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Unconditional discrete optimization of linear-fractional function “-1”-order

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Abstract. The solution of many applied problems is reduced to solving optimization problems with the objective function of a linear-fractional functional of “-1” order. For example, economic tasks, tasks of placing objects, tasks about assignments, layout, production planning, an informative description of objects, medical tasks, etc. The article developed a new method for solving the unconditional optimization problem with the objective function of the discrete-fractional-linear functional of the “-1” order. As well as certain optimality conditions for the chosen solution, a theorem is formed and proved, which is the basis of the proposed method.

1. Introduction

Many applied problems are solved by casting into a discrete optimization problem. And the applied task itself requires obtaining effective solutions in minimum time and cost. Moreover, with an increase in the number of variables, the solution of the problem becomes more complicated. In practice, obtaining such an optimal solution to the problem is time consuming. Some existing optimization methods do not always give the expected solution, but they provide close solutions to the expected.

In addition, if there are errors in the initial data of the problem, the estimate of the optimal solution is reduced, and the mathematical model itself, as a rule, is only an approximate expression of the original problem. Good feasible solutions to the optimization problem are not only limited by practical value, they are also an important element for many methods of finding the optimal solution, for example, the method of boundaries and branches.

Most of the practical tasks of human activity relate to discrete optimization problems, and also in decision-making practice one has to deal with problems, many of which are NP-tasks. Currently, many methods, algorithms and software have been developed for solving such problems [1-6].

Many applied problems are solved on the basis of continuous optimization, and many are reduced to solving linear fractional discrete optimization problems. For example, tasks about assignments, layout, production planning, an informative description of objects, etc.

For an informative description of objects use the indicators of the object, time and cost of measuring signs and others. To obtain an informative set of signs, it is necessary to maximize the criteria of informativeness, and to find non-informative ones, minimize it. Fractional-linear programming problems play a special role not only in the informative description of objects, but also



in planning, layout, etc. For example, in planning problems, the relevance of linear-fractional programming problems lies in the possibility of finding the optimal production solution in any limited time and volume of production. Among the planning tasks, individual economic indicators that need to be extremized, for example, cost, profitability, etc., become more important. In such cases, the admissible conditions of such problems and the admissible conditions of an ordinary linear problem may not differ, and the objective function is expressed as a fraction, i.e. linear-fractional functionals of the “-1” order, in the numerator and denominator of which are linear algebraic sums of variables x_1, x_2, \dots, x_n - the desired plans for the release of manufactured goods [8].

The article proposes the fastest method for solving such problems in which the objective function is a linear-fractional functional of “-1” order.

2. Basic concepts and notations

The discrete optimization problem consists of finding the maximum (or minimum) of the function φ , which is defined on a finite (or countable) discrete set \mathfrak{S} .

$$\varphi(x) \rightarrow \text{extr}, x \in \mathfrak{S}$$

where the function φ is the objective function, and the elements of the set \mathfrak{S} are feasible solutions.

Let a discrete set be given: $\mathfrak{S} = \mathfrak{S}_1 \cup \mathfrak{S}_2 \cup \dots \cup \mathfrak{S}_N$, $p_i = |\mathfrak{S}_i|$, $i = \overline{1, N}$, $|\mathfrak{S}| = \prod_{i=1}^N p_i$ and

$$\varphi(\alpha, \beta, \gamma, x) = \frac{\sum_{i=1}^N \alpha_i x_i}{\sum_{i=1}^N \beta_i x_i \sum_{j=1}^N \gamma_j x_j} \text{ a linear-fractional function of the “-1” order, where}$$

$$x_i \in \mathfrak{S}_i, i = \overline{1, N}, \sum_{i=1}^N \beta_i x_i \sum_{j=1}^N \gamma_j x_j \neq 0.$$

Based on the functionals defined as simple homogeneous zero-order functionals, an optimization method was proposed in [8]. However, at present, only brute-force and gradient methods exist for solving discrete optimization of linear fractional functionals of various orders. But to determine informative features based on similar functionals, methods were proposed in [9–14]. To fill this gap, the method of discrete optimization of linear fractional functionals of “-1” order is proposed below.

