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Analytical Solution of the Nonlinear Vibration of the Steel Skeleton Supported Membrane Structure

Mengfei Wang[1], Changjiang Liu[2,3,*], Haibing Xie[1], Su Jiang[1], Mengjia Zhang[2] and Yaozeng Su[2]

[1]College of Environment and Civil Engineering, Chengdu University of Technology, Chengdu, China
[2]School of Civil Engineering, Guangzhou University, Guangzhou, China
[3]Guangdong Engineering Technology Research Center for Complex Steel Structures, Guangzhou University, Guangzhou, 510006, China

*Corresponding author email: cjliu@gzhu.edu.cn

Abstract. Steel skeleton supported membrane structure is the major structural form of membrane structure, accounting for more than 50%. The structural characteristics of membrane structure itself determine that the analysis of its self-vibration characteristics is very important in the design process. According to the von Karman's large deflection theory and the D'Alembert's principle, the governing equations of nonlinear viscous damping vibration of the skeleton supported orthotropic membrane structure were proposed. By applying the Galerkin method and the KBM perturbation means of obtaining the analytical solutions of frequency function, displacement function and mode function of damped nonlinear and linear free vibration of simply supported orthotropic membrane structures on four sides were obtained. In addition, it can be known by example analysis that the transverse stiffness of membrane structure can be improved by increasing prestress and rise span ratio in the practical engineering, and while the influence of geometrically nonlinear, orthotropic, damping and prestress on the dynamic feature of membrane structure must be fully considered. This paper provides some theoretical references for the wind-induced dynamic stability calculation and wind-resistant design of the steel skeleton supported membrane structure.
Keywords: Membrane structure; Orthotropic; Geometric nonlinearity; Linear; Free vibration.
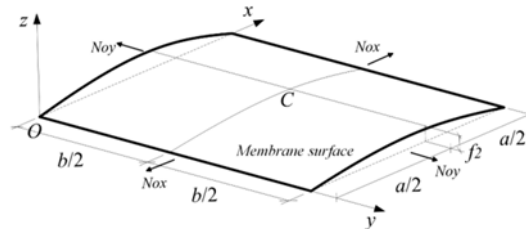
1. Introduction

Matched with the traditional rigid structure, the structural characteristics of the membrane structure determine that it is essential to analyze the shape-finding and self-vibration characteristics during the design process[1]. A.H.Sofiyev[2], Li[3]and other scholars have studied the nonlinear vibration of membrane. However, in the vibration analysis of steel skeleton supported membrane structure, there are few studies that consider both the geometric nonlinearity of membrane surface deformation and the orthotropic characteristics of material. L.V.Swpanova[4], Ali.A.Yazdi[5] and other scholars proved the effectiveness of the perturbation method and the Galerkin method in solving nonlinear problems. In this paper, the KBM perturbation method and the Galerkin method were used to solve the partial differential governing equations of nonlinear free vibration of supported membrane structures. The nonlinear and linear vibration frequencies of skeleton supported membrane structure under different parameter variables are analyzed and calculated through calculation examples. The results of this paper provide a handy analytical way for calculating the natural frequency and transverse displacement of the skeleton supported orthotropic membrane structure with viscous damping.

2. Structural Models and Governing Equations

The structural model studied in this paper is cylindrical shell-shaped steel framework supported membrane structure, it's shown in figure 1. The membrane material is orthotropic, and the boundary condition is four edges clamped, x and y are two orthogonal directions. a and b denote the lengths of the membranes in the x and y directions; N_{ox} and N_{oy} represent the pretension in x and y directions; f_2 is the rise span ratio in x direction. The coordinate system is xyz, point C is the center of the horizontal projection of the membrane surface to the plane xoy.

Figure 1. Skeleton supported membrane structural model.

