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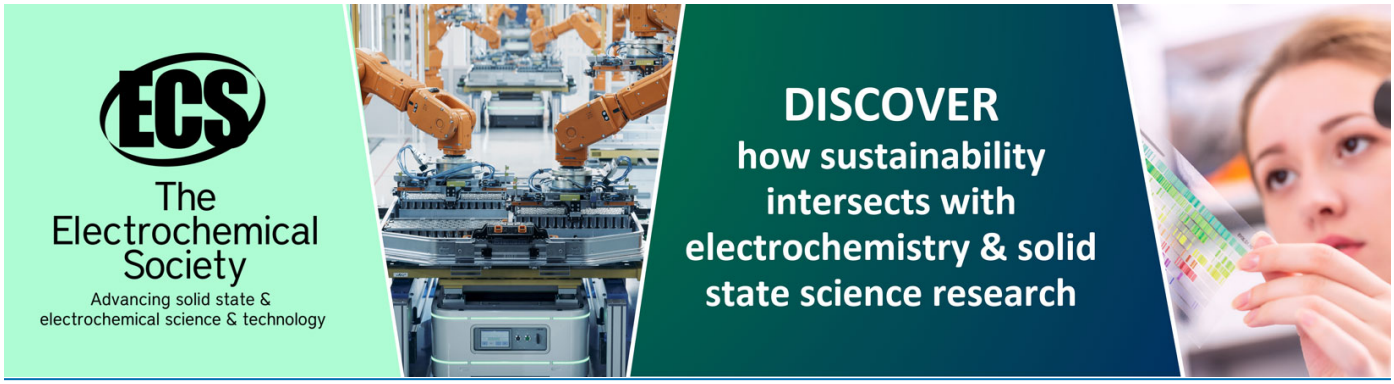

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Numerical Solution of Single and Two Bubbles Moving in Quiescent Liquid Using Euler-Lagrange Approach

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Abstract

It is known that the bubbles dynamics are very important and dominant property in multiphase airlift systems. The major benefit came from understanding visually and numerically these dynamics that lead to improvement of bubbles distribution and a well-mixed mixture inside reactors. Hence, Euler-Lagrange approach has been used to study bubbles dynamics. By using the two-dimensional Reynolds-averaged Navier–Stokes equations that reinforces turbulence model the continuous phase velocity is calculated. The coupling between the phases was take into consideration through source terms of momentum and the source terms in the equations (ε and k), which include the effect of wake-generated turbulence by means consistent Lagrangian-similar terms. by utilizing the motion equations taking into account added mass, drag, pressure, gravity, wall, and the transverse lift force the Bubble motion was calculated. So as to determine the importance relative for the different physical phenomena's included in the model., a modified particle source in cell (PIC) is introduced, where the effect of bubble is accounted not only where the center of bubble is located but also to all the cells that containing bubble.

Keyword: Euler-Lagrange Approach; Particle Tracking; Two phase Flow; Bubbles,Hydrodynamics; CFD

1. Introduction

Multiphase reactors are of vital importance to chemical and oil industry. The reactions between gas and liquid are found in a variety of chemical and biochemical processes.

Computational Fluid Dynamics (CFD), which has been common for many decades in several fields, are gaining increasing acceptance in chemical engineering community. CFD aims to solve the transport equations that can successfully describe the conservation of mass, momentum, energy and other properties pertaining to flow, using numerical techniques for a given geometry with certain boundary conditions, employing models for turbulence or mass and heat transfer relevant for the assignment under consideration. It was an significant tool in aerospace and auto motives industry for a long time, that it has mostly replaced the time-consuming expensive wind tunnel experiment where in these applications single-phase flows are prevalent. Applications in most reactors involve multiphase flows, where the numerical treatment and modeling encounter extra challenges. Therefore, theoretical and numerical studies of hydrodynamics, mass and heat transfer for the optimization and design of

multiphase reactors using CFD have acquired wide interest through the latest decade, due to increasing computational power available has enabled computations previously considered unfeasible[1].

2. Approach Of Lagrange

Approach of Euler-Lagrange is a hybrid technique, where the continuous phase are calculated on a fixed grid in an Eulerian frame and the dispersed phase are simulated in a Lagrangian fashion by tracking a large number of computational particles through the flow field. A converged fully-coupled solution of the two-phase flow system is reached by sequentially solving the Eulerian and Lagrangian part, accounting for the source- or coupling-terms in the conversation equations of the fluid phase; i.e., the effect of particles on the fluid flow. Calculations of turbulent flow fields can be done by applying direct numerical simulation DNS, large eddy simulation LES or Reynolda average Navier-Stokes RANS Incorporation with a suitable model of turbulence. Based on that and the considered flow configuration, the coupled Euler-Lagrange calculations are done fully unsteady, quasi-unsteady or steady. Approach of Euler-Lagrange is only applicable to multiphase flows with dispersed particles (bubbles or droplets, solid particles) which are treated as point-masses. The great advantage of the Lagrangian approach is that the separated nature of the particles is maintained, allowing a detailed modelling of all pertinent elementary processes (e.g., collision of particle, collisions of inter-particle, agglomeration or coalescence) and in addition, the particle size distribution can be easily resolved [2].

3. Case Study

Studied single and two bubbles numerically using volume of fluid approach. This approach is based on volume tracking method. Where the interface between liquid and +--*gas should be marked with a function and tracked with time. It includes of two intimately coupled parts : a part that tracks the gas-liquid interface through the Eulerian mesh and maintains an accurate and sharp representation of this interface, and a part that solves for the gas and liquid phase flow field [3] .

