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# Research and development of a trajectory-position controller for an underwater glider

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**Abstract.** In this article, development of glider control system for movement in a three-dimensional based on position-trajectory control laws is considered. To ensure control, a model of spatial motion of the glider is used, taking into account the component of the variable buoyancy. As a result of the simulation, the trajectories of the glider were obtained at points set in the horizontal plane, considering the parameters of the trajectory for movement in the vertical plane. The results obtained make it possible to evaluate the adequacy of the developed control system. The deviation of the system when the glider reaches the set point is minimal, and using the trajectory parameters in the vertical plane, it is possible to control the depth of immersion and speed. The presented results of the simulation of coating elements prove the possibility of practical use of the control system when planning glider-type AUV missions.

## 1. Introduction

In this work we review the synthesis of a controller for a glider-type underwater vehicle based on a position-trajectory controller described in [1, 2]. This controller is widely used for autonomous control of various vehicles [3].

The main advantages of this controller make it possible to control non-linear objects that consider external environmental disturbances as well as consider interconnected and non-linear mathematical models of a plant.

## 2. Underwater glider technology

An underwater glider is an autonomous underwater vehicle that uses variable-buoyancy device for its propulsion. Compared to conventional submarine-type vehicles that employ propellers, gliders dive and float up alternately that, with use of underwater wings (hydrofoils), causes the vehicle to follow a sinewave or sawtooth-like trajectory. While gliders are much slower than most submarines, they offer significantly more range, making them suitable for various continuous research missions, lasting weeks or even months and covering thousands of kilometers of range.

The variable-buoyancy system usually consists of two parts: ballast reservoir and center of mass displacement device. The former allows to change the overall buoyancy of the vehicle, causing it to dive or float up, and the latter moves the center of vehicle's mass, thus creating pitch angle. The above principles force the glider to follow the signature sawtooth trajectory, propelling it forward.

However, in case of the vehicle model we're reviewing in this paper, the mass displacement device is excluded. Thereby, the ballast reservoir has to be located in a compartment located in the bow of the vehicle. In this case, the reservoir combines the two glider propulsion principles described above. In empty state, the glider is floating on water surface with slightly positive pitch angle (with its nose up).

As the reservoir is getting filled, the buoyancy reduces and the center of mass moves in direction of the glider's bow. That forces the vehicle to dive and move forward. As the reservoir is getting emptied, the process reverses and the glider floats up.

The proposed scheme may reduce the power consumption of onboard systems even more, greatly extending mission time and overall durability and endurance. In order to perform turns, we propose to split the ballast compartment into two parts, applied for left and right turns respectively.
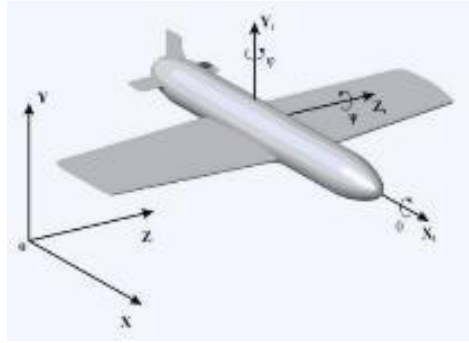
## 3. Mathematical model definition



**Figure 1.** Coordinate systems and positive directions of Euler angles.

The cited works [4, 5] present the structure of a mathematical model that considers the glider's movement. To derive the mathematical model of a glider, we use the following rectangular coordinate systems, shown in Figure 1.

In general, the mathematical model of the glider's motion in a three-dimensional environment may be represented by Eq. (1.1–1.6):

