IOP Conference Series: Materials
Science and Engineering

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To cite this article: Asaad M. B. Al-Gharrawi and Husain Ali Abdul-Husain 2020 *IOP Conf. Ser.: Mater. Sci. Eng.* **888** 012010

IOP Conf. Series: Materials Science and Engineering **888** (2020) 012010 doi:10.1088/1757-899X/888/1/012010

Monte carlo simulation for stability of finite slope subjected to earthquake loading

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Abstract. Analysis of finite slopes is a common practical problem which faces the geotechnical engineering. The aim of the slope analysis is to predict the safety factor against shear failure of the slope which is the ratio between the resisting and the disturbing forces developed along an assumed slip surface. Presence of seismic forces due earthquake lead to more critical situation. The deterministic approaches are employed for this purpose. These approaches do not take into account the uncertainty related to soil properties and/or seismic loads in their procedures. In the present work, a probabilistic approach is adopted using the Monte Carlo simulation technique to study the effect of uncertainty due to soil properties (unit weight, angle of internal friction and cohesion) and seismic accelerations (vertical and horizontal). The results of the work demonstrate that the use of probabilistic approach will yield more accurate results for estimating the safety factor and then economic design of the slope. Also, the results reveal that the statistical properties for the soil shear strength parameters have significant impact on the standard deviation of the output (factor of safety), while the soil unit weight and horizontal acceleration have small effect. However, the vertical acceleration has no effect on it. On other hand, the mean of the output distribution does not affect by the statistical parameters of all input variables. All cases simulated in this study give reliability index (RI) less than 3. Hence, RI values indicate that there are unsatisfactory level of safety for the slope subjected to seismic loading. Finally, the soil shear strength parameters have the same positive significant impact on the safety factor. But, the soil properties and the coefficient of the horizontal acceleration should be selected carefully in the stability analysis due to the effect of their uncertainty on the analysis results.

1. Introduction

Geotechnical engineer, in the site, may face some problems about the stability of earth slopes (natural slopes or artificial slopes). These practical problems appear in the construction of railways, highways, earth dams or river-training works. However, natural and artificial slopes are classified into infinite and finite slopes (5).

Due to the gravitational forces affect on the soil mass, progressive disintegration of the soil mass structure will occur. Also, slide movements of the slope may be developed due to the earthquake forces in the seismic areas. This disintegration may cause slowly or suddenly slide failure of the earth slope (7).

The stability analysis of slopes is represented by computing the available margin of safety. This process needs to predict the potential slip surface and compare the disturbing and resisting forces on the assumed slip surface. However, the goal of the geotechnical engineer focused to determine the safety factor based on the slope geometry (height of the slope and inclination angle) and the soil properties (unit weight, cohesion and angle of internal friction).

The assumed slip surface may be planer or curved. The curved slip surface represents the actual failure surface and gives more accurate results when compared with the planer surface. Different procedures of

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slope analysis have been suggested in the literatures and they are, mainly, divided into mass procedures and slices methods (5).
Analysis of geotechnical problems have high uncertainty due to different sources. In general, three main sources of uncertainty are established. The first source is based on the uncertainty of the soil behavior and in turn the slip surface. The soil properties determined based on limiting soil samples and/or low quality of testing data represent the second source. While, the third one relates to the soil properties and its location (12).
All analysis procedures (mass procedures or slices methods) are deterministic methods. It is well known that any variable depends on stochastic variables will be a stochastic one. However, the probability distribution of the dependent variable not necessary to be same as that of the independent variable. Analytical methods, approximate methods or simulation methods can be used to simulate the uncertainty of the input variables. In the stochastic simulation process of the input variables there are different method such as: Monte Carlo method, Latin hypercube sampling method and Rosenblueth's method (9).
The aim of the current study is to simulate the stability problem of finite slope subjected to seismic loading using Monte Carlo simulation method. In order to perform the simulation process, distributions of the input variables have been assumed as proposed in the literatures. Also, the slices method has been used in the analysis process to predict the safety factor against slope failure. The impact of coefficients of variation for all input variables (soil properties and earthquake acceleration) on the resulted probability distribution of the safety factor and the reliability index have been investigated. Finally, sensitivity analysis of the safety factor to the values of input variables has been studied in this study.

