namely (i) a shaker ($\beta_2 = \infty$), (ii) a strong puller ($\beta_2 = +1$), (iii) a weak puller ($\beta_2 = +0.1$), (iv) a neutral swimmer ($\beta_2 = 0$), (v) a weak pusher ($\beta_2 = -1$), and (vi) a strong pusher ($\beta_2 \leq -3$). All of them are initially located at $\alpha = 0.27$, slightly displaced upwards from the channel centerline and oriented at an angle $\theta = 60^\circ$ as shown in the inset of figure 6. Consequently, all of them move towards the left (negative x -axis). (The choice of initial conditions is completely arbitrary.) Two different types of trajectories can be seen, a straight line trajectory either close to the wall or at the center of the channel and an oscillatory trajectory spanning the channel width.

A shaker has $B_1 = 0$. Irrespective of the initial conditions, it goes towards the wall and gets trapped near the wall. The wall induced U_x continues to propel it, but it does not move away from the wall and maintains a constant orientation. A similar behavior is observed for a strong puller but with the difference that it gets trapped at an α slightly smaller than that of a shaker. The weak puller also has a straight line trajectory, however, it stays at the center of the channel and not close to the wall. Qualitatively different behavior is observed for a neutral swimmer and a pusher. They make oscillatory trajectories between the two walls of the channel. Compared to a neutral squirmer, a pusher spends more time in contact with the wall during the hydrodynamic collisions. Slowly, these oscillations get restricted to regions near the walls and strong pushers ($\beta_2 \leq -3$) get trapped near the wall and slide along the wall.

We now contrast the oscillatory trajectories reported in figure 6 with those found in literature. Using a numerical methodology similar to that used in this work, Lintuvuori *et al* (2016) found hydrodynamic trapping of pullers and oscillations of pushers near a plane wall. They showed that the competition between near field hydrodynamics and the wall repulsion potential results in such effects. However the hydrodynamic oscillations reported in their work is restricted to a single wall. In other words the amplitude of oscillations are close to channel width ($>$ squirmer size) in figure 6 while it is smaller than channel size in the study by Lintuvuori *et al* (2016). Oscillations of swimmers are also observed by de Graaf *et al* (2016) but near the center line of the channel. We also observe these oscillations during the onset of swimming, which is resulting from the presence of two confining walls. In de Graaf *et al* (2016) the authors investigated the onset of swimming while figure 6 describes the long time behavior of the squirmer. The helical trajectories of pushers and neutral swimmers reported in Zhu *et al* (2013) are also different from the trajectories reported here. In the former, the helical trajectories are along the channel walls that resulted from the three dimensionality and curvature of the confining cylinder while figure 6 shows the two dimensional trajectories between planar walls.

Since the trajectory is invariant or periodic in x -direction, such trajectories can be conveniently represented in a θ - α plane where θ represents the instantaneous squirmer orientation and α represents the instantaneous scaled lateral position. The trajectories of pullers on this reduced space are shown in figure 7(a). Pullers traveling on a straight trajectory have constant orientation and thus they reach a *fixed point* on this phase diagram. Started from the same initial conditions, they follow different paths depending upon the value of β_2 . Pullers with $\beta_2 \geq 1$ finally slide along the wall ($\alpha \approx 1, \theta < 90^\circ$) and others move at the center of the channel ($\alpha = 0, \theta = 90^\circ$ thus oriented parallel to the channel). It may be noticed that weak pullers ($\beta_2 \leq 0.1$) show oscillations before finding their fixed point at the channel center, consistent with the observations by de Graaf *et al* (2016). Thus, leaving out the transient path the trajectory of a puller can be represented by a fixed point on θ - α plane.