Where $p_w$ is water density; $V$ is displacement; $v_{x1}$, $v_{y1}$, $v_{z1}$ are projections of object's velocity on axis OX1, OY1, OZ1 (or surge, heave and sway respectively); $v_0$ is the absolute value of velocity; $\omega_{x1}$, $\omega_{y1}$, $\omega_{z1}$ are angular velocities around axes OX1, OY1, OZ1; $J_{x1}$, $J_{y1}$, $J_{z1}$ are moments of inertia relative to axis OX1, OY1, OZ1; $\lambda_{11}$, $\lambda_{22}$, $\lambda_{33}$ are added masses of glider's hull, $\lambda_{26}$, $\lambda_{35}$ are added static moments of the hull; $\lambda_{44}$, $\lambda_{55}$, $\lambda_{66}$ are added moments of inertia; $c_{x1}$, $c_{y1}$, $m_{z1}$ are coefficients of positional hydrodynamic forces and moments; $c_{y1}^{\omega z1}$, $m_{z1}^{\omega z1}$ are coefficients of rotational hydrodynamic forces and moments; $p$ is excess buoyancy; $x_p$, $y_p$, $z_p$ are the arms of excess buoyancy in glider's principal axes; $c_{z1}^{\beta}$, $m_{x1}^{\beta}$, $m_{y1}^{\beta}$ are coefficients of derivative positional hydrodynamic forces and moments; $c_{z1}^{\omega z1}$, $m_{x1}^{\omega x1}$, $m_{x1}^{\omega x1}$, $m_{y1}^{\omega x1}$, $m_{y1}^{\omega y1}$ are coefficients of derivative rotational forces and moments; $h$ is metacentric height; $\psi$ is pitch angle; $\theta$ is roll angle; $\varphi$ is yaw angle.

$$(p_w V + \lambda_{11})\dot{v}_{x1} = c_{x1}\frac{p_w V_0^2}{2}V^{2/3} + (p_w V + \lambda_{22})V_{y1}\omega_{x1} + \lambda_{26}\omega_{z1}^2 + +p\sin\psi\,; \tag{1.1}$$

$$(p_W V + \lambda_{22})\dot{v}_{y1} + \lambda_{26}\dot{\omega}_{z1} = c_{y1}\frac{p_w v_0^2}{2}V^{2/3} + c_{y1}^{\omega z1}\frac{p_w v_0}{2}\omega_{z1}V - (p_w V + +\lambda_{11})v_{x1}\omega_{z1} + +p\cos\psi\,; \tag{1.2}$$

$$(J_{z1} + \lambda_{66})\dot{\omega}_{z1} + \lambda_{26}\dot{v}_{y1} = m_{z1}\frac{p_w v_0^2}{2}V + m_{z1}^{\omega z1}\frac{p_w v_0}{2}\omega_{z1}V^{4/3} - -p_w Vgh\sin\psi - \lambda_{26}v_{x1}\omega_{z1} +$$
$$+p(x_p\cos\psi - y_p\sin\psi)\,; \tag{1.3}$$

$$(p_w V + \lambda_{33})\dot{v}_{z1} + \lambda_{35}\dot{\omega}_{y1} = c_{z1}^{\beta}\frac{p_w v_0}{2}V^{2/3}v_{z1} + c_{z1}^{\omega z1}\frac{p_w v_0}{2}\omega_{y1}V - -\omega_{x1}(p_w V + \lambda_{22})v_{y1} +$$
$$+\omega_{y1}(p_w V + \lambda_{11})v_{x1} - p\cos\psi\sin\theta\,; \tag{1.4}$$

$$(J_{x1} + \lambda_{44})\dot{\omega}_{x1} + \omega_{y1}\lambda_{26}v_{y1} + v_{y1}\lambda_{35}\omega_{y1} = m_{x1}^{\beta}\frac{p_w v_0}{2}Vv_{z1} + +m_{x1}^{\omega x1}\frac{p_w v_0}{2}V^{4/3}\omega_{x1} +$$
$$m_{x1}^{\omega y1}\frac{p_w v_0}{2}V^{4/3}\omega_{y1} - p_w gVh\sin\theta\cos\psi - p(z_p\cos\theta + +y_p\sin\theta)\cos\psi\,; \tag{1.5}$$

$$(J_{y1} + \lambda_{55})\dot{\omega}_{y1} + \lambda_{35}\dot{v}_{z1} = m_{y1}^{\beta}\frac{p_w v_0}{2}Vv_{z1} + m_{y1}^{\omega x1}\frac{p_w v_0}{2}\omega_{x1}V^{4/3} + +m_{y1}^{\omega y1}\frac{p_w v_0}{2}\omega_{y1}V^{4/3} +$$
$$\omega_{x1}\lambda_{26}v_{y1} + v_{x1}\lambda_{35}\omega_{y1} + p(x_p\cos\psi\sin\theta + z_p\sin\psi)\,. \tag{1.6}$$

