IOP Conference Series: Materials Science and Engineering

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To cite this article: Yu I Dimitrienko *et al* 2020 *IOP Conf. Ser.: Mater. Sci. Eng.* **934** 012017

Asymptotic stress analysis of multilayer composite thin cylindrical shells

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Abstract. Using the asymptotic theory, explicit analytical expressions were obtained that allow one to calculate the stress-strain state of multilayer thin cylindrical anisotropic elastic composite shells for small deformations. Forenamed expressions allow us to obtain the distribution of all components of the stress-strain state without any assumptions about their nature of the distribution over the thickness. An algorithm was proposed that makes it possible to obtain the total stress tensor in the shell, and all components of the stress tensor are numerically calculated for the case of a cylindrical composite shell with axisymmetric bending by pressure. The character of the distribution of characteristics of the stress-strain state of the shell was analyzed and the corresponding graphs were constructed.

1. Introduction

Thin shells are widely used in various technical industries, while the use of cylindrical thin shells is most widespread, for example, such a shell may be a rocket igniter charge body, aircraft fuselage, satellite power frame, etc. For calculating the bearing capacity of structures made of composite materials it is necessary to sufficiently accurately calculate the distribution of microstresses in the components of a composite material, for example, for layered structures, the stresses in each individual layer, especially interlaminar and trough-thickness stresses, due to low strength of composites in these directions. [1-8]

The classical theories of calculating thin composite shells of the Kirchhoff–Love type, Timoshenko and their numerous modifications are based on certain assumptions regarding the nature of the distribution of stresses and displacements over the thickness of the structure, the adequacy of which must be confirmed for each class of tasks. [9-17] For example, the classical assumption of the linear nature of the distribution of longitudinal displacements over the thickness and the independence of the transverse displacement from the thickness leads to a linear dependence of the interlayer shear stresses on the transverse coordinate. [12,13] At the same time, these same interlayer stresses can be calculated using three-dimensional equations of the theory of elasticity, and a quadratic dependence of these stresses on thickness arises, which indicates the inconsistency of the assumptions made. In [14,15], a method was developed for the asymptotic averaging of equations of the three-dimensional theory of elasticity for thin multilayer shells [16,17], which, without any hypotheses regarding the distribution of displacements or stresses over the thickness, allows one to obtain an explicit, mathematically founded expression for all 6 components of the stress tensor. The aim of this work is the application of the developed general asymptotic theory of shells for the case of cylindrical thin multilayer shells and the analysis of obtained results.

2. The equations of three-dimensional theory of elasticity in curvilinear coordinates

Let's consider a thin cylindrical shell for which the following relation holds: $æ = h/L << 1$: where h is the shell thickness, L is the shell length, $æ$ is a small parameter. [12-13] We introduce $q_k = \{z, s, r\}$ – the cylindrical coordinates (z is directed along the axis of the cylinder, s is the length of the arc in the circumferential direction, r is directed along the radius of the shell), measured from the middle surface of the shell and $\bar{q}_k = q_k / L$ – the dimensionless coordinates, and the local coordinate $\xi = \bar{q}_3 / æ$ along the thickness of the shell. The Lame parameters H_α for a cylindrical thin shell in cylindrical coordinates and their derivatives have the following form [13]: $H_3 = 1$, $\frac{\partial H_\alpha}{\partial \bar{q}_\beta} = A_{\alpha,\beta} + æ(A_\alpha \bar{k}_\alpha)_{,\beta} \approx A_{\alpha,\beta}$, $\frac{\partial H_\alpha}{\partial \xi} = æ H_{\alpha 3}$.

Let's consider the quasistatic problem of the linear theory of elasticity in a general form [18]:

$$\nabla \cdot \sigma = 0, \tag{1}$$

$$\sigma = C \cdot\cdot \varepsilon, \tag{2}$$

$$\varepsilon = \frac{1}{2}(\nabla \otimes \mathrm{u} + \nabla \otimes \mathrm{u}^{\mathrm{T}}) \tag{3}$$

$$\Sigma_{3\pm} : \sigma_{i3} = -\tilde{p}_\pm \delta_{i3} \qquad \Sigma_T : u_i = u_{ei} \qquad \Sigma_S : [\sigma_{i3}] = 0 \qquad [u_i] = 0 \tag{4}$$

where (1) is the equilibrium equations, (2) is the generalized Hooke's law, (3) is the Cauchy relations, (4) are the boundary conditions, $\Sigma_{3\pm}$ - the outer and inner surfaces of the shell ($\xi = \pm 0.5$), Σ_T - the end surface on which the displacement u_{ei} is specified, Σ_S - the interface between the layers of the shell, on which the conditions for the ideal contact of the layers are specified, $[u_i]$ - the jump in the functions, u_i - the dimensionless displacements, $\tilde{p}_\pm$ are dimensionless pressures related to the characteristic value of the modulus of elasticity E_0 .

