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To cite this article: Wisam K. Irhaif and Hussein B. Al Hussein 2020 *IOP Conf. Ser.: Mater. Sci. Eng.*
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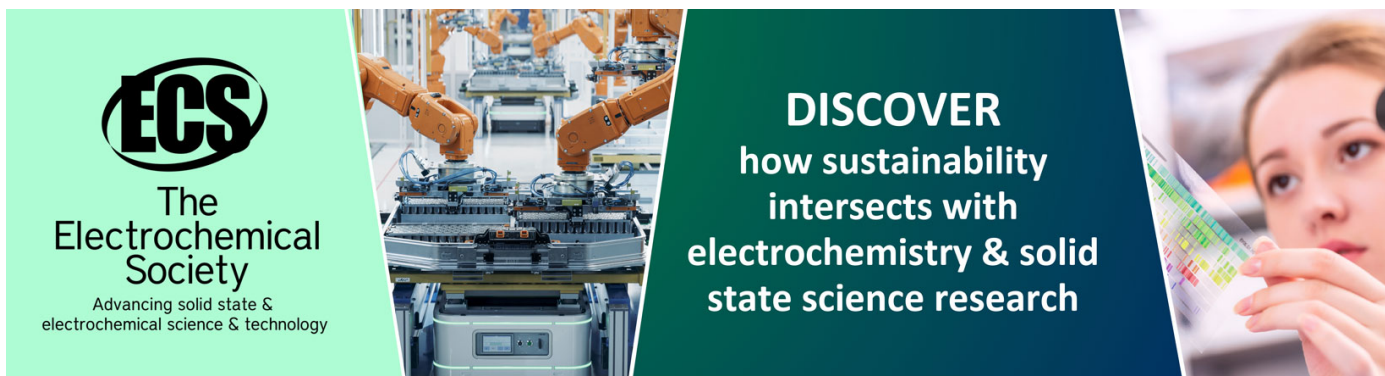
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Stability Analysis and Bifurcation in External Cavity Quantum Dot Semiconductor Laser

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Abstract. A simplified mathematical model to describe the nonlinear dynamics of a quantum dot laser (QDLs) coupled with external cavity was modified. This system is currently modeled by very complex equations which are intractable analytically and numerically. The model derived is simple, and efficient to provide full insight of the dynamics of the QDLs while compared with the well-known classical models. The equilibrium points and stability analysis of critical points is carried out. Various bifurcation scenarios are obtained numerically showing several striking routes to chaos.

Keywords: QDLs, stability, bifurcation, chaos, coexisting behavior.

1. Introduction

The relevant case provides information linked to the dynamic behavior of many chaotic electronic devices [1-3]. Concerning the modeling, design and analysis of chaotic electric generators, we can mention the most used models through a Perform an appropriate reconfiguration (e.g. simple modification) of derivation models to explain their complex nonlinear and chaotic dynamics and some of the traditional circuit oscillators [4-8]; Design the global analog computing programs to inquiry chaos in several kinds of models (e.g. Chua, Rossler, and Lorenz equations); Adjust classic electronic circuits (e.g. Clapp, Hartley, and



Colpitts) in hardness mode to increase their chaotic capabilities [3]; Introducing additional nonlinearities (in the nonlinear circuit of the traditional structures of the oscillators) through analogue devices (e.g. diodes, transistors, and Zener diodes): bifurcation analysis and stability [9-10]; In conventional circuit oscillators the investigation of chaos is due to nonlinear coupling [11], and designing pulse generators to produce broadband chaotic waves [12].

Based on what was mentioned in above, semiconductor lasers provide one of the best physical systems for studying nonlinear dynamic phenomena. Many researchers have investigated the chaotic behavior of several laser systems [13]. However, the suggested models in which the nonlinear terms appear extremely complex is therefore not easy to use or flexible to verify the amazing and complex bifurcation structures that these specific kinds of oscillators exhibit. The aim in this paper is to utilization the same impression to derive a simple model of a laser coupled with optical feedback. The suggested model is useful as it appears very flexible when dealing with the chaos control, bifurcation analysis and synchronization issues.

Quantum dot semiconductor laser (QDSL) has many advantages, such as small size, easy integration, compactness and convenience of

operation. Therefore, they are preferable to any other types of lasers in the field of optical telecommunications.