Coefficients $p$ and $z_p$ are used as main control parameters. We synthesize the controller according to position-trajectory control law considering changing the arm length of excess buoyancy $z_p$. The above control parameters and their directions are shown in Figure 2.
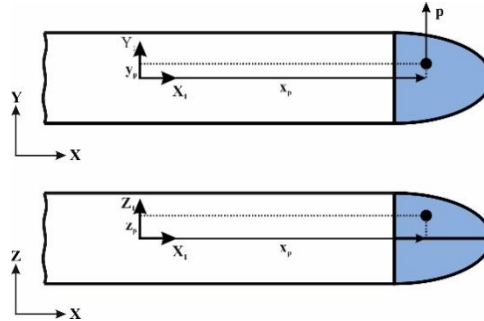


**Figure 2.** Coordinate systems and control parameters.

## 4. Control system synthesis

The process of developing a controller for an underwater glider has a few main steps:

Defining the target path function y(x) and z(x). That function must represent the target of control as an error of close loop system;

Defining the trajectory error based on difference between coordinates of real and desired trajectories;

Evaluating control parameters $p$ and $z_p$.

To solve the problem of synthesizing the control system we will divide it into two parts:

Control system for moving along the plane $OX_1Y_1$ which matches surge and heave motions;

Control system for moving along the plane $OX_1Z_1$ which matches the sway motion and turning.

We define an error of closed-loop system along the plane $OX_1Y_1$ (2):

$$e_y = y - y_0, \tag{2}$$

Where $y_0$ is function of a desired trajectory, $y_0 = A\sin(\omega t) + A_m$, where A is dive amplitude, $\omega$ is dive frequency, $A_m$ is average depth of trajectory.

By substituting the desired path function into (2), we get the value of an error (3):

$$e_y = y - A\sin(\omega t) - A_m. \tag{3}$$

According to the position-trajectory method of control, we introduce an equation that satisfies the condition of asymptotic stability (4):

$$\ddot{e}_y + a_1\dot{e}_y + a_2 e_y = 0, \tag{4}$$

Where $e_y$ is an error that includes the difference between real and desired trajectories along the plane $OX_1Y_1$; $a_1, a_2$ are tuning parameters of the controller.

Due to control being implemented by change of buoyancy, we need to express the excess buoyancy function p from equation (4).

By deriving eq. $(1.1 - 1.6)$ and substituting the required parameters from mathematical model into eq. (4), it is possible to express the value of variable buoyancy p (5):

$$P_{num} = -A\omega^2\sin(\omega t) - a_1\big[(a_{12}v_{x1} + a_{22}v_{y1} + a_{32}v_{z1}) - A\omega\cos(\omega t)\big] - a_2[y - $$
$$-A\sin(\omega t) + A_m] - \frac{a_{12}A1}{\rho_w V + \lambda_{11}} - a_{22}\left[\frac{J_{z1}+\lambda_{66}}{\det O}A2 + \frac{-\lambda_{26}}{\det O}A3\right] - a_{32}\left[\frac{J_{y1}+\lambda_{22}}{\det O_2}(A4 - A5) + \right.$$
$$\left. + \frac{-\lambda_{35}}{\det O_2}(A6 + A7)\right] - \dot{a}_{12}v_{x1} - \dot{a}_{22}v_{y1} - \dot{a}_{32}v_{z1};$$

$$P_{denom} = $$
$$\frac{\cos\psi}{\rho_w V + \lambda_{11}} + a_{22}\left[\frac{J_{z1}+\lambda_{66}}{\det O}\cos\psi + \frac{-\lambda_{26}}{\det O}(x_p\cos\psi - y_p\sin\psi)\right] - -a_{32}\left[\frac{J_{y1}+\lambda_{55}}{\det O_2}\cos\psi\sin\theta - \right.$$
$$\left. \frac{-\lambda_{35}}{\det O_2}(x_p\cos\psi\sin\theta - z_p\sin\psi)\right];$$

$$p = {P_{num}}/{P_{denom}}, \tag{5}$$

where $a_{12} = \sin\psi$, $a_{22} = \cos\psi\cos\theta$, $a_{32} = -\cos\psi\sin\theta$, $\dot{a}_{12} = \cos\psi\,(\omega_{z1}\cos\theta + \omega_{y1}\sin\theta)$,

