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Method of studying the dynamics of the controlled mechanical continuously variable transmission with an elastic element and one-way clutch.

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Abstract. In comparison with other types of transmissions currently used, mechanical continuously variable transmissions are capable of providing significant technical and economic advantages, which makes research in this area relevant. This paper presents a mechanical and mathematical method that allows for studying the dynamic processes that take place during operating a controlled mechanical continuously variable transmission with an elastic element and a one-way clutch which provides for transmitting torque to the driven shaft in one direction only.

1. Introduction

The purpose of the research is to determine the kinematic parameters of the transmission elements, the magnitudes of the dynamic loads arising from the periodic engagement and disengagement of the one-way clutch.

The idea of developing continuously variable transmissions arose at the beginning of the 20th century due to the known imperfections of the gearbox transmissions. There are three main types of continuously variable transmissions: mechanical, electrical, hydraulic. Mechanical gearbox transmission compared to others have the advantage that the transformation of mechanical energy by mechanical devices without intermediate power conversion can be carried out with less losses. The studies of mechanical gearbox transmissions are described in the works of Alyukov S.V. [1,2]. The works of Sharkov O. V. are devoted to the issues of the development of one-way clutches and their dynamic loading. [3,4].

This article discusses the developed mechanical continuously variable transmission (torque converter) with oscillatory motion of an intermediate member. The transmission can be used in drives of low power machines, with thermal engines or with asynchronous motors, in drives of various technological equipment, such as belt conveyors, mixers, etc. [5].

2. Main part

The computational loading scheme for the continuously variable transmission under study with one path is shown in Figure 1. The scheme introduces the follows: J_1 is the reduced moment of engine inertia and of associated driving parts of the system, J_2 is the moment of inertia of the converter output

member and of the driving parts of the OWC, J_3 is the reduced moment of inertia of the OWC driven parts, J_4 is the reduced moment of inertia of the mechanical system driven parts, a, b, d and h are the geometrical parameters of the pulse mechanism, c is the torsional shaft stiffness.
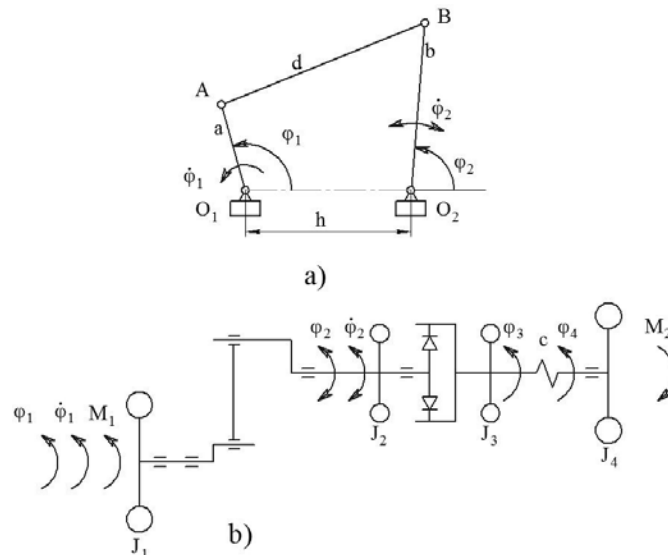

Figure 1. The computational loading scheme for continuously variable transmission.

This mechanical continuously variable transmission includes:

- a pulse mechanism, the crank O_1A of which is connected to the flywheel of the engine J_1;

- a one-way clutch (OWC) with additional working surfaces [6] installed between the output member (BO$_2$ rocker arm) of the pulse converter with the reduced inertia moment J_2 and the torsion shaft input end J_3;

- Transmission driven shaft with reduced moment of inertia J_4.

In the given transmission, the torque is transformed by using the oscillatory motion of the intermediate member (BO$_2$ rocker arm), which is then converted into rotation of the output shaft by means of a one-way clutch. The OWC is engaged and transmits a momentum of impulse to the driven shaft J_4, provided that the rocker arm BO$_2$ moves in the direction of torsion shaft input end rotation J_3 (direct pulse). Reverse pulse, in which the direction of rocker arm BO$_2$ movement is opposite to the torsion shaft input end rotation J_3, is not transmitted to the driven shaft J_4 due to the fact that the OWC is disengaged at this moment.

