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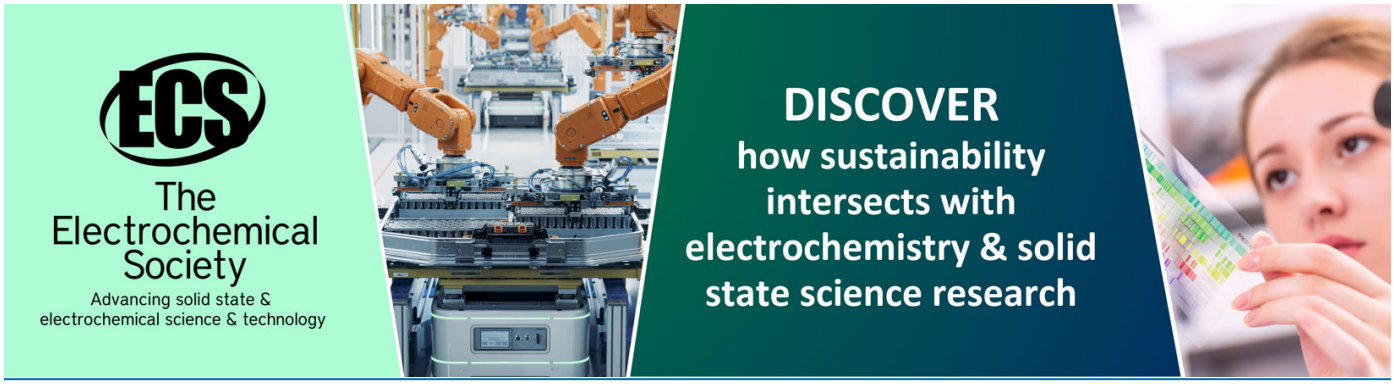

IOP Conf. Series: Materials Science and Engineering **700** (2019) 012030 doi:10.1088/1757-899X/700/1/012030

Inspection planning for fatigue cracks in offshore structures by Monte Carlo simulations

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Abstract. Probability-informed inspection planning has traditionally a tendency to produce longer inspection intervals for older structures in contrast with the intuitive assumption that shorter inspection intervals are needed. In this paper an alternative method for probability-informed inspection planning is suggested based on a parameter updating rather than the general updating used in most cases. The proposed method produces constant inspection intervals after the theoretical first inspection. Further, the method produces reasonable inspection intervals more or less in line with what is used in practise in the offshore industry at present.

1. Introduction

The vast majority of the infrastructure (offshore platforms, buildings, bridges, etc) in the world is existing and now ageing. This large number of existing structures is of vital importance for society, but they are in many cases degrading and need significant investments for maintenance. Replacing such structures with new ones might be both economically and environmentally unsound. Still, most universities educate students in civil and structural engineering merely in the design of new structures. The University of Stavanger, however, has established a course and research work into the continued use of ageing structures.

Using ageing structures beyond their original design life can raise several concerns. Structures in operation are exposed to conditions of stress and environment that ultimately will degrade them from their initial state and damage will accumulate until the structures may be judged to be no longer fit-for-service. If not withdrawn from further service or being repaired, failure of some kind will eventually occur. Also, the cost of the required maintenance, inspection and repair needed to cope with this deterioration and damage will at some stage become unacceptable compared to the revenue that can be gained from the use of these structures. Hence, it becomes vital to know:

1. how structures change with age,
2. how their condition and other aspects influencing their safety can be determined,
3. how their capacity (strength and fatigue life) can be determined as a result of ageing mechanisms,
4. how any anomalies in the structures should be evaluated,
5. how anomalies found in an existing structure can be repaired and mitigated, and
6. how the integrity management of ageing structures should be performed.

This paper is work performed under item 6 as listed above and is specifically aimed at providing insight into the inspection planning for old structures. The work is based on the master thesis performed by Neeraas [1].

The integrity management of existing structures requires that changes to all aspects that influence the safety of structures are identified, registered, evaluated and if needed mitigated or repaired. One such change that will influence the safety of a structure in operation is the development of fatigue cracks and an inspection regime to identify such fatigue cracks are needed for any structure that is experiencing cyclic loading. Historically the inspection for fatigue cracks typically have followed a calendar-based programme with inspection intervals in the range of every 4-5th year for critical parts of the structure. Probability-based inspection planning (sometimes called risk-based inspection planning or RBI) was developed in the 1980'ties, see e.g. Madsen et al [2]. In general, the probability-based inspection planning was introduced to reduce the amount of inspections by evaluating the probability of the presence of a crack at different positions and evaluating the consequence of having the crack in these positions. These evaluations would then be used to prioritize where to inspect by choosing to inspect the positions where the combination of probability and consequence of having a crack has the largest influence on safety level.

In addition to prioritization of where to inspect, the probability-based inspection analysis also allowed for taking into account the results of inspections performed. Most importantly, if an inspection were performed and no cracks were determined, the probability of finding a fatigue crack in the following years could be reduced by probabilistic updating methods (Bayesian updating).

An example of such updating is shown in Figure 1. In the example, inspection has been determined to be needed after approximately 18 years. After the first inspection, which is modelled as "no cracks found", the probability is updated and the next inspection is determined to be needed after an additional 12 years. The inspection intervals indicated in this example figure, assuming that all inspection will turn out negative, is then by reading the figure approximately 18, 12, 20 and 25 years.