External cavity semiconductor lasers (ECSLs) or QDSL with optical feedback (OFB) are an integral part of high speed chaos based communication systems [8,9]. Hence, the ECSLs have been a subject of extensive research [10–12]. Understanding the influence of delayed optical feedback on the behavior of the ECSLs is of great relevance for technological applications.

The aim of this paper can be brief in three key steps. The first is to contribute to understanding the amazing dynamic behavior shown by the QDSL coupled with OFB and complete the results got so far. The second is to provide both analyses and numerical tools, which might assist for control and design purpose. The third is to point out some anonymous behavior of ECSLs.

This paper can be arranged as follows. Section (1) is a theoretical model. To describe the dynamic behavior of an oscillator, a set of ordinary nonlinear differential equations is derived, and the electronic structure is also processed. For ease we use a dimensionless differential equations based on the structure of the ECSLs. Section (2) is concerned with investigating the equilibrium point presented by the QDLs without optical feedback as well as analyzing the stability of the equilibrium

points (in the linear response). Section (3) is considered numerical study. The influence of feedback strength on the dynamic oscillator behavior is checked. Different stage images and bifurcation diagrams related with their corresponding charts of largest numerical bifurcation are got to define the nature of changes to chaos.

2. Theoretical model

A complex stochastic differential equation is defined as the field equation. The purpose is to transform the complex stochastic differential equation from field equation (E) into two real stochastic differential equations for the phase Φ and the photon density $S = |E|^2$. Without the stochastic term, this is just a transformation to polar coordinates [14]. Averaging over the stochastic term, the final rate equation for the photon density S , the phase of the electric field Φ , and the three equations for the occupation probability of a ground and excited states in the quantum-dot laser (ρ_{gs} and ρ_{es}) and carrier density in the wetting layer (N_{wl}) read,

$$\dot{S} = [vg_0(2\rho_{gs} - 1) - \gamma_s]S + k\sqrt{SS_\tau} \cos(\Phi - \Phi_\tau) \quad (1a)$$

$$\dot{\Phi} = -\frac{\alpha}{2}vg_0(2\rho_{gs} - 1) - \frac{\gamma}{2}\sqrt{S_\tau/S} \sin(\Phi - \Phi_\tau) \quad (1b)$$

$$\dot{\rho}_{gs} = \gamma_{ces}\rho_{es}(1 - \rho_{gs}) - \gamma_d\rho_{gs} - g_0(2\rho_{gs} - 1)S \quad (1c)$$

$$\dot{\rho}_{es} = \gamma_{cwl}N_{wl}(1 - \rho_{es}) - \gamma_d\rho_{es} - \gamma_{ces}\rho_{es}(1 - \rho_{gs}) \quad (1d)$$

$$N_{wl}^{\bullet} = \frac{J}{e} - \gamma_n N_{wl} - 2\gamma_{cwl} N_{wl} (1 - \rho_{es}) \quad (1e)$$

In our approach, the carrier-light interaction is summarized in the photon density S , which includes all longitudinal modes. In Eq. (1e) factor 2 calculates for the twofold spin degeneracy in the quantum dot energy levels. Another factor similar to Factor 2 in Eq. (1a) is included in the definition of the differential gain factor g_o [11]. and time t is scaled with w_r^{-1} , where w_r is the frequency of the relaxation oscillation, an intrinsic resonance of the optical mode. γ_s is the photon decay rate in the cavity. The parameter α is the linewidth enhancement factor, $g_o = \sigma v_g$, where σ is the cross section of interaction of carriers in the dots with photons; v_g is the group velocity; and $v = 2N_d\gamma/d$, where γ is the confinement factor and d is the thickness of the dot layer. N_d is the two-dimensional density of dots. Although being completely determined by w_o and τ the feedback phase ϕ is treated as an independent parameter since small variations of the external cavity length cause a variation of the phase ϕ over its full range $[0; 2\pi]$ while the external roundtrip time τ is hardly affected by these fluctuations. This is a well-established procedure in the analysis of semiconductor lasers subject to optical feedback [15-16]. The parameter k measures the injected field strength. The phase shift of the light during one round trip in the external cavity ($\tau = 2L/c$) is

given by $\phi = w_o \tau$, c is the speed of light, with w_o denoting the frequency of the solitary laser at the lasing threshold. The photon labeled by the subscript τ , S_τ , and phase with ϕ_τ , are the electric field amplitude and the optical phase taken at the delayed time $(t - \tau)$.

