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Improving finite element modelling of the dynamic behaviour of a car trunk lid with modal based updating method

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Abstract. Predicted results of a finite element model used to develop physical test structures are often found to be not in a good agreement with the experimental results due to the invalid assumptions made in the finite element modelling. This study will focus on improving the correlation level between the finite element model and experimental model of a car trunk lid with the presence of resistance spot welded joints using modal based updating method. The natural frequencies and mode shapes computed from the updated finite element model of the car trunk lid are compared with the corresponding experimental results. HyperWorks and Leuven Measurement System (LMS) test are used for the modelling and modal testing. MSC Nastran SOL200 is used to identify potential updating parameters and to improve the initial finite element model in the light of corresponding experimental results which are obtained from modal testing. The comparison of the results showed that the total error of 58.59 per cent predicted from the initial finite element model of the trunk lid has been successfully reduced to 3.62 per cent. In conclusion, modal based updating has been successfully used to improve the correlation level between the initial finite element model and the physical model of the trunk lid.

Keywords. Dynamic Behaviour; Modal Based Updating; CWELD; Resistance Spot Welds; Finite Element Method; Experimental Modal Testing

1. Introduction

Dynamic analysts are striving to study more on the dynamic behaviour of automotive structure as the application of light weight-based materials is new and becoming crucial in the automotive industry due to the environment and social pressure. A car body in white, covering a large number of structural components that are made mostly from thin metal sheets. In practice, the structural components are usually assembled by joints such as rivets, bolts and welds [1]. Therefore, modelling and analysis of the dynamic behaviour of a large complex automotive structure are often found to be very challenging and essentially require to be further fundamentally explored [2].

In mass production of automotive structures, spot welds are used extensively to join thin metal sheets and beams; a car or truck body-in-white commonly contains several thousand spot welds. Thus far, the use of RSW and laser spot welding for joining the structural components has received much attention in the industry [3]. To point out, a car trunk lid, which is the structure under investigation of this study, is one of the important automotive structures that is made from thin metal sheets connected



with resistance spot welds and adhesive joints. A shorter time progression and no additional mass [4] give the main advantages of resistance spot welds (RSW) as a superior joint technique of assembling the automotive structure. Nevertheless, modelling RSW in a finite element model is difficult and problematic.

This is because, the welded joints do not only provide strong connections between structural components but them also significantly contribute to the dynamic behaviour of the structures [5]. Thus, the selection of element connectors in finite element software must be carefully chosen based on the characteristics of the RSW.

Although a large number of studies on spot welded joints have been carried out, very little work has been found on the effort to establish a proper procedure for scheming identifying and verifying spot welded structures via the finite element method [6]. For example, Nurul et al. al. al. [7] used CWELD element connectors to represent laser spot welds in the structure under study. Palmonella [8] proposed the guidelines on how to use the ACM2 and CWELD element connectors in modelling welded joints using finite element software packages. However, spot welded joints are found to be complex and have many uncertainties when it comes to analytical modelling [9] due to the presence of local effect in the welded areas.

Fang et al. [10] proposed the model known as CWELD element, which is available in NASTRAN. The CWELD element is featured with a special shear flexible beam-type element consisting of two nodes and twelve degrees of freedom. If the plate meshes were congruent, with nodes at the spot weld location, then the beam element could be used to directly connect the two plates (providing the plate elements provide sufficient rotational stiffness). For general meshes, the nodes of the beam are connected to a chosen set of nodes of the plate it belongs to. The area enclosed by the nodes of each plate that are attached to the beam element is called “patches”. Figure 1 shows a typical patch associated with a spot weld, where each patch consists of 4 elements. The six degrees of freedom of each beam node are connected to the three translational degrees of freedom of each plate node with constraints from Kirchoff shell theory. These connections yield 12 constraint equations that may be used to generate multipoint constraints explicitly, or the constraints may be used to obtain the beam stiffness matrix in terms of the plate degrees of freedom.

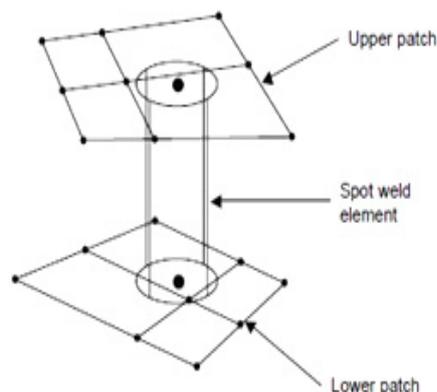


Figure 1. A schematic of the patch associated with the CWELD spot weld.