$\dot{a}_{22} = -\sin\psi\cos\theta\,(\omega_{z1}\cos\theta + \omega_{y1}\sin\theta) - \cos\psi\sin\theta\,(\omega_{x1} -- \tan\psi\,(\omega_{x1}\cos\theta - \omega_{z1}\sin\theta))$;

$\dot{a}_{22} = \sin\psi\sin\theta\,(\omega_{z1}\cos\theta + \omega_{y1}\sin\theta) - \cos\psi\cos\theta\,(\omega_{x1} -- \tan\psi\,(\omega_{x1}\cos\theta - \omega_{z1}\sin\theta))$; $\det O = (\rho_w V + \lambda_{22})(J_{z1} + \lambda_{66}) - \lambda_{26}^2$;

$\det O_2 = (\rho_w V + \lambda_{11})(J_{y1} + \lambda_{55}) - \lambda_{35}^2$;

$A1 = c_{x1}\frac{p_w V_0^2}{2}V^{2/3} + (p_w V + \lambda_{22})V_{y1}\omega_{x1} + \lambda_{26}\omega_{z1}^2$;

$A2 = c_{y1}\frac{p_w v_0^2}{2}V^{2/3} + c_{y1}^{\omega_{z1}}\frac{p_w v_0}{2}\omega_{z1}V - (p_w V + \lambda_{11})v_{x1}\omega_{z1}$;

$A3 = m_{z1}\frac{p_w v_0^2}{2}V + m_{z1}^{\omega_{z1}}\frac{p_w v_0}{2}\omega_{z1}V^{4/3} - p_w Vgh\sin\psi - \lambda_{26}v_{x1}\omega_{z1}$;

$A4 = c_{z1}^{\beta}\frac{p_w v_0}{2}V^{2/3}v_{z1} + c_{z1}^{\omega_{z1}}\frac{p_w v_0}{2}\omega_{y1}V$; $A5 = \omega_{x1}(p_w V + \lambda_{22})v_{y1} + \omega_{y1}(p_w V + \lambda_{11})v_{x1}$;

$A6 = m_{y1}^{\beta}\frac{p_w v_0}{2}Vv_{z1} + m_{y1}^{\omega_{x1}}\frac{p_w v_0}{2}\omega_{x1}V^{4/3} + m_{y1}^{\omega_{y1}}\frac{p_w v_0}{2}\omega_{y1}V^{4/3}$;

$A7 = \omega_{x1}\lambda_{26}v_{y1} + v_{x1}\lambda_{35}\omega_{y1}$

are shortened expressions introduced to simplify the calculation.

Next, we will define the control system for maneuvering. In that case, the goal of control system is to provide movement through a set of defined points. In order to create the roll motion, we will change the $z_p$ parameter that allows glider to turn. That principle can be used for control system of glider in coordinate plane $OX_1Z_1$. We define the desired path as a straight line, described as a linear equation (6):

$$z = kx + b, \tag{6}$$

Where k is a slope, b is an intercept (for simplicity we will define it as 0, in that case the line will run through the origin).

As known, the slope is $\frac{\Delta z}{\Delta x}$. Assume k as desired slope and $k_s$ as real:

$$k_s = \frac{\Delta z}{\Delta x} = \frac{\dot{z}}{\dot{x}} = \frac{(a_{13}v_{x1} + a_{23}v_{y1} + a_{33}v_{z1})}{(a_{11}v_{x1} + a_{21}v_{y1} + a_{31}v_{z1})}, \tag{7}$$

Where $a_{11} = \cos\varphi\cos\psi$, $a_{13} = -\sin\varphi\cos\psi$; $a_{21} = \sin\varphi\sin\theta - \cos\varphi\sin\psi\cos\theta$, $a_{23} = \cos\varphi\sin\theta + \cos\varphi\sin\psi\sin\theta$, $a_{31} = \sin\varphi\cos\theta + \cos\varphi\sin\psi\sin\theta$, $a_{33} = \cos\varphi\sin\theta - \sin\varphi\sin\psi\sin\theta$.