To study the time-dependent behavior of large gas bubbles in a more fundamental way, a CFD model based on the volume of fluid method VOF concept was developed [4]. This two-dimensional, finite difference volume tracking model resolves the time-dependent movement of the liquid and gas phases, and of the interface separating the two phases. Due to its advanced interface tracking scheme, the model is able to account for substantial changes in the topology of the gas-liquid interface induced by the relative liquid motion. This particular capability allows a detailed study of bubble formation, coalescence and breakup.

A. For single bubble the settings are shown in Table (1).

Table 1. Single bubble properties and current [3]

Property	(Delnoij, 2001) specifications VOF approach	Current specifications Euler-Lagrange approach
Mesh size	(0.005 × 0.005) m	(0.004 × 0.004) m
DT(time step)	1.0×10^{-5} s	0.01 s
Water liquid density	1000 kg /m^3	1000 kg /m
Air gas density	1.2 kg /m^3	1.2 kg /m

Water liquid viscosity	8.9×10^{-4} kg / m.s	8.9×10 kg / m.s
Height	0.5 m	1.6 m
Width	0.25 m	0.2 m
Bubble size	5 cm	0.004 mm

B. ***For two adjacent bubbles Delnoij's setting is*** shown in Table (2).

Table 2. Two bubble properties [3]

Property	(Delnoij, 2001) specifications VOF approach	Current specifications Euler-Lagrange approach
Mesh size	(0.001 × 0.001) m	(0.004 × 0.004) m
DT(time step)	1.0×10^{-5} s	0.01 s
Water liquid density	1000 kg /m^3	1000 kg /m^3
Air gas density	1.2 kg /m^3	1.2 kg /m^3
Water liquid viscosity	8.9×10^{-4} kg / m.s	8.9×10^{-4} kg / m.s
Bubble size	5 cm	0.004 mm
Height	0.5 m	1.6 m
Width	0.25 m	0.2 m

Delnoij's simulation to single and two adjacent bubbles was accurate and precise, and it was possible to capture the bubble motion and liquid response to the bubble existence; however, liquid velocity profile is the most important property that was looked for. Moreover, Velocity profile was used to validate Euler-Lagrange model, where the main goal of this comparison is to check the liquid response to the bubble motion and two- phase interaction. A general picture about the liquid velocity profile is what we are looking for to be compared with that gained from Euler-Lagrange.

4. Euler-Lagrange Approach Modeling

The approach of Euler-Lagrange depended on the mixture-theory is utilized where in the bubble-fluid interactions are captured through inter-phase exchange of momentum as well as difference in the local void fractions of fluid. The formula of mathematical for the disperse and continuum phases was described as follows [5].

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4.1 Continuous phase governing equations

By using the unsteady Reynolds averaged conservation equations the flow of fluid is determined. These equations are utilizing the well-known ε–k turbulence model [6], extended by accounting for the influence of the dispersed phase. The time-dependent conservation equations for the fluid in a two-dimensional flow can be written in the general form :

$$\frac{\partial}{\partial t}(\epsilon\rho\phi) + \nabla(\epsilon\rho u\emptyset) = \nabla\left(\epsilon\Gamma_{\phi}\nabla\emptyset\right) + \epsilon\, S_{\emptyset} + S_{\emptyset p} \qquad [1]$$

Where; , Γ_{ϕ} , $S_{\emptyset}$, $S_{\emptyset p}$ can be predicted from Table (3) below

Table 3. Summary of terms in the general equation for the different variables that describe the fluid phase in Cartesian flow (Mahdavimanesh, et al., 2013)

	$\emptyset$	Γ	S_ϕ	$S_{\phi p}$
Continuity	1	0	0	0
X-Momentum	u	$\mu + \mu_t$	$\frac{\partial}{\partial x}\left(\Gamma\frac{\partial \mathrm{u}}{\partial \mathrm{x}}\right) + \frac{\partial}{\partial y}\left(\Gamma\frac{\partial \mathrm{v}}{\partial \mathrm{x}}\right)$	$S_{up} = -\frac{1}{V_{cv}\,\Delta t_E}\sum_{k=1} m_k \sum_{n=1}\left([u_b]_k^{n+1} - [u_b]_k^n\right)$
Y-Momentum	v	$\mu + \mu_t$	$\frac{\partial}{\partial x}\left(\Gamma\frac{\partial \mathrm{u}}{\partial \mathrm{y}}\right) + \frac{\partial}{\partial y}\left(\Gamma\frac{\partial \mathrm{v}}{\partial \mathrm{y}}\right) - \frac{\partial p}{\partial y} + \rho g$	$S_{vp} = -\frac{1}{V_{cv}\,\Delta t_E}\sum_{k=1} m_k \sum_{n=1}\left\{\left([u_b]_k^{n+1} - [u_b]_k^n\right) - g\left(1 - \frac{\rho_l}{\rho_b}\right)\Delta t_L\right\}$
Turbulent Kinetic Energy	k	$\mu + \frac{\mu_t}{\sigma_k}$	$G_k - \rho\varepsilon$	$S_{kp} = u_b\,S_{ub} + v_b\,S_{vb} - (u_L\,S_{ub} + v_L\,S_{vb})$
Dissipation Rate	ε	$\mu + \frac{\mu_t}{\sigma_\varepsilon}$	$\frac{\varepsilon}{k}(C_1 G_k - C_2\rho\varepsilon)$	$S_{\varepsilon p} = C_3\,\frac{\varepsilon}{k}\,S_{kp}$

Where:

$$G_k = \mu_t\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2\right\}$$

$$\mu_t = \rho\,C_\mu\,\frac{k^2}{\varepsilon}$$

C_1 =1.44 , C_2 = 1.44 , C_3 =1.87 , C_μ= 0.09 , σ_k =1.0 , σ_ε = 1.3

4.1 Disperse phase governing equations

The approach of Lagrangian for the simulation of the disperse phase is depended on Newton's motion equation . The motion of each individual bubble is calculated by using the motion equations in the frame of Lagrangian.. The position and momentum equations [7] are given as:

$$\frac{d}{dt}x_b = u_b \qquad [2]$$

$$m_b \frac{d}{dt}u_b = \sum F \qquad [3]$$

Where : $\sum F$ is the net forces acting on single bubble

This set of equations are in x-direction, and same set in y-direction should be solved also to determine the (y position and v velocity) for each bubble.