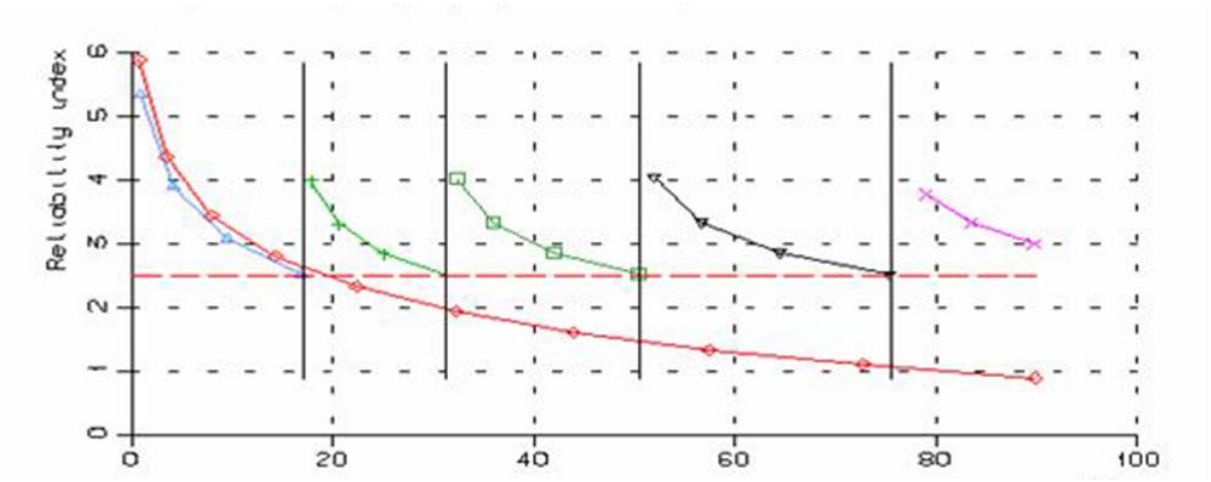

Figure 1. Updated probability of fatigue crack in structure (www.dnvgl.com).

Although the example in Figure 1 is purely an illustration it indicates the trend that has motivated this study, namely that the inspection intervals for ageing structures seems to increase with age rather than to decrease as one would intuitively expect.

2. Overview of simulations performed

The simulations presented in this paper studies the crack growth and the inspection need for a node in an offshore jacket. The study includes evaluating the effect of these parameters:

- three different calculated fatigue lives (20, 40 and 60 years)
- uncertainties in the initial crack size (a_0)

- uncertainties in the applied load
- level of acceptable failure probability corresponding to 0.01 and 0.001 respectively.

An overview of the simulation process is shown in Figure 2.

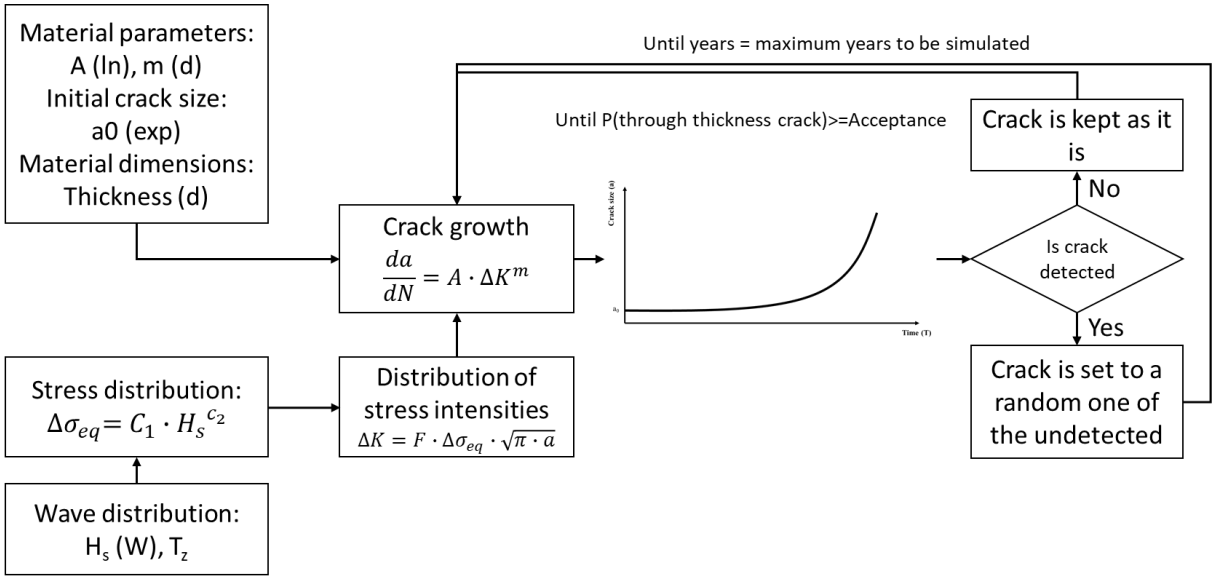

Figure 2. A proposed process for probability-based inspection planning based on Ersdal and Oma [12]. In the figure ln indicates a lognormal distributed parameter, exp indicates an exponential distributed parameter, W indicates a Weibull distributed parameter and d indicates a deterministic parameter.

The first step in this process is to define the input parameters: material parameters, initial crack size, wave height and period distribution. The initial crack size in the welded detail is assumed to follow and exponential distribution with an mean initial crack size of $\mu_{a0} = 0.11mm$ [3] and the probability distribution function and cumulative distribution function described by:

$$f(x) = \lambda \cdot e^{-\lambda \cdot x} \quad (1)$$

$$F(x) = 1 - e^{-\lambda \cdot x} \quad (2)$$

where $\lambda = \frac{1}{\mu_{a0}}$.