The transmission is controlled by changing the crank length O_1A, which, respectively, leads to a change in the rocker arm oscillations amplitude BO$_2$. The crank control mechanism O_1A is not considered in this paper. Due to the compliance of the elastic element (torsion shaft), the torque convertor acquires the property of internal automaticity when the loading mode of the driven shaft changes. The operation principle of such a transmission was discussed earlier in the publications [5, 6].

Thus, to study the dynamics of pulse transmission, a real mechanical system is represented as a 4 mass computational physical model (Figure 1).

A mathematical model of the mechanical system motion, based on the accepted physical model, will be compiled on the basis of the analytical Lagrange equations of the second order with undetermined multipliers. The need to include undetermined Lagrange multipliers is caused by the difficulty of obtaining an explicit analytical expression showing the change in the a rocker arm rotation angle (generalized coordinate φ_2) depending on the angle of motor shaft rotation φ_1 (i.e., in the form $\varphi_2 = f(\varphi_1)$). The relationship of the generalized coordinates φ_1 and φ_2 is determined by the following holonomic constraint equation:

$$f(\varphi_1, \varphi_2) = a^2 + b^2 + h^2 - d^2 - 2 \cdot a \cdot h \cdot \cos(\varphi_1) + 2 \cdot b \cdot h \cdot \cos(\varphi_2) -$$
$$- 2 \cdot a \cdot b \cdot \cos(\varphi_1 - \varphi_2) = 0. \tag{1}$$

The holonomic constraint equation (1) is obtained by representing all the members of the mechanism in the form of a closed loop vector with the further development of the equations of the loop vector projections, which after the corresponding algebraic transformations leads to the desired expression.

For holonomic systems, the Lagrange equations with undetermined multipliers have the form [3]:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i + \sum_{k=1}^{m} \lambda_k \cdot \frac{\partial f_{ki}}{\partial q_i} \tag{2}$$

where: L - is the Lagrange function;

q_i and $\dot{q}_i$ are the generalized coordinate and its derivative with respect to time, respectively;

f_k is the equation of the kth geometric constraint imposed on the ith generalized coordinate;

λ_κ is the Lagrange undetermined multiplier of the kth geometric constraint;

Q_i is the component of the generalized force acting in the direction of generalized coordinate δq_i variation.

The elementary work of all external forces applied to the points of the system under consideration on the virtual displacements δq_i of these points is determined by the equation,

$$\delta A = Q_1 \delta q_1 + Q_2 \delta q_2 + ... + Q_n \delta q_n$$

For the given mechanical system, the expression of virtual work will be

$$\delta A = M_1 \delta \varphi_1 - M_2 \delta \varphi_4 \tag{3}$$

where: $Q_1 = M_1$ and $Q_4 = -M_2$ are the components of the generalized force (having the dimension of the moment), equal respectively to the engine torque and the drag torque and applied respectively to the driving and driven shaft of the pulse transmission;

$\delta \varphi_1$ and $\delta \varphi_4$ are virtual variations of generalized coordinates.

The Lagrange function is the difference of the kinetic T and potential Π energies of the system [7] $L = T - \Pi$.

Considering the above, for the mechanical system under study, the Lagrange function will have the following form:

$$L = \frac{1}{2} \cdot J_1 \cdot \dot{\varphi}_1^2 + \frac{1}{2} \cdot J_2 \cdot \dot{\varphi}_2^2 + \frac{1}{2} \cdot J_3 \cdot \dot{\varphi}_3^2 + \frac{1}{2} \cdot J_4 \cdot \dot{\varphi}_4^2 - \frac{1}{2} \cdot c \cdot \left(\varphi_3 - \varphi_4\right)^2 \tag{4}$$