ρ_{gs} and ρ_{es} are the occupation probability in a ground and excited states in the quantum dots; N_{wl} is the carrier density in the well; γ_n and γ_d are the non-radiative decay rates for carriers in the WL and dot respectively; γ_{cwl} and γ_{ces} are the capture rate from wetting layer into an empty excited state and from excited into ground states respectively. J is the electrically injected pump current per dot, e is elementary charge. The last terms in Eqs. (1.d) and (1.e) describe the rate of exchange of carriers between a ground and excited states in the dots and between the well and the excited state in the dots. Here we show that the mechanism for the capture of carriers into the dots can significantly alter the damping rate of the relaxation oscillations and, as a result, reduce the sensitivity to optical feedback. Carrier escape from the dots can be ignored because it is a temperature-dependent function controlling. This leads to a carrier capture time from the well that is dependent on the occupation probability of the dots.

For numerical purposes, we introduce the new variables and rewrite the system in Eqs. (1) in the dimensionless formula. These variables are:

$$x = \frac{g_0}{\gamma_d} S, \Phi \equiv \Phi, y = \frac{g_0 v}{\gamma_s} (2\rho_{gs} - 1), z \equiv \rho_{es}, w = \frac{\gamma_{cwl}}{g_0 v} N_{wl},$$

$$\Gamma = \frac{\gamma_{ces}}{\gamma_s}, \Gamma_1 = \frac{g_0 v}{\gamma_s}, \Gamma_2 = \frac{\gamma_d}{\gamma_s}, \Gamma_3 = \frac{\gamma_{cwl}}{\gamma_s}, \Gamma_4 = \frac{\gamma_n}{\gamma_s}, \delta_0 = \frac{J}{g_0 v q}$$

and the time scale $t' = \gamma_s t$. The rate

$$x^\bullet = x(y - 1) + \varepsilon \sqrt{x x_\tau} \cos(\Phi - \Phi_\tau) \quad (2a)$$

$$\Phi^\bullet = -\frac{\alpha}{2} y - \frac{\varepsilon}{2} \sqrt{x_\tau / x} \sin(\Phi - \Phi_\tau) \quad (2b)$$

$$y^\bullet = \Gamma z (\Gamma_1 - y) - \Gamma_2 y (1 + 2x) - \Gamma_1 \Gamma_2 \quad (2c)$$

$$z^\bullet = \Gamma_1 w (1 - z) - \Gamma_2 z - \Gamma z (1 - y/\Gamma_1)/2 \quad (2d)$$

$$w^\bullet = \Gamma_3 \delta_0 - \Gamma_4 w - 2\Gamma_3 w (1 - z) \quad (2e)$$

where $\varepsilon = k/\gamma_s$. The well-established assumptions here are that the laser round-trip time within the active region is less than the delay time τ .

3. Equilibrium points and linear stability analysis

It should be worth noticing the relative simplicity of our model in Eqs. (2). The nonlinearity appears twice in each equation and five state variables are involved. This simplicity of the state equations might allow an in-depth analysis of the dynamics of the QDLs. Further, with such a simplified model, investigations based on chaos control and synchronization of these types of oscillators might be performed in a

systematic manner. It is also necessary to mention that ϕ is the phase of the electric field of the photon and consequently did not take into account analytical calculations because her study is linked to an external addition.

An important stage towards the stability analysis could be estimating the volume contraction of the equations modeled in order to obtain preliminary insights of the kinds of attractors (stable or unstable) which is might coexist in the system. We will rewrite Eqs. (2) without using the phase, and we will derive each of the equations that we obtained for x , y , z and w to find the values of the variables mentioned above, we will use the values of the parameters in Table (1):

Table (1): Numerical parameter used in the simulation.