Next, we will define the desired slope k. Because the input of the control system will accept the coordinates of a point, we define the slope k as (8):

$$k = \frac{\Delta z}{\Delta x} = \frac{z_1 - z_0}{x_1 - x_0}, \tag{8}$$

Where $x_1, z_1$ are the coordinates of a set point, $x_0, z_0$ are the current glider's coordinates.

These values were obtained by integrating equations (1). With use of this method, the slope k will be constantly recalculated, which will allow the control system to steer the glider more precisely.

Next, we'll define the error of control system in coordinate plane $OX_1Z_1$ (9):

$$e_z = k_s - k. \tag{9}$$

Substituting eq. (7) and (8), we get the error equation (10):

$$e_z = \frac{(a_{13}v_{x1} + a_{23}v_{y1} + a_{33}v_{z1})}{(a_{11}v_{x1} + a_{21}v_{y1} + a_{31}v_{z1})} - \frac{z_1 - z_0}{x_1 - x_0}. \tag{10}$$

From modelling the glider's motion in works [5], it is known that the best stability of a glider with attributes used in this research is achieved by a change of $z_p$ parameter in the limits $-0,445 < z_p < +0,445$, where the negative values provide the left roll and positive values are the right roll. Therefore, if the glider is passing to the left from the desired path, the values of the error are negative and otherwise if passing to the right. Accordingly, the values of $z_p$ parameter can be defined as system (11):

$$\begin{cases} z_p = 0 \; if \; e_z = 0; \\ z_p = 0{,}445 \; if \; e_z > 0; \\ z_p = -0{,}445 \; if \; e_z < 0. \end{cases} \tag{11}$$

To make the tuning of $z_p$ smoother, we use a logistic function (sigmoid curve) $f(x) = \frac{L}{1+e^{-k\,(x-x_0)}}$, where L is the curve's maximum value, $x_0$ the $x$-value of the curve's midpoint,
K is the steepness of the curve. In case of the control system, function will be defined as (12):

$$z_p = 0{,}445 \left( \frac{2}{1+e^{-b\,e_z}} - 1 \right), \tag{12}$$

Where b is the steepness of the curve. The $z_p$-$e_z$ graph with $b = 20$ is shown in Figure 3.
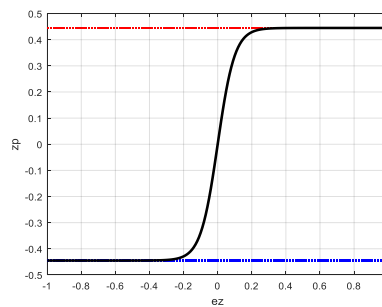


