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Bar structures calculation by the method of discrete elements with generalized unknowns in aggressive environments

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Abstract. The article considers the application of the method of discrete elements with generalized unknowns for calculating rod systems located in an aggressive environment. The object of research was a spatial rod truss. The basic formulas for determining the reactions of bonds at unit displacements are shown. The equation of displacements for an arbitrary node is derived. The system of equations was solved in partial finite differences. Reactions from external loads were represented as trigonometric series. An algorithm for calculating a statically indeterminate spatial truss by the method of discrete elements with generalized unknowns under the action of an aggressive environment is given. As a result of the calculation, the values of node point movements are obtained.

1. Introduction

Regular spatial rod structures are widely used in civil and industrial construction. They are used as coverings for large spans in sports facilities, trade pavilions, concert halls, etc [1,2]. They are also used as supporting structures of process equipment in industrial buildings in non-ferrous metallurgy, petrochemical and chemical industries. Often in such industries, technological processes include the use of various aggressive environments [3,4]. Therefore, the usual strength calculations for such structures cannot take into account the additional wear. In such cases, it is necessary to calculate the stress-strain state of structures, taking into account the development of corrosion processes over time. This is possible when using mathematical models that take into account corrosion wear [5,6].

Calculation of complex rod structures, taking into account the impact of a corrosive environment, is complicated due to the need to conduct a continuous analysis of changes in the stiffness characteristics of all structural elements during the calculation process.

2. Main part

We introduce the assumption that the structure, being in a certain aggressive environment, experiences a uniform effect of this environment on all elements. The corrosion wear of the rods' cross sections proceeds according to the same dependence, but with the coefficients that takes into account the orientation of the rod in space. The authors suggest using for the calculation the Discrete Element Method (DEM) with generalized unknowns [7-9]. This method allows calculating the systems with any boundary conditions, a complex contour, and non-homogeneous effects of an aggressive environment on various elements.

Consider a spatial truss (Figure 1) having square or rectangular cells, with lattices shifted relative to each other by half the size of the cell. This shift ensures the uniformity of length of all struts between the grates and spatial operation of the entire structure. The connection in the truss nodes is supposed to be hinged [10]. The calculation of the considered hinge-rod system is possible using the displacement method. To do this, we introduce connections in the directions x, y and z to all nodes of the upper and lower lattices. The numbering of the nodes of the upper and lower lattices is shown in Figure 2.

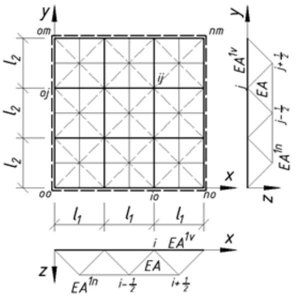

Figure 1. Spatial truss.

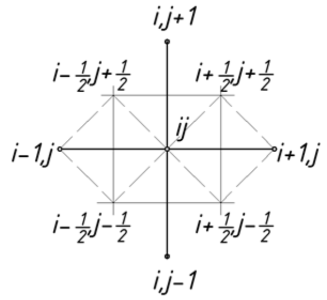

Figure 2. The numbering of the nodes of the upper and lower lattices.

For the rod of the main system with unit displacements of the terminal connections, the reactions in the bonds are determined by the following dependencies, taking into account the influence of an aggressive environment [11,12]:

$$
\begin{aligned}
&r_{xx} = r_{x'x'} = \frac{EA_{(t)}}{l}\cos^2\alpha\cos^2\gamma, \quad r_{xy} = r_{x'y'} = \frac{EA_{(t)}}{2l}\sin 2\alpha\cos^2\gamma,\\
&r_{xz} = r_{x'z'} = -\frac{EA_{(t)}}{2l}\cos\alpha\sin 2\gamma, \quad r_{yy} = r_{y'y'} = \frac{EA_{(t)}}{l}\sin^2\alpha\cos^2\gamma,\\
&r_{yz} = r_{y'z'} = -\frac{EA_{(t)}}{2l}\sin\alpha\sin 2\gamma, \quad r_{zz} = r_{z'z'} = \frac{EA_{(t)}}{l}\sin^2\gamma,\\
&r_{xx'} = r_{x'x} = -r_{xx}, \quad r_{xz'} = r_{x'z} = -r_{xz}, \quad r_{xy'} = r_{x'y} = -r_{xy},\\
&r_{yy'} = r_{y'y} = -r_{yy}, \quad r_{yz'} = r_{y'z} = -r_{yz}, \quad r_{zz'} = r_{z'z} = -r_{zz}.
\end{aligned}
\tag{1}
$$

The values of the other coefficients are obtained from the reciprocity conditions, thus $r_{ab} = r_{ba}$.