parameter	value
Γ_1	1.79
Γ_2	0.07
Γ_3	5.32
Γ_4	0.037
Γ	8.12
δ_0	0.281

Let $\dot{x} = \dot{y} = \dot{z} = \dot{w} = 0$, be solution of system (2) with initial conditions, $x_0, y_0, z_0, w_0 > 0$. It is noted from Eq. (2.a) that the solution is either $x = 0$ or $y = 1$. Then, if we select $x = 0$, it is observed that, system (2)

has at most three possible groups of the equilibrium points, namely $z_1=0.9957$, $z_2=2.6002$ and $z_3=-0.0119$, where

$$y = \frac{\Gamma\Gamma_1 z - \Gamma_1\Gamma_2}{\Gamma z + \Gamma_2} \quad (3a)$$

$$w = \frac{\Gamma_3\delta_0}{\Gamma_4 + 2\Gamma_3(1-z)} \quad (3b)$$

$$\begin{aligned} 0 = & 2\Gamma\Gamma_1\Gamma_3\delta_0 z - 2\Gamma\Gamma_1\Gamma_3\delta_0 z^2 + 2\Gamma_1\Gamma_2\Gamma_3\delta_0 - 2\Gamma_1\Gamma_2\Gamma_3\delta_0 z + \\ & \Gamma^2\Gamma_4 z^2 - \Gamma\Gamma_2\Gamma_4 z + 2\Gamma^2\Gamma_3 z^2 - 2\Gamma\Gamma_2\Gamma_3 z - 2\Gamma^2\Gamma_3 z^3 + \\ & 2\Gamma\Gamma_2\Gamma_3 z^2 - 2\Gamma\Gamma_4 z^2 \left(\Gamma_2 + \frac{\Gamma}{2}\right) - 2\Gamma_2\Gamma_4 z \left(\Gamma_2 + \frac{\Gamma}{2}\right) - \\ & 4\Gamma\Gamma_3 z^2 \left(\Gamma_2 + \frac{\Gamma}{2}\right) - 4\Gamma_2\Gamma_3 z \left(\Gamma_2 + \frac{\Gamma}{2}\right) + 4\Gamma\Gamma_3 z^3 \left(\Gamma_2 + \frac{\Gamma}{2}\right) + \\ & 4\Gamma_2\Gamma_3 z^2 \left(\Gamma_2 + \frac{\Gamma}{2}\right) \end{aligned} \quad (3c)$$

this led to the following third order polynomial

$$A_1 z^3 + A_2 z^2 + A_3 z + A_4 = 0 \quad (3d)$$

The roots of the last equation can be obtained using Matlab. The existence conditions for each of these equilibrium points are discussed below:

- 1- The trivial equilibrium points $x = 0$ exists.

2- The laser free equilibrium point that is denoted by w and z in all above groups. Whereas, the necessary condition for sustaining the laser action is limited to positive equilibrium values.

In the other case, from Eq. (2a) when $y = 1$, equilibrium points can be obtained, write in Table (2), which we will discuss successively.

Table (2): equilibrium points in case $y=1$.

$z_1=0.9933$	$z_2=0.1360$
$x_1=44.03$	$x_2=4.826$
$w_1=13.85$	$w_2=0.161$

Where

$$(1 + 2x) = \frac{[\Gamma z(\Gamma_1 - 1) - \Gamma_1 \Gamma_2]}{\Gamma_2} \quad (4a)$$

$$\begin{aligned}
 0 = & \Gamma_1 \Gamma_3 \delta_0 - \Gamma_1 \Gamma_3 \delta_0 z - \Gamma_2 \Gamma_4 z - \frac{\Gamma \Gamma_4 z}{2} + \frac{\Gamma \Gamma_4 z}{2 \Gamma_1} - 2 \Gamma_2 \Gamma_3 z \\
 & + 2 \Gamma_2 \Gamma_3 z^2 - \Gamma \Gamma_3 z + \Gamma \Gamma_3 z^2 + \frac{\Gamma \Gamma_3 z}{\Gamma_1} \\
 & - \frac{\Gamma \Gamma_3}{\Gamma_1} z^2 \quad (4b)
 \end{aligned}$$

this led to the following two order polynomial

$$A_1 z^2 + A_2 z + A_3 = 0 \quad (4c)$$

The existence conditions for each of the equilibrium points in Table (2) are discussed below:

- 1- Clearly, system (2) has two positive roots represented by z_1 and z_2 , if the set of conditions holds as in Table (1).
- 2- Each of the solutions depends on the initial values that determine the course of the analytical solution.