2. Analysis of finite slope

As mentioned previously, the goal of geotechnical engineer in the slope analysis is to predict the value of the safety factor against sliding failure of the soil mass within failure surface. The analysis process consists of the following steps (8):
1. Assume a trial slip surface. In the past, the plan slip surface was assumed which is referred to it as Culmann's approximation. It should be noticed that the analysis using this approximation gives good results for approximately vertical slopes. But the extensive researches revealed that the actual failure surface is curve and can be assumed to be as cylindrical one (5).
2. Predict the developed forces on the slip surface which are tends to slid the soil mass downward the slope (to the toe of the slope).
3. Determine the resisting shear strength of the soil which works to prevent the sliding of the soil mass. The soil shear strength is determine based on Mohr-Coulomb criteria.

Generally, in the case of treating the soil mass in the slip surface as a unit, the analysis procedure is called "mass procedure" while the "slices method" is the name of the analysis procedures where the soil mass divided into several vertical parallel slices as illustrated in Figure (1). Each slice in the slices method is treated as an isolated body. The stability of the slice is determined separately.
Analysis of the finite slope subjected to seismic (earthquake) load is calculated by pseudo-static method, plastic deformation method or finite element method. In this study pseudo-static approach has been used. In the this method, the earthquake forces (horizontal and vertical) are converted to equivalent static forces by multiplying the slice's weight by horizontal and vertical seismic coefficients α_h and α_v, respectively. The concept of this method is that a failure surface is assumed and then a factor of safety is determined by comparing the strength necessary to maintain the limiting equilibrium conditions with the available soil strength (11). The analysis steps are as follows:

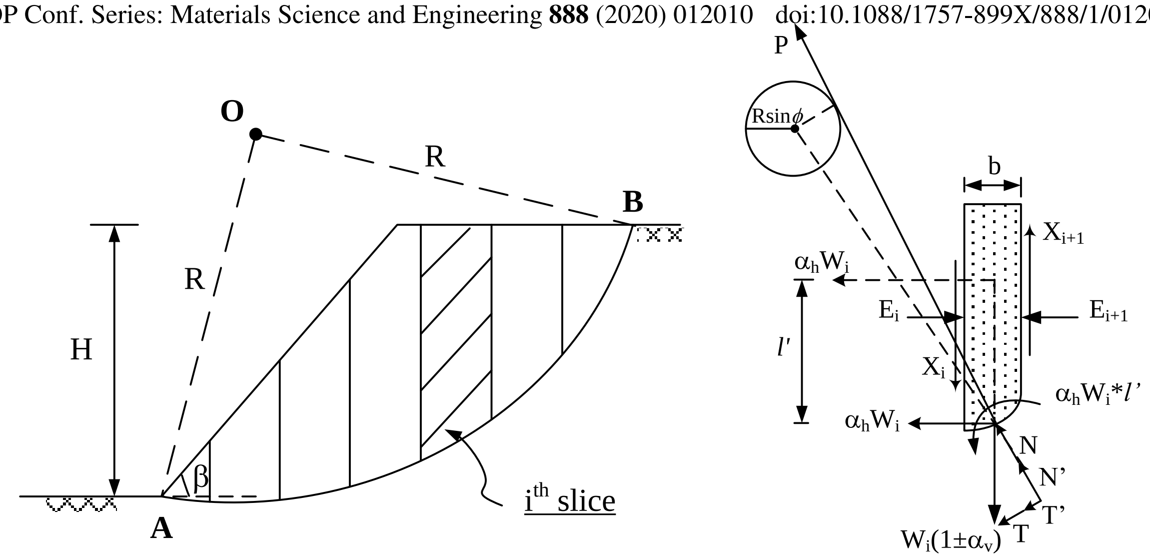

Figure 1. Finite slope with slices within the sliding surface and forces on the ith slice (as adopted by Saran 2010).

1. The soil mass within the failure surface is divided into number of vertical slices usually between six and twelve slices so that the slices have the same width.
2. Determine the forces acting on an ith slice as:
   - Slice's weight = Wi
   - Shear forces on the vertical sides of the slice = Xi and Xi+1
   - Normal forces on the vertical sides of the slice = Ei and Ei+1
   - Force due to soil cohesion along the curved portion of the sliding surface within the slice = c(Δs), where Δs is the curve length.
   - Horizontal force due to seismic acceleration = α_h.Wi
   - Vertical force due to seismic acceleration = $\pm\alpha_v$.Wi
   - Resultant of the friction and reaction forces on the slice base = P which makes an angle ϕ with the normal line.
3. The analysis assumption when the sliding wedge is considered as a unit is:

$$\sum_{i=1}^{n}\left(E_i - E_{i+1}\right) = 0 \text{ and } \sum_{i=1}^{n}\left(X_i - X_{i+1}\right) = 0 \tag{1}$$