2.1. Governing Equations and Boundary Conditions

According to the governing equation derived by Liu[6]et al., and introduce stress function $\varphi = \varphi(x,y,t)$ [7]. The motion equation and compatible equation of the skeleton supported membrane structure can be derived as follows:

$$\rho_0 \frac{\partial^2 w}{\partial t^2} + c\frac{\partial w}{\partial t} - (h\frac{\partial^2 \varphi}{\partial y^2} + N_{0x})\frac{\partial^2 w}{\partial x^2} - k_{0x}h\frac{\partial^2 \varphi}{\partial y^2} - \left(h\frac{\partial^2 \varphi}{\partial x^2} + N_{0y}\right)\frac{\partial^2 w}{\partial y^2} = 0 \tag{1}$$

$$\frac{1}{E_1}\frac{\partial^4 \varphi}{\partial y^4} + \frac{1}{E_2}\frac{\partial^4 \varphi}{\partial x^4} = \left(\frac{\partial^2 w}{\partial x\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - k_{0x}\frac{\partial^2 w}{\partial y^2} \tag{2}$$

Where: ρ_0 represents the membrane material surface density; c represents the damping coefficient; w represents deflection $w(x,y,t)$; h represents the membrane material thickness; E_1 and E_2 represent Young's modulus in x and y direction; μ_1 and μ_2 denote Poisson's ratio in x and y directions; G represents shear modulus; k_{0x} denotes initial principal curvature in the x direction.

The corresponding displacement and stress boundary conditions are shown as follows:

$$\begin{cases} w(0,y,t)=0, \dfrac{\partial^2 w}{\partial x^2}(0,y,t)=0 \\ w(a,y,t)=0, \dfrac{\partial^2 w}{\partial x^2}(a,y,t)=0 \end{cases}, \begin{cases} w(x,0,t)=0, \dfrac{\partial^2 w}{\partial y^2}(x,0,t)=0 \\ w(x,b,t)=0, \dfrac{\partial^2 w}{\partial y^2}(x,b,t)=0 \end{cases}, \begin{cases} \dfrac{\partial^2 \varphi}{\partial x^2}(0,y,t)=0 \\ \dfrac{\partial^2 \varphi}{\partial x^2}(a,y,t)=0 \end{cases}, \begin{cases} \dfrac{\partial^2 \varphi}{\partial y^2}(x,0,t)=0 \\ \dfrac{\partial^2 \varphi}{\partial y^2}(x,b,t)=0 \end{cases} \tag{3}$$

2.2. Solution of Governing Equation

The functions satisfying the boundary condition equation (3) are separated as follows[8,9]:

$$w(x,y,t) = T_{mn}(t)\cdot W_{mn}(x,y) = T(t)\cdot \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{4}$$

Where, $W_{mn}(x,y)$ is the given deformation function; $T_{mn}(t)$ is a function of time; m and n represent the order of vibration mode in x and y directions respectively.

The substitution of equation (4) into equation (3) yields, assuming that the stress $\varphi(x,y,t)$ satisfying the stress boundary condition equation (3) is:

$$\varphi(x,y,t)=T^2\left(\alpha\cos\frac{2m\pi x}{a}+\beta\cos\frac{2n\pi y}{b}+\delta_1 x^2+\delta_2 y^2\right)+T(t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\xi \tag{5}$$

$$\alpha=\frac{E_2 n^2 a^2}{32m^2 b^2},\beta=\frac{E_1 m^2 b^2}{32n^2 a^2},\quad \xi=\frac{k_{0x}(n\pi/b)^2}{\left((n\pi/b)^4/E_1\right)+\left((m\pi/a)^4/E_2\right)},\quad \delta_1=\frac{\pi^2 E_2 n^2}{16b^2},\quad \delta_2=\frac{\pi^2 E_1 m^2}{16a^2}$$

Equation (3) and equation (5) are substituted into Equation (1), and the Bubnov-Galerkin method [10] is used to transform it into homogeneous differential equation. Then, by using KBM perturbation method[11] to solve the equation, the approximate expression of nonlinear free vibration frequency can be obtained:

$$\omega_n=\omega_0-\frac{3\varepsilon\alpha_1 F^2 e^{\varepsilon\alpha_3 t}}{8\omega_0} \tag{6}$$

$$A_0=\frac{ab}{4}\rho_0, B_0=\frac{ab}{4}c, E_0=h\pi^4\frac{3E_1 m^4 b^4+3E_2 n^4 a^4}{64a^3 b^3}, C_0=\frac{m^2\pi^2 b^2 N_{0x}+n^2\pi^2 a^2\left(N_{0y}+k_{0x}h\xi\right)}{4ab},$$

$$\varepsilon=\frac{h^2}{ab}<<1, \omega_0^2=\frac{C_0}{A_0}, \alpha_1=-\frac{E_0}{\varepsilon A_0},\quad \alpha_3=-\frac{B_0}{\varepsilon A_0},$$

Where, F represents the initial displacement.

