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The method of structural schemes transformation

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Abstract. The structures of modern automatic control systems often have many contours. The method of structural schemes automated transformation to obtain the transfer functions of the system with an arbitrary combination of external input and output actions of the blocks has been proposed. The inputs of the blocks are connected to the outputs of the previous blocks. Therefore, the analysis of output actions is simultaneously considered to be the analysis of input actions. But there is also a need to analyze separately the actions at inputs and outputs of the blocks. The result of the manipulation is matrix of transfer functions of the system. It can be for outputs and inputs of blocks' actions. The presented method can be used in the controllers' synthesis.

1. Introduction

The structures of modern automatic control systems often have many contours [1, 2, 3, 4, 5, 6, 7, 8]. The method of structural schemes transformation is suggested which is considered further by the example of a specific system (figure 1):

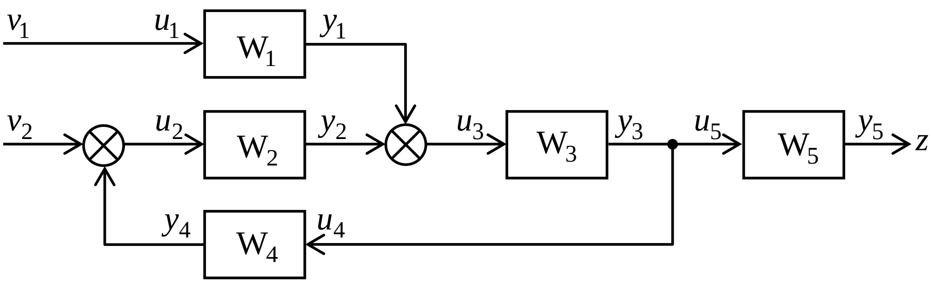

Figure 1. System block diagram.

2. The task of the system structure

The matrix equation of blocks is as follows:

$$\bar{y} = \mathbf{S}_{yu}\bar{u} ,$$

where $\bar{u}=\begin{bmatrix} u_1 \\ u_2 \\ \dots \\ u_n \end{bmatrix}$ — input actions of blocks; $\bar{y}=\begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_n \end{bmatrix}$ — output actions of blocks;

$\mathbf{S}_{\text{yu}}=\begin{bmatrix} \mathrm{W}_1 & 0 & \dots & 0 \\ 0 & \mathrm{W}_2 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \mathrm{W}_n \end{bmatrix}$ — blocks transfer functions.

The matrix equation of connections between blocks is as follows:

$$\bar{u}=\mathbf{S}_{\text{uy}}\bar{y}, \tag{1}$$

where $\mathbf{S}_{\text{uy}}=\begin{bmatrix} h_{1,1} & h_{1,2} & \dots & h_{1,n} \\ h_{2,1} & h_{2,2} & \dots & h_{2,n} \\ \dots & \dots & \dots & \dots \\ h_{n,1} & h_{n,2} & \dots & h_{n,n} \end{bmatrix}$, $h_{k,j}=\begin{cases} 1, \text{ if there is the connection between } y_j \text{ and } u_k; \\ 0 \text{ there is no connection} \end{cases}$

The matrix equation of the external actions' connection is:

$$\bar{u}=\mathbf{S}_{\text{uv}}\bar{v},$$

where $\bar{v}=\begin{bmatrix} v_1 \\ v_2 \\ \dots \\ v_n \end{bmatrix}$ — external actions; $\mathbf{S}_{\text{uv}}=\begin{bmatrix} g_{1,1} & g_{1,2} & \dots & g_{1,n} \\ g_{2,1} & g_{2,2} & \dots & g_{2,n} \\ \dots & \dots & \dots & \dots \\ g_{n,1} & g_{n,2} & \dots & g_{n,n} \end{bmatrix}$,

$g_{k,j}=\begin{cases} 1, \text{ if there is the connection between } v_j \text{ and } u_k; \\ 0 \text{ there is no connection.} \end{cases}$

Matrix equations system is as follows:

$$\bar{y}=\mathbf{S}_{\text{yu}}\bar{u}, \quad \bar{u}=\mathbf{S}_{\text{uy}}\bar{y}, \quad \bar{u}=\mathbf{S}_{\text{uv}}\bar{v}.$$

Dependence of output actions $\bar{y}$ blocks on external input actions $\bar{v}$:

$$\bar{y}=(\mathbf{E}-\mathbf{S}_{\text{yu}}\mathbf{S}_{\text{uy}})^{-1}\mathbf{S}_{\text{yu}}\mathbf{S}_{\text{uv}}\bar{v}, \tag{2}$$

where $\mathbf{E}$ is the unity matrix.