We now turn our attention to the origin of this fixed point. This is a point where the squirmer (i) experiences no net attraction or repulsion towards the wall (ii) does not experience an angular velocity. Mathematically, if (θ_f, α_f) represents a fixed point then,

$$U_y^{B_1}(\theta_f, \alpha_f) + \beta_2 U_y^{B_2}(\theta_f, \alpha_f) = 0 \quad (19)$$

$$\Omega^{B_1}(\theta_f, \alpha_f) + \beta_2 \Omega^{B_2}(\theta_f, \alpha_f) = 0. \quad (20)$$

These two equations can be simultaneously solved with the help of data in figure 7(a) to determine all fixed points for any β_2 . The solutions so obtained are shown in figure 7(b). Three clusters of solutions can be identified in this figure. (i) $\alpha = 0, \theta = 90^\circ$ always emerges

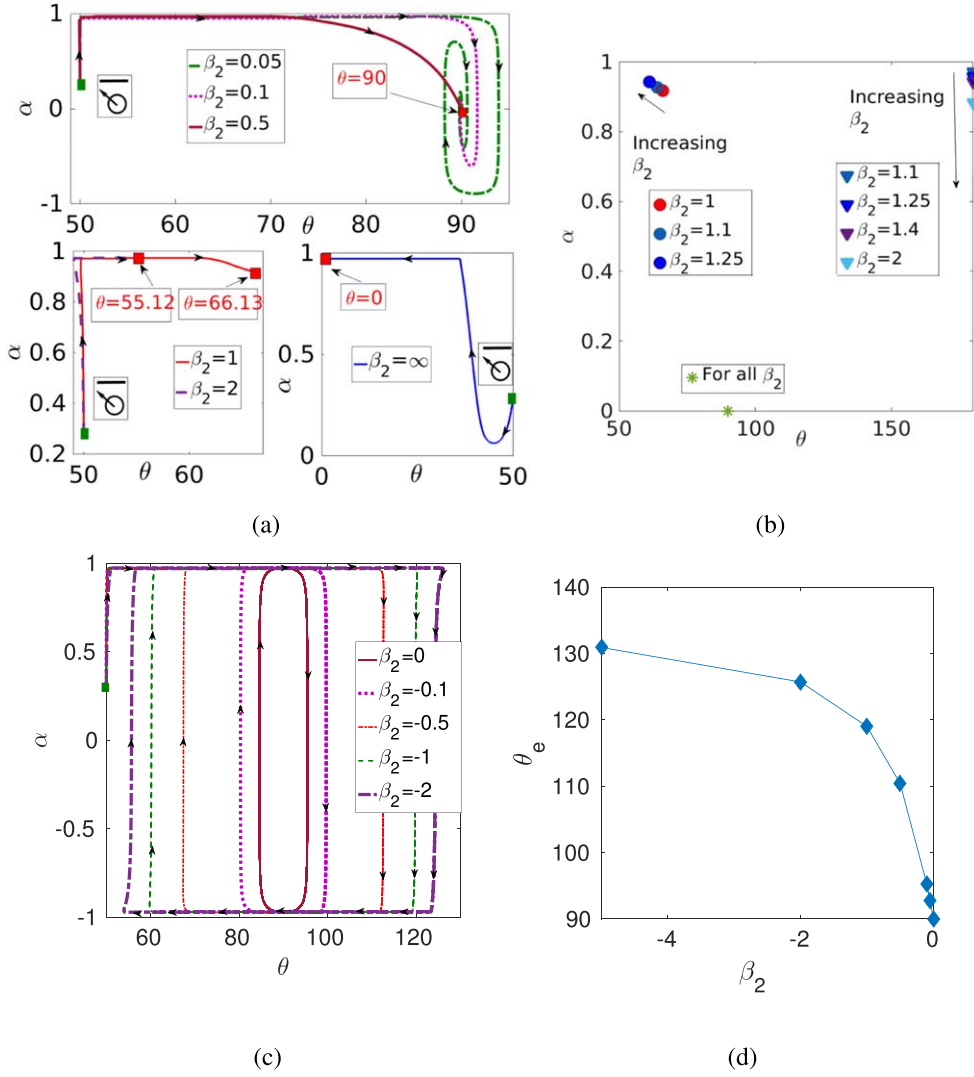


Figure 7. (a) Trajectories of pullers initially located at $\alpha = 0.27$ and $\theta = 50^\circ$ (marked with green square) for different β_2 . (b) Fixed points of puller for different β_2 . Solid circles represent stable nodes and solid triangles represent saddle points. Symbol * shows that stability depends on the value of β_2 . (c) Trajectories of pushers initially located at $\alpha = 0.27$ and $\theta = 50^\circ$ for different β_2 (d) Escape angle of pusher plotted as a function of β_2 .