The slope of the Paris equation m (or m_a) is in these analyses fixed to 3.0. The material parameters lnA is determined by calibration of the crack growth curve to provide similar probability of failure as a SN-fatigue analysis as shown in Figure 3.

In the SN-fatigue analysis the design value of log(a) is 11.764 and the slope of the SN curve is set to $m_{SN} = 3$. Mean value of log(a) is assumed to be 2 standard deviations larger than the design value and the standard deviation is assumed to be 0.25. Hence, the mean value of log(a) is 12.264. Based on the calibration performed the material parameters of the Paris equation has been modelled by a mean value of lnA is 29.1 and a standard deviation of lnA is 0.64.

The long-term distribution of significant wave heights is given by:

$$F_{H_s}(h) = 1 - exp\left[-\left(\frac{h - H_0}{H_C - H_0}\right)^{\gamma}\right] \quad (3)$$

where $H_C = 2.895m$, $H_0 = 0.198m$ and $\gamma = 1.499$.

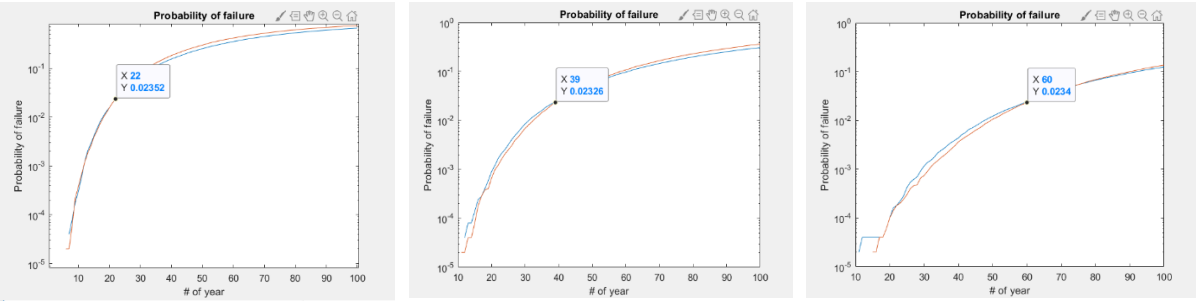

Figure 3. Result of calibration the fracture mechanics parameters to the SN-curve.

Mean zero up-crossing period of the sea-state is given by:

$$T_Z = 3.3 \cdot \sqrt{H_s} \qquad \text{with a minimum value of } T_Z = 5.72 \text{ s.} \tag{4}$$

Based on the wave height and period the width of the stress intensity, ΔK, is defined by the equivalent stress range $\Delta\sigma_{eq}$ implemented as described in Ersdal [4], given by the wave distribution and the geometric function of a tubular jacket node.

$$\Delta\sigma_{eq} = \frac{C_1}{1.702 - 0.138 \cdot m_a} \cdot (H_s)^{C_2 - 0.03} \tag{5}$$

where C_1 and C_2 are coefficients, H_s is the significant wave height and m_a is the slope of the crack-growth curve in the Paris equation.

Crack-growth is modelled by the Paris equation:

$$\frac{da}{dN} = A \cdot \Delta K^{m_a} \tag{6}$$

where A and m_a are material parameters and:

$$\Delta K = \Delta\sigma \cdot F(a,t) \cdot \sqrt{\pi \cdot a} \tag{7}$$

where ΔK is the stress intensity range, $\Delta\sigma$ is the stress range of each cycle, $F(a,t)$ is the geometric stress function for the detail and a is the present crack depth. For a tubular member detail, the geometric stress function may be set to (Dalane, 1993):

$$F(a,t) = \left(1.08 - 0.7 \cdot \frac{a}{t}\right) \cdot \left(1.0 + 1.24 \cdot e^{-22.1\frac{a}{t}} + 3.17 \cdot e^{-357\frac{a}{t}}\right) \tag{8}$$

where a is the present crack depth and t is the thickness of the material. For each sea-state the crack growth can be calculated as [5]:

$$da_i = A \cdot \left[\Delta\sigma_{eq} \cdot F(a_{i-1}, t) \cdot \sqrt{\pi \cdot a_{i-1}}\,\right]^{m_a} \cdot N_w \tag{9}$$

where N_w is the number of cycles in the sea-state.

As sea-states are simulated by a Monte-Carlo simulation the crack size increases. At the end of each year the number of cracks calculated to be larger than the thickness (through-thickness cracks) is counted and if the number of through thickness cracks divided by the total number of simulations exceeds the acceptance requirements (0.01 and 0.001) an inspection is performed.

The inspection is simulated by drawing a random number and checking if the probability of detection for the size of the simulated crack is larger than this random number. The probability of detection (PoD) curves are given by DNVGL [6]:

$$PoD(d) = 1 - \frac{1}{1 + \left(\frac{d}{x_0}\right)^b} \tag{10}$$

where d is the dimension of the crack (depth or length), X_0 and b is parameters for individual inspection methods.

The ratio between crack depth and crack length $\frac{a}{c}$ is assumed to be 0.15 [5]. In the simulations the crack depth is simulated and where crack length is used this is transferred to crack depth by $a = \frac{1}{2} \cdot 0.15 \cdot (2 \cdot c)$.

PoD-curves is dependent on the NDT method used, the environment they are used in and the accessibility of inspection. DNVGL-RP-C210 (DNVGL 2015) gives the values for X_0 and b for MPI and CVI respectively for various environments and accessibilities. In these simulations under water conditions and low accessibility are selected for both cases. For MPI X_0 = 1.16 and b = 0.90 are used with depth of crack as dimension. For CVI X_0 = 83.03 and b = 1.079 are used with length of crack as dimension in the PoD curve.