It can be seen from the above equation that the frequency of nonlinear free vibration is about the amplitude. In the case that only linear vibration is considered, the high-order trace are omitted. The approximate expression of linear free vibration frequency can be obtained as follows:

$$\omega_l=\omega_0=\sqrt{\frac{C_0}{A_0}} \tag{7}$$

The analytic expressions of nonlinear and linear $T(t)$ functions can be obtained and substituted into equation (3) respectively, the displacement functions of damped nonlinear and linear free vibration of the skeleton supported prestressed membrane structure are obtained:

$$w_n(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\left(Fe^{\frac{1}{2}\alpha_3\varepsilon t}\cdot\cos\left(\left(\omega_0-\frac{3\alpha_1\varepsilon F^2 e^{\alpha_3\varepsilon t}}{8\omega_0}\right)t\right)\right) \tag{8}$$

$$w_l(x,y,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\left(Fe^{\frac{1}{2}\alpha_3\varepsilon t}\cdot\cos(\omega_0 t)\right) \tag{9}$$

By superposing the initial surface function of the skeleton supported membrane with its displacement equation (8) and equation (9), the modes S_n and S_l of the skeleton supported membrane can be obtained:

$$S_n=-\frac{f_2(x-(a/2))^2}{(a/2)^2}+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\left(Fe^{\frac{1}{2}\alpha_3\varepsilon t}\cdot\cos\left(\left(\omega_0-\frac{3\alpha_1\varepsilon F^2 e^{\alpha_3\varepsilon t}}{8\omega_0}\right)t\right)\right) \tag{10}$$

$$S_l=-\frac{f_2(x-(a/2))^2}{(a/2)^2}+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\left(Fe^{\frac{1}{2}\alpha_3\varepsilon t}\cdot\cos(\omega_0 t)\right) \tag{11}$$

3. Analysis of Example

Take the orthotropic membrane materials commonly used in engineering practice as an case: $E_1=1.4\times10^6$kN/m^2, $E_2=0.9\times10^6$kN/m^2[6], the membrane density $\rho_0=1.7$kg/m^2, the damping $c=120$Ns/m, the thickness of the membrane $h=1.0$m, the length $a=1$m and width $b=1$m, the prestress $N_{0x}=N_{0y}=1000$ N/m. According to equation (6) and (7), assuming that $F=0.05$m and $t=0.01$s. The first three