as a solution from the symmetry of system for which $U_y^{B_1, B_2}(\alpha_f, \theta_f) = 0$, $\Omega^{B_1, B_2}(\alpha_f, \theta_f) = 0$. In addition, (ii) a cluster near the wall but with $\theta_f < 90^\circ$ and (iii) a cluster representing squirmers oriented away from the wall ($\theta_f = 180^\circ$) at all α , emerge when the wall response of the two modes cancel each other as per the above equations (19) and (20).

However, it may be noted that it is not sufficient for a fixed point to satisfy equations (19) and (20) to represent a trajectory, it must be stable as well. In order to determine the stability, we define a coefficient matrix A (Strogatz 1994),

$$A = \begin{bmatrix} \frac{dU_y}{dy} & \frac{dU_y}{d\theta} \\ \frac{d\Omega}{dy} & \frac{d\Omega}{d\theta} \end{bmatrix} \quad (21)$$

for the fixed point. Elements of matrix A are calculated using a finite difference method from the known values of U_y and Ω at any location and orientation. Let $\Delta = \lambda_1 \lambda_2$ and $\tau = \lambda_1 + \lambda_2$ are the determinant and trace of matrix A respectively where λ_1 and λ_2 are the eigenvalues of the matrix A . If $\Delta < 0$, the fixed point is a saddle point. If $\Delta > 0$ and $\tau^2 - 4\Delta > 0$ then the fixed point is a node. The node is stable when $\tau < 0$ and unstable when $\tau > 0$.

Based on this criteria, three families of fixed points identified in figure 7(b) are classified. The first family at $\theta = 90^\circ$, $\alpha = 0$ is a stable fixed point if $\beta_2 < 1$ and a saddle point otherwise. The second family of fixed points that exist close to the wall are stable. They act to trap a puller near the wall. As β_2 increases, the fixed point gets closer to the wall and squirmer gets oriented more towards the wall. The third family of fixed points at $\theta = 180^\circ$ are all saddle points. Thus equations (19) and (20) and the stability criteria mentioned above explains the origin of fixed points noted in figure 7(a).

The oscillatory trajectories of neutral squirmers and pushers in figure 6 transform to closed trajectories called *limit cycles* on θ - α plane as shown in figure 7(c). Unlike pullers, these have no stable fixed points, they continuously reorient and escape from the channel wall. Subsequently, they cross the centerline and reach the other wall, only to repeat the process. As $|\beta_2|$ increases the angle at which they escape from the wall, called escape angle, increases. The increase in escape angle yields wider limit cycles.

Escape angle can be used as a sole measure to characterize a limit cycle corresponding to each β_2 . It may be calculated as the angle θ_e , at which wall interaction changes from attraction to repulsion at the location closest to the wall (α_e). Mathematically,

$$U_y^{B_1}(\theta_e, \alpha_e) + \beta_2 U_y^{B_2}(\theta_e, \alpha_e) = 0. \quad (22)$$

Using the data in figure 3 and choosing $\alpha_e = 0.972$ (the closest point to the wall in simulations) the above equation was solved to determine θ_e as a function of β_2 . The result is shown in figure 7(d). θ_e increases with increase in $|\beta_2|$. This suggests that dominant B_2 mode results in wider limit cycles as concluded earlier.

Very large β_2 or strong pushers will have the width of the limit cycle going to 2π . In other words, limit cycle disappears, and fixed point emerges in the phase plot, representing their trapping on the wall.