If the simulations indicate that a crack is detected by the procedure described above it is set to the largest of 1) a random one of the undetected ones and 2) a random value of the initial distribution of cracks. The undiscovered cracks are retained as they are. Both discovered and undiscovered cracks are then simulated further until the maximum life cycle is achieved.

In order to obtain a reasonable accuracy in the simulations the number of Monte-Carlo simulations is set according to the formula [7]:

$$s = \sqrt{\frac{\hat{P}_f\left(1 - \hat{P}_f\right)}{N}} \tag{11}$$

where s is the standard error of the failure probability, $\hat{P}_f$ is the failure probability, and N is the number of simulations. In order to obtain a 5% accuracy for a problem with a $\hat{P}_f \sim 10^{-4}$ the number of simulations need to be:

$$N = \frac{P_f(1 - P_f)}{s^2} = \frac{P_f(1 - P_f)}{(0{,}05 \cdot P_f)^2} = \frac{(1 - P_f)}{P_f \cdot (0{,}05)^2} \approx 4\,000\,000$$

In these simulations the acceptable failure probability is set to 0.01 and 0.001 respectively. This implies that 40 000 and 400 000 simulations are needed for each case respectively.

The parameter sensitivity study includes the model uncertainty (load and crack growth model) and the initial crack size. The model uncertainty is included by stochastic variables with mean value of 1.0 and a coefficient of variation (CoV) in the crack growth model as shown in Equation 12.

$$\frac{da}{dN} = A \cdot \left[\alpha_G \cdot \alpha_Y \cdot \Delta\sigma_{eq} \cdot F \cdot \sqrt{\pi \cdot a}\right]^{m_a} \tag{12}$$

The model uncertainty in the load model is modelled by a stochastic variable that is kept for the full life of the simulation (GCoV) and a stochastic variable that is changed every year (YCoV). Each of these variables are modelled with a CoV in the range of 0.0-0.3. The model uncertainty in the initial crack size is modelled by a stochastic variable with a CoV in the range of 0.0-0.2.

In addition, the effect of two probability levels for when inspections should be performed is studied ($P_{acceptance}$ equal to 0.01 or 0.001). The $P_{acceptance}$ values are based on normally used acceptance criteria for important and medium important structural nodes and members.

3. Results of the simulations

Figure 4 and Figure 5 show the simulated initial crack sizes and the crack growth parameter lnA respectively. Figure 6 shows the crack growth of 20 random simulations of the detail. The updates after inspection can be detected by a discrete reduction of the crack size. Also, some examples of deeper cracks can be seen. These are a result of that the updated crack size is selected as a random value between the undetected cracks, and there will always be a likelihood that a deeper crack is un-detected and can be selected as the new crack size of a detected crack.

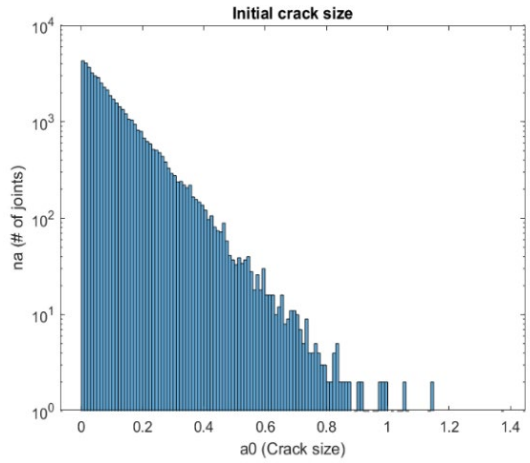

Figure 4. Example of simulated initial crack size.

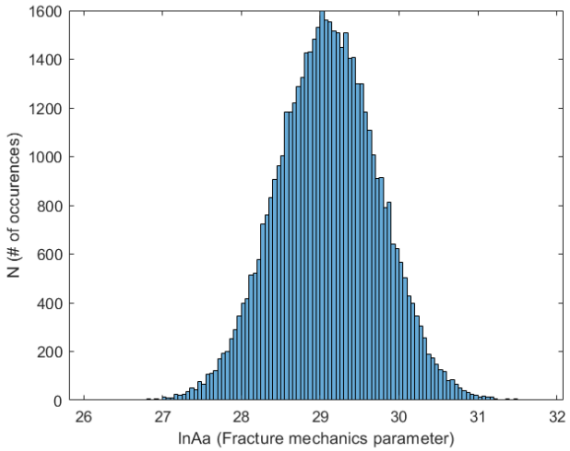

Figure 5. Example of simulated fracture mechanics material parameter.

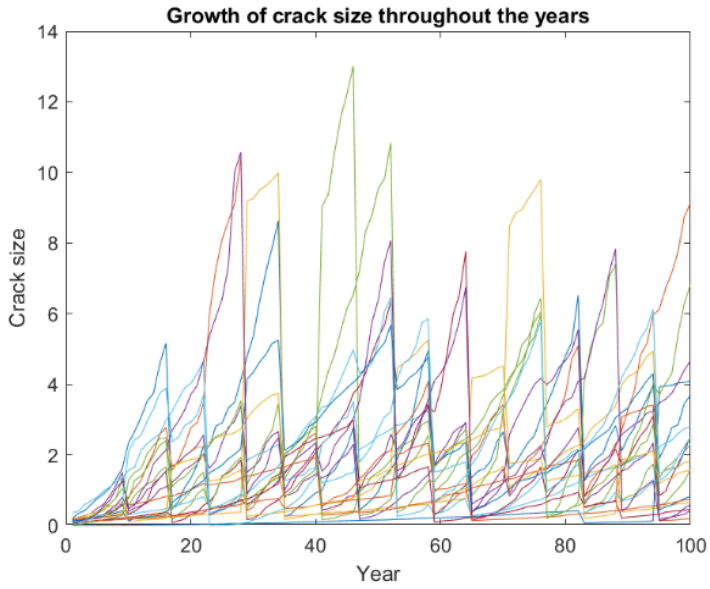

Figure 6. Example of crack growth and updating after inspection for 20 random simulations.