In curvilinear coordinates the system takes form:

$$\begin{aligned}
&\varepsilon_{\alpha\alpha} = O_\alpha u_{\alpha,\alpha} + O_1 O_2 H_{\alpha,\beta} u_\beta + O_\alpha H_{\alpha,3} u_3 \\
&2\varepsilon_{12} = H_1 O_2 \left(u_1 O_1\right)_{,2} + H_2 O_1 \left(u_2 O_2\right)_{,1} \\
&\varepsilon_{33} = \frac{1}{æ} u_{3/3} \\
&2\varepsilon_{\alpha 3} = \frac{1}{æ} u_{\alpha/3} + O_\alpha \left(u_{3,\alpha} - H_{\alpha 3} u_\alpha\right) \\
&\sigma_{IJ} = C_{IJKL} \varepsilon_{KL} + C_{IJ33} \varepsilon_{33}. \\
&\sigma_{I3} = 2C_{I3K3} \varepsilon_{K3}; \\
&\sigma_{33} = C_{33KL} \varepsilon_{KL} + C_{3333} \varepsilon_{33};
\end{aligned} \tag{5}$$

Here $C_{ijkl}(\xi)$ are the dimensionless elastic moduli, ε_{ij} are small deformations. The shell layers are assumed to be elastic, with a cylindrical monoclinic symmetry of elastic properties [18].

3. Asymptotic solution

IOP Conf. Series: Materials Science and Engineering **934** (2020) 012017 doi:10.1088/1757-899X/934/1/012017

Let us consider the case when the pressure $\tilde{p}_{\pm}$ on the outer and inner surfaces of the shell is of the order of smallness $O(æ^3)$, $\tilde{p}_{\pm} = æ^3 p_{\pm}$, where $p_{\pm}$ is a finite value. This order of pressure, as a rule, corresponds to the real conditions of loading of thin shells. The solution of the problem (6) is sought in the form of asymptotic expansions over small parameter $æ$:

$$u_k = u_k^{(0)}(\overline{q}_I) + \sum_{n=1}^{\infty} æ^n u_{ij}^{(n)} = u_k^{(0)}(\overline{q}_I) + æ u_k^{(1)}(\overline{q}_I, \xi) + æ^2 u_k^{(2)}(\overline{q}_I, \xi) + \ldots$$

$$\varepsilon_{ij} = \sum_{n=0}^{\infty} æ^n \varepsilon_{ij}^{(n)} = \varepsilon_{ij}^{(0)} + æ\varepsilon_{ij}^{(1)} + æ^2 \varepsilon_{ij}^{(2)} + \ldots$$

$$\sigma_{ij} = \sum_{n=0}^{\infty} æ^n \sigma_{ij}^{(n)} = \sigma_{ij}^{(0)} + æ\sigma_{ij}^{(1)} + æ^2 \sigma_{ij}^{(2)} + \ldots$$

Using these asymptotic expansions, explicit expressions were obtained for the longitudinal σ_{IJ}, shear σ_{I3}, and transverse σ_{33} components of the stress tensor through deformations $\varepsilon_{KL}^{(0)}$ and curvatures η_{KL} of the shell:

$$\begin{gathered}
\sigma_{IJ} = \hat{C}_{IJKL}^{(1)} \varepsilon_{KL}^{(0)} + \hat{C}_{IJKL}^{(0)} \xi \eta_{KL}; \\
-\sigma_{\alpha 3} = \hat{C}_{\alpha 3KL}^{(2)} \varepsilon_{KL}^{(0)} + \hat{R}_{\alpha 3KLJ}^{(2)} \varepsilon_{KL,J}^{(0)} + \hat{N}_{\alpha 3KL} \eta_{KL} + \hat{V}_{\alpha 3KLJ} \eta_{KL,J}; \\
\sigma_{33} = \hat{C}_{33KL}^{(3)} \varepsilon_{KL}^{(0)} + \hat{R}_{33KLJ}^{(3)} \varepsilon_{KL,J}^{(0)} + \hat{E}_{33KLJM}^{(3)} \varepsilon_{KL,JM}^{(0)} + \hat{N}_{33KL}^{(3)} \eta_{KL} + \hat{V}_{33KLM}^{(3)} \eta_{KL,M} + \\
+ \hat{U}_{33KLMN}^{(3)} \eta_{KL,MN} + \hat{W}_{KL}^{(3)} \eta_{KL}^{(2)} - æ^3 (p_{\pm} + \Delta p(\xi + 0.5))
\end{gathered} \tag{6}$$