Definition 1. φ is called a homogeneous function of k -order if for $\forall \beta \in R, (\beta \neq 0)$ the equality $\varphi(\beta x_1, \beta x_2, \dots, \beta x_k) = \beta^k \varphi(x_1, x_2, \dots, x_k)$ holds.

3. Formulation of the problem

Let the objective function be given

$$\varphi(\alpha, \beta, \gamma, x) = \frac{\sum_{i=1}^N \alpha_i x_i}{\sum_{i=1}^N \beta_i x_i \sum_{j=1}^N \gamma_j x_j}, \quad (1)$$

which is a homogeneous “-1” order, i.e. here $k = -1$.

Let us consider in the set \mathfrak{S} the following optimization problem

$$\begin{cases} \varphi(\alpha, \beta, \gamma, x) = \frac{\sum_{i=1}^N \alpha_i x_i}{\sum_{i=1}^N \beta_i x_i \sum_{j=1}^N \gamma_j x_j} \rightarrow \max, \\ x_i \in \mathfrak{I}_i, i = \overline{1, N}. \end{cases} \quad (2)$$

Let real numbers a, b, c be given as well as satisfying the conditions $d, e, f > 0$ and $e + b > 0, f + c > 0$. Then one of the following properties holds.

1^o. If $\frac{a}{d} \geq \frac{b}{e} + \frac{c}{f} + \frac{bc}{ef}$, then $\frac{d}{ef} \leq \frac{d+a}{(b+e)(c+f)}$ is executed.

Proof.

$$\begin{aligned} \frac{a}{d} \geq \frac{b}{e} + \frac{c}{f} + \frac{bc}{ef} &\Rightarrow aef \geq dbf + dce + dbc \Rightarrow aef + def \geq dbf + dce + dbc + def \Rightarrow \\ &\Rightarrow ef(a+d) \geq d(b+e)(c+f) \Rightarrow \frac{a+d}{(b+e)(c+f)} \geq \frac{d}{ef}. \end{aligned}$$

2^o. If $\frac{a}{d} < \frac{b}{e} + \frac{c}{f} + \frac{bc}{ef}$, then $\frac{d}{ef} > \frac{d+a}{(b+e)(c+f)}$ is executed.

Proof.

$$\begin{aligned} \frac{a}{d} < \frac{b}{e} + \frac{c}{f} + \frac{bc}{ef} &\Rightarrow aef < dbf + dce + dbc \Rightarrow aef + def < dbf + dce + dbc + def \Rightarrow \\ &\Rightarrow ef(a+d) < d(b+e)(c+f) \Rightarrow \frac{a+d}{(b+e)(c+f)} < \frac{d}{ef}. \end{aligned}$$

Let $\forall x_i, y_i \in \mathfrak{I}_i, x_i \neq y_i, i = \overline{1, N}$ be selected.

We introduce the following notation:

$$A = \sum_{i=1}^N \alpha_i x_i, B = \sum_{i=1}^N \beta_i x_i, C = \sum_{i=1}^N \gamma_i x_i, \begin{cases} \Delta \alpha_t = \alpha_t (y_t - x_t) \\ \Delta \beta_t = \beta_t (y_t - x_t), t = \overline{1, N} \\ \Delta \gamma_t = \gamma_t (y_t - x_t) \end{cases}.$$

If in the properties, accept $a = \Delta \alpha_t, b = \Delta \beta_t, c = \Delta \gamma_t, d = A, e = B, f = C$, then for $\forall t \in \overline{1, N}$ taking into account

$$\begin{cases} A + \Delta \alpha_t \geq 0; \\ B + \Delta \beta_t > 0, \\ C + \Delta \gamma_t > 0, \end{cases}$$

one of these properties will take place.

Given vector $x = (x_1, x_2, \dots, x_N), x_i \in \mathfrak{I}_i, i = \overline{1, N}$.

Theorem 1. In order for the selected vector x to provide an optimal solution to problem (2), it is necessary and sufficient that there are $a = \Delta\alpha_t$, $b = \Delta\beta_t$ and $c = \Delta\gamma_t$ ($t = \overline{1, N}$) that satisfy the conditions of property 1.