It is also interesting to analyze the swimmer behavior by reversing the order, namely as β_2 is increased from large negative value, to large positive value. Then the change in collision behavior can be interpreted in terms of the reorientation dynamics of the swimmer. Strong pushers do not escape from the wall. Weak pushers undergo reorientation dynamics near the wall over a large range of θ before eventually escaping from the wall. On the other hand, for a neutral squirmer ($\beta_2 = 0$) the escape angle is 90° . Further increase in β_2 (namely for pullers) would change the escape angle to $\theta_e < 90^\circ$. This implies that pullers are drawn towards the wall but not escaped from the wall which had resulted in their fixed points.

4.5. Effect of higher squirming modes

Squirmer is just a model for micro-swimmers, no real micro-organisms is strictly a squirmer. Usually only the first two modes of squirming (B_1 and B_2) are considered in modeling a micro-swimmer, which is a good approximation in an unbounded fluid. Recent experiments

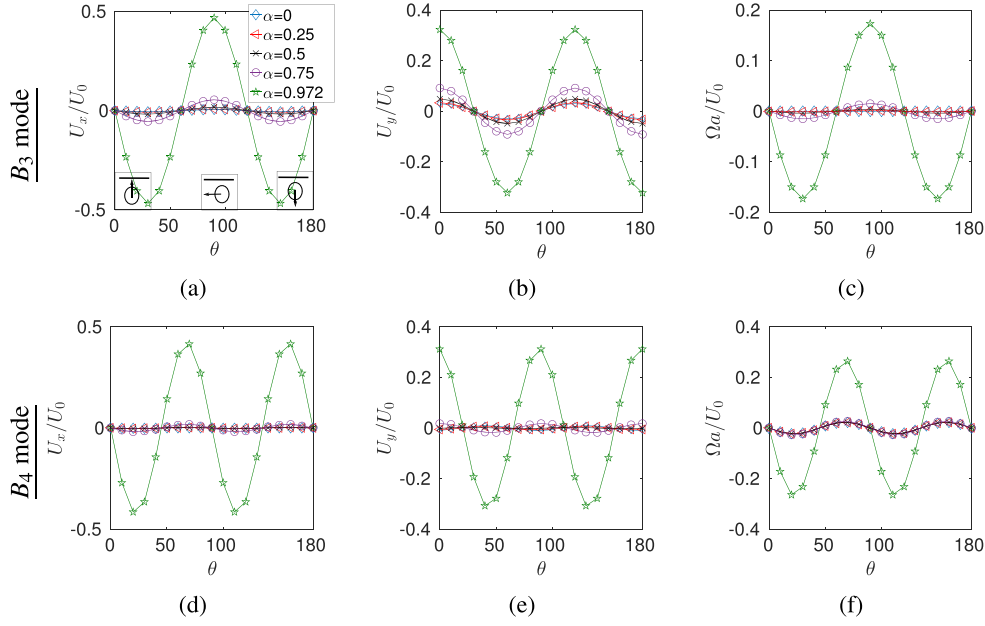


Figure 8. Normalized translational (U_x , U_y) and angular (Ω) velocities of (a)–(c) a B_3 and (d)–(f) a B_4 squirmer in confinement as a function of orientation of the squirmer at various α . Legend for all graphs are same as that shown in (a). The insets of (a) show three orientations of the squirmer schematically $\theta = 0^\circ$, 90° and 180° near the upper channel wall.

(Gilpin *et al* 2016) showed the relevance of higher modes in the natural behavior of micro-organisms, and therefore it is interesting to investigate the effect of these higher modes.

Figure 8 shows the instantaneous translational and rotational velocities of the squirmer due to B_3 and B_4 modes. As in the case of first two modes the wall induced velocities are larger as the squirmer approaches the wall. Moreover, both translational and rotational velocities change sign more often, every 60° in θ for B_3 and every 45° in θ for B_4 squirmer. This sign change is important because, mathematically, it is this sign change in U_y and Ω that resulted in fixed points of pullers and limit cycles of pushers in the previous analysis. Therefore, the presence of either of these higher modes can potentially change the qualitative behavior of hydrodynamic collisions with the wall.