The tensors included in these expressions $\hat{C}_{IJKL}^{(1)}$, $\hat{C}_{IJKL}^{(0)}$, $\hat{C}_{\alpha 3KL}^{(2)}$, $\hat{R}_{\alpha 3KLJ}^{(2)}$, $\hat{N}_{\alpha 3KL}$, $\hat{V}_{\alpha 3KLJ}$ at other depend only on the elastic moduli C_{IJKL} of the composite and local coordinate ξ.

4. Example of calculation of stresses in a composite cylindrical shell

Consider the case of axisymmetric external pressure loading of a 4-layer composite cylindrical shell, cantilever fixed at the ends. For this type of loading the following system was obtained for longitudinal σ_{IJ}, shear σ_{I3}, and transverse σ_{33} components of the stress tensor through displacements $u_3^{(0)}$ of the shell:

$$\begin{gathered}
\sigma_{IJ} = \hat{C}_{IJ22}^{(1)} \frac{1}{R} u_3^{(0)}(X^1) - \hat{C}_{IJ11}^{(0)} \xi u_{3,11}^{(0)}, \\
-\sigma_{\alpha 3} = \hat{C}_{\alpha 322}^{(2)} \frac{1}{R} u_3^{(0)}(X^1) + \hat{R}_{\alpha 3221}^{(2)} \frac{1}{R} u_{3,1}^{(0)}(X^1) - \hat{N}_{\alpha 311} u_{3,11}^{(0)} - \hat{V}_{\alpha 3111} u_{3,111}^{(0)}, \\
\sigma_{33} = \hat{C}_{3322}^{(3)} \frac{1}{R} u_3^{(0)}(X^1) + \hat{R}_{33221}^{(3)} \frac{1}{R} u_{3,1}^{(0)}(X^1) + \hat{E}_{332211}^{(3)} \frac{1}{R} u_{3,11}^{(0)}(X^1) - \\
\hat{N}_{3311}^{(3)} u_{3,11}^{(0)} - \hat{V}_{33111}^{(3)} u_{3,111}^{(0)} - (p_{\pm} + \Delta p(\xi + 0.5))
\end{gathered} \tag{7}$$

Each layer of the shell was a unidirectional fiberglass, the system of corners α_i of the reinforcement of the composite layers changed during the program experiment.

Following figures show the distribution of the flexural stresses σ_{11}, σ_{22} along the thickness of the composite sheath with the system of corners $\alpha_1 = 30^\circ,\ \alpha_2 = 45^\circ,\ \alpha_3 = 45^\circ,\ \alpha_4 = 30^\circ$:

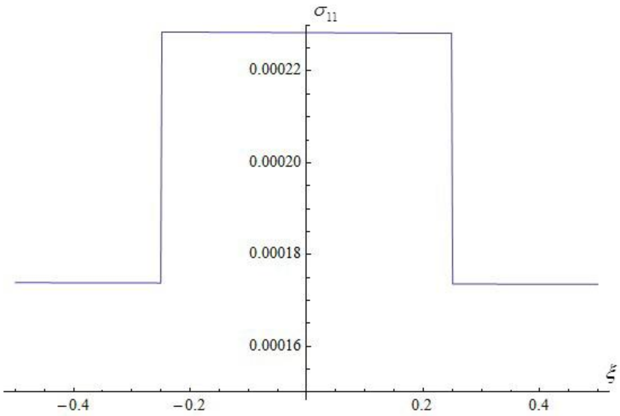

Figure 1 - Distribution of stress σ_{11}, GPa across the thickness of the composite shell under axially symmetric bending