The stability analysis is carried out by transforming Eqs. (2) into the following form

$$f_1 = \dot{x}, f_2 = \dot{y}, f_3 = \dot{z}, \text{ and } f_4 = \dot{w} \quad (5)$$

Now we will find the values of the derivatives that we created from Eqs. (5) namely . Using the values of the variables that we obtained in Table (2), as well as the values of the parameters in Table (1), we will find eigenvalue.

$$Df = \begin{bmatrix} y-1 & x & 0 & 0 \\ -2\Gamma_2 y & -\Gamma z - \Gamma_2(1+2x) & \Gamma(\Gamma_1 - y) & 0 \\ 0 & \frac{\Gamma z}{2\Gamma_1} & -\Gamma_1 w - \Gamma_2 - \frac{\Gamma}{2\Gamma_1}(\Gamma_1 - y) & \Gamma_1(1-z) \\ 0 & 0 & 2\Gamma_3 w & -\Gamma_4 - 2\Gamma_3(1-z) \end{bmatrix}$$

Df is a 4×4 Jacobian matrix, our analysis is restricted to the case of small amplitude of the steady state. The stability of the periodic motion is got according to the real parts of the roots of the following characteristic equation ($V_i = \det[Df - \lambda I] = 0$) for two sets respectively in Table (2)

$$V_1 = \begin{bmatrix} 0-\lambda & 44.03 & 0 & 0 \\ -0.14 & -14.3-\lambda & 6.4 & 0 \\ 0 & 2.25 & -26.62-\lambda & 0.01 \\ 0 & 0 & 147.36 & -0.43-\lambda \end{bmatrix}$$

Then the characteristic equation of V_1 is given by :

$$\lambda^4 + 41.35\lambda^3 + 388.55\lambda^2 + 303.16\lambda + 61.47 = 0 \quad (6)$$

$$V_2 = \begin{bmatrix} 0-\lambda & 4.82 & 0 & 0 \\ -0.14 & -1.85-\lambda & 6.4 & 0 \\ 0 & 0.3 & -2.14-\lambda & 1.54 \\ 0 & 0 & 1.71 & -9.22-\lambda \end{bmatrix}$$

$$\lambda^4 + 13.21\lambda^3 + 36.86\lambda^2 + 21.59\lambda + 11.53 = 0 \quad (7)$$

Clearly, all roots of Eqs. (6) and (7) have negative real parts. So, V_i is locally asymptotically stable if the above two conditions hold. .

4. NUMERICAL STUDY

In this section, the global dynamics of system (2) is investigated numerically. The objectives are first confirming our obtained analytical results and second specify the control set of parameters that control the

dynamics of the system. Consequently, system (2) is solved numerically using the set of hypothetical parameters with different sets of initial points and then the resulting trajectories are drawn in the form of phase portrait and time series figures. With initial values $x_o = 0.04, y_o = 0.8, z_o = 0.51$ and $w_o = 0.049$.

It is observed in Fig. 1(a) with the parameters values in table (1) the system satisfy to the positive second equilibrium point. Fig. 1(b) is shown a new approaches with changing value of gamma to 0,1. The positive equilibrium values indicate that the system operates in the long run according to the operating parameters that ensure continuity. In general, the parameters values have qualitative effect on the dynamical behavior of system (2) but the system still approaches to a positive equilibrium point.