frequencies of different parameters (initial displacement, prestress, damping, aspect span ratio, etc.) of nonlinear and linear free vibration at the rise span ratio 1/10 and 1/12 were respectively calculated, the calculation results are shown in figures 2-6 and table 1 (1NR10 represents f_2=1/10, 1NR12 represents f_2=1/12 . According to equation (10), the first three modes of the first three orders are plotted at t=0s and t=0.0035s, as shown in figure 7.
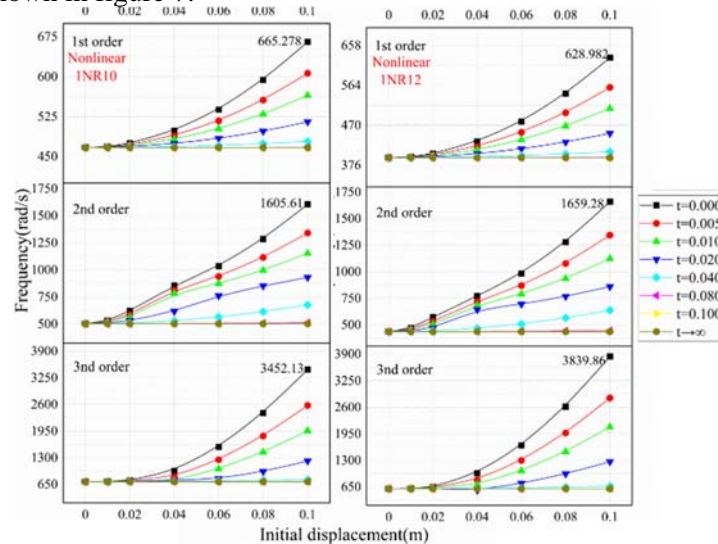
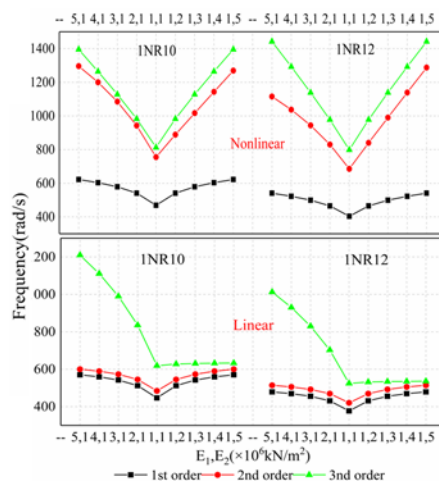**Figure 2.** Nonlinear vibration frequencies of each order under different initial displcements.
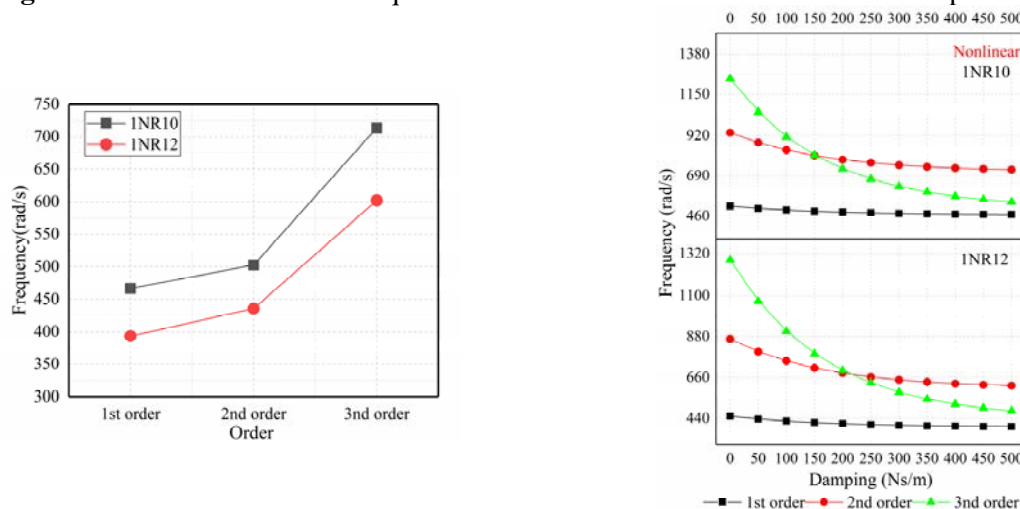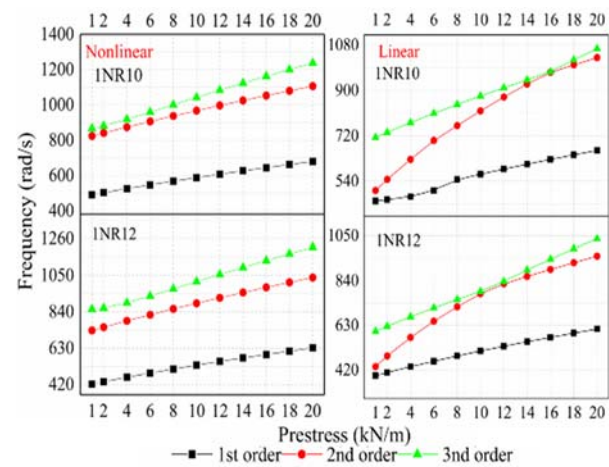

Figure 3. Linear vibration frequencies of each order .

Figure 4. Nonlinear vibration frequencies of each order under different damping.

According to equation (7), it can be seen that the value of linear vibration frequency is independent of the initial displacement and time. Analysis of figures 2-4 and figure 6 show that: When considering the damping and geometric nonlinearity of membrane vibration, the frequency of free vibration increases as initial displacement and order increases, decreases as damping enhances. The frequency of nonlinear and linear vibration increase with the increase of f_2, prestress and order. As can be seen from figure 5: When $E_1 > E_2$ and value of E_2 is fixed, the frequencies of nonlinear and linear free vibration decrease as the elastic modulus ratio decreases, and increases with the order aggrandize. When $E_1 < E_2$ and value of E_1 is fixed, the law is opposite. As can be seen from table 1: If the a and b values of the two orthogonal directions is interchanged or the rise span ratio is changed, the frequency of linear free vibration is smaller than that of nonlinear free vibration. From the results of figure 7: It can be seen that equation (10) can be used to calculate the vibration modal equation of each order conveniently.