Figure 7 gives the probability of failure calculated for one of the simulations. As indicated by the Figure 7 this method of updating the failure probability after inspection provides a constant inspection intervals after the first inspection. Hence, we have in the main result part of this paper looked exclusively on the number of years until first inspection is needed and the inspection interval after the first inspection.

As an additional benefit the likelihood of a crack being detected (in Figure 8 called “repair required”) can be calculated using this simulation method. The trend indicated in Figure 8 is quite similar in all cases simulated. For CVI the curve flattens out at a plateau of 0.4 and for MPI on a plateau value of 0.6-0.67 as also shown in the example in Figure 8.

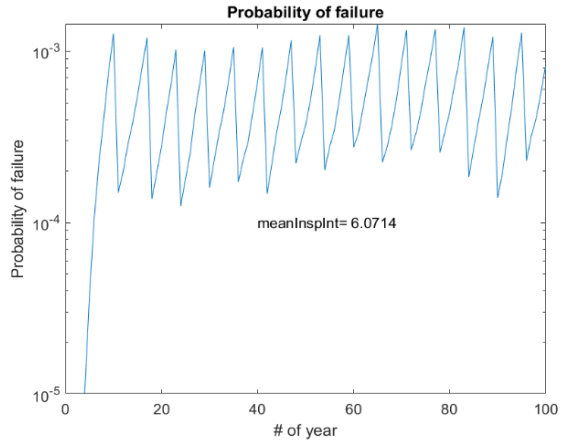

Figure 7. Example of simulated probability of failure.

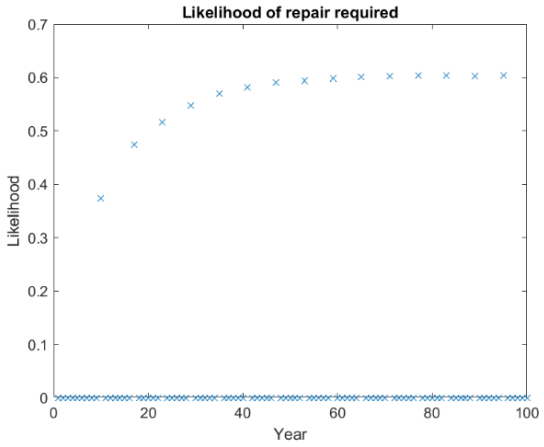

Figure 8. Simulated probability of detection of crack.

The core of the updating is the distribution of cracks prior to and after inspections. An example of two such distributions are shown in Figure 9 and Figure 10. The distribution of cracks prior to updating is shown in red, and the distribution of cracks after updating is shown in blue (note that blue over red becomes purple in these figures). It can easily be seen that the number of large cracks is reduced, and it is also possible to see that the number of small cracks is increased after the updating.

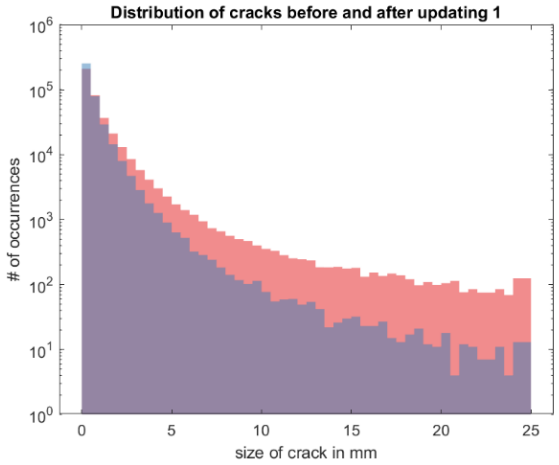

Figure 9. Distribution of simulated crack sizes before and after first inspection.

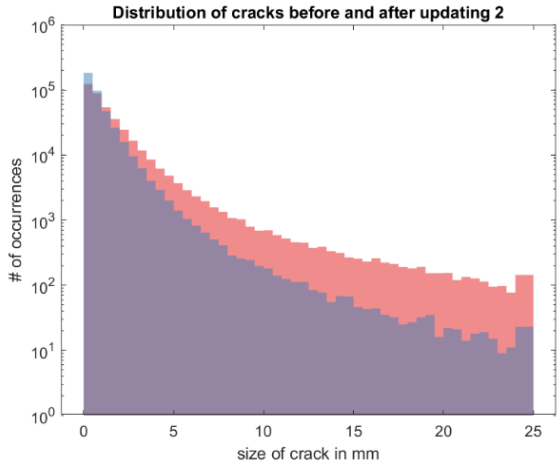

Figure 10. Distribution of simulated crack sizes before and after second inspection.

4. Results of the simulations

A key part of this study was to investigate the effect of the parameters representing the model uncertainties and uncertainty in the initial crack growth. As expected, an increase in the model uncertainty both for the GCoV and YCoV decreases the inspection intervals as shown in Figure 11 and Figure 12. The decrease in calculated inspection interval is more prominent for the GCoV case.