4. Taking moments of all mentioned forces about point (O) which represents the center of the assumed failure surface of radius (R), then the resisting moment (MR) and disturbing moment (MD) are:

$$M_R = c(\Delta s)R + P.R.\sin\phi + W_i.\alpha_h.l' \tag{2}$$

$$M_D = (T + T')R \tag{3}$$

For n-slices within the sliding surface and by neglecting ($W_i.\alpha_h.l'$) due to its small value, then:

$$FS = \frac{M_R}{M_D} = \frac{c.S + \left\{\sum_{i=1}^{n}\left(N - N'\right)\right\}\tan\phi}{\sum_{i=1}^{n}\left(T + T'\right)} \tag{4}$$

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Where: N and N' represent the normal components of $W_i(1 \pm \alpha_v)$ and $W_i.\alpha_h$ along the curved portion of slip surface within the slice while T and T' represent the tangential components.

5. Repeat the steps (1) to (4) with different center and radius of slip surface to predict the minimum factor of safety of the slope.

In the present study computer program has been used to carry out the process of finding the minimum factor of safety. However, the program can solve using several methods, the Bishop's method has been select throughout this study (11).

3. Simulation by Monte Carlo Technique

Several techniques can be used in the simulation of stochastic behavior of the input variables. The basic concept of simulation is to simulate some phenomenon and then observe the number of times that some event of interest takes place (12).

Monte Carlo simulation tries to create a random set of values from known or assumed probability distributions of some inputs variables occupied in a certain problem to find out the probability distribution of the output variable. The steps of Monte Carlo simulation in the current work (slope stability analysis) are as follows (1):

1. Define all input variables (deterministic or stochastic) which are used to predict the slope factor of safety. In the present study, slope geometry (slope height and angle of inclination) are considered as deterministic variables. In other hand, soil properties (unit weight, cohesion and friction angle) and coefficients of earthquake acceleration (horizontal and vertical) are considered as stochastic variables which are called input variables of simulation process. The input variables are modeled by their statistical parameters (mean and standard deviation) in addition to probability distribution. Table (1) shows all necessary statistical data for input variables used in the present study.

Table 1. Parameters of the input variables used in the current work.

Variable	Type	Statistical parameters: Mean (μ)	Statistical parameters: Coefficient of variation (COV)	Distribution	Value	Ref.
Unit weight (γ)	Stochastic	16 kN/m^3	5% - 10%	Normal	-	Baecher and Christian (2003)
Friction angle	Stochastic	20^o	5% - 15%	Normal	-	
Cohesion	Stochastic	35 kN/m^2	10% - 30%	Normal	-	
Horizontal acceleration	Stochastic	0.2 g	5% - 15%	Normal	-	Abramson et. al (2002)
vertical acceleration	Stochastic	0.1 g	5% - 15%	Normal	-	
Slope height	Deterministic	-	-	-	20 m	
Slope angle	Deterministic	-	-	-	1(V):2(H)	

2. Assign the output variable of the slope stability analysis which is the factor of safety against sliding failure as demonstrated in the previous section. As it is mentioned previously, the factor of safety is considered as a stochastic variables because it depends on some stochastic variables.

3. Specify the deterministic function used to compute the output variable (factor of safety).

4. Generate randomly a set of input variables and use the deterministic formula to compute the output variable.

5. Repeat the above step (step-4) many times (iterations).
6. The output values for all iterations are used to predict the mean, standard deviation and the probability density function of it.

The main factor related to the simulation process is to assign the number of iterations used in the simulation process. In order to choose the suitable number of iterations throughout this study, four different cases have been analyzed for 100, 1000, 10000 and 100000 iterations. The statistical parameters (mean, standard deviation, coefficient of variation, kurtosis and skewness) of the output variable for each case were computed as shown in Table (2). It can be noted that there is no significant changes in the mean, standard deviation and in turn the coefficient of variation. Kurtosis and skewness can help to establish an initial understanding of the data. Data that followed a normal distribution perfectly would have kurtosis and skewness values of zero. So, the case of 1000 iterations gives the smallest values of kurtosis and skewness coefficients. This conclusion coincides with that stated by Husain (2016) (Abdul-Husain 2016). The histogram and probability distribution of the output variable (factor of safety) for 1000 iterations are illustrated in Figure (2).

Table 2. Statistical parameters of the model output for different number of iterations.