The first part of the transmission operation cycle is characterized by the fact that the OWC is engaged and its driving and driven parts move together. Wherein $\varphi_2 = \varphi_3$, $\dot{\varphi}_2 = \dot{\varphi}_3$. With this in mind, the geometric constraint equation will also be transformed

$$f_1(\varphi_1, \varphi_3) = a^2 + b^2 + h^2 - d^2 - 2 \cdot a \cdot h \cdot \cos(\varphi_1) + 2 \cdot b \cdot h \cdot \cos(\varphi_3) -$$
$$- 2 \cdot a \cdot b \cdot \cos(\varphi_1 - \varphi_3) = 0. \tag{5}$$

Using the Lagrange equations (2), expressions (3), (4), as well as the constraint equation (5), we obtain the following system of differential equations of motion, describing the dynamics of the transmission in the first part of the cycle, when the OWC is engaged:

$$\begin{cases} J_1 \cdot \ddot{\varphi}_1 = M_1 + 2 \cdot A_1 \cdot \lambda; \\ \left(J_2 + J_3\right) \cdot \ddot{\varphi}_3 + c(\varphi_3 - \varphi_4) = -2 \cdot A_2 \cdot \lambda; \\ J_4 \cdot \ddot{\varphi}_4 - c(\varphi_3 - \varphi_4) = -M_2, \end{cases} \tag{6}$$

where: A_1, A_2, are auxiliary functions,

$$A_1 = a \cdot h \cdot \sin(\varphi_1) + a \cdot b \cdot \sin(\varphi_1 - \varphi_3);$$

$$A_2 = b \cdot h \cdot \sin(\varphi_3) + a \cdot b \cdot \sin(\varphi_1 - \varphi_3).$$

To eliminate the undetermined Lagrange multiplier λ, we multiply the right and left sides of the first equation of system (6) by the coefficient A_3, the second equation of system (6) by the coefficient A_1 and sum them up. Then we get a system of two equations:

$$\begin{cases} \ddot{\varphi}_1 \cdot J_1 \cdot A_2 + \ddot{\varphi}_3 \cdot \left(J_2 + J_3\right) \cdot A_1 = M_1 \cdot A_2 - A_1 \cdot c(\varphi_3 - \varphi_4); \\ J_4 \cdot \ddot{\varphi}_4 - c(\varphi_3 - \varphi_4) = -M_2, \end{cases} \tag{7}$$

But two equations in system (7) are not enough, since the number of generalized coordinates is three. The second derivative with respect to time of the geometric constraint equation (5) will be used as the missing third equation:

$$\ddot{\varphi}_1 \cdot \left[a \cdot h \cdot \sin(\varphi_1) + a \cdot b \cdot \sin(\varphi_1 - \varphi_3)\right] - \ddot{\varphi}_3 \cdot \left[b \cdot h \cdot \sin(\varphi_3) + a \cdot b \cdot \sin(\varphi_1 - \varphi_3)\right] =$$
$$= -\dot{\varphi}_1^2 \cdot a \cdot h \cdot \cos(\varphi_1) + \dot{\varphi}_3^2 \cdot b \cdot h \cdot \cos(\varphi_3) - \left(\dot{\varphi}_1 - \dot{\varphi}_3\right)^2 \cdot a \cdot b \cdot \cos(\varphi_1 - \varphi_3). \tag{8}$$

We introduce the notations:

$$B_1 = M_1 \cdot A_2 - A_1 \cdot c(\varphi_3 - \varphi_4);$$

$$B_2 = -\dot{\varphi}_1^2 \cdot a \cdot h \cdot \cos(\varphi_1) + \dot{\varphi}_3^2 \cdot b \cdot h \cdot \cos(\varphi_3) - \left(\dot{\varphi}_1 - \dot{\varphi}_3\right)^2 \cdot a \cdot b \cdot \cos(\varphi_1 - \varphi_3).$$