Proof.

Adequacy. Let $\forall x \in \mathfrak{S}_1 \times \mathfrak{S}_2 \times \dots \times \mathfrak{S}_N$ be selected. Then the expressions (arbitrary amounts)

$$\begin{cases} A = \sum_{i=1}^N \alpha_i x_i, \\ B = \sum_{i=1}^N \beta_i x_i, \\ C = \sum_{i=1}^N \gamma_i x_i, \end{cases}$$

can be reduced to the following form:

$$\begin{cases} A^* = A + \sum_{t=1}^p \Delta\alpha_t^{(k)}, \\ B^* = B + \sum_{t=1}^p \Delta\beta_t^{(k)}, \\ C^* = C + \sum_{t=1}^p \Delta\gamma_t^{(k)}. \end{cases}$$

For A^* , B^* , C^* we have the corresponding equalities

$$\begin{cases} A^* = A + A_1 + A_2, \\ B^* = B + B_1 + B_2, \\ C^* = C + C_1 + C_2. \end{cases}$$

Here A_m, B_m and C_m are the sums of $\Delta\alpha_t^{(k)}, \Delta\beta_t^{(k)}$ and $\Delta\gamma_t^{(k)}$, respectively, satisfying the conditions of the m-property ($m = 1, 2$).

By the hypothesis of the theorem, the sums A_1, B_1 and C_1 are equal to zero. So,

$$A^* = A + A_2, B^* = B + B_2, C^* = C + C_2.$$

Since each summand of the sums of $A_2 = \sum_{t=1}^p \Delta\alpha_t^{(k)}, B_2 = \sum_{t=1}^p \Delta\beta_t^{(k)}$ and $C_2 = \sum_{t=1}^p \Delta\gamma_t^{(k)}$ satisfies the conditions of property 2, there is:

$$\frac{\sum_{t=1}^p \Delta\alpha_t^{(k)}}{A} < \frac{\sum_{t=1}^p \Delta\beta_t^{(k)}}{B} + \frac{\sum_{t=1}^p \Delta\gamma_t^{(k)}}{C} + \frac{\sum_{t=1}^p \Delta\beta_t^{(k)} \Delta\gamma_t^{(k)}}{BC}.$$

It follows that

$$\frac{A + \sum_{t=1}^p \Delta \alpha_t^{(k)}}{\left(B + \sum_{t=1}^p \Delta \beta_t^{(k)}\right) \left(C + \sum_{t=1}^p \Delta \gamma_t^{(k)}\right)} < \frac{A}{BC} \text{ and } \frac{A + A_2}{(B + B_2)(C + C_2)} < \frac{A}{BC}.$$

Necessity. Suppose that there exist $\Delta \alpha_t^{(k)}, \Delta \beta_t^{(k)}$ and $\Delta \gamma_t^{(k)}$ satisfying the conditions of property 1. Then, in accordance with this property, we obtain

$$\frac{A + \Delta \alpha_t^{(k)}}{(B + \Delta \beta_t^{(k)})(C + \Delta \gamma_t^{(k)})} \geq \frac{A}{BC}.$$

Since each term of the sums $A_1 = \sum_{t=1}^p \Delta \alpha_t^{(k)}, B_1 = \sum_{t=1}^p \Delta \beta_t^{(k)}$ and $C_1 = \sum_{t=1}^p \Delta \gamma_t^{(k)}$ satisfies the conditions of property 1, then there is:

$$\frac{\sum_{t=1}^p \Delta \alpha_t^{(k)}}{A} \geq \frac{\sum_{t=1}^p \Delta \beta_t^{(k)}}{B} + \frac{\sum_{t=1}^p \Delta \gamma_t^{(k)}}{C} + \frac{\sum_{t=1}^p \Delta \beta_t^{(k)} \Delta \gamma_t^{(k)}}{BC}.$$

It follows that

$$\frac{A + \sum_{t=1}^p \Delta \alpha_t^{(k)}}{\left(B + \sum_{t=1}^p \Delta \beta_t^{(k)}\right) \left(C + \sum_{t=1}^p \Delta \gamma_t^{(k)}\right)} \geq \frac{A}{BC}.$$

Thus, we have

$$\frac{A + A_1}{(B + B_1)(C + C_1)} > \frac{A}{BC}$$

and the value $\varphi(\alpha, \beta, \gamma, x) = \frac{A}{BC}$ corresponding to the selected x is not optimal.