Statistical parameter	No. of iterations			
	100	1000	10000	100000
Mean	1.14	1.12	1.13	1.13
Standard deviation	0.136	0.126	0.131	0.129
COV	11.93 %	11.25 %	11.59 %	11.42 %
Kurtosis	0.384	-0.115	0.064	0.012
Skewness	0.248	0.004	0.159	0.163

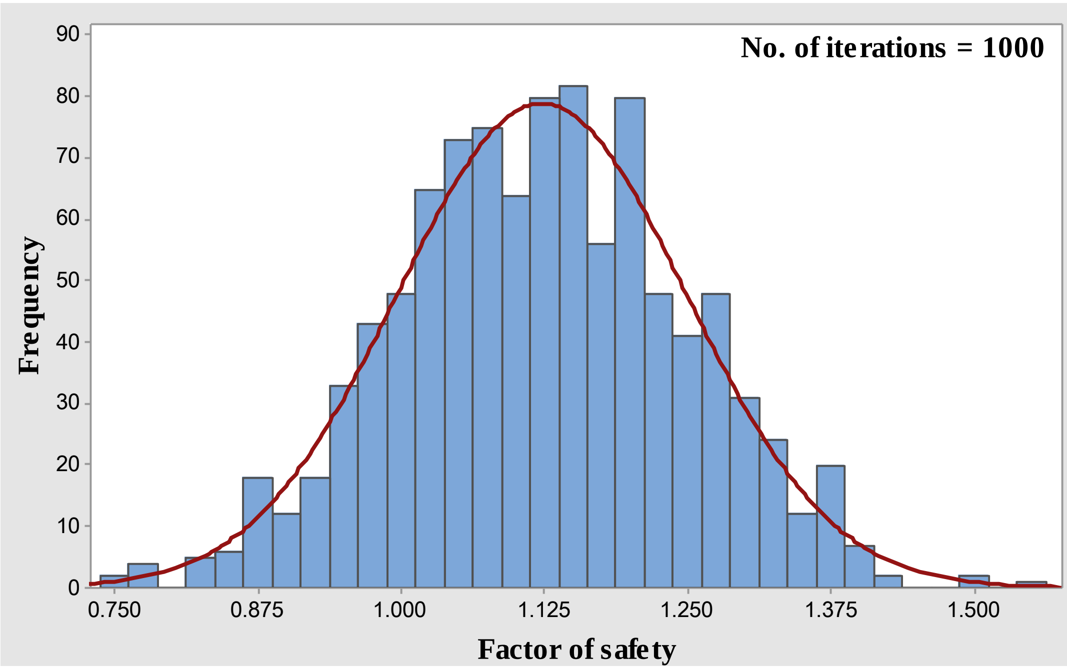

Figure 2. Histogram and probability distribution of model output (factor of safety) for simulation with 1000 iterations.

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4. Results and Discussion

In the present work, the effect of the statistical parameters of the input variables on the resulted probability distribution of the safety factor of the finite slope have been carried out. Also, sensitivity analysis and the influence on the reliability index for different situations have been investigated.

4.1. Effect of coefficient of variations

As it is mentioned in the previous sections, the statistical parameters that affect the results of the simulation process are means, standard deviations and the assumed probability distributions of the input variables. The mean values and the probability distributions have been fixed through the present work. The values of standard deviation have been changed by changing the values of coefficients of variation (COV) of all stochastic variables to check their effect on the resulted slope factor of safety. Several problems have been solved stochastically by changing the values of the coefficients of variations of the input variables (stochastic variables only). The resulted probability distribution of the slope factor of safety have been determined in addition to the statistical parameters of the distribution (mean and standard deviation). Table (3) demonstrates the results of the study. Also, Table (3) contains the percentage of changing in the mean and standard deviation for the factor of safety. It can be noted that the mean of the factor of safety distribution does not affect significantly by changing the coefficient of the variation for all input variables. Also, the standard deviation of the distribution does not change by changing of the coefficient of variations for soil unit weight, coefficient of horizontal acceleration and coefficient of vertical acceleration. While, the standard deviation is affected by changing in the coefficient of variations for soil friction angle and cohesion. The maximum change in safety factor standard deviation is about 20% in the case of increasing or decreasing the soil cohesion coefficient of variation by 10%. While, it is changed by about 12% and 18% when the coefficient of variation of soil friction angle decreased and increased by 5% respectively.
In order to assign the type of the distribution for the safety factor resulting from simulation model, Kolmogorov-Smirnov has been used to check the compatibility of the distribution with the normal distribution. The test has a null hypothesis (H_o) which is stated as the data follows the normal distribution. While, the alternative hypothesis (H_A) is that the data does not follow the normal distribution. Table (3), also, shows the results of the test for all cases. It can be noted that the normal distribution is a suitable distribution for the safety factor at a significant level of 5%. In other words, there is 95% confident that the safety factor of the finite slope subjected to seismic loading follows normal distribution.
Other measures in the probabilistic analysis are the probability of failure (PF) and the reliability index (RI). The probability of failure is defined as the ratio of the failed cases to the total number of cases analyzed in the simulation study .(6)