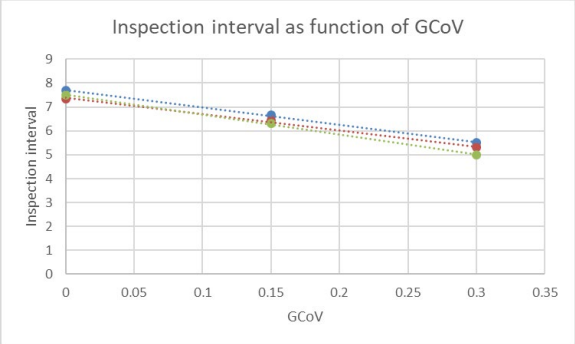

Figure 11. Inspection interval as a function of GCoV.

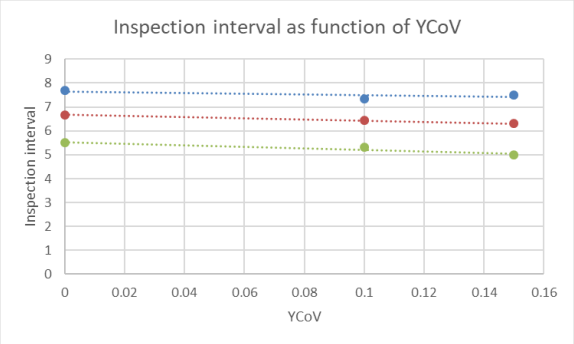

Figure 12. Inspection interval as a function of YCoV.

The effect of the CoV on the initial crack size is shown in Figure 13. Essentially, the CoV on the initial crack size have surprisingly very low influence on the calculated inspection intervals. However, this can be explained by the very narrow distribution of initial crack sizes used as basis in these simulations. The effect of the "acceptable probability of failure" Pacc on the inspection interval is shown in Figure 14. As expected, a lower Pacc result in more frequent inspection intervals.

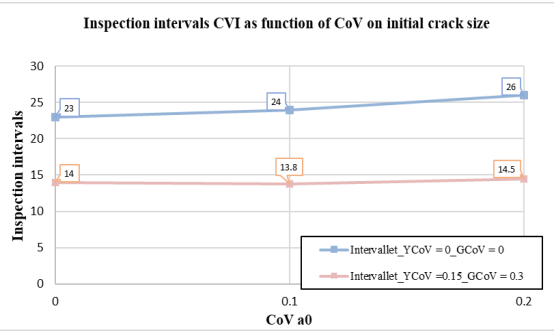

Figure 13. Inspection interval as a function of CoV on initial crack size.

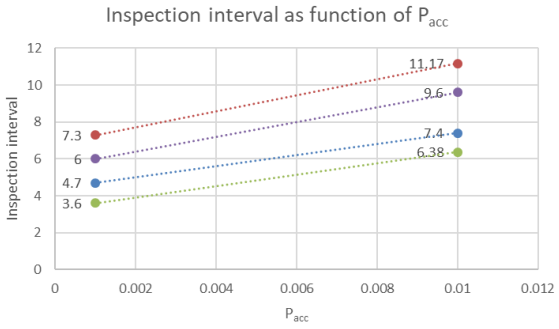

Figure 14. Inspection interval as a function of Pacc for MPI and CVI for CoV's equal to zero and CoV's equal to 0.15.

Figure 15 shows the inspection interval (as fraction of the calculated fatigue life) as function of the calculated fatigue life. A certain tendency for a longer inspection interval as fraction of the calculated fatigue life is seen. However, the increase is minor in the case with model uncertainty.

Table 1 gives an overview of a selected number of simulations performed in the master thesis of Neeraas [1]. As shown in Table 1 the inspection intervals for a detail with a calculated fatigue life of 20 years are as low as 3.6-5.2 years for CVI inspections and 6-8 years for MPI inspections. Such inspection intervals are not far from the present practice in the offshore industry. However, these suggested inspection intervals are a result of a master thesis without the quality assurance required in projects in the offshore industry. Hence, the results should be seen as indications of the capability of the method rather than actual suggestions for inspection intervals to be used in practice.