In this paper, the CWELD is used to represent the RSW joints in the car trunk. The decision was made based on the outcomes of previous studies that showed CWELD has better capability as compared to other element connectors. The initial CWELD finite element model can be further improved using model updating.

Model updating is a powerful method of increasing the predicted results to be as close to the measured counterparts [11]. In other words, modal updating is an inverse problem whereby it is more

complex for identifying the most suitable changes required to obtain the desired dynamic behaviour given a set of frameworks of possible modification [12]. Predefined properties of the structure will be updated using engineering tools to get the actual properties of the structure.

There are two main categories of modal updating methods, which are modal based updating, and frequency response function (FRF) based updating [13]. Both methods have been widely used by structural dynamicists to improve the initial finite element models over the physical models of structures. Recently, the frequency response function (FRF) based updating was adopted by Syazwan et al. [14] to improve the predicted results of the laser spot welded structure. It was found that the method adopted led to a strong correlation between the predicted and measured frequency response function, particularly for lower modes. Furthermore, model updating methods instantaneously utilise the structural response obtained by the finite element method and the measured structural response to calibrate mathematical modelling [15].

The influential and successful model updating in improving the predicted results, however, remains to be very challenging and have many fundamental issues to be handled properly. This is because the use of the common updating properties of model and joints in updating the FE models is found to be no longer effective, especially for updating an assembled structure with different types of joints. Therefore, other potential updating properties can be fundamentally explored for a better correlation between the physical test structure and analytical model. This study is attempted to improve the CWELD element connectors based finite element model of a car trunk lid in the light of experimental data using the modal-based updating.

2. Experimental work of the trunk lid

In this experimental work, experimental modal analysis was performed on a car trunk lid using impact testing with roving accelerometers. There are in total of 56 excitations points, all labelled on the trunk lid after conducting pre-processing in the finite element modal to determine the nodal points of the structure. The trunk lid was suspended using four springs and strings to simulate free-free boundary conditions. An impact hammer was used to excite the trunk lid while accelerometers were used to measure the dynamic response. LMS SCADAS system was used to process the input and output responses of the trunk lid under the test. The frequency bandwidth of interest was 0 to 150 Hz which is the bandwidth used in the automotive industry. The details of the test set up is shown in figure 2.

3. Finite element modelling of the trunk lid

The finite element model of the car trunk lid, as shown in figure 4 was constructed using CATIA software and imported into HyperMesh software packages for the modelling of the trunk lid. Figure 3 shows the physical trunk lid used in this study. In the modelling, all components were modelled using 2D shell elements based on the nominal thickness of the structure, which is less than 1mm. The FE model consisted of 31299 elements and 31481 nodes.

The size of the elements used for the modelling was 7 mm. the CWELD element connector, as shown in figure 5, was used to represent the resistance spot welds in the trunk lid. It was found that the number of the spot welds used to assemble the structural components to form the car trunk lid was 68 in total. The dynamic behaviour (natural frequencies and mode shapes) of the finite element model of the trunk lid was calculated using the normal modes solution with the frequency of interest between 0 to 150 Hz. The modal properties of the initial FE model defined are tabulated in table 1.

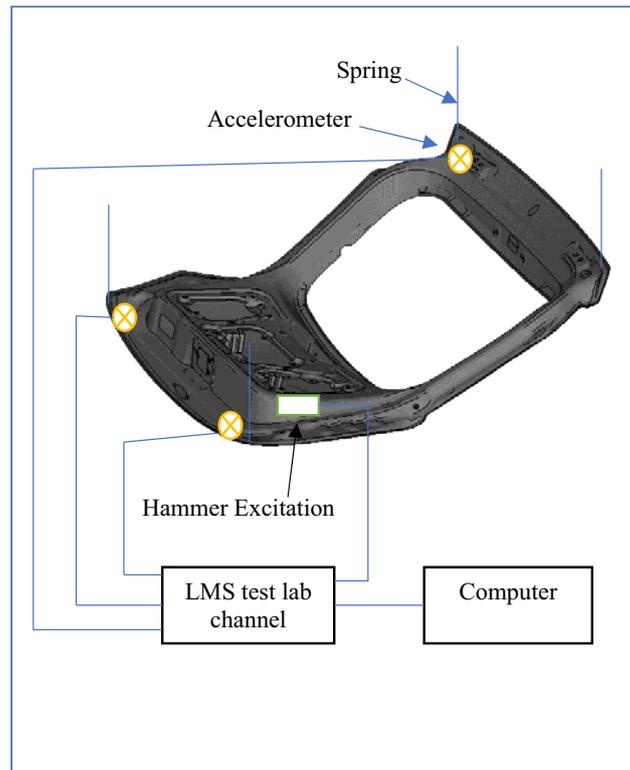


Figure 2. Schematic diagram of the experimental setup of the car trunk lid.