4.2 Interacting forces

The net force acting on each individual bubble is computed by considering all the relevant forces. The net force is composed of separate, uncoupled contributions because of the forces of pressure, gravity, lift, drag, virtual mass and wall forces, respectively as in Fig.(1) [8,9].

$$\sum F = F_p + F_g + F_l + F_D + F_{vm} + F_w \qquad [4]$$

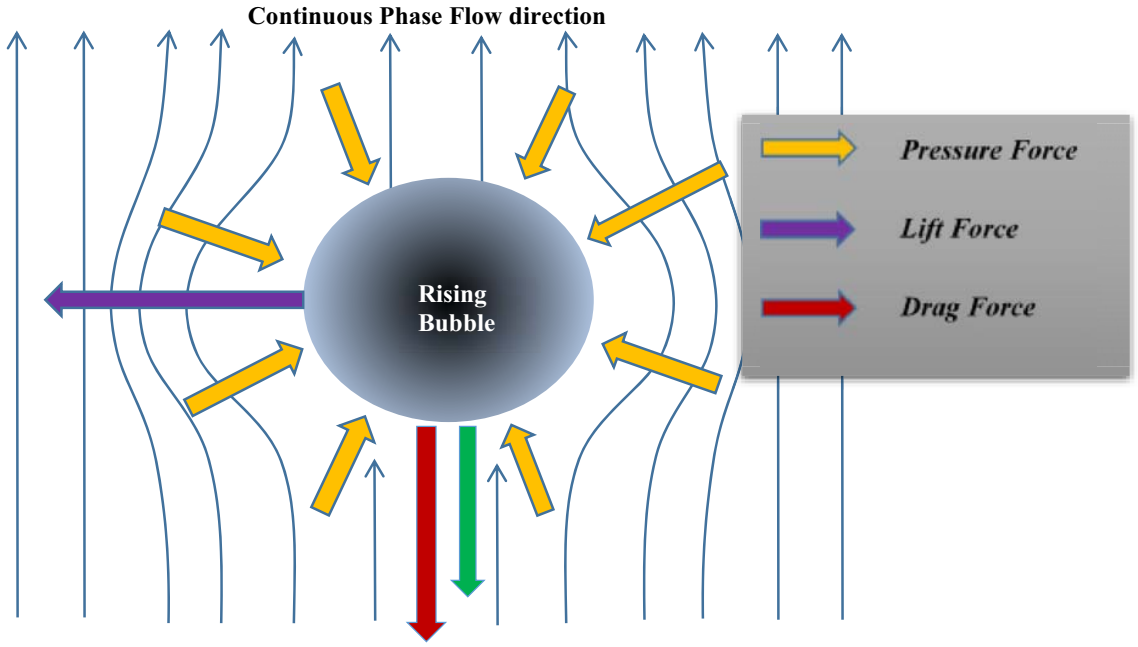


Figure 1. Net forces acting on single bubble in continuous liquid

Where,

Force of gravity; $F_g = (1 - \frac{\rho_l}{\rho_b})\, m_b\, g$ [5]

Pressure force; $F_p = \frac{\rho_l}{\rho_b} m_b \frac{Du}{Dt}$ [6]

Lift force; $F_l = -\frac{\rho_l}{\rho_b}\, m_b\, C_L\, (u_b - u_l) \times \omega$ [7]

Drag force; $F_D = -\frac{3}{4}\frac{\rho_l}{\rho_b D_b} m_b\, C_D (u_b - u_l)|u_b - u_l|$ [8]

Virtual Mass; $F_{vm} = -\frac{\rho_l}{\rho_b D_b}\, m_b\, C_{vm}\, \left(\frac{du_b}{dT_b} - \frac{Du_l}{DT_l}\right)$ [9]

Wall force; $F_w = -\frac{3}{\pi}\frac{\rho_l}{\rho_b D_b^2} m_b C_w \left[\frac{1}{y^2} - \frac{1}{(R-y)^2}\right]\left|(u_b - u_l).n_y\right|^2 n_x$ [10]

Several models have been proposed to model forces coefficients by several researchers. In this paper, [10] model had been used. Moreover, all these models depend mainly on dimensionless numbers that are[5]:

Reynolds number $Re = \frac{\rho_l D_b |u_l - u_b|}{\mu_l}$ [11]

Morton number $Mo = \frac{g\,\mu_l^4\,(\rho_l - \rho_b)}{\rho_l^2\,\sigma^3}$ [12]

Eotvous number $Eo = \frac{g\,{D_b}^2\,(\rho_l - \rho_b)}{\sigma}$ [13]

Coefficients model are as follows [10]:

- *Drag coefficient* $C_D = \max\left\{min\left[\frac{16}{Re}(1 + 0.15Re^{0.687}), \frac{48}{Re}\right], \frac{8}{3}\frac{Eo}{Eo+4}\right\}$ [14]
- *Lift coefficient;* for 1.39 < Eo < 5.74,5.5 < log (Mo) < -2.8 and depends on the Eo No. rang is as follows:

$$C_L = \begin{cases} \min[0.288\tanh(0.121\mathrm{Re}), f(\mathrm{Eo}_d)], & \mathrm{Eo}_d < 4 \\ f(\mathrm{Eo}_d), & 4 < \mathrm{Eo}_d \le 10 \\ -0.29, & \mathrm{Eo}_d > 10 \end{cases} \quad [15]$$

$$f(\mathrm{Eo_d}) = 0.00105\,\mathrm{Eo_d^3} - 0.0159\,\mathrm{Eo_d^2} - 0.0204\,\mathrm{Eo_d} + 0.474 \quad [16]$$

$$\mathrm{Eo_d} = \frac{\mathrm{Eo}}{\mathrm{E}^{2/3}}, \quad \mathrm{E} = \frac{1}{1+0.163\,\mathrm{Eo}^{0.757}} \quad [17]$$

- Wall coefficient C_W is calculated according to a correlation derived by [10] and is given by:

$$C_w = \begin{cases} \exp(-0.933\,Eo + 0.179), & 1 \le Eo \le 5 \\ 0.007Eo + 0.04, & 5 < Eo \le 33 \end{cases} \quad [18]$$

4.3 Model of Coalescence

The mechanism of bubble coalescence was combination into Euler–Lagrange modeling by Van den Hengle et al.,2005 and Sommerfeld et al., 2003[11, 2]. Sommerfeld, et al., 2003[2] predicted the Coalescence through compare the film drainage time with the contact time. In the approach adopted by Van den Hengle et al.,2005[11], the process of coalescence was predicted utilizing a stochastic way depend on the model of [12, 13]. In this search, the coalescence process is predicted by dividing Lagrange time step to smaller time steps $\delta t_{a,b}$ (i.e. $\delta t_{a,b} = \delta t_{bub}/10$), in each time step ($\delta t_{a,b}$) the bubbles velocity will be calculated and the bubbles location will advanced according to Eq. (4.3) and Eq. (4.4). after the bubbles position are calculated through coalescence time step a position test will carried out, if the distance between the new bubbles position is less than the sum of the bubbles radius then the coalescence will occur otherwise, no coalescence will occur and the bubbles velocity and position will be calculated in the next coalescence time step. If there is no coalescence through the current time step then the bubbles velocity and positions will advanced according to Lagrange time step and so on to the next test. The coalescence test is represented in the following algorithm:

Initialize by setting $t_{a,b}$ (coalescence) = t_{bub}(Lagrange).

Calculate interphase bubbles forces and bubbles velocity$\left[\sum F_y, \sum F_x, v_y, v_x\right]$.

Update the bubbles position$[y_b , x_b]$.
Test the new bubbles position.
If $|y_b - x_b| \leq D_b$ then.
Coalescence will occur and the new bubble properties are predicted from Table (4). Proceed to the next time step δt_{bub}(Lagrange).
If $|y_b - x_b| > D_b$ then.
There is no coalescence, proceed to the next time step $\delta t_{a,b}$ (coalescence).
For simplicity, the contact time and film breakage time is neglected also this process is accounted for two bubbles simulation.

The formed bubble diameter is calculated from the formed bubble volume form Table (4) where, the bubble diameter will be :

Table 4. Bubble properties that change through a coalescence event [2]

$r = \sqrt[3]{\frac{3}{\pi 4}\, v_c}$ [19]

Parameters	Before the coalescence	After the coalescence
Index	a,b	c
Mass	ma , mb	Mc = ma + mb
Volume	Va , Vb	Vc = Va + Vb
Position	Xa , Xb	Xc=$\frac{X_a m_a + X_b m_b}{M_c}$
Velocity	Ua , Ub	Uc =$\frac{u_a m_a + u_b m_b}{M_c}$

4.4 Coupling of interphase
The coupling between the liquid and gas phases show through the volume fraction of liquid and the transfer of interphase momentum. Because the phase of liquid and the bubbles are determined in various frames of reference (Eulerian and Lagrangian, respectively), a mapping method that couples the two reference frames is required. This mapping method transfer the bubble quantities of Lagrangian to the grid of Eulerian, that are in demand as closing of the phase liquid equations and correct versa (Lagrange to Euler and Euler to Lagrange).
Kitagawa, et al., 2001 [14] gave the following criteria for the mapping function:

A. It must be a smooth function, i.e. the first derivatives must be continuous.
B. It must have an absolute maximum around the position where the variable is transferred.
C. It must have a finite domain, for practical causes. The function must be zero at the boundaries of the domain.
D. The integral of the function over the entire domain must equal to unity to secure the conservation of variable being transferred

Kitagawa, et al., 2001[14] suggest to utilize a template function of Lagrangian that transform the dispersed phase volume fraction to a spatially differentiable distribution utilizing functions of goniometric . After the successful application of the template technique for Lagrangian in the Euler–Lagrange frame the sin wave function has been used as follows:

$$w(l) = w(y - y_b) = \frac{sin\left(\frac{2\pi}{\Delta y}.(y - y_b) - \frac{\pi}{2}\right) + 1}{2} \qquad [20]$$

Figure (2) schematically demonstrations how the Euler-Lagrange two-way coupling is executed. at the bubble center the template function was constructed (l). This template is moving along with the bubble. In any computational cell (j) the integral of this function ∫_Ωj▒w(l) dΩ represent the effect of bubble (l) on cell (j) or the effect of the Eulerian value in cell (j) on bubble (). Note that in 2D space the integral is calculate as follows:

$$\int_{\Omega j} w(l)\, d\Omega = \iint_{x,y} w(x - x_b) w(y - y_b)\, dx\, dy \qquad [21]$$