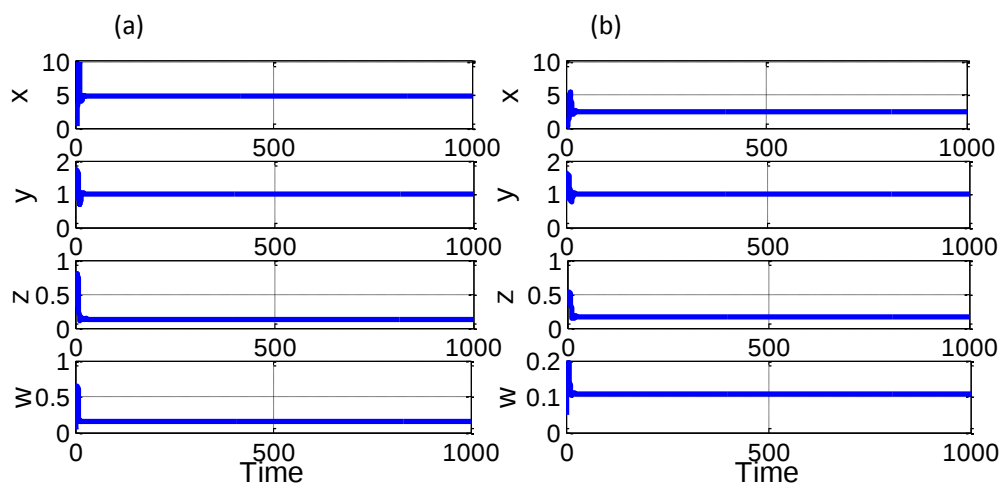


Fig.1: Time series of the photon number (x), carrier number in GS (y), ES(z) and WL(w), respectively, (a) with the parameters as in Table (1), (b) by changing $\Gamma = 4.12403$ and $\delta_o = 0.181$.

5. The bifurcation parameters

In dynamical systems, a bifurcation occurs when a small smooth change made to the parameter values (the bifurcation parameters) of a system causes a sudden “qualitative” or topological change in its behaviour. Generally, at a bifurcation, the local stability properties of equilibria, periodic orbits or other invariant sets changes . To highlight the effects of feedback strength on the dynamics of this particular oscillator, I_1 was chosen as control parameter. The extreme sensitivity of the dynamical behaviour of the oscillator to tiny changes in I_1 was shown.

In Fig. 2 (a) we observe the general behavior of the change of photons number (x) by the change of the feedback strength (η) from the note of the behavior, the change is under the influence of ($I_1 = 1.5$), the steady state and period from 0.2 to 0.25, it represents a clear periodic behavior, and then it turns into a double periodic from 0.25 to 0.26. After that, the dynamic becomes a four periodic leading to chaos (C_1), which is limited to 0.27 to 0.29. As for its maximum capacity, it is from 0.9 to 3.3.

We notice in Fig.2 (b) that there is a clear change in the dynamic compared to Fig. 2 (a) and this change is the result of a change in the value of ($I_1=1.58$) so it will be a mixture of periodic to double periodic

then to chaos (C_1) and then to double periodic and finally again to chaos (C_2). As for capacity, there is a slight increase, which is limited to the period 1.2 to 4.3 .

Fig.2 (c) is differ much from Fig. 2 (b), as the influencing factor is the ($\Gamma_1=1.48$) at the beginning, the steady state and periodic, which ends at 0.25 and then a double periodic, which ends at point 0.26 and this change is followed by a four periodic, the last change in this dynamic is chaos (C_2) up to 0.35, but as for capacity, it is relatively small.

The most important observations can be summarized from the Γ_1 effect; general behavior remains constant up to 0.29 from the coupling external cavity (feedback strength), the steady state, periodic, and double periodic areas were not affected by change (differential gain), the change is relatively large at the η . Of the three figures, we also notice that the difference in the region of region C_1 and C_2 changes so that the region C_2 expands at the expense of C_1 and notice coexisting behaviors. To explain by the presence of coexisting attractors both in their regular and chaotic regime, it is because of the extreme sensitivity of our model to initial conditions. This overlay or mixing patterns is unique behavior in the quantum dot lasers [17].