Taking into account the accepted notations, the system of equations (6), the second derivative with respect to time of the constraint equation (8), the mathematical model of the motion of a pulse transmission in a part of the cycle, when the OWC is engaged, is as:

$$\begin{cases} \ddot{\varphi}_1 \cdot J_1 \cdot A_2 + \ddot{\varphi}_3 \cdot \left(J_2 + J_3\right) \cdot A_1 = B_1; \\ \ddot{\varphi}_1 \cdot A_1 - \ddot{\varphi}_3 \cdot A_2 = B_2; \\ \ddot{\varphi}_4 \cdot J_4 - c(\varphi_3 - \varphi_4) = -M_2. \end{cases} \tag{9}$$

The OWC provides the kinematic condition according to which the speed of their driven parts $\dot{\varphi}_3$ can be greater, but cannot be less than the speed of the driving parts $\dot{\varphi}_2$ associated with the rocker arm BO₂, i.e. $\dot{\varphi}_3 \geq \dot{\varphi}_2$.

In this case, torsion shafts are twisted to a certain maximum value at the beginning, then untwist to zero at the end of the first part of the cycle. But in any case, the angle of twist of the torsion shafts

$\varphi_{3AK} = \varphi_3 - \varphi_4$ cannot be negative, i.e. $\varphi_3 \geq \varphi_4$. Thus, the condition for the ending of the first part of the cycle is torsion shaft twisting absence, i.e. $\varphi_{3AK} = 0$.

In the second part of the cycle, the OWC is disengaged. The mechanical system is divided into two independent subsystems:

- the subsystem at the input, including the flywheel of the engine J_1, the rocker arm of BO_2 and the driving parts of the OWC J_2;

- the subsystem at the output, which includes the driven parts of the OWC J_3 and the driven shaft of the mechanical system J_4.

The system of motion equations of the subsystem at the input into the pulse transmission will take the form:

$$\begin{cases} J_1 \cdot \ddot{\varphi}_1 = M_1 + 2 \cdot A_3 \cdot \lambda; \\ J_2 \cdot \ddot{\varphi}_2 = -2 \cdot A_4 \cdot \lambda. \end{cases} \quad (10)$$

where: A_3, A_4, are auxiliary functions,

$$A_3 = a \cdot h \cdot \sin(\varphi_1) + a \cdot b \cdot \sin(\varphi_1 - \varphi_2);$$
$$A_4 = b \cdot h \cdot \sin(\varphi_2) + a \cdot b \cdot \sin(\varphi_1 - \varphi_2).$$

The first equation of the mathematical model of the subsystem at the input is obtained by excluding the Lagrange undetermined multiplier λ from the system of equations (10) as described above. The second equation is obtained by double differentiation with respect to time of the equation of the geometric constraint (3):

$$\begin{cases} \ddot{\varphi}_1 \cdot J_1 \cdot A_4 + \ddot{\varphi}_2 \cdot J_2 \cdot A_3 = M_1 \cdot A_4; \\ \ddot{\varphi}_1 \cdot A_3 - \ddot{\varphi}_2 \cdot A_4 = C_2, \end{cases} \quad (11)$$

Where

$$C_2 = -\dot{\varphi}_1^2 \cdot a \cdot h \cdot \cos(\varphi_1) + \dot{\varphi}_2^2 \cdot b \cdot h \cdot \cos(\varphi_2) - (\dot{\varphi}_1 - \dot{\varphi}_2)^2 \cdot a \cdot b \cdot \cos(\varphi_1 - \varphi_2).$$

The mathematical model of the subsystem at the output will be as follows:

$$\begin{cases} J_3 \cdot \ddot{\varphi}_3 = -c(\varphi_3 - \varphi_4); \\ J_4 \cdot \ddot{\varphi}_4 = c(\varphi_3 - \varphi_4) - M_2. \end{cases} \quad (12)$$

The second part of the cycle ends under the kinematic condition $\dot{\varphi}_3 \geq \dot{\varphi}_2$.

Numerical simulation of the system.