3. The example for the block diagram in figure 1

For the block diagram in figure 1:

$$\bar{u}=\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix},\ \bar{y}=\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix},\ \bar{v}=\begin{bmatrix} v_1 \\ v_2 \end{bmatrix},\ \mathbf{S}_{\text{yu}}=\begin{bmatrix} \mathrm{W}_1 & 0 & 0 & 0 & 0 \\ 0 & \mathrm{W}_2 & 0 & 0 & 0 \\ 0 & 0 & \mathrm{W}_3 & 0 & 0 \\ 0 & 0 & 0 & \mathrm{W}_4 & 0 \\ 0 & 0 & 0 & 0 & \mathrm{W}_5 \end{bmatrix},$$

$$\mathbf{E}=\begin{bmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix},\ \mathbf{S}_{\mathrm{uv}}=\begin{bmatrix}0&0&0&0&0\\0&0&0&1&0\\1&1&0&0&0\\0&0&1&0&0\\0&0&1&0&0\end{bmatrix},\ \mathbf{S}_{\mathrm{uv}}=\begin{bmatrix}1&0\\0&1\\0&0\\0&0\\0&0\end{bmatrix}.$$

Explanations:

$$\mathbf{S}_{\mathrm{uy}}=\begin{array}{c|c|c|c|c|c|}
 & y_1 & y_2 & y_3 & y_4 & y_5\\ \hline
u_1 & & & & & \\ \hline
u_2 & & & & 1 & \\ \hline
u_3 & 1 & 1 & & & \\ \hline
u_4 & & & 1 & & \\ \hline
u_5 & & & 1 & & \\ \hline
\end{array},\quad
\mathbf{S}_{\mathrm{uv}}=\begin{array}{c|c|c|}
 & v_1 & v_2\\ \hline
u_1 & 1 & \\ \hline
u_2 & & 1\\ \hline
u_3 & & \\ \hline
u_4 & & \\ \hline
u_5 & & \\ \hline
\end{array}.$$

Matrix of transfer functions of the system is as follows:

$$\mathrm{W}_{\mathrm{y}}=(\mathbf{E}-\mathbf{S}_{\mathrm{yu}}\mathbf{S}_{\mathrm{uy}})^{-1}\mathbf{S}_{\mathrm{yu}}\mathbf{S}_{\mathrm{uv}}=\begin{bmatrix}\mathrm{W}_{1,1}&\mathrm{W}_{1,2}\\\mathrm{W}_{2,1}&\mathrm{W}_{2,2}\\\mathrm{W}_{3,1}&\mathrm{W}_{3,2}\\\mathrm{W}_{4,1}&\mathrm{W}_{4,2}\\\mathrm{W}_{5,1}&\mathrm{W}_{5,2}\end{bmatrix}=\begin{bmatrix}\mathrm{W}_1 & 0\\ \dfrac{\mathrm{W}_1\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4}{1-\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4} & \dfrac{\mathrm{W}_2}{1-\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4}\\ \dfrac{\mathrm{W}_1\mathrm{W}_3}{1-\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4} & \dfrac{\mathrm{W}_2\mathrm{W}_3}{1-\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4}\\ \dfrac{\mathrm{W}_1\mathrm{W}_3\mathrm{W}_4}{1-\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4} & \dfrac{\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4}{1-\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4}\\ \dfrac{\mathrm{W}_1\mathrm{W}_3\mathrm{W}_5}{1-\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4} & \dfrac{\mathrm{W}_2\mathrm{W}_3\mathrm{W}_5}{1-\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4}\end{bmatrix}.$$

The combination of external actions and output actions of the blocks:

for v_1	for v_2
$y_1=\mathrm{W}_{1,1}v_1=\mathrm{W}_1 v_1$	$y_1=\mathrm{W}_{1,2}v_2=0\cdot v_2=0$
$y_2=\mathrm{W}_{2,1}v_1=\dfrac{\mathrm{W}_1\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4}{1-\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4}v_1$	$y_2=\mathrm{W}_{2,2}v_2=\dfrac{\mathrm{W}_2}{1-\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4}v_2$
$y_3=\mathrm{W}_{3,1}v_1=\dfrac{\mathrm{W}_1\mathrm{W}_3}{1-\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4}v_1$	$y_3=\mathrm{W}_{3,2}v_2=\dfrac{\mathrm{W}_2\mathrm{W}_3}{1-\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4}v_2$
$y_4=\mathrm{W}_{4,1}v_1=\dfrac{\mathrm{W}_1\mathrm{W}_3\mathrm{W}_4}{1-\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4}v_1$	$y_4=\mathrm{W}_{4,2}v_2=\dfrac{\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4}{1-\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4}v_2$
$y_5=\mathrm{W}_{5,1}v_1=\dfrac{\mathrm{W}_1\mathrm{W}_3\mathrm{W}_5}{1-\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4}v_1$	$y_5=\mathrm{W}_{5,2}v_2=\dfrac{\mathrm{W}_2\mathrm{W}_3\mathrm{W}_5}{1-\mathrm{W}_2\mathrm{W}_3\mathrm{W}_4}v_2$