The volume fraction of liquid in computational cell(j) is determined utilizing the formula below;

$$\varepsilon_l(j) = 1 - \frac{\sum V_b(l) \int_{\Omega j} w(l) d\Omega}{V_{cell}} \qquad [22]$$

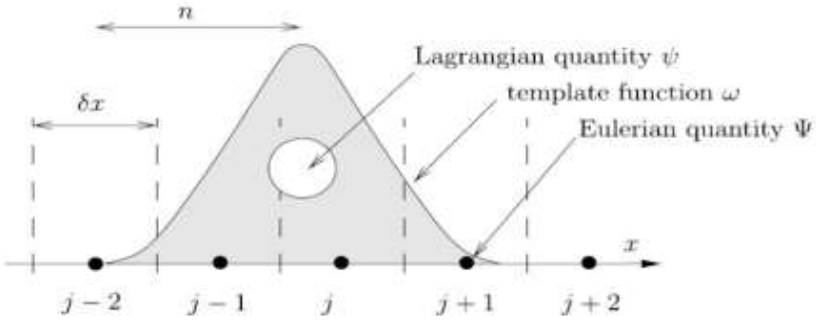


Figure 2. Lagrangian and Eulerian two-way coupling utilizing a template window function.

5.1 Single Bubble Simulation

For single gas bubble moving in a quiescent liquid, Delnoij, 2001[3] studied the behavior of a large single bubble moving in a column containing liquid. Using (interface tracking method) volume of fluid approach, which is known as "VOF", to simulate the bubble shape inside the column taking into account the surface tension as a dominant force between two phases, which represent the momentum transfer. Delnoij, 2001[3] obtained the shape of bubble and respond of liquid that containing the gas phase accurately.

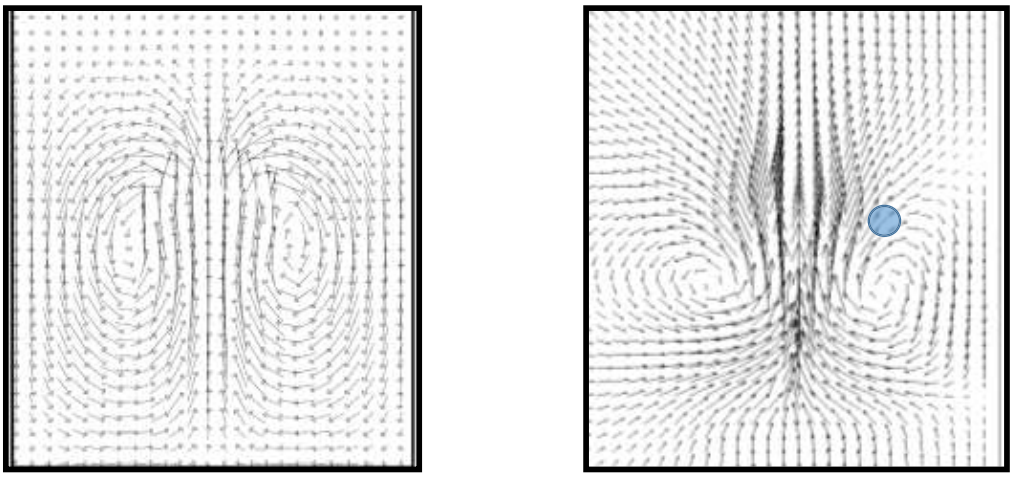
In present work, Euler-Lagrange (PIC) method was used to simulate a single gas bubble moving in a containing liquid. The momentum exchange was represented by the forces acting on both phases, these forces are pressure force, gravity force, virtual mass force, drag force and lift force.

"VOF" approach tracks the interface between bubbles(air) and liquid(water) for that, time step was small and thus it cost in time of calculation, where his calculation takes about sixteen hours to obtain the desired results (Note: the workstation that used in Delnoij, 2001[3] calculations was not in the efficiency and ability of nowadays computers).

While in present work, simulation gives the same accuracy. Where, a parcel of bubbles are considered as a single bubble moving together. This way reduces the number of bubbles that should be tracked and it is an efficient technique.

There is a huge difference in time of calculation between two approaches. The reason for this difference is for computer developments, and the procedure of each approach, where VOF in fact needs to track the interface between two phases along the surface. Moreover, if a breakup happens, that's mean an extra tracking and thus more time consumptions.

However, the VOF approach can simulate a large bubble size and in some cases it may represent a number of bubbles as one computational bubble, in this case, the calculation time is reduced. This assumption is applicable for a high void fraction of gas and for dense dispersed phase.



Delnoij simulation for single bubble VOF (Interface Tracking Method) at time t=0.6 s