Using the dependencies (1), we create an equation of the displacement method for an arbitrary node ij of the upper lattice and an adjacent node $\left(i-\frac{1}{2}, j-\frac{1}{2}\right)$ of the lower lattice of considered system:

$$
\begin{aligned}
&R_{ij}^{(x)} = L_{xx}(x_{ij}) + L_{xy}(y_{ij}) + L_{xz}(z_{ij}) + L_{xx'}(x'_{ij}) + L_{xy'}(y'_{ij}) + L_{zy'}(z'_{ij}) + L_x(F_{ij}) = 0,\\
&R_{ij}^{(y)} = L_{yx}(x_{ij}) + L_{yy}(y_{ij}) + L_{yz}(z_{ij}) + L_{yx'}(x'_{ij}) + L_{yy'}(y'_{ij}) + L_{yz'}(z'_{ij}) + L_y(F_{ij}) = 0,\\
&R_{ij}^{(z)} = L_{zx}(x_{ij}) + L_{zy}(y_{ij}) + L_{zz}(z_{ij}) + L_{zx'}(x'_{ij}) + L_{zy'}(y'_{ij}) + L_{zz'}(z'_{ij}) + L_z(F_{ij}) = 0,\\
&R_{i-\frac{1}{2},j-\frac{1}{2}}^{(x')} = L_{x'x}(x_{ij}) + L_{x'y}(y_{ij}) + L_{x'z}(z_{ij}) + L_{x'x'}(x'_{ij}) + L_{x'y'}(y'_{ij}) + L_{x'z'}(z'_{ij}) + L_{x'}(F_{ij}) = 0,\\
&R_{i-\frac{1}{2},j-\frac{1}{2}}^{(y')} = L_{y'x}(x_{ij}) + L_{y'y}(y_{ij}) + L_{y'z}(z_{ij}) + L_{y'x'}(x'_{ij}) + L_{y'y'}(y'_{ij}) + L_{y'z'}(z'_{ij}) + L_{y'}(F_{ij}) = 0,\\
&R_{i-\frac{1}{2},j-\frac{1}{2}}^{(z')} = L_{z'x}(x_{ij}) + L_{z'y}(y_{ij}) + L_{z'z}(z_{ij}) + L_{z'x'}(x'_{ij}) + L_{z'y'}(y'_{ij}) + L_{z'z'}(z'_{ij}) + L_{z'}(F_{ij}) = 0.
\end{aligned}
\tag{2}
$$