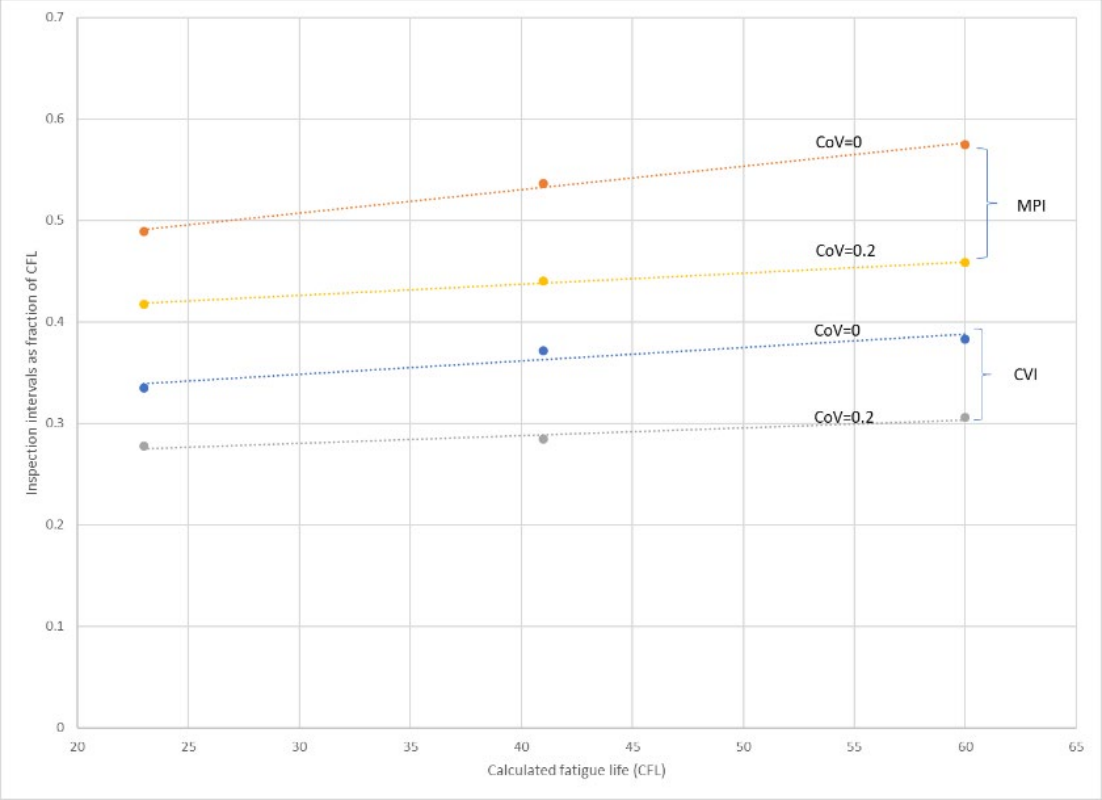

Figure 15. Inspection intervals as fraction of calculated fatigue life (CFL) as a function of the calculated fatigue life.

Table 1. Overview of results for inspection intervals for a jacket structure [1].

FL	P_{ACC}	Y_{COV}	G_{COV}	$A0_{COV}$	CVI INT	MPI INT	$A0_{COV}$	CVI INT	MPI INT
23	0.01	0	0	0	7.4	11.17	0.1	8	11.33
23	0.01	0.15	0	0	7.4	10.57	0.1	7.6	10.57
23	0.01	0	0.15	0	6.67	10.71	0.1	6.67	10.86
23	0.01	0.15	0.15	0	6.38	9.6	0.1	6.23	9.6
23	0.01	0	0.3	0	5.18	7.9	0.1	5.8	7.9
23	0.001	0	0	0	4.7	7.3			
23	0.001	0.15	0.15				0.1	3.6	6
41	0.01	0	0	0	14.5	22	0.1	16	22
41	0.01	0.15	0	0	14	19.7	0.1	13.5	20.7
41	0.01	0	0.15	0	13.2	17.75	0.1	12.8	18.3
41	0.01	0.15	0.15	0	11.33	18.3	0.1	12	17.8
41	0.01	0	0.3	0	9.4	14.6	0.1	9.3	14
41	0.001	0	0	0	8.75	13.4			
41	0.001	0.15	0.15				0.1	6.75	10.25
60	0.01	0	0	0	22	35	0.1	24	34
60	0.01	0,15	0	0	22.5	32	0.1	22	33
60	0.01	0	0.15	0	20	31	0.1	20.5	32
60	0.01	0.15	0.15	0	18.7	27.5	0.1	18	27.5
60	0.01	0	0.3	0	14	21	0.1	15	21.3
60	0.001	0	0	0	13.25	23			

Similarly, a study to establish guidance on inspection intervals were developed during the update of NORSOK N-005 [8] as shown in Table 2. The inspection intervals here is based on a generic reliability based analysis assuming no crack detection in all sub-sequent inspections. The inspection method assumed is above water Eddy Current inspection (EC) with probability of detection (PoD) as defined in DNV GL RP-C210 [6]. A wall thickness of 25 mm is used and the detail is assumed analysed with a DFF =1.0. SN curve for tubular sections in air and pure membrane stress are used. Based on DNV GL RP-C210 (with Weibull shape parameter 0.8, ln(A) = 2.3114 and coefficient of variation (CoV) 0.3, 0.2 and 0.15 respectively). The table provides generic structural reliability inspection intervals comparable with the results of the simulations in this paper with P_{acc} = 0.001. From the 4th inspection and onwards the requirement of NORSOK N-006 [9] of a maximum inspection interval of 5 years was used to overrule the probabilistic inspection intervals in this work.

Table 2. Generic reliability-based inspection intervals prepared during the development of NORSOK N-005.

Fatigue life (years)	Uncertainty of fatigue and hot spot stress calculation	1st (years)	2nd (years)	3rd (years)	4th and onward (years)
20	High (simplified)	4.7	4.3	5.0	5
20	Medium (parametric)	6.7	5.3	6.0	5
20	Low (Finite element calculation)	7.5	4.5	6.0	5

The inspection intervals presented in Table 2 is based on a long-term stress range distribution rather than the method used elsewhere in this paper. The case calculated in this paper with YCoV = 0.15, GCoV = 0.15 and Pacc = 0.001 giving MPI inspection interval of 6 years is comparable with the "Medium (Parametric)" case in Table 2 indicating an inspection interval of 6.7, 5.3 and 6. Although the results in ***Table 2*** are performed with SN-curves for air and with a long term distribution of stress ranges (and some other minor differences) the comparison might be described as a relatively good match.