**Figure 3.** The $z_p$-$e_z$ graph.

## 5. Research of the control system
For researching the synthesized controller, we use attributes of the Neptune glider [1, 6, 7]. According to the variable buoyancy control law (5), we'll consider an example simulation of longitudinal motion along the coordinate plane $OX\_1\,Y\_1$. Using the function (5) in motion model 1, we will obtain the following motion graphs, shown in Figure 4.
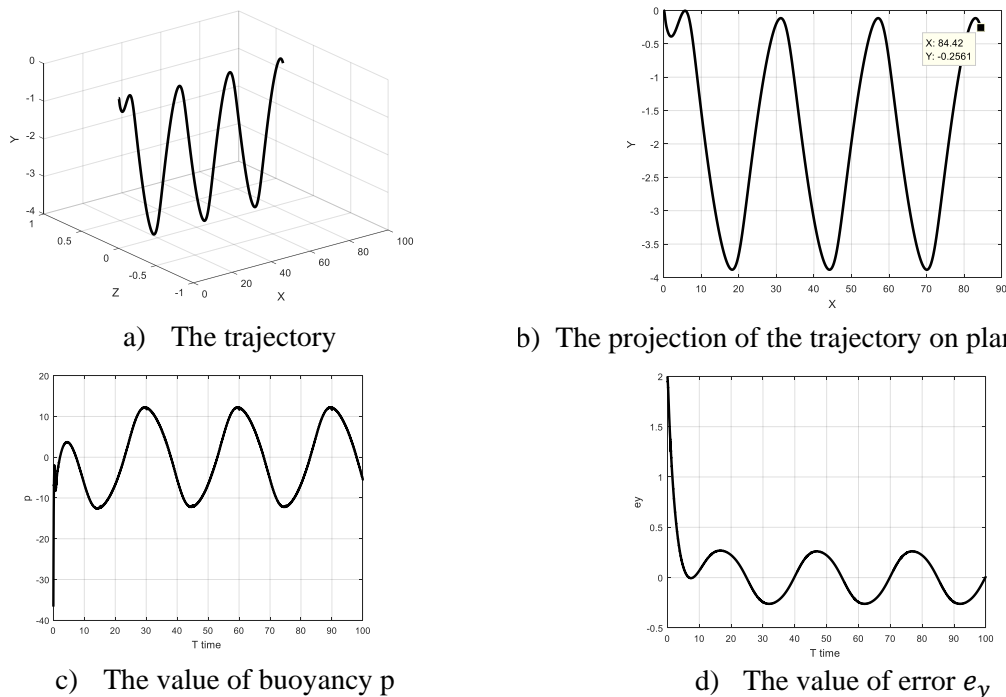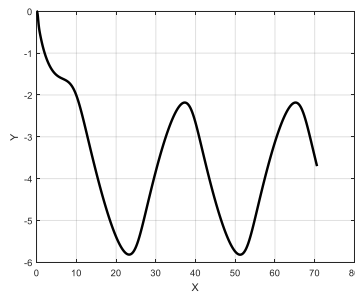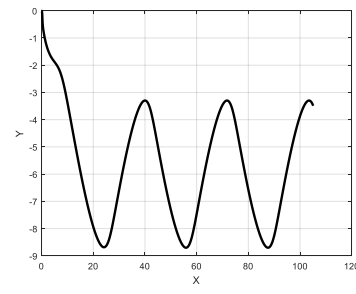


a)   The trajectory



b)  The projection of the trajectory on plane $OXY$



c)   The value of buoyancy p



d)   The value of error $e_y$

**Figure 4.** Graphs of various glider parameters.

For simulating the trajectory above we did not consider the tuning of $z_p$, its value was set to 0. We have chosen the following parameters of the sine wave: amplitude $A = 2$ meters, average depth $A_m = 2$ meters, period $T = 30$ seconds. In 100 seconds, glider makes 84 meters with average speed $v_{x1} = 0,84\ m/s$ and does 3 full cycles of diving/ascending. Error $e_y$, after establishing the set mode, does not exceed 0.26 meters modulo.

The control system allows to change the depth, amplitude and frequency of dives, as shown in Figure5.

a) The trajectory with average depth $A_m = 4$ m, amplitude $A = 2$ m and period $T = 40$ s

b) The trajectory with average depth $A_m = 6$ m, amplitude $A = 2$ m and period $T = 30$ s