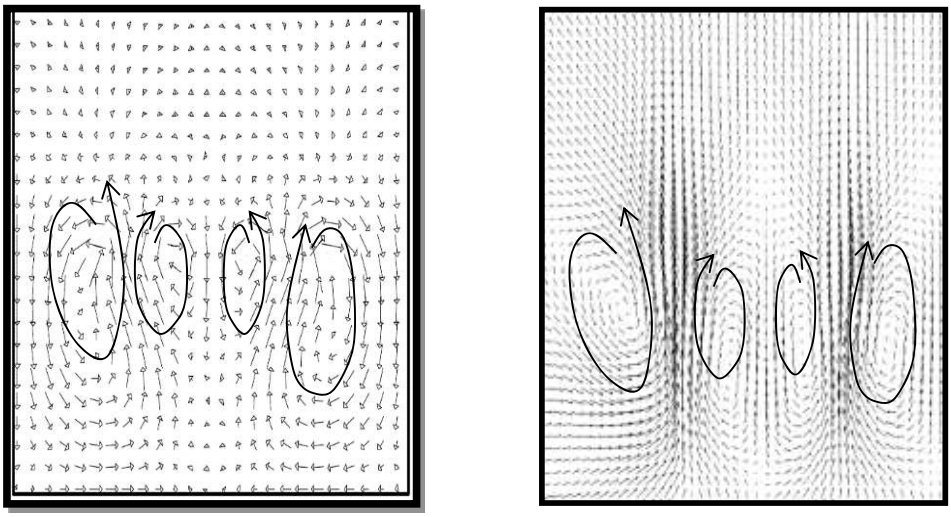
Current simulation for single bubble E-L (PIC method) at time t=0.6 s

Figure 3. Single gas bubble moving in quiescent liquid

Figure (3) shows the two -phase interaction and the effect of each phase on other while the bubble travelling along the trajectory. The liquid is respond to this motion through the momentum transfer from bubble to liquid, however the bubble (speed, position and shape) is effected by the liquid through momentum transfer from the liquid to the bubble. Forces that bubble exerted on the liquid are accounted and hold in the momentum equation as source terms, which is acting as a driving force that makes the liquid moves. Vortices construct due to the forces that bubble exerted on liquid and computed as source terms added to the equations of turbulent kinetic energy and dissipation of kinetic energy. The main objective is to compare between the two simulations and the way that the liquid respond to bubble existence. It is obvious there is a vortex to the right and lift of the bubble center in both approaches these vortexes are duo to the bubble motion. The vortexes in E-L approach are more denes and clear than the VOF approach because there are different in bubble size and the way that each approach treat the momentum exchange between two phases. In general, both approaches have captured the same direction of the liquid motion and the vortexes position and direction duo to motion of bubble (see Fig. (3)), and it seems to be agree to each other in general.

5.2 Two Bubbles Simulation

The main purpose of studying two bubbles moving in the liquid is to take a close look on the mechanism of interaction of bubbles with the liquid and bubble with bubble, where the present of more than one bubble will affect not only on the containing liquid, but also on neighboring bubbles. This effect would not be noticed clearly in multi-bubble system. Moreover, an easier type of coalescence will be studied and discussed. Delnoij, 2001[3] also studied two bubbles moving in a quiescent liquid. Once again, VOF was used in the simulation, considering the surface tension as a driving force for momentum transfer between two phases. The present investigation on two bubbles moving in a quiescent liquid extends to validate the quality of solution with the numerical and experimental results of Delnoij, 2001 and to study the effect of bubbles on each other's.



Delnoij simulation for two bubbles VOF (Interface Tracking Method) at time t=0.18 s

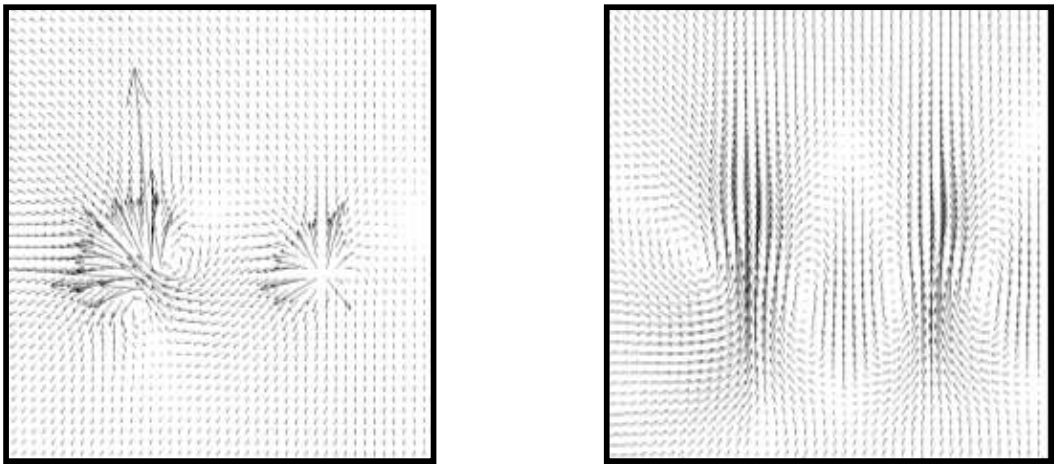
Current simulation for two bubbles E-L (PIC method) at time t=0.18 s

Figure 4. Two gas bubbles moving in a quiescent liquid

Figure (4) shows the validation of the Euler-Lagrange approach in the simulation of bubbles moving in liquid. The two bubbles are adjacent and moving in the same direction and their effect on the carrying fluid is obvious from the vortex that created due to the momentum and kinetic energy exchange. Four vortexes are created due to the motion of two bubbles. The outside and inside vortexes, the outside mean the vortexes that located in the lift side of left bubble and in the right side of the right bubble. The outside vortexes are bigger than the inside vortexes, that because the fluid that exist outside of outer vortexes has more space and freedom to move while the high turbulence, less space and the bubbles effect have reduced the size of the inside vortexes. Once again, the main objective of this comparison is to capture the vortexes position and direction also direction of fluid motion that created due to bubbles motion. It seems that the two simulations have gave the same respond of fluid to the bubbles motion.

5.3Bubbles-liquid interphase affects

The investigation takes also the influence of kinetic energy exchange and momentum, the following figures show the influence of bubbles present on the liquid, these effects is a result to the forces that bubbles exerted to the liquid and to the size of bubbles (void fraction). In this section the forces effect are studied and tested as in Fig. (5a) and Fig. (5b).