Using the procedure mentioned in section 4.4 the effect of the presence of B_3 and B_4 modes on the puller and pusher dynamics is analyzed and described next. The phase plots are shown in figures 9(a) and (b) for a fixed value of β_2 (-1 for pusher and 1 for puller). Having $\pm B_3$, $\pm B_4$ do not change the fixed point of the puller substantially till $|\beta_3| \geq 1$ or $|\beta_4| \geq 0.25$. The stable fixed point which represents the trapping of the puller shifts close to the wall as $|\beta_3|$ increases but this set disappears for $\beta_3 > 1$. This set shifts away from the wall as β_4 increases and disappears for $\beta_4 > 0.25$. However, the stable fixed point at the channel center when the squirmer is oriented parallel to the wall ($\theta_f = 90^\circ$, $\alpha_f = 0$) exists for all values of β_3 and β_4 . The family of saddle points at $\theta = 180^\circ$ are relatively unaffected by B_3 or B_4 mode and does not have any consequences.

The escape angle which characterizes the limit cycle of a pusher is plotted as a function of β_3 and β_4 in figure 9(c) and (d). Unlike previous cases, we observe two sets of escape angles: $\theta_e > 90^\circ$ and $\theta_e < 90^\circ$. The escape angle which is larger than 90° is found to increase