where

$$
\begin{aligned}
&L_z(F_{ij}) = -F_{ij}, \quad L_{z'}(F_{ij}) = -F_{i-\frac{1}{2},j-\frac{1}{2}}, \quad L_x(F_{ij}) = L_y(F_{ij}) = L_{x'}(F_{ij}) = L_{y'}(F_{ij}) = 0,\\
&L_{xx}(x_{ij}) = x_{ij}\left(\frac{4EA_{(t)}}{l}\cos^2\alpha\cos^2\gamma + \frac{2EA_{(t)}^{1v}}{l_1}\right) - \left(x_{i+1,j} + x_{i-1,j}\right)\frac{EA_{(t)}^{1v}}{l_1},\\
&L_{xy}(y_{ij}) = 0, \qquad L_{xz}(z_{ij}) = 0, \qquad L_{yz}(z_{ij}) = 0,\\
&L_{xx'}(x'_{ij}) = -\left(x_{i-\frac{1}{2},j-\frac{1}{2}} + x_{i+\frac{1}{2},j-\frac{1}{2}} + x_{i+\frac{1}{2},j+\frac{1}{2}} + x_{i-\frac{1}{2},j+\frac{1}{2}}\right)\frac{EA_{(t)}}{l}\cos^2\alpha\cos^2\gamma,\\
&L_{xy'}(y'_{ij}) = -\left(y_{i-\frac{1}{2},j-\frac{1}{2}} + y_{i+\frac{1}{2},j-\frac{1}{2}} + y_{i+\frac{1}{2},j+\frac{1}{2}} + y_{i-\frac{1}{2},j+\frac{1}{2}}\right)\frac{EA_{(t)}}{2l}\sin 2\alpha\cos^2\gamma,\\
&L_{xz'}(z'_{ij}) = -\left(z_{i-\frac{1}{2},j-\frac{1}{2}} + z_{i+\frac{1}{2},j-\frac{1}{2}} + z_{i+\frac{1}{2},j+\frac{1}{2}} + z_{i-\frac{1}{2},j+\frac{1}{2}}\right)\frac{EA_{(t)}}{2l}\cos^2\alpha\cos 2\gamma,\\
&L_{yy}(y_{ij}) = y_{ij}\left(\frac{4EA_{(t)}}{l}\sin^2\alpha\cos^2\gamma + \frac{2EA_{(t)}^{2v}}{l_2}\right) - \left(y_{i+1,j} + y_{i-1,j}\right)\frac{EA_{(t)}^{2v}}{l_2},\\
&L_{yx'}(x'_{ij}) = -\left(x_{i-\frac{1}{2},j-\frac{1}{2}} - x_{i+\frac{1}{2},j-\frac{1}{2}} + x_{i+\frac{1}{2},j+\frac{1}{2}} - x_{i-\frac{1}{2},j+\frac{1}{2}}\right)\frac{EA_{(t)}}{2l}\sin^2\alpha\cos^2\gamma,\\
&L_{yy'}(y'_{ij}) = -\left(y_{i-\frac{1}{2},j-\frac{1}{2}} + y_{i+\frac{1}{2},j-\frac{1}{2}} + y_{i+\frac{1}{2},j+\frac{1}{2}} + y_{i-\frac{1}{2},j+\frac{1}{2}}\right)\frac{EA_{(t)}}{2l}\sin 2\alpha\cos^2\gamma,\\
&L_{yz'}(z'_{ij}) = -\left(z_{i-\frac{1}{2},j-\frac{1}{2}} + z_{i+\frac{1}{2},j-\frac{1}{2}} - z_{i+\frac{1}{2},j+\frac{1}{2}} - z_{i-\frac{1}{2},j+\frac{1}{2}}\right)\frac{EA_{(t)}}{2l}\sin^2\alpha\sin 2\gamma,\\
&L_{zz}(z_{ij}) = z_{ij}\left(\frac{4EA_{(t)}}{l}\sin^2\gamma\right),\\
&L_{zx'}(x'_{ij}) = -\left(x_{i-\frac{1}{2},j-\frac{1}{2}} - x_{i+\frac{1}{2},j-\frac{1}{2}} - x_{i+\frac{1}{2},j+\frac{1}{2}} + x_{i-\frac{1}{2},j+\frac{1}{2}}\right)\frac{EA_{(t)}}{2l}\cos\alpha\sin\gamma,\\
&L_{zy'}(y'_{ij}) = -\left(y_{i-\frac{1}{2},j-\frac{1}{2}} + y_{i+\frac{1}{2},j-\frac{1}{2}} - y_{i+\frac{1}{2},j+\frac{1}{2}} - y_{i-\frac{1}{2},j+\frac{1}{2}}\right)\frac{EA_{(t)}}{2l}\sin\alpha\sin 2\gamma,\\
&L_{zz'}(z'_{ij}) = -\left(z_{i-\frac{1}{2},j-\frac{1}{2}} + z_{i+\frac{1}{2},j-\frac{1}{2}} + z_{i+\frac{1}{2},j+\frac{1}{2}} + z_{i-\frac{1}{2},j+\frac{1}{2}}\right)\frac{EA_{(t)}}{2l}\sin^2\gamma,\\
&L_{x'x'}(x'_{ij}) = x_{i-\frac{1}{2},j-\frac{1}{2}}\left(\frac{4EA_{(t)}}{l}\cos^2\alpha\cos^2\gamma + \frac{2EA_{(t)}^{1n}}{l_1}\right) - \left(x_{i-\frac{3}{2},j-\frac{1}{2}} + x_{i+\frac{1}{2},j-\frac{1}{2}}\right)\frac{EA_{(t)}^{1n}}{l_1},\\
&L_{x'y'}(y'_{ij}) = 0, \qquad L_{x'z'}(z'_{ij}) = 0, \qquad L_{y'z'}(z'_{ij}) = 0,\\
&L_{y'y'}(y'_{ij}) = y_{i-\frac{1}{2},j-\frac{1}{2}}\left(\frac{4EA_{(t)}}{l}\sin^2\alpha\cos^2\gamma + \frac{2EA_{(t)}^{2n}}{l_2}\right) - \left(y_{i-\frac{3}{2},j+\frac{1}{2}} + y_{i-\frac{1}{2},j-\frac{1}{2}}\right)\frac{EA_{(t)}^{2n}}{l_1},\\
&L_{z'z'}(z'_{ij}) = z_{i-\frac{1}{2},j-\frac{1}{2}}\left(\frac{4EA_{(t)}}{l}\sin^2\gamma\right).
\end{aligned} \tag{3}
$$