$$PF = \frac{No.of\ cases\ with\ FS < 1}{Total\ No.of\ cases} * 100 \tag{5}$$

The reliability index measures how many standard deviations the mean safety factor take apart from the critical one. Usually it is recommended that the value of RI not less than 3 for assurance of safety for the slope design.

$$RI = \frac{\mu_{FS} - 1}{\sigma_{FS}} \tag{6}$$

Where: μ_{FS}: is the mean safety factor.

σ_{FS}: is the standard deviation of the safety factor.

Table (4) contains the values of PF and RI for different values of the coefficient of variations for the input variables. It can be seen that PF does not affect by changing the COVs for the soil unit weight and the coefficient of vertical acceleration. While COV for the coefficient of the horizontal acceleration has small effect on it. On the other hand, the COVs for the soil shear parameters (soil friction angle and soil cohesion) have significant impact on the PF value. All cases simulated by Monte Carlo method give RI less than 3. Hence, RI values indicate that there are unsatisfactory level of safety for the slope subjected to seismic loading. (10).

Table 3. Effect of variation for coefficient of variations for all input variables and the results of goodness of fit for the model output.

Variable	COV	μ	% Δμ	σ	% Δσ	K–S (statistic)	Accept H_o*
Soil unit weight	5 %	1.122	0.12	0.123	2.69	0.018	Yes
	7.8 %	1.123	-	0.126	-	0.017	Yes
	10 %	1.124	0.14	0.130	3.18	0.024	Yes
Soil Friction Angle	5 %	1.121	0.17	0.111	12.30	0.020	Yes
	10 %	1.123	-	0.126	-	0.017	Yes
	15 %	1.125	0.21	0.148	17.56	0.013	Yes
Soil Cohesion	10 %	1.123	0.02	0.098	22.48	0.020	Yes
	20 %	1.123	-	0.126	-	0.017	Yes
	30 %	1.123	0.02	0.163	28.87	0.021	Yes
Horizontal acceleration coefficient	5 %	1.124	0.07	0.122	3.30	0.016	Yes
	10 %	1.123	-	0.126	-	0.017	Yes
	15 %	1.123	0.02	0.133	5.19	0.022	Yes
Vertical acceleration coefficient	5 %	1.123	0	0.126	0	0.018	Yes
	10 %	1.123	-	0.126	-	0.017	Yes
	15 %	1.123	0	0.126	0	0.017	Yes

* Critical value of the test is 0.043 for n = 1000.

Table 4. Effect of variation for coefficient of variations for all input variables on the probability of failure and reliability index.

	Coefficient of variation for soil properties								
	Unit weight			Friction angle			Cohesion		
	5%	7.8%	10%	5%	10%	15%	10%	20%	30%
PF (%)	16.5	16.5	16.5	15.4	16.5	20.8	9.7	16.5	23.7
RI	0.989	0.972	0.955	1.091	0.972	0.843	1.254	0.972	0.753

	Coefficient of variation for acceleration coefficients					
	Horizontal coefficient			Vertical coefficient		
	5%	10%	15%	5%	10%	15%
PF (%)	16	16.5	18.1	16.5	16.5	16.5
RI	1.012	0.972	0.922	0.972	0.972	0.972

4.2. Sensitivity analysis

In order to investigate the effect of the input variables on the safety factor, sensitivity analysis has been performed. All variables are constant except one which is varied between its minimum and maximum values with uniform increment and safety factors are computed in each case.(12)

Figure (3) illustrates the results of the sensitivity analysis. It can be noticed that the coefficient of the vertical acceleration has no effect on the safety factor of the finite slope. Also, the coefficient of the horizontal acceleration and soil unit weight have approximately the same moderate inverse effect on the safety factor of the slope. Finally, the soil shear strength parameters have the same positive significant impact on the safety factor. So, the soil properties and the coefficient of the horizontal acceleration should selected carefully in the stability analysis due to their effect on the results.