In view of the essential nonlinearity of the obtained equations (9), (11) and (12), their solution is possible only with computer assistance. Applying the Runge-Kutta numerical method, a solution of these systems of equations for the transmission steady-state mode is obtained. A characteristic was obtained that determines the change in the torsion twist angle per cycle, shown in Figure 2 with the transformer ratio i=$\dot{\varphi}_4 / \dot{\varphi}_1$=0,25, Figure 3 shows the graphs of the angular speeds of the transmission members.
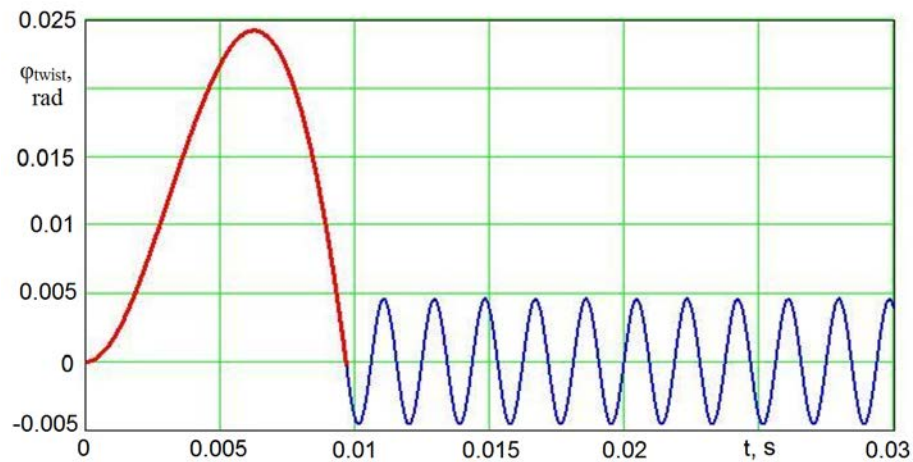
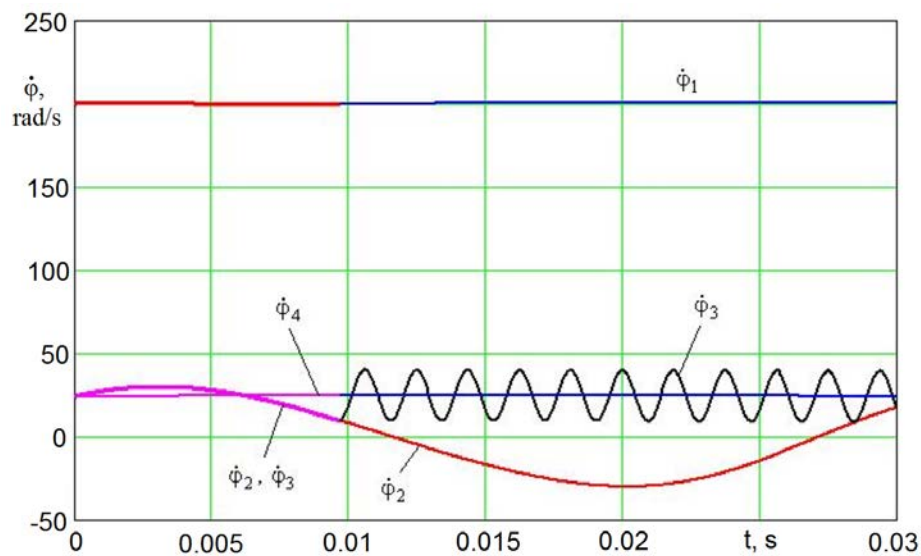**Figure 2.** Change in the torsion twist angle per cycle.

Figure 3. Angular speeds of the transmission members per cycle.

3. Conclusion

1) A mathematical model of the motion of a controlled mechanical continuously variable transmission with an elastic element, represented by a system of differential equations (9), (11) and (12), is solved by the Runge – Kutta numerical method with a minimum error. The adequacy of the mathematical model can be checked by comparing the results of pulse transmission experimental studies.

2) Currently, there are still some problems in the field of pulse transmission theory and design. The solution of individual problems is proposed to be presented in subsequent articles.

Acknowledgements

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