**Figure 5**. Glider trajectories with different parameters.

As seen in Figure 4, in the second case, glider travels a greater distance in the same amount of time. Consider the movement taking into account the control over the course using the $z_p$ parameter. We will set the end point as (100; -70). Sine wave parameters are $A_m = 4\ m$, $A = 2\ m$, $T = 40\ s$. Simulation results are shown in Figure 6.
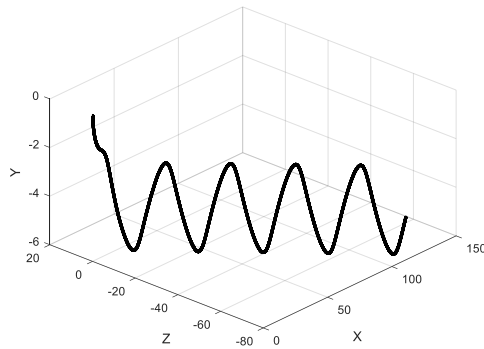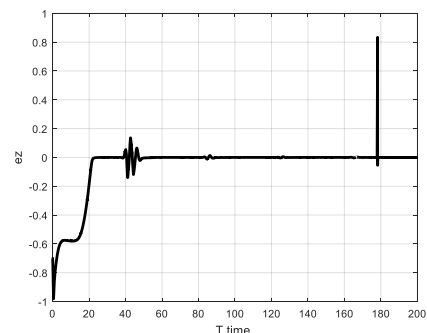
a) the trajectory
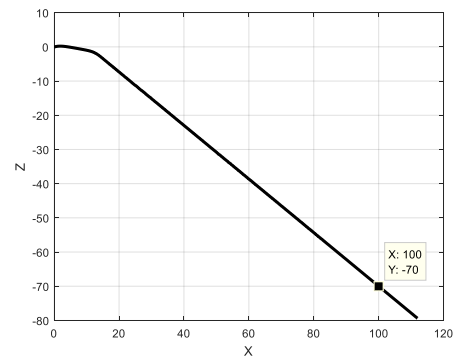
b) the projection of trajectory on plane $OXZ$
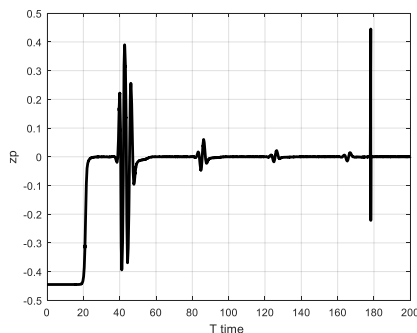
c) the value of $z_p$ over time

d) the value of error $e_z$

**Figure 6.** Graphs of trajectory through point (100; -70).

The proposed control system allows to set several points for glider to move through. In the next example 3 points were set: (50; -20), (150; -30), (200; 0). Graphs of the trajectory is shown in Figure 7.
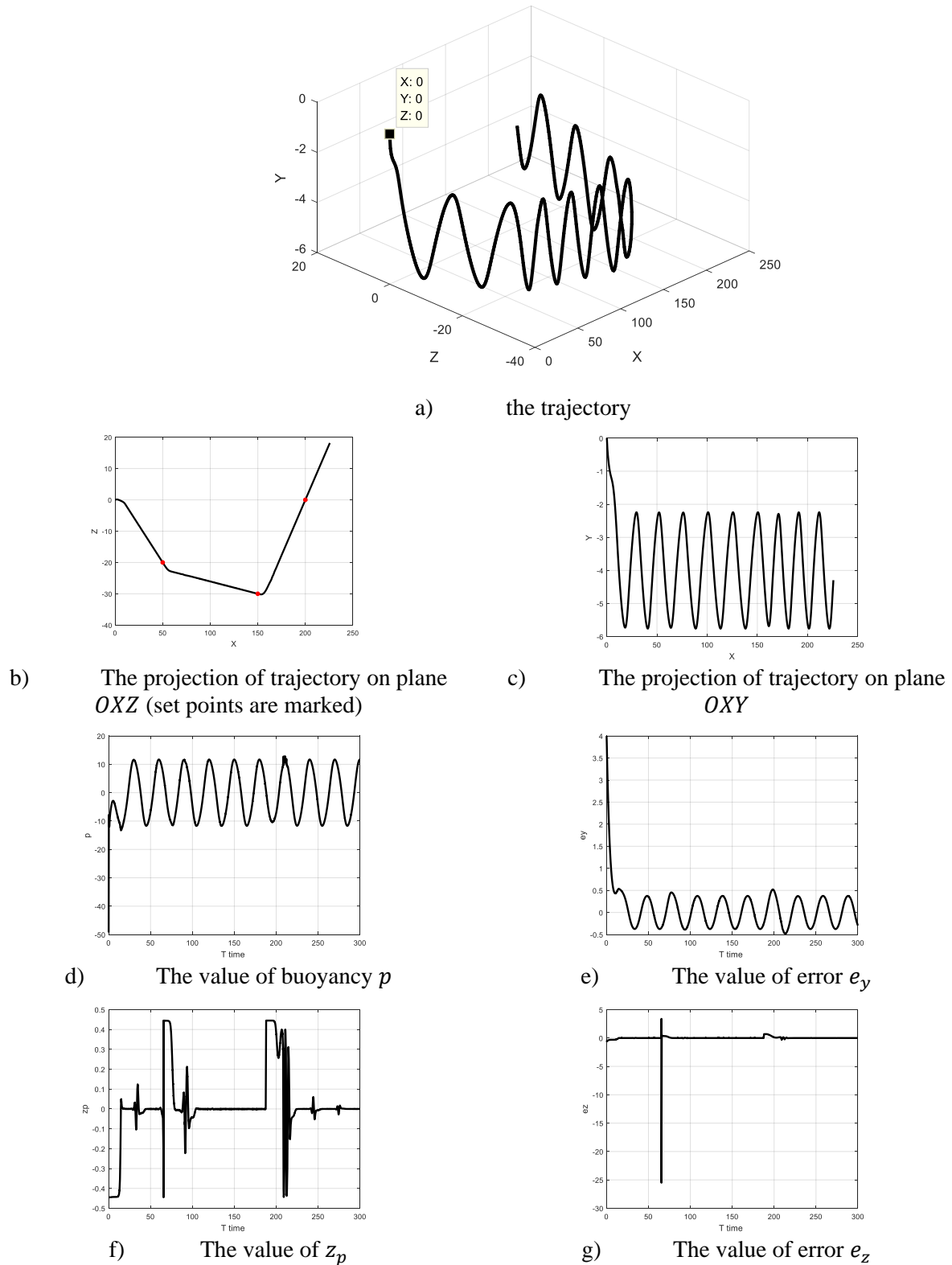