Figure 5. Nonlinear and linear free vibration frequencies under different $E_1{:}E_2$.

Figure 6. Nonlinear and linear free vibration frequencies under different prestress.
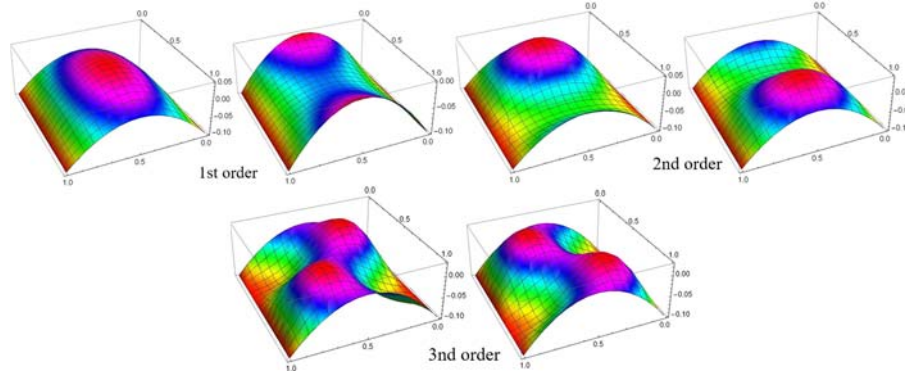
Figure 7. The first three modes of nonlinear free vibration.(m).

Table 1. The nonlinear and linear free vibration frequency(rad/s) of each order with different aspect span ratios under different rise span ratios.

	f_2	a,b (m)	1,1	1,2	2,1	1,3	3,1	1,4	4,1	1,5	5,1
Nonlinear	1/10	1st order	491.25	209.75	218.51	171.11	153.55	167.18	140.31	167.02	137.03
		2nd order	823.79	491.25	540.25	290.47	585.64	209.75	607.44	180.83	615.30
		3nd order	866.69	743.56	566.30	821.74	586.17	853.79	604.16	866.67	612.14
	1/12	1st order	422.55	195.77	196.94	169.24	148.05	167.08	138.80	167.10	136.51
		2nd order	732.64	422.55	548.58	257.99	594.00	195.77	611.05	175.41	616.92
		3nd order	855.67	765.40	575.11	832.22	593.57	857.97	607.48	868.53	613.67
Linear	1/10	1st order	466.75	166.18	193.08	102.96	113.29	86.54	90.64	81.11	82.94
		2nd order	502.68	222.23	243.00	173.07	179.41	161.23	163.46	157.14	158.09
		3nd order	713.71	466.75	239.62	260.33	174.26	166.18	160.14	123.56	155.88
	1/12	1st order	393.49	146.27	167.65	96.61	104.33	84.17	87.12	80.09	81.38
		2nd order	435.51	207.77	223.33	169.37	173.89	159.97	161.54	156.61	157.28
		3nd order	602.17	393.49	217.74	222.77	168.47	146.27	158.16	112.52	155.05

4. Conclusion

(1) The rise span ratio, prestress and aspect span ratio have an effect on the linear and nonlinear free vibration of membrane, while the nonlinear free vibration of membrane is also affected by the initial displacement and damping parameters. (2) With the same initial displacement, the frequency of nonlinear free vibration decreases with the aggrandize of time and viscous damping, and increases with the aggrandize of prestress. The frequency of linear free vibration increases with the aggrandize of prestress and order. In the practical engineering, the transverse stiffness of membrane structure can be improved by increasing prestress. (3) In consideration of the geometrical nonlinearity and linearity of the membrane, the values of nonlinear and linear vibration frequencies increase as the f_2 increase. Therefore, the transverse stiffness of membrane structure can be improved by increasing the rise span ratio. (4) When the aspect span ratio of the orthogonal direction changes, the frequency values of the nonlinear and linear free oscillations will change. (5) The research results of this paper provide a theoretical reference for the subsequent research on the wind and rain resistance dynamic response of the skeleton supported membrane structure.

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