4. The example using the program MathCAD

It is given:

$$\mathrm{W}_1:=\frac{\mathrm{K}_1}{\mathrm{T}_1\cdot s+1},\quad \mathrm{W}_2:=\frac{1}{\mathrm{T}_2\cdot s+1},\quad \mathrm{W}_3:=\frac{\mathrm{K}_3}{\mathrm{T}_3\cdot s+1},\quad \mathrm{W}_4:=-\mathrm{T}_4\cdot s,\quad \mathrm{W}_5:=\frac{\mathrm{K}_5}{\mathrm{T}_5\cdot s+1}.$$

Matrix of transfer functions is:

$$W_y := \left(E - S_{yu} \cdot S_{uy}\right)^{-1} \cdot S_{yu} \cdot S_{uv} \rightarrow \begin{pmatrix} W_1 & 0 \\ -\frac{W_1 \cdot W_2 \cdot W_3 \cdot W_4}{W_2 \cdot W_3 \cdot W_4 - 1} & -\frac{W_2}{W_2 \cdot W_3 \cdot W_4 - 1} \\ -\frac{W_1 \cdot W_3}{W_2 \cdot W_3 \cdot W_4 - 1} & -\frac{W_2 \cdot W_3}{W_2 \cdot W_3 \cdot W_4 - 1} \\ -\frac{W_1 \cdot W_3 \cdot W_4}{W_2 \cdot W_3 \cdot W_4 - 1} & -\frac{W_2 \cdot W_3 \cdot W_4}{W_2 \cdot W_3 \cdot W_4 - 1} \\ -\frac{W_1 \cdot W_3 \cdot W_5}{W_2 \cdot W_3 \cdot W_4 - 1} & -\frac{W_2 \cdot W_3 \cdot W_5}{W_2 \cdot W_3 \cdot W_4 - 1} \end{pmatrix}.$$

For external action v_2 and output action y_5:

$$W_{y.s} := W_{y.s} \left| \begin{matrix} \text{simplify} \\ \text{collect}, s \end{matrix} \right. \rightarrow \frac{K_3 \cdot K_5}{T_2 \cdot T_3 \cdot T_5 \cdot s^3 + \left(T_2 \cdot T_3 + T_2 \cdot T_5 + T_3 \cdot T_5 + K_3 \cdot T_4 \cdot T_5\right) \cdot s^2 + \left(T_2 + T_3 + T_5 + K_3 \cdot T_4\right) \cdot s + 1}.$$

The parameters of transfer functions are:

$$K_1 := 1,\ K_2 := 1,\ K_3 := 1,\ K_5 := 1,\ T_1 := 0.05,\ T_2 := 0.02,\ T_3 := 0.01,\ T_4 := 0.08,\ T_5 := 0.02.$$

The transient response is:

$$y_t(t) := y_s \left| \begin{matrix} \text{invlaplace} \\ \text{float}, 4 \end{matrix} \right. \rightarrow 0.25 \cdot e^{-50.0 \cdot t} + -0.001772 \cdot e^{-540.7 \cdot t} + -1.248 \cdot e^{-9.246 \cdot t} + 1.0.$$

The transient response allows to evaluate the quality of the process at the input action on a Heaviside function (figure2).

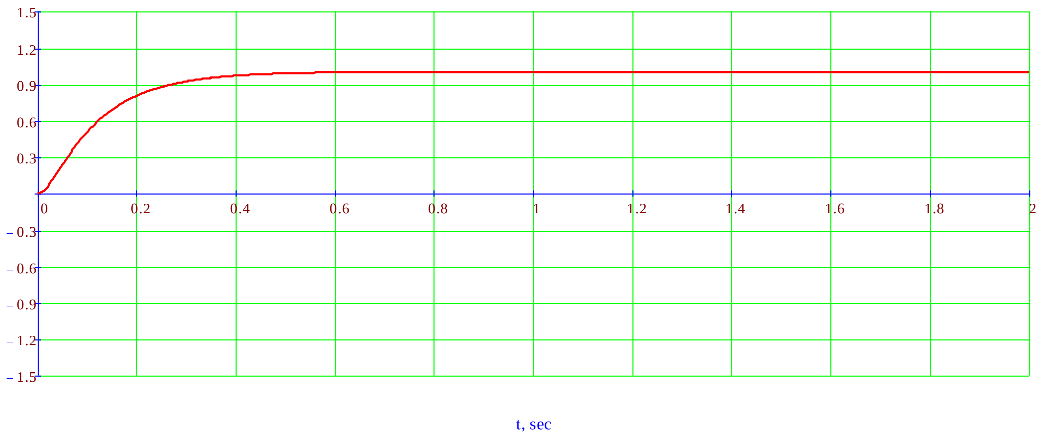

Figure 2. The transient responseon a Heaviside function (output action y_5).