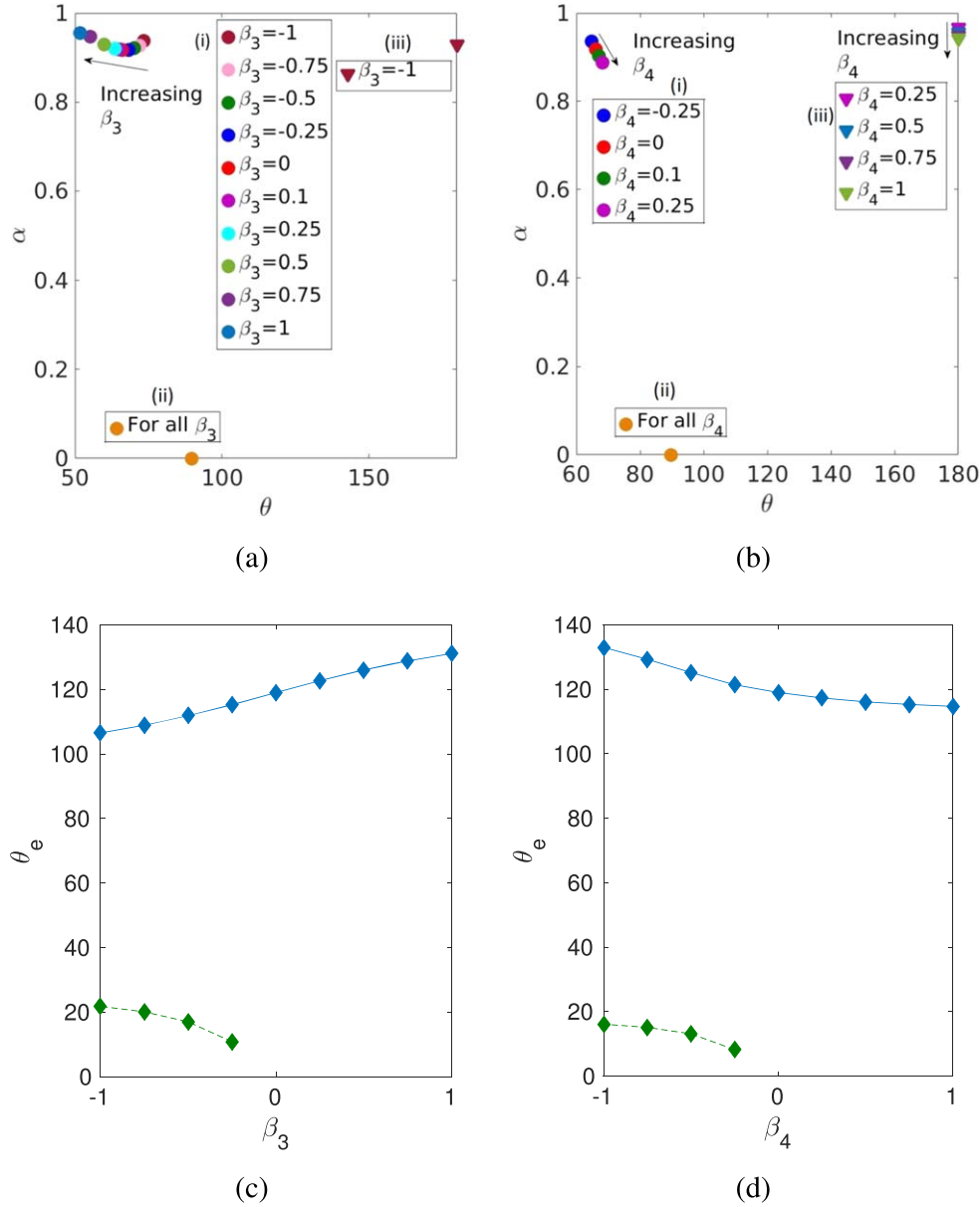


Figure 9. Fixed points and escape angles for squirmers with higher order squirming modes, B_3 and B_4 . Shift in the location of fixed points with addition of (a) B_3 mode and (b) B_4 mode for a puller ($\beta_2 = 1$). Solid circles represent stable nodes and solid triangles represents saddle points. Different legends correspond to different characteristic behaviors: (i) a set of stable fixed points representing the trapping of puller near the wall and that slightly change with change in $|\beta_3|$, $|\beta_4|$ (ii) a second set, namely a stable fixed point at the channel center representing the squirmer oriented parallel to the wall ($\theta_f = 90^\circ$, $\alpha_f = 0$) and (iii) a third set of saddle points at $\theta = 180^\circ$. Change in the escape angle with addition of (c) B_3 mode and (d) B_4 mode for a pusher ($\beta_2 = -1$).

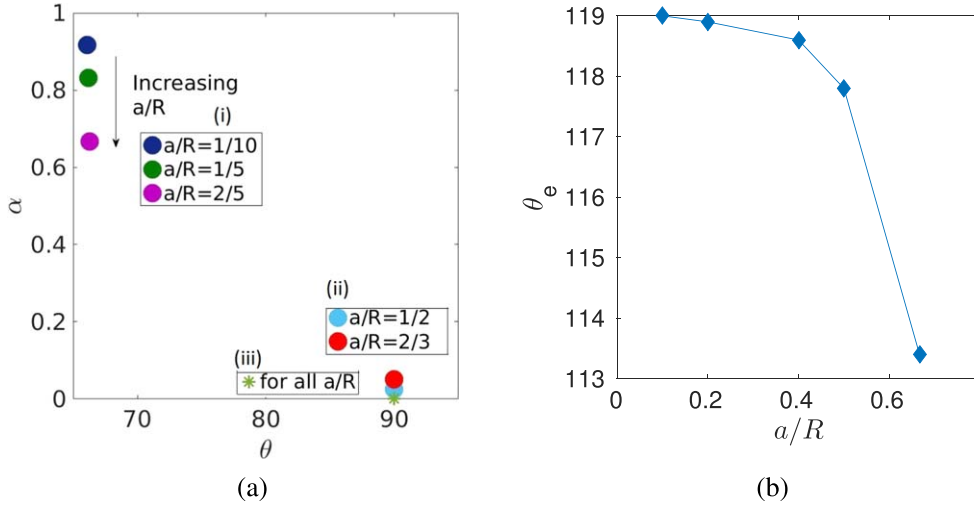


Figure 10. Fixed points and escape angles of squirmers for different confinement widths: (a) fixed points of puller ($\beta_2 = 1$) (b) escape angle of pusher ($\beta_2 = -1$) as a function of a/R . Different legends in (a) correspond to different characteristic behavior: (i) a set of stable fixed point near the wall that move towards the channel center for stronger confinements upto $a/R = 2/5$, (ii) a new set of fixed points that appears near the channel center for $a/R > 2/5$ and (iii) a third set always present at the channel center.