a)         the trajectory



b)       The projection of trajectory on plane $OXZ$ (set points are marked)



c)       The projection of trajectory on plane $OXY$



d)       The value of buoyancy $p$



e)       The value of error $e_y$



f)       The value of $z_p$



g)       The value of error $e_z$

**Figure 7.** Graphs of trajectory via 3 points.

The most prevalent types of tasks for AUVs include search and inspections. Such objectives are conducted by surveys of selected areas of bottom surface and subsequent research of discovered objects. Usually, a typical mission of an AUV consists of area and point surveys. In order to conduct such surveys, the trajectory of a glider can be set as one of the special shapes, for example, meander or zigzag [8].

The meander shape provides an optimal coverage of researched area, because it does not contain overlaps and returns. An example of a meander is shown in Figure 8.
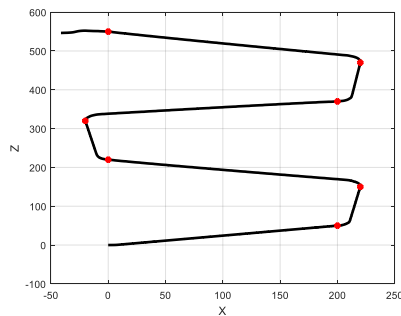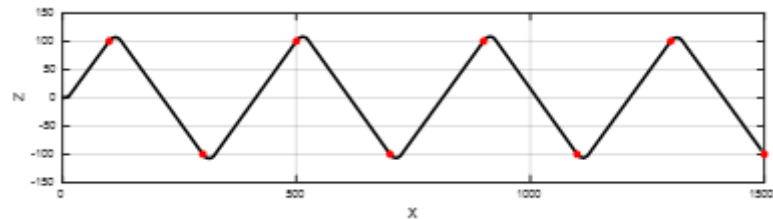


**Figure 8.** A meander-like shape.



**Figure 9.** A zigzag shape.

If it is necessary to examine underwater infrastructure, such as cables and tubes, a zigzag trajectory can be used. An example is shown in Figure 9.

The developed control system can follow the shapes above, which makes it possible to conduct various underwater curves.

## 6. Conclusions

To sum up our work, the trajectories of the glider were obtained at points set in the horizontal plane, taking into account the parameters of the trajectory for movement in the vertical plane. The results obtained make it possible to evaluate the adequacy of the developed control system. The deviation of the system when the glider reaches the set point is minimal, and using the trajectory parameters in the vertical plane, it is possible to control the depth of immersion and speed. The above results of the simulation of coating elements prove the possibility of practical use of the control system when planning glider-type AUV missions. The simulation results are similar to the works [9, 10] and confirm the possibility of using the circuit without the center of mass displacement apparatus.

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