The other operators are calculated from the reciprocity condition: $L_{ab}(i,j) = L_{ba}(i,j)$.

The border equations of the system (2) are obtained automatically [13,14] from the intermediate equations (for nodes ij; $i-\frac{1}{2}$, $j-\frac{1}{2}$) for the case of the hinge support of the truss along the contour, that is, when the structure is periodically continued.

Since the conditions of periodic continuation are met, we will look for a solution to the system of equations (4.2) in the form of decompositions by proper solutions of individual equations in partial finite differences:

$$L_{xx}(x_{ij}) = \lambda^{(1)} x_{ij},\ L_{yy}(y_{ij}) = \lambda^{(2)} y_{ij}. \tag{4}$$

Substituting proper solutions of these equations in the decompositions of the desired functions, we obtain:

$$\begin{aligned}
&q_{ij}^{(1)} = x_{ij} = \sum_k \sum_r q_{kr}^{(1)} \cos\frac{k\pi i}{n} \sin\frac{r\pi j}{m}, \quad q_{ij}^{(2)} = y_{ij} = \sum_k \sum_r q_{kr}^{(2)} \sin\frac{k\pi i}{n} \cos\frac{r\pi j}{m}, \\
&q_{ij}^{(3)} = z_{ij} = \sum_k \sum_r q_{kr}^{(3)} \sin\frac{k\pi i}{n} \sin\frac{r\pi j}{m}, \\
&q_{ij}^{(4)} = x'_{i-\frac{1}{2},j-\frac{1}{2}} = \sum_k \sum_r q_{kr}^{(4)} \cos\frac{k\pi}{n}\left(i-\tfrac{1}{2}\right) \sin\frac{r\pi}{m}\left(j-\tfrac{1}{2}\right), \\
&q_{ij}^{(5)} = y'_{i-\frac{1}{2},j-\frac{1}{2}} = \sum_k \sum_r q_{kr}^{(5)} \sin\frac{k\pi}{n}\left(i-\tfrac{1}{2}\right) \cos\frac{r\pi}{m}\left(j-\tfrac{1}{2}\right), \\
&q_{ij}^{(6)} = z'_{i-\frac{1}{2},j-\frac{1}{2}} = \sum_k \sum_r q_{kr}^{(6)} \sin\frac{k\pi}{n}\left(i-\tfrac{1}{2}\right) \cos\frac{r\pi}{m}\left(j-\tfrac{1}{2}\right).
\end{aligned} \tag{5}$$

Similarly

$$F_{ij}^{(z)} = \sum_k \sum_r F_{kr}^{(3)} \sin\frac{k\pi i}{n} \sin\frac{r\pi j}{m}, \quad F^{(z)}_{i-\frac{1}{2},j-\frac{1}{2}} = \sum_k \sum_r F_{kr}^{(6)} \sin\frac{k\pi}{n}(i-\tfrac{1}{2}) \sin\frac{r\pi}{m}(j-\tfrac{1}{2}), \tag{6}$$

where

$$\begin{aligned}
F_{kr}^{(3)} &= \frac{4}{mn} \sum_i \sum_j F_{ij}^{(3)} \sin\frac{k\pi i}{n} \sin\frac{r\pi j}{m}, \\
F_{kr}^{(6)} &= \frac{4}{mn} \sum_i \sum_j F^{(6)}_{i-\frac{1}{2},j-\frac{1}{2}} \sin\frac{k\pi}{m}(i-\tfrac{1}{2}) \sin\frac{r\pi}{m}(j-\tfrac{1}{2}).
\end{aligned}$$