with an increase in β_3 (figure 9(c)), and decrease with increase in β_4 (figure 9(d)). The additional set of $\theta_e < 90^\circ$ is observed only for $\beta_3 < 0$, $\beta_4 < 0$. However, these escape angles less than 90° are found to be corresponding to unstable limit cycles. Thus the presence of moderately strong B_3 or B_4 modes does not change the fixed points of pullers and escape angles of pushers substantially.

4.6. Effect of confinement on squirmer trajectories

In this section we discuss the effect of channel width on the squirmer dynamics. To investigate the same, we varied a/R , the ratio of size of the squirmer to the channel width and the results are shown in figure 10. For pullers, the stable fixed point moves away from the wall towards the channel center as the channel size is reduced. Thus ‘trapping’ is no more at the confining walls. Moreover, when the channel width is smaller than 5 times the radius of the squirmer ($2R < 5a$) this set of stable nodes disappears. Instead a set of stable fixed points at the channel center ($\theta = 90^\circ$, $\alpha = 0$) appears which represent a straight line trajectory at the center of the channel. The escape angle characterizing the limit cycles for pushers also change with change in a/R . However, as shown in figure 7(d) this change is weak; decreasing a/R increases the escape angle slightly and consequently the width of the limit cycles.

The difference in the effect of confinement on the behavior of pullers and pushers as seen in figure 10 is mainly due to the wall interactions of B_2 mode. When the confinement ratio a/R is large, the angular velocity induced by the B_2 mode changes its direction close to the wall (not shown). For a puller, this change results in larger angular velocity ($\Omega^{B_1} + \Omega^{B_2}$) near the wall. It also results in zero angular velocity occurring much closer to the channel center and consequently the fixed point of pullers shift towards the center. However, U_y velocity is relatively unaffected by the changes in a/R . Therefore, the escape angle which is determined

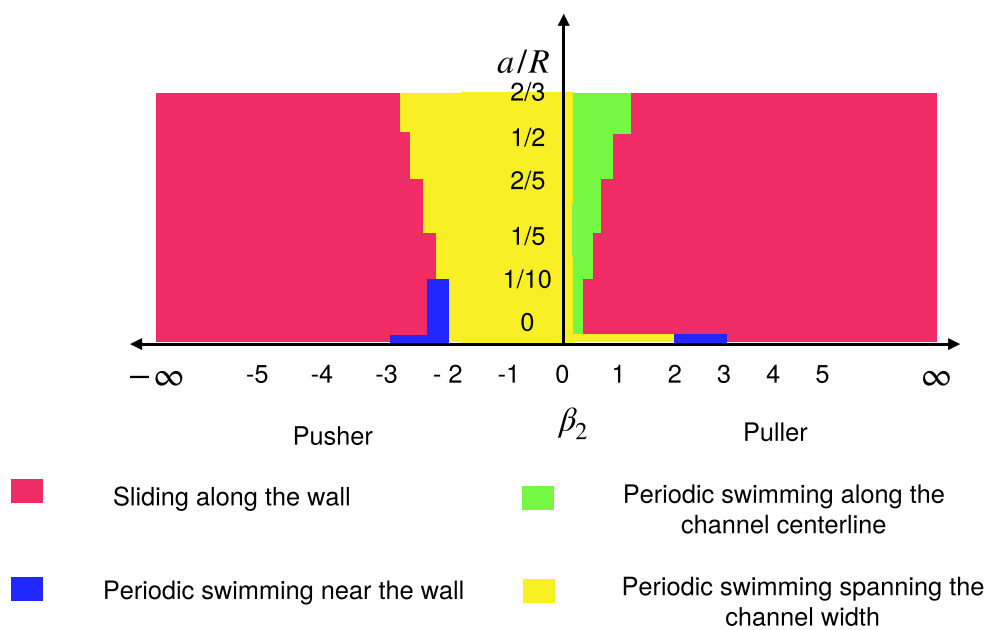


Figure 11. Four types of swimming behavior in a channel as a function of β_2 and ratio of the squirmer radius to channel width, a/R —(i) sliding along the wall, (ii) swimming along the channel centerline, (iii) periodic swimming near the wall and (iv) periodic swimming spanning the channel width.

as the angle at which U_y changes sign does not change significantly with changes in a/R for pushers.