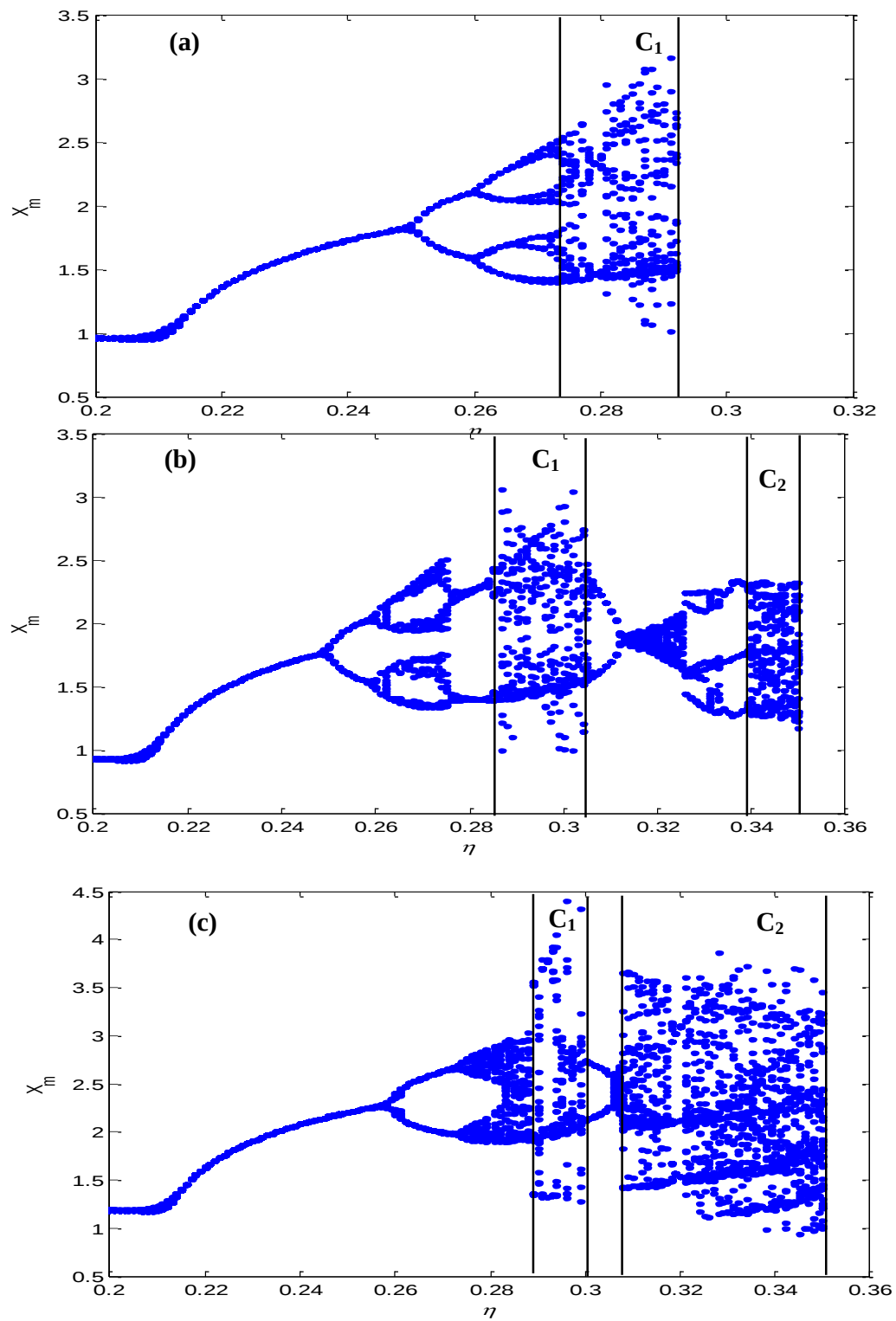


Fig.2: Bifurcation diagrams of the photon number (x) by the change of feedback strength(η) when (a) $\Gamma_1 = 1.5$ (b) $\Gamma_1 = 1.58$ (c) $\Gamma_1 = 1.4881$

In Fig. 3.(a), the changing of the number of photons is related to feedback strength, and this change is under the influence of the value of ($\Gamma_2=0.08$). the behavior confined from 0.2 to 0.23 represents a steady state, and this is followed by a periodic behavior, which is limited to the period 0.23 to 0.28, then the next behavior, which is a double periodic and confined to the period 0.28 to 0.3, and finally the chaos (Hopf bifurcation), which is confined to the period 0.3 to 0.35, As for the capacity, it starts from 1.3 and ends with 5.5.

As shown in Fig. 3 (b), the steady state takes a longer period than in the previous figure, and this results from a decrease in the value of (τ) to (5.68021), as well as the presence of another difference, which is the absence of a double periodic behavior and that the Hopf bifurcation is relatively larg.