(a)Two bubbles E-L (PIC method) No Exchange in Momentum and Kinetic Energy

(b)Two bubbles E-L (PIC method) Momentum and kinetic Energy Exchange

Figure 5. Two bubbles moving in a quiescent liquid due to effect of momentum and kinetic energy

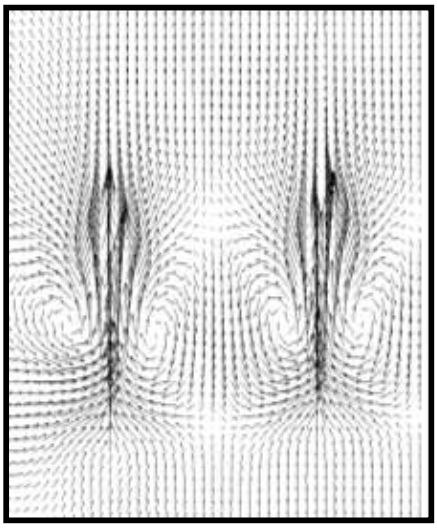
Figure (5) depicts the effect of forces that bubbles apply on the liquid; where Fig. (5a) shows no forces affect on the liquid while in Fig. (5b), the forces and kinetic energy have the major effect on the fluid that containing the bubbles. There are an obvious difference between the simulations (i.e. Fig. (5a) and Fig. (5b)). in case (a), where the forces that bubbles apply on the liquid are neglected, the bubbles size (void fraction) was the only effect of the bubbles existence on the liquid for this liquid respond to the motion of bubbles were not understandable. Moreover, the bubbles look like in rest and do not moving that is clear from liquid respond to the bubbles. In case (b), when the forces were accounted, the results shows the motion of bubbles have a major effect on liquid through the respond of liquid to the motion of bubbles. In addition, vortexes are formed around the bubbles when these forces are accounted. These vortexes did not appear in case (a) when the forces were neglected.

5.4 Bubbles-liquid and bubbles-bubbles interphase effects

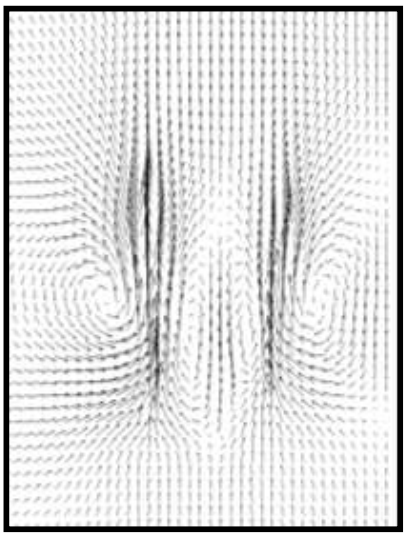
It is very important in two-phase flow system to understand the mechanism and nature of flow fields inside the system. Since it is difficult to do this in the running processes and if it happens, it will not give a full picture on the system because of the presence of capturing device. From this point, the CFD technique becomes an important tool to understand that. Even in CFD methods and when multi-bubbles are simulated, the influence of bubbles on the liquid and other bubbles will be hard to notice. To show this effect clearly, two adjacent moving bubbles have been simulated for different distances between their centers.

1. Figure.(6a). It shows the simulation of two adjacent bubbles with a distance of 10 cm between their centers and the effect of each bubble on the liquid is obvious.
2. Figure.(6 b). It illustrates the overlapping effect of two adjacent moving bubbles. As the distance getting smaller as the disturbance becomes higher due to the accumulation in the source terms of kinetic energy equation and dissipation of kinetic energy equation, where the distance between the centers of the adjacent bubbles is 5 cm.
3. Figure.(6 c). It manifests the increase in the overlapping influence of two bubbles on the liquid with the decrease in the distance between the centers of the adjacent bubbles.
4. Figure.(6 d). In this figure, the two bubbles look like they had coalescence, where their effects united and the disturbance between the bubbles became small due to the decrease in the distance between the centers.

5. Figure.(6 e). In this figure, the two bubbles reaches the coalescence distance so that they coalescence and become one bubble, it is noticeable that their effect after coalescence become greater than one bubble effect because of the bubble size.
6. Figure.(6 f). It shows the single bubble moving in a quiescent liquid to compare its effect with the bubble after coalescences.

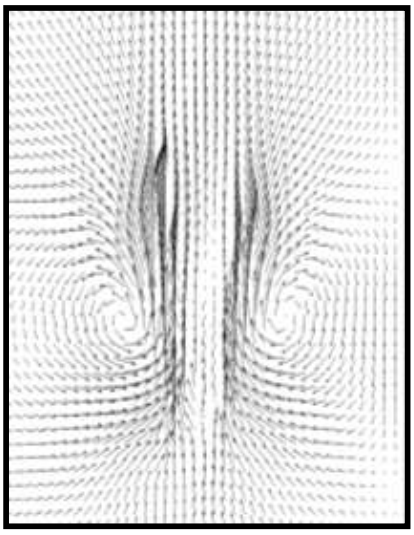


(a): Two adjacent Bubbles moving in stagnant liquid 10 cm distance between their centers