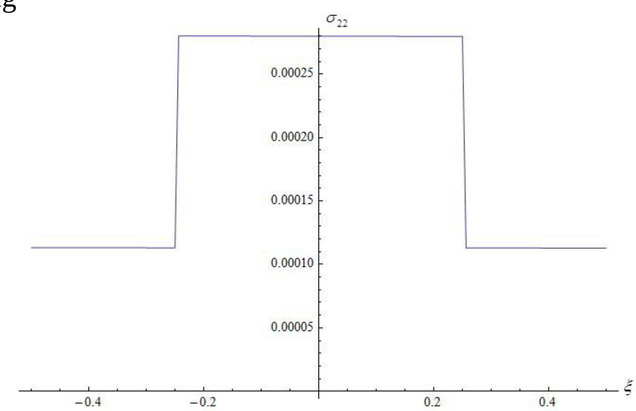

Figure 2 - Distribution of stress σ_{22}, GPa across the thickness of the composite shell under axially symmetric bending

It can be seen that the graphs of the components σ_{11}, σ_{22} of stress tensor have a piecewise linear appearance, which is due to the absence of conditions on the nature of the graph in all directions except the transverse, since the normal is directed along coordinate 3 (see boundary conditions (4)).

Following figures show the distribution of the stresses σ_{13}, σ_{33} along the thickness of the composite sheath with the system of corners $\alpha_1 = 30^\circ,\ \alpha_2 = 45^\circ,\ \alpha_3 = 45^\circ,\ \alpha_4 = 30^\circ$:

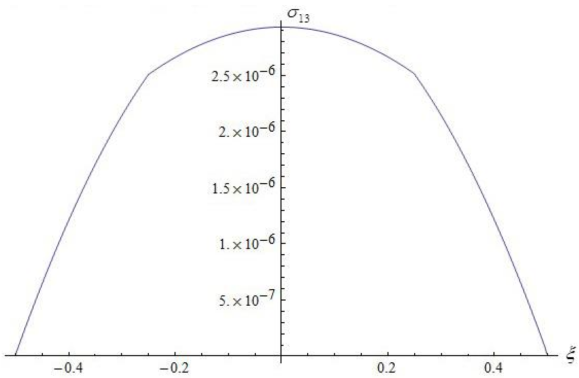

Figure 3 - Distribution of stress σ_{13}, GPa across the thickness of the composite shell under axially symmetric bending

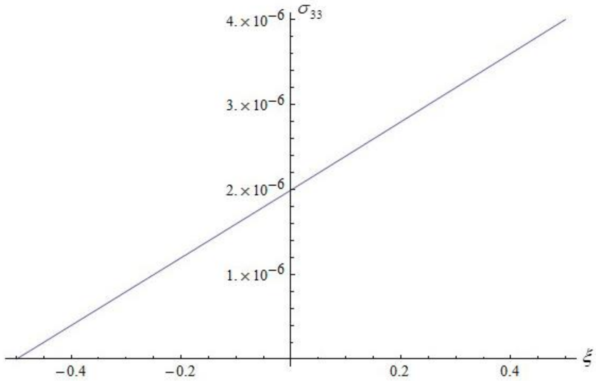

Figure 4 - Distribution of stress σ_{33}, GPa across the thickness of the composite shell under axially symmetric bending

It can be seen that the absolute values for the graphs of the components σ_{13}, σ_{33} of the stress tensor are significantly less than normal stresses, which correlates with the Timoshenko theory, but at the same time, the Timoshenko theory does not allow obtaining expressions for these stresses.

5. Conclusions

A theory of thin cylindrical composite shells is developed, based on the asymptotic analysis of three-dimensional equations of the theory of elasticity without additional assumptions about the distribution of stresses and displacements along the thickness of the shell. Explicit, mathematically accurate analytical formulas are obtained for calculating the distribution of the components of all 6 components of the stress tensor over the thickness of a cylindrical shell. An example is given of calculating stresses in a cylindrical shell under axisymmetric bending of pressures

Thus, local problems of the theory of thin shells were formulated and solved; explicit expressions were obtained for all six components of the stress tensor, including transverse normal stresses and interlayer shear stresses in the shell; a special case of the problem of a loaded axisymmetric cylindrical shell was considered; explicit expressions were obtained for all six components of the stress tensor, including transverse normal stresses and interlayer shear stresses in the shell for the case of a loaded axisymmetric cylindrical shell; a software package was implemented for calculating displacements and stresses using separate procedures that implement all the mathematical apparatus necessary to solve the problem; displacements and all 6 components of the stress tensor were calculated and visualized for the case of axisymmetric pressure loading of a cantilevered cylindrical shell.

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