The inputs of the blocks are connected to the outputs of the previous blocks. Therefore, the analysis of output actions is simultaneously considered to be the analysis of input actions. But the reis also a need to analyze separately the actions at inputs and outputs of the blocks.

For this, the obtained expression (2) is already used, into which (1) is inserted:

$$\bar{u} = \mathbf{S}_{uy}\bar{y} = \mathbf{S}_{uy}(\mathbf{E} - \mathbf{S}_{yu}\mathbf{S}_{uy})^{-1}\mathbf{S}_{yu}\mathbf{S}_{uv}\bar{v}\ .$$

Matrix of transfer functions for input actions is:

$$\mathrm{W}_u = \mathbf{S}_{uy}(\mathbf{E} - \mathbf{S}_{yu}\mathbf{S}_{uy})^{-1}\mathbf{S}_{yu}\mathbf{S}_{uv}\ .$$

For the system (figure 1) the program MathCAD is used:

$$W_u := S_{uy}\cdot W_y \rightarrow \begin{pmatrix} 0 & 0 \\ -\dfrac{W_1\cdot W_3\cdot W_4}{W_2\cdot W_3\cdot W_4 - 1} & -\dfrac{W_2\cdot W_3\cdot W_4}{W_2\cdot W_3\cdot W_4 - 1} \\ W_1 - \dfrac{W_1\cdot W_2\cdot W_3\cdot W_4}{W_2\cdot W_3\cdot W_4 - 1} & -\dfrac{W_2}{W_2\cdot W_3\cdot W_4 - 1} \\ -\dfrac{W_1\cdot W_3}{W_2\cdot W_3\cdot W_4 - 1} & -\dfrac{W_2\cdot W_3}{W_2\cdot W_3\cdot W_4 - 1} \\ -\dfrac{W_1\cdot W_3}{W_2\cdot W_3\cdot W_4 - 1} & -\dfrac{W_2\cdot W_3}{W_2\cdot W_3\cdot W_4 - 1} \end{pmatrix}.$$

For external action v_2 and block input action u_3:

$$W_{u.s} := W_{u_{k,j}} \rightarrow -\frac{W_2\cdot W_3\cdot W_4}{W_2\cdot W_3\cdot W_4 - 1},\quad u_s := W_{u.s}\cdot v_s \rightarrow -\frac{0.08}{0.11\cdot s + 0.0002\cdot s^2 + 1}.$$

The transient response is:

$$u_t(t) := u_s \left|\begin{array}{l}\text{invlaplace} \\ \text{float}, 4\end{array}\right. \rightarrow 0.7526\cdot e^{-540.8\cdot t} + -0.7526\cdot e^{-9.246\cdot t}.$$

The transient response allows to evaluate the quality of the process at the input action on a Heaviside function (figure3).

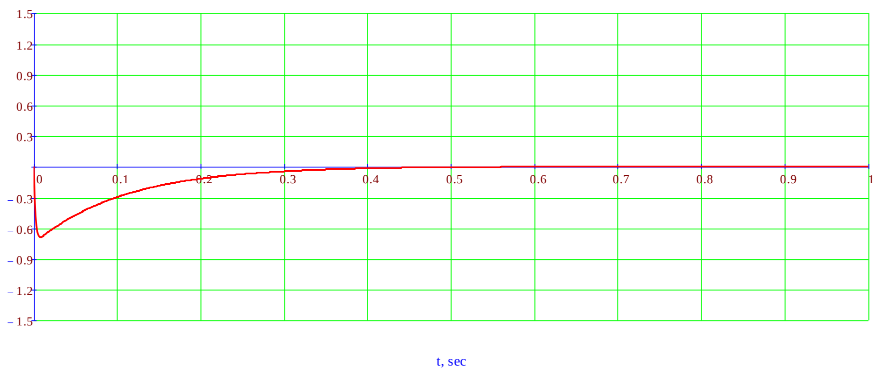

Figure 3. The transient response on a Heaviside function (input action u_2).

5. Conclusion

The presented method of forming the mathematical model of the system which is defined by the block diagram and functions of the blocks can be used in the controllers' synthesis.

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