5. Conclusions and discussion

The aim of the simulations presented here is to:

- Investigate the trend of the inspection intervals for older structures to see if lower inspection intervals were obtained with age by this method of updating. Rather than giving lower intervals with age, the method is predicting a constant inspection interval onwards from the theoretically first inspection. This is an interesting but also understandable conclusion and fully in line with the inspection intervals that have been used in shipping and offshore industry for several years.
- Use the Monte-Carlo simulation method to illustrate the probability-informed crack growth model and updating in a way that were easier to follow and understand. The method has shown that it is capable of producing results that are easy to follow. However, the way the method is programmed at the moment is rather slow and are not necessarily applicable for practical engineering purposes.
- Perform a parameter study to gain insight in how the inspection intervals change with variation in model uncertainty (modelled by the CoV of the model uncertainty). The uncertainty on the load model (YCoV and GCoV) behaved as expected and gave shorter intervals between inspection when the CoV values increased. Somewhat surprising the model uncertainty on the initial crack distribution had little influence on the resulting inspection intervals.
- Establish inspection intervals for calculated fatigue lives in the range between 20-60 years. These results are shown in Table 1.
- Evaluate if the proposed method gave reasonable inspection intervals. The analysis here is based on several simplifications (e.g. the model for equivalent stress and the stress intensity function used) and the analysis is performed as a part of a master thesis with no or little quality assurance. However, the inspection intervals produced by the method seems reasonable and comparable with

inspection intervals developed by the expert committee during the 2015 revision of NORSOK N-005 [8].

The calibration of fracture mechanics crack growth parameters to match SN-fatigue curves is a topic of debate in the industry and is not necessarily well standardized. In DNVGL RP-C210 some guidance on this subject is given, but this guidance is not fully followed in this paper. For example, DNVGL RP-C210 highlights the importance of achieving calibrated material parameters that produces reasonable crack growth curves. In addition, it may be more correct to calibrate the material parameters to the SN-curve at a 50% probability level rather than the 2.3% probability level as used herein (see ***Figure 3***).

6. Recommendation and further work

The updating process presented in this work is somewhat synthetic and do not follow a Bayesian parameter updating as described in e.g. Ang and Tang [10, 11]. A more correct Bayesian parameter updating is needed in further work and would require that a probability distribution of the crack size at the time of inspection is established. The updated probability distribution functions for the crack size after inspection can then be established by use of the likelihood function. As an example, if X is the random variable where updated information is available (e.g. the crack size at a location) with a density function $f(X)$, the updated density function for the variable after observing the experimental outcome ε is then given by:

$$f^{U}(X) = \frac{P(\varepsilon \mid X) \cdot f(X)}{\int_{-\infty}^{\infty} P(\varepsilon|X) \cdot f(X) \cdot dX} \tag{13}$$

where the function $P(\varepsilon|X)$ is commonly referred to as the likelihood function of X and denoted $L(X)$. $L(X)$ is the likelihood of observing the experimental outcome ε assuming a given X.

For this method to be used in practical engineering a version requiring less computation time is needed. For example, further work to establish a method for parameter updating using first or second order reliability method (FORM and SORM). Monte-Carlo simulation may still be an option if the code is streamlined to minimize simulation time.

The effect of the calibration of fracture mechanics crack growth parameters to match SN-fatigue curves is a topic of debate in the industry and need further investigation. The calibration of fracture mechanics crack growth parameters to match SN-fatigue curves should be done in accordance with DNVGL RP-C210. In addition, further work to study the effect on inspection intervals by using "in-air", "free-corrosion" and "cathodic protection" SN-curves would be beneficial for practical engineering.

Acknowledgement

The results of the simulations presented in this paper is the product of the master thesis by Håkon Neeraas [1] and have limitations that imply that they cannot necessarily be used for inspection planning for jacket structures. The results presented herein cannot in any way be construed to be recommendation by the employers of the co-authors (Petroleum Safety Authority and Aker Solutions).

The authors would like to acknowledge Magnus G. Ersdal for the development of the simulation program used in this paper.

References

[1] Neeraas H 2019 Sprekkvekst og inspeksjonsplanlegging / crack growth and inspection planning University of Stavanger (Stavanger, Norway)
[2] Madsen H O, Krenk S and Lind N C 1986 Methods of Structural Safety: (Prentice Hall Inc. Englewood Cliffs, New Jersey, USA)
[3] DNV 1992 Classification notes 30.6 Structural reliability analysis of marine structures (DNV, Høvik, Norway)
[4] Ersdal G 2005 Assessment of existing offshore structures for life extension University of

Stavanger (Stavanger, Norway)
[5] Dalane J I 1993 System reliability in design and maintenance of fixed offshore structures Dept. of marine structures (Norwegian Institute of Technology, Trondheim, Norway)
[6] DNVGL 2015 DNVGL-RP-C210 Probabilistic methods for planning of inspection for fatigue cracks in offshore structures (DNVGL, Høvik, Norway)
[7] Bury K V 1975 Statistical Models in Applied Science (Wiley London)
[8] Standard Norge 2017 In-service integrity management of structures and marine systems. Edition 2 (Standard Norge, Lysaker, Norway)
[9] Standard Norge 2015 Assessment of structural integrity for existing offshore load-bearing structures. Edition 2 (Standard Norge, Lysaker, Norway)
[10] Ang A H S and Tang W H 1975 Probability Concepts in Engineering Planning and Design, Volume I – Basic Principles (John Wiley and sons New York, USA)
[11] Ang A H S and Tang W H 1984 Probability Concepts in Engineering Planning and Design, Volume II – Decision, Risk, and Reliability (John Wiley and sons New York, USA)
[12] Ersdal G and Oma N 2019 Investigation of updating methods for probability-informed inspection planning for offshore structures, COTech2019, University in Stavanger, Norway.