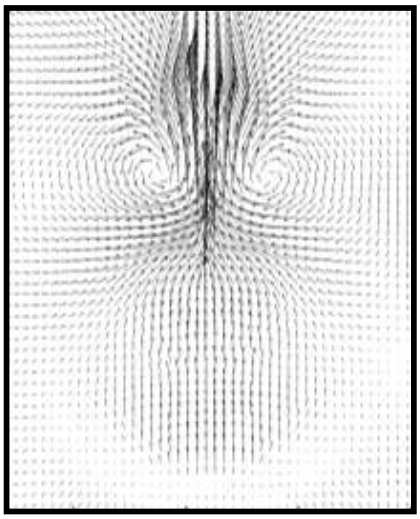


(b): Two adjacent Bubbles moving in stagnant liquid 5 cm distance between their centers

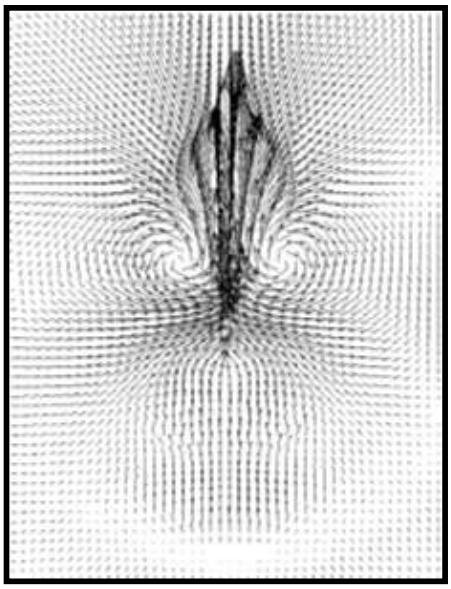
Figure 6. Two adjacent bubbles moving in a quiescent liquid with different distances between centers (continued …)



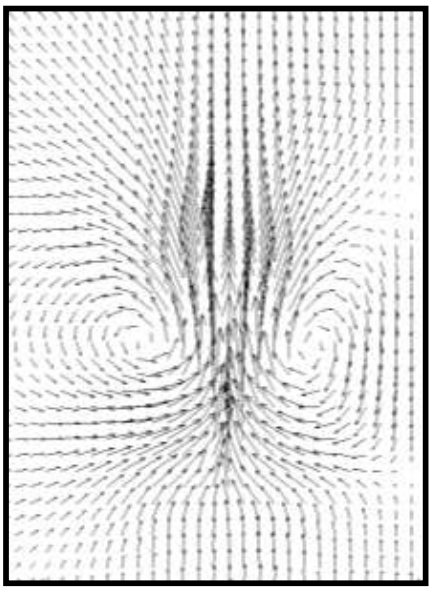
(c): Two adjacent Bubbles moving in stagnant liquid 3 cm distance between their centers



(d): Two adjacent Bubbles moving in stagnant liquid 1 cm distance between their centers



(e): Two adjacent Bubbles moving in stagnant liquid Coalescence of two bubbles



(f): Single Bubble moving in stagnant liquid

Figure 6. Two adjacent bubbles moving in a quiescent liquid with different distances between centers Continued

From the simulation of two gas bubbles moving in a quiescent liquid it give an agreement to the results obtained by Delnoij, 2001[3]. The effect of momentum and kinetic energy exchange is very important in the calculation that leads to reasonable results and if that exchange was neglected, the solution wouldn't have a meaning at all. This effect proved in the two adjacent bubbles, where the disturbance that created by the neighborhood's bubbles is increased with the distance between the centers of adjacent bubbles decrease. This disturbance keeps increasing until the distance between the bubbles centers reaches the critical distance, after that, the disturbance vanishes, and the bubble coalescence appears, and the two bubbles coalescences. In addition, the new bubble diameter is greater than the mother bubble and hence, the effect of formed bubble on the containing liquid becomes greater than single bubble moving in the liquid. Bubble – bubble interaction is a very effective property on the whole system, where bubbles motion, position and coalescence depend on the interaction and effect of each bubble on the liquid and other bubbles. Delnoij, 2001[3]found these results as an accurate and promising simulation for gas-liquid system. From this point, the simulation for single and two bubbles moving in a quiescent liquid is considered as a reasonable simulation for such systems using Euler-Lagrange PIC approach.

6. Conclusion

Simulations were successfully carried out for the prediction of flow patterns in single, two and multi-bubbles models. The following conclusions are drawn from the present study:

1) A mapping function technique was used to translate the Eulerian framework to Lagrangain framework and vice versa using a sinwave of Kitagawa, et al., 2001[14]. This function has distributed the effect of each bubble on the containing cells (maximum four cells). Without this function, it is not possible to solve the cases.
2) The 2D Euler-Lagrange model prediction agrees with the results obtained byDelnoij, 2001[3] in qualitative manner for single and two air bubbles, where he used the volume of fluid approach to simulate the effect of large air bubbles motion in a quiescent liquid (water), taking into account the momentum exchange between two phases. This means that the physical and numerical scheme is properly capable of predicting flow patterns in bubble column reactors. Where, the main goal from this comparison is to capture the liquid (water) velocity profile and the effect of air bubbles in the carrying liquid.
3) Sommerfeld's coalescence model used in two adjacent moving bubbles was simple, accurate and very reasonable to describe the coalescence procedure.
4) The study and simulation of two adjacent moving bubbles in a quiescent liquid showed that the disturbance that bubbles created in the carrying fluid is increased as the distance between the bubbles centers decreased until it reaches a critical length after that length, the disturbance of two bubbles are united and started vanishing.
5) Particle source in cell method (PSIC), [7,2], was used in Euler-Lagrange approach. Where, this model states," that the effect of each bubble accounted where the bubble center lay" while in the present work, a modification was made, and the bubble effect was distributed on the containing cells (maximum four cells). This modification showed a very good representation and decreased the relative total error.

7. References

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