If problem (2) is solved by exhaustive search, then the number of interchanges is $\prod_{i=1}^N m_i$, and in the proposed method the number of interchanges is much less, i.e. $\sum_{i=1}^N m_i$.

4. Experimental research and solving a practical problem

A computational experiment was carried out for 5 discrete sets $\mathfrak{I}_1, \mathfrak{I}_2, \dots, \mathfrak{I}_5$. Elements of these sets are real and integer numbers generated on a computer.

The cardinalities of these sets are:
 $p_1 = |\mathfrak{I}_1| = 100, p_2 = |\mathfrak{I}_2| = 200, p_3 = |\mathfrak{I}_3| = 150, p_4 = |\mathfrak{I}_4| = 400, p_5 = |\mathfrak{I}_5| = 150,$ and the coefficients of the linear fractional functional are given as

$$\begin{aligned}\alpha_1 &= 3, \alpha_2 = 5, \alpha_3 = -7, \alpha_4 = 8, \alpha_5 = 7, \\ \beta_1 &= 9, \beta_2 = 1, \beta_3 = -1, \beta_4 = 2, \beta_5 = 3, \\ \gamma_1 &= 7, \gamma_2 = 2, \gamma_3 = -3, \gamma_4 = 4, \gamma_5 = 6.\end{aligned}$$

Based on the data presented, the following discrete optimization problem is solved:

$$\begin{cases} \varphi(\alpha, \beta, x) = \frac{\sum_{i=1}^5 \alpha_i x_i}{\sum_{i=1}^5 \beta_i x_i \sum_{j=1}^5 \gamma_j x_j} \rightarrow \max, \\ x_i \in \mathfrak{I}_i, i = \overline{1, 5}. \end{cases}$$

The solution of the problem was carried out on the basis of the proposed method and exhaustive search and compared their results. In the proposed method, the optimal solution was found in 1050 steps and the results of the proposed method and the exhaustive search method completely coincided. In the exhaustive search method, the number of steps was $6 \cdot 10^{10}$. In addition, some of the problems presented and solved in [7] were solved on the basis of the proposed method by converting them into a discrete optimization problem, and the results completely coincided with the results of the known methods of unconditional discrete optimization.

Using the proposed method, the following practical problem was solved, associated with finding informative features of objects based on the relationship of features. In this problem, we studied vital clinical informative signs of coronary heart disease, where the object of the study is the pathology of the ventricle of the heart, the condition of which is divided into 3 groups: myocarditis infarction - the first class of objects (P_1), middle condition - the second class of objects (P_2), angina pectoris - third class of objects (P_3).

The initial data were formulated in the form of 3 classes. A class is a type of disease, in each class the number of signs is the same, i.e. equal to 82.

To determine the informative features, it is necessary to solve the following optimization problem

$$\begin{cases} \varphi(\alpha, \beta, x) = \frac{\sum_{i=1}^{\ell} \alpha_i x_i}{\sum_{i=1}^{\ell} \beta_i x_i \sum_{j=1}^{\ell} \gamma_j x_j} \rightarrow \max, \\ \ell = \overline{1, 82}, x_i \in \mathfrak{I}_i, i = \overline{1, 82}. \end{cases}$$

On the basis of the “ k -nearest neighbours” method, each object belongs to its class. When analysing the results, the optimal ℓ was found, which is $\ell = 58$. The selected informative set of features ensured 100% accuracy in classifying the objects of the training sample.

5. Conclusion

To solve the problem of unconditional discrete optimization of linear-fractional functionals of the “-1” order, the optimality conditions for the chosen solution are determined. Based on these optimality conditions, a theorem is formed and proved that is the basis for the proposed method. The proposed method provided the best results for the selection of an informative set of features. In addition, the proposed method can be applied to solve many applied problems that can be reduced to problems where the objective function is a linear-fractional functional of the “-1” order.

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