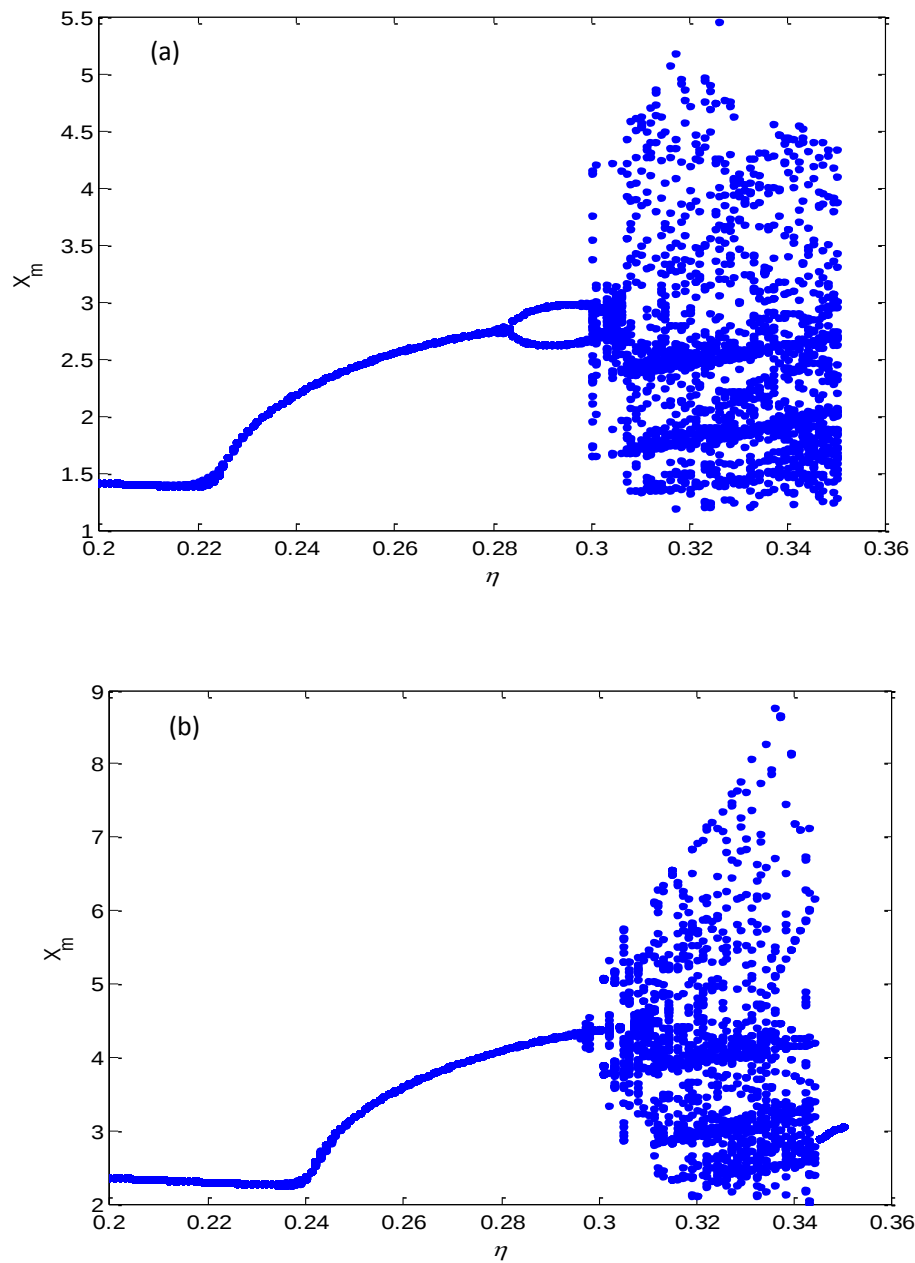


Fig. 3. Bifurcation diagram of the ECSLs in terms of the control parameter Γ and δ_o .

6. Conclusion

In this paper, we have introduced and analysed a simplified mathematical model to describe the nonlinear dynamics of a quantum dot laser (QDLs)

coupled with external cavity . The electronic trasmtion of the structure was proposed and the modelling process was performed to derive sets of ordinary differential equations describing the behaviour of the system. The analytical criteria for the occurrence of equilibrium points were derived and stability of the system was investigated. It was shown that the equations of the model considered in this work can exhibit positive and negative equilibrium points as well, and we neglected the latter because this case is not matched by a true physical meaning because there are no unoccupied cases or a negative ratio of works at the semi-separate levels in the nanostructure. Various bifurcation diagrams associated were obtained showing period doubling, period adding and abrupt/sudden transition routes to chaos. Analytical and numerical results were compared and a good agreement was observed.

Acknowledgements. This work is supported by the Nassiriya Nanotechnology Research Laboratory (NNRL), Science College, University of Thi Qar, Iraq.

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