Figure 3. Physical car trunk lid.

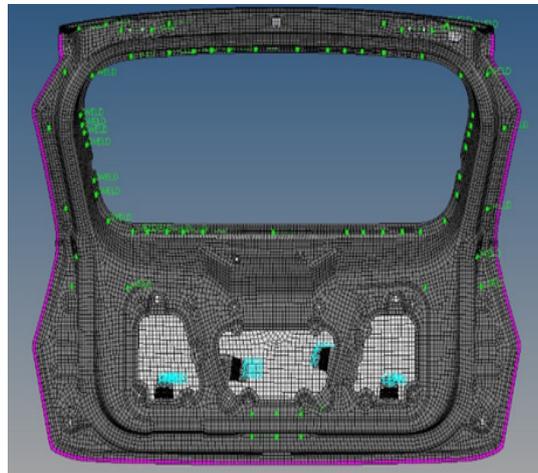


Figure 4. The finite element modelling of the car trunk lid.

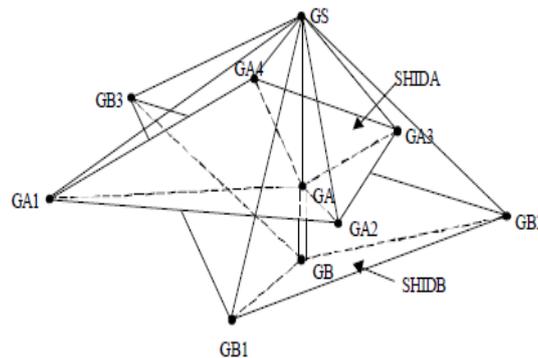


Figure 5. CWELD connectivity.

Table 1. Initial modal properties of the car trunk lid.

The name of the property	Nominal Value
Young's Modulus	210GPa
Poisson's Ratio	0.3
Density	7890 kg/m ³

4. Modal based model updating of the trunk lid

Model updating has been widely used to improve the accuracy of finite element models by correcting the invalid assumptions about the finite element models [16-20]. This method also allows a wide choice of potential updating parameters to be corrected using the sensitivity analysis method. The enhancement is carried out by systematically changing predefined potential updating parameters which have a large influence in improving the accuracy of the finite element models. The NASTRAN optimisation code (SOL 200) is used to perform modal updating in this study. Modal updating can be

derived as shown below. The equation of motion of a multiple-degree-of-freedom system in a discretised form can be written as:

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = f(t) \quad (1)$$

The free vibration equation can be expressed as

$$(-\omega^2 [M] + i\omega[C] + [K])\{U\} = 0 \quad (2)$$

Modal parameter of the system can be calculated from

$$[K - \omega^2 M] \phi_{FE} = 0 \quad (3)$$

Differentiate equation (3) with respect to updating parameters, \mathbf{P} .

$$\frac{\delta}{\delta \mathbf{P}} [K - \omega^2 M] \phi_{FE} = 0 \quad (4)$$

$$\left[\frac{\delta K}{\delta \mathbf{P}} - \frac{\delta \omega^2}{\delta \mathbf{P}} M - \omega^2 \frac{\delta M}{\delta \mathbf{P}} \right]_{FE} = 0 \quad (5)$$

Then,

$$\frac{\delta \omega^2_{FE}}{\delta \mathbf{P}} = \left[\frac{\delta K}{\delta \mathbf{P}} - \omega^2 \frac{\delta M}{\delta \mathbf{P}} \right]_{FE} / M \quad (6)$$

Where,

$$\mathbf{Z} = \mathbf{K} - \omega^2 \mathbf{M} \quad (7)$$

Hence,

$$\frac{\delta \omega^2}{\delta \mathbf{P}}_{FE} = \frac{1}{M} \left[\frac{\delta \mathbf{Z}}{\delta \mathbf{P}} \right]_{FE} \quad (8)$$

Express the experimental natural frequency ω^{2EXP} as a Taylor expansion in term of parameter, \mathbf{P}

$$\omega^{2EXP} = \omega^2_{FE} + \frac{\delta \omega^2_{FE}}{\delta \mathbf{P}} \mathbf{P} \quad (9)$$