5. Deterministic or probabilistic approach

The distribution of cumulative probability and the safety factor of the simulation process is illustrated in Figure (4). A sample problem of a finite slope of the same mean properties shown in Table (1) has been analyzed deterministically. The safety factor of the slope is found to be 1.24. While, the probabilistic safety factor for 90% and 95% confidence levels with the aid of the Figure (4) are 1.42 and 1.48 respectively. From Figure (4), the confidence level corresponding to the computed deterministic safety factor is approximately 50%. It is clearly noticed that the use of deterministic approach gives more conservative result than that of probabilistic approach. Hence, the use of probabilistic approach in the analysis of the finite slope subjected to seismic loading yields economical design of slope compared with the deterministic approach.

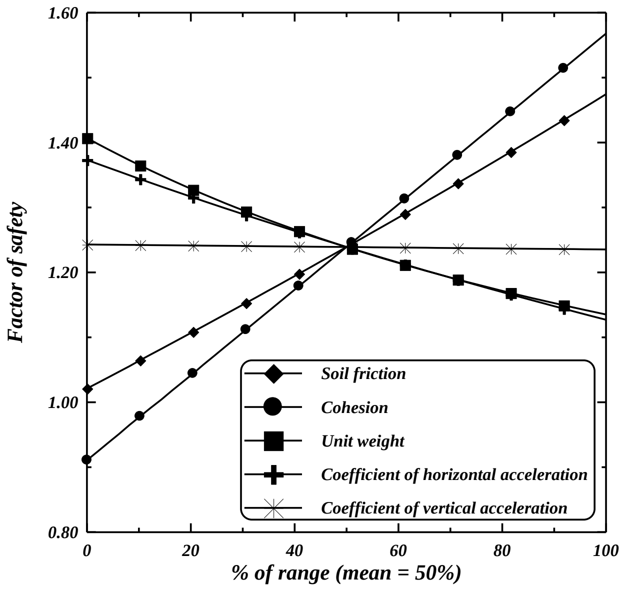

Figure 3. Results of sensitivity analysis of the factor of safety for finite slope.

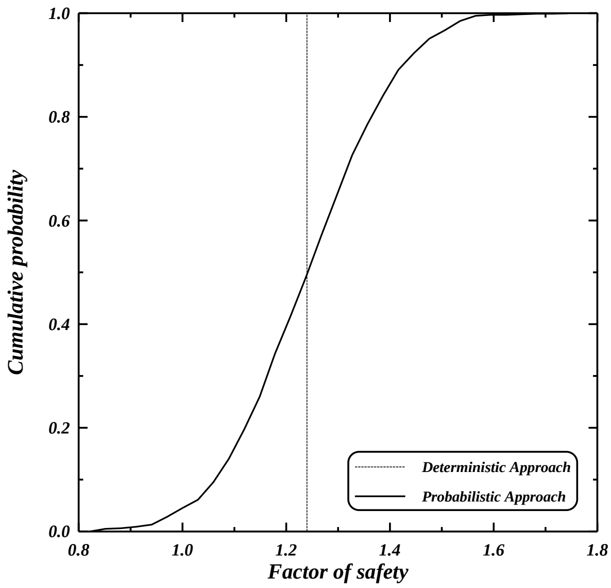

Figure 4. Cumulative probability distribution incorporating parameter uncertainty for the numerical example.

6. Conclusions

One of the important problem in the geotechnical engineering is the stability of the finite slopes specially when an seismic loads are associated. All proposed methods of slope satiability are deterministic methods while there are uncertainties associated in this problem for some input variables. The present work dealt with the soil properties and earthquake acceleration as stochastic variables. Based on the results of the study the following points can be stated:

1. Using of deterministic approach will give underestimate amount of safety factor. The use of probabilistic approach is more appropriate to conduct safe and economic design.
2. The variation of the coefficient of variations of the soil shear strength parameters have significant impact on the properties (statistical parameters) of the safety factor of the slope. While the other variable have small effect and the horizontal acceleration has no effect.
3. The value of safety factor is more sensitive to the variation of shear strength parameters and relates with them positively while it is less sensitive to the variation of the unit weight and horizontal acceleration and relates with them negatively. And the vertical acceleration has no effect on the safety factor.
4. All cases simulated in this study give reliability index (RI) less than 3. Hence, RI values indicate that there are unsatisfactory level of safety for the slope subjected to seismic loading.

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