A summary of the effect of confinement on the squirmer trajectories in a channel is shown in figure 11. As in the case of a single wall, both strong pushers and pullers are trapped near the wall and they slide along the wall. However the symmetry between pushers and pullers is lost if their dipole strength is weak (small β_2). Weak pushers generate trajectories that span across the channel width while weak pullers swim along the channel centerline. The range of β_2 in which this channel width spanning oscillations of pushers and centerline swimming of pullers is observed increases with increased confinements (reducing a/R). Thus the symmetry in the behavior of a weak puller and a pusher that was observed in the case of a swimmer near a single wall (figure 5(b)) is absent for a channel confined squirmer. However, the strong pullers and pushers maintain the symmetry in behavior even in the channel.

5. Conclusions

In this work, we have presented a full-scale simulation study of hydrodynamics of a squirmer—a model micro-swimmer for ciliated organisms—confined in a channel. LBM was used to solve for fluid flows. Through the coupled dynamics of the squirmer and the fluid, we determined the instantaneous translational and rotational velocities of the confined squirmer that arise from the hydrodynamic interaction with the channel walls. Swimming velocities corresponding to different modes of squirming were determined separately to understand the role of each mode. Trajectories of different classes of squirmers such as pullers, neutral swimmers and pushers were then determined. It was shown that strong pullers and strong

pushers are always trapped and slide near one of the channel walls and thus their trajectory can be completely represented by a fixed point in a phase space spanned by the squirmer orientation and the height from the centerline. Neutral swimmers, and weak pushers which show an oscillatory trajectory spanning the entire channel width are represented as limit cycles, which can be solely characterized by the escape angle from the wall.

Presence of a repulsive potential on the channel wall can be decisive in the hydrodynamic collision process of a swimmer. Depending upon the range of the potential and the strength and swimming modes of the swimmer, diverse behavior can be observed near a wall. It is interesting to note that 2D disk squirmers behave qualitatively similar to 3D spherical squirmers near a repulsive, planar wall. Effects of higher modes of squirming were also analyzed to find that the instantaneous velocities change mildly, and these changes have a relatively weaker effect on the trajectory of a swimmer in the channel. On the other hand, it was found that stronger confinements resulted in fixed points of pullers moving towards the channel center—a feature that is absent in the highest order singularity representation of squirmers. Thus, we systematically analyzed different factors that govern the trajectory dynamics of swimmers in confinement. The trajectories are found to be robust to swimmer model and fluid flow parameters.

Compared to the behavior of a pusher in a channel, fixed points of pullers are found to be more sensitive to variation in parameters. This generic behavior originates from the fact that a fixed point is determined by two constraints, namely (1) a zero attraction or repulsion with the channel walls and (2) a zero channel wall induced angular velocity. On the other hand, pushers and neutral swimmers which exhibit a limit cycle, and thus represented by an escape angle is less sensitive to changes in parameters. This is because, escape angle of a swimmer is determined by only one constraint, namely the U_y velocity (attraction/repulsion) of the swimmer near the wall.

Our work illustrates the role of complex hydrodynamic collision process of a swimmer in determining its trajectory in a microchannel. Having established the ground works with a single swimmer, future investigations may be done to analyze effect of the curvature of the walls, presence of multiple swimmers and non-Newtonian characteristics of the fluid. Such a comprehensive picture further advances the understanding of the dynamic behavior of microswimmers in complex and crowded environments.

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