Substituting equation (8) into (9) yields

$$\omega^{2EXP} = \omega^2_{FE} + \frac{1}{M} \left[\frac{\delta \mathbf{Z}}{\delta \mathbf{P}} \right]_{FE} \mathbf{P} \quad (10)$$

$$\omega^{2EXP} - \omega^2_{FE} = \left[\frac{\delta \mathbf{Z}}{\delta \mathbf{P}} \right]_{FE} \mathbf{P} \quad (11)$$

$\omega^2_{FE} - \omega^{2EXP}$ needs to be corrected by the minimization of the error using the objective function:

$$\min \sum_{i=1}^M \left(\frac{\omega^i_{FE}}{\omega^i_{EXP}} - 1 \right)^2 \quad (12)$$

Furthermore, the optimisation algorithm in equation (13) is a sensitivity based iterative-based procedure, let the objective function (J) to be minimised by correcting the eigenvalues of the initial finite element model until the objective function is converged. The objective function based on eigenvalues is defined as follows [21]:

$$J = \min \sum_{i=1}^M \left(\frac{\omega_{FE}^i}{\omega_{EXP}^i} - 1 \right)^2 \quad (13)$$

Where ω_{EXP}^i is the experimental eigenvalue and ω_{FE}^i is the predicted eigenvalue from the finite element model and m is the number of eigenvalues used in the updating procedure. In model updating, the selection of updating parameters is not a straight-forward task but in fact, the selection is a crucial and most important task [22]. A wrong selection of updating parameters will lead to ill-condition. The ill-condition can be avoided by performing sensitivity analysis method that permits an extensive choice of the updating parameters to be corrected. To illustrate in detail, the procedure of the modal-based updating is shown in figure 6.

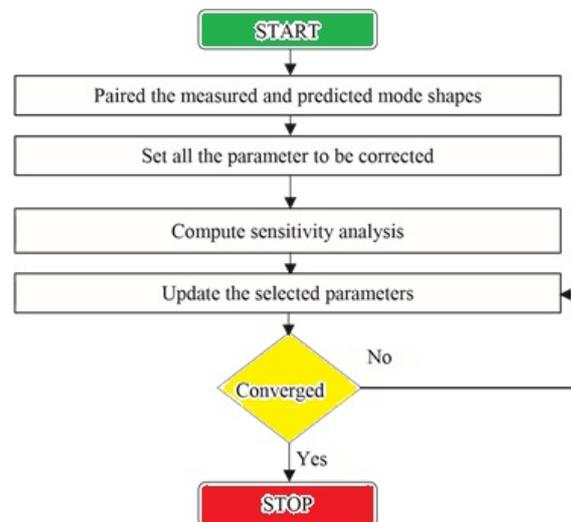


Figure 6. Modal updating flowchart.

Additionally, the modal based method uses the modal test parameters as targets in the correlation process [23]. A correlation between the finite element analysis and experimental modal analysis mode shapes is quantified using Modal Assurance Criterion (MAC) [24] before updating. The value of MAC must be in the range from 0.7 to 1 for every mode shape. In the modal-based, the predicted natural frequencies will be altered as close as measured ones iteratively based on chosen parameters. Normally, the damping value is ignored because the value of damping is too small and does not affect the results.

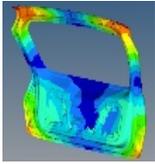
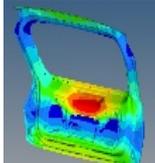
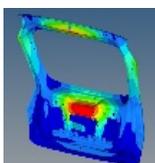
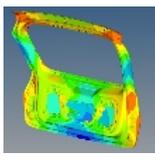
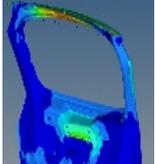
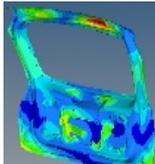
5. Results and discussion

Model updating method has been effectively used by researchers [25-29] to accurately determine modal parameters of engineering structures in the light of experimental results which are usually obtained from experimental modal analysis. In this study, the predicted and measured modal parameters of the test structure were obtained from the finite element method and experimental modal analysis. CQUAD4 shell elements and CWELD element connectors were used in modelling the

physical car trunk lid. The level of correlation between the predicted and measured results of the trunk lid was evaluated by frequency deviation and Modal Assurance Criterion (MAC) value.