Substituting the values of generalized unknowns q_{kr} and loads F_{kr} into the system of equations (2) shows that the functional similarity conditions are met for both intermediate and border equations of this system. Therefore, the canonical equations system of the generalized displacements method are divided into separate blocks of equations with relative unknowns with indexes *kr*

$$r_{kr,kr(t)} q_{kr} = R_{kr,kr}^{(p)}. \tag{7}$$

After calculating the values q_k from (4.7) and substituting them into expressions (4.5) for nodal displacements, stresses can be found in any of the considered construction rods:

$$S = \frac{EA_{(t)}}{l}\Big[\Delta x\sqrt{r_{xx}^2 + r_{xy}^2 + r_{xz}^2} + \Delta y\sqrt{r_{yx}^2 + r_{yy}^2 + r_{yz}^2} + \Delta z\sqrt{r_{zx}^2 + r_{zy}^2 + r_{zz}^2}, \tag{8}$$

where r_{xx}, r_{yy}. are defined by expressions (4.1).

The matrix of physical and geometric structure characteristics $r_{kr,kr(t)}$ contains information about the influence of the corrosion process on the values of deflections and stresses in the structure elements.

The general scheme for calculating a statically indeterminate spatial hinge-rod structure by the method of generalized unknowns under conditions of an aggressive environment are reduced to the following algorithm:

1. the values of the matrix coefficients $r_{kr,kr(t)}$ are calculated at the *i*-th step in time, taking into account the corrosion effect;
2. the system of equations (7) is solved and the values of generalized displacements $q_{kr}(t_i)$ are determined at the *i*-th step;
3. using (7), we get the values of the displacements of all nodes of the structure at the *i*-th step in time in the direction X, Y, Z;
4. using the formula (8), we calculate the values of internal stresses in each rod at the i-th step in time;
5. transition to a new (*i*+1) time step, taking into account the accumulation of displacements and internal stresses to point 1 of this algorithm.

As an example, we consider a hinged-rod system under the influence of a uniform load $F_{ij} = F_{const} = 2000N$ and under the action of an aggressive environment $E = 2{,}1 \cdot 10^{11} N / m^2$, $A = A^n = A^v = 0{,}218 \cdot 10^{-3} m^2$, number of cells 25x25, cell size 2x2m. The calculation was carried out at a time step of 2 years and 6 months over a period of time of 10 years.

The following assumptions were made in the calculation:

- the rod elements connecting the system nodes are considered to be perfectly straight during the entire deformation process;
- consideration of the possible loss of stability of compressed rods is made by understating the limit stresses for compressed rods;
- deformations of elements are small, so the equilibrium equations are made for an undeformed scheme and a linear problem is solved at each step in time.

The numerical values of the results of calculating the spatial truss, taking into account the action of the corrosive environment, are shown in table 1. The results are shown for the Central cell of the spatial truss.

Table 1. The results of calculating the spatial truss.

Locations of the elements	Bar forces in elements at different time periods (N)				
	t=0	t=2.5 years	t=5 years	t=7.5 years	t=10 years
Top Chord	-53711.68	- 75731.39	-88887.42	-950650.1	-100047.4
Bottom Chord	53276.38	74955.54	88085.99	94834.39	99236.64
Web	-2462.40	-3465.75	-4645.94	-5258.07	-5741.55

3. Conclusions

The method of discrete elements with generalized unknowns is highly effective in calculating of spatial rod systems subjected to corrosion wear. Analysis of the calculation results showed that ignoring the influence of aggressive environment on the stress-strain state of the structure is unacceptable. So, in some nodes, the strain, and consequently the stress, grow almost twice as compared to the initial values [15-20].

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