Table 2 demonstrates that all modes are paired in the correct sequence without having the issues of swapped or missed modes. MAC values of corresponding mode shapes (mode pairing) between EMA and FEM are used to quantify the mode shapes calculated from of the finite element model. The MAC analysis of the initial FE model with CWELD element connectors and the findings are tabulated in table 2. In general, any mode with MAC value more than 0.7 is regarded as a well-correlated mode shape while MAC value is less than 0.7 is considered as a poor correlated mode shape.

Table 2. Mode shapes and MAC value of the initial FE.

Mode	Experimental modal analysis	Finite element analysis	MAC value
1			0.98
2			0.82
3			0.98
4			0.55
5			0.76
6			0.95

In table 2, the lowest MAC value has been recorded from the 4th mode with a value of 0.55. Due to the complexity of the structure under study, the range of 0.5 and above is regarded to be still within the acceptable level. Moreover, local deformations arising from the assembly process may contribute to the low MAC value.

Table 3 shows the comparison of the natural frequencies obtained from the experimental modal analysis (column II) and the initial finite element model with CWELD element connector (column III). Therefore, from a direct comparison, particularly, between the model with CWELD element connector (column III) and EMA (column II), in column IV it was found that the largest percentage error has been recorded from the 4th mode with 18.93%, while the lowest error which is 4.77% has been registered from the 6th mode. The accumulated error registered from all 6 modes is 58.59%.

From the comparison between the experimental data, initial FE and updated FE natural frequencies tabulated in table 3, it shows that errors registered at column IV are proportionally inconsistent between modes. This is due to the invalid assumptions of the material properties used in modelling the finite element model of the trunk lid. Modal based updating of the finite element model has been carried out using NASTRAN (SOL 200), and a significant improvement has been achieved. Sensitivity analysis has been performed on the initial finite element model of the trunk lid to identify potential updating parameters before performing the updating. The updating parameters identified are the Young's Modulus, the thickness of the model and density.

Table 3. Comparison of the natural frequencies of the car trunk lid between the experimental and initial FE.

Modes (I)	EMA (Hz) (II)	Initial FE (Hz) (III)	Error % (IV)
1	40.28	42.68	6.11
2	55.29	63.25	14.39
3	89.48	97.63	9.11
4	93.76	111.50	18.93
5	137.05	144.30	5.29
6	142.03	148.80	4.77
Total			58.59
error			

Table 4 shows the results of the updated natural frequencies of the trunk lid. The total error registered from all modes has been dramatically reduced from 58.5 % to 3.62% (column VI). Meanwhile, table 5 shows the perturbation of the initial properties of the Young's Modulus, thickness and density. The successful updating of the initial finite element model shown in table 4 does not only prove the powerful tool of modal-based updating, but it also proves that CWELD is better than ACM2 in representing the RSW in the physical trunk lid, as compared with the findings obtained by Syazwan et al [30].

Table 4. Comparison of the natural frequencies of the car trunk lid between the experimental and initial FE.

Modes (I)	EMA (Hz) (II)	Updated FE (Hz) (V)	Error % (VI)
1	40.28	40.11	0.26
2	55.29	55.86	1.03
3	89.48	89.43	0.06
4	93.76	93.89	0.14
5	137.05	136.54	0.37
6	142.03	139.52	1.76
Total error			3.62

Table 5. Perturbations to the updating parameters.

Properties	Component	Initial properties	Updated properties
Young's modulus	Inner surface	210 GPa	195 GPa
	Outer surface	210 GPa	191 GPa
Density	Inner surface	7900 kg/m ³	8620 kg/m ³
	Outer surface	7900 kg/m ³	8050 kg/m ³
Thickness	Inner surface	1 mm	0.70 mm
	Outer surface	0.65 mm	0.61 mm

6. Conclusions

This work gives a justification for the study of modelling and analysing the dynamic behaviour of the car trunk lid structure using CWELD element connectors in representing the RSW using the finite element method. The attempt to predict and correct the dynamic behaviour of the car trunk lid using the modal-based updating method have been successfully carried out.

In the attempt, it was found that all modes, starting from the 1st to 6th mode were very sensitive to the updating parameters of the Young's Modulus, density and thickness of the trunk lid. The use of the parameters in updating the initial FE model of the trunk lid has led to a very good correlation between the physical and the finite element model with a dramatic reduction in the total error from 58.59 to 3.62 per cent. This achievement proves that modal based updating is a powerful tool to enhance the correlation level between a finite element model and experimental model. Furthermore, it was found that CWELD element connectors can better represent RSW in the physical trunk lid. The updated FE model of the trunk lid can then be confidently used in evaluating the performance of the structure before the physical prototypes are manufactured and verified.

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