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Optimal sea floor placement of the oil/gas production equipment

S S Arsenyev-Obraztsov, A I Ermolaev and A M Kuvichko

Russian State Gubkin University of Oil and Gas, Russia

Corresponding author: arseniev@gubkin.ru

Abstract. In the construction designing of oil field surface facilities, exists a problem of the seabed production equipment optimal placement. As an initial approximation, we can represent this task in the form of the linear Boolean programming problem. An application of routine methods of discrete programming can theoretically give the desired solution. However, these methods do not take into account the specific nature of the problem. This problem belongs to the NP class. So we can run into significant computational difficulties. This situation is typical for construction of the ground surface or subsea located facilities for real oil/gas fields. To overcome it, we propose to replace a discrete programming model with a linear programming one, which takes into account problem-specific properties. Also, we present examples of the application of the proposed parallel optimization algorithms. Input data for them: the seabed profile, geometry, and space distribution of oil/gas reserves. Optimization problem objective function is the penalty for the irrational placement of seabed production equipment elements.

1. Introduction

The problems and tasks associated with the construction and placement of the seabed systems for the development of offshore oil/gas fields are of great importance for the economic effectiveness of production projects. There are several reasons for the subsea field development: proximity of deposit to the seacoast, deep enough seabed, the high total amount of initial reservoir energy and absence of need for processing and storing facilities preceding the transportation stage.

The development of offshore fields is carried out with the usage of expensive Seabed Production Complexes (SPC). Therefore, the losses from irrational facilities design and equipment placement dramatically increase. It is one of the main reasons for the usage of three-dimensional multi-phase/multi-component fluid flow simulators during the field development plan design. They are implanted into the computational comparison cycle using different optimization algorithms. In this paper, we analyze the procedures for optimal placement of the SPC units: wells, subsea production elements, and central gathering station. Under the term - subsea production element, we mean sub water platform with placed wellhead equipment and facilities for connecting wellheads to the oil/gas gathering network.

There is a significant number of articles devoted to well-placement optimization [1, 2, 3], and many others. For several years we develop an approach to the well-placement problem [4, 5, 6], which differs from similar research works in:

- (1) We do not use technical and economic indicators: NPV, cumulative production, and some other effectiveness estimators. As optimization criteria, we select heuristic rules used for the reasonable deposit development, and tested by long-term field practice;



- (2) This task can be set as the discrete programming problem which we further replace with the solution of a series of linear programming problems. reasonable

Both items increase the computational efficiency of optimization procedures, especially for the solution of high-dimensional problems common for designing development plans for oil and gas fields. Without fundamental changes, we can use proposed well-placement algorithms for the optimal placement of other elements of the SPC: subsea production facilities, a central gathering unit, and some other.

2. Optimization of the SPC elements placement

The following sequence of operations can approximately express the algorithm for SPC elements allocation on the seabed:

- (a) determine a set of sections of the pay area where it is reasonable to locate bottom holes (the number of wells is predefined);
- (b) determine the geometrical configuration (chart) of the horizontal wellbore (the length of the horizontal wellbore is preset);
- (c) determine the allocation of the subsea production units (the number of units is pre-specified);
- (d) determine central gathering unit allocation.

Initial information for the well-placement problem includes reservoir geometrical parameters, data about the deposit reserves spatial distribution, porous medium permeability, and reservoir fluids saturation. The generation of the initial information for optimization algorithms begins with the subdivision of the pay area into square blocks of equal size. Block sizes should be suitable for placing a full-sized horizontal wellbore in it. With the usage of geological modeling software for each block, we estimate the hydrocarbon reserves and other characteristics affecting well-placement. Then for each block, we evaluate the "expediency indicator" in terms of possible bottom hole placement in it.

Parameters for well-placement problem are:

- n and s - are numbers of blocks and wells, respectively, $n > s$, and $n/s -$ is an integer (if this condition violated, then "dummy" blocks with zero reserves must be added so that ratio of the changed number of blocks to the number of wells equals to some integer);
- c_{ij} - "remote production fine" - a penalty for production of oil/gas from block j by well bottom hole placed in the block i , ($i, j = 1, 2, \dots, n$). It also can be interpreted as an "inclusion price for block j to be in the block i pay zone".

Problem solution is a set of variables $\{x_{ij}\}$ such that $x_{ij} = 1$ if block j included in the area of influence of the well located in the block i , and $x_{ij} = 0$ otherwise. The term "area of influence" means a set of blocks closest (adjacent) to the block with well. We assume that these blocks provide production well by the main part of the inflow of formation fluids. From the definition of solution it follows: if $x_{ii} = 1$, then the block i contains the bottom of the well i , if $x_{ii} = 0$, then there is no well in the block i .

Taking into account proposed assumptions and notations, the solution of this problem is the set of variables $\{x_{ij}\}$, satisfying the following system of terms and conditions [4]:

$$\sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij} \rightarrow \min_x, \quad (1)$$

$$\sum_{i=1}^n x_{ii} = s, \quad (2)$$

$$\sum_{j=1}^n x_{ij} = \left(\frac{n}{s}\right) x_{ii}, \quad i = \overline{1, n}, \quad (3)$$

$$\sum_{i=1}^n x_{ij} = 1, \quad j = \overline{1, n}, \quad (4)$$

$$x_{ij} \in \{0, 1\}, \quad i = \overline{1, n}, \quad j = \overline{1, n}. \quad (5)$$

The objective function (1) is a total penalty for the irrational well-placement. The equality (2) is a limitation for the number of wells placed on the oil field. Restrictions (3) are equivalent to the condition that the area of influence of each well contains the same number of blocks. The constraints (4) mean that any block can belong to only one area of influence. If (n/s) is an integer, then from the constraints (3) and (4) follows fulfillment of the constraint (2) [4]. If any block (for example, l -th) is forbidden for well placement, then the following conditions are added to the problem (1)-(5): $x_{lj} = 0$, ($j = 1, \dots, n$).

In [4, 5, 6] for the calculation of feasibility indicators, it is proposed to use the following heuristic rules for rational well-placement. The minimum size list of them looks like:

- (a) it is desirable to guarantee the shortest possible distance from the well bottom to any point of the reservoir;
- (b) also it is desirable to bring wells closer to the blocks with larger reserves.

Later this set of rules can be extended or changed.

The optimality criterion (1) expresses the desire to diminish the total penalty for violation of these rules. According to them, we can define values of c_{ij} as:

$$c_{ij} = \begin{cases} 0, & j = i \\ \lambda_j^{1-\gamma} \cdot r_{ij}^\gamma & j \neq i \end{cases}, \quad (6)$$

where

$$\lambda_j = \frac{V_j}{\max_{1 \leq k \leq n} \{V_k\}}, \quad r_{ij} = \frac{R_{ij}}{\max_{1 \leq k \leq n, 1 \leq l \leq n} \{R_{kl}\}}, \quad \gamma \in [0; 1].$$

Let V_j - oil/gas reserves of the block j ; R_{ij} - distance between the centers of blocks i and j ; γ - expert estimation of the "distance" indicator importance ($0 \leq \gamma \leq 1$). From (1) it follows, that when we enlarge a distance from the well to the block with large reserves penalty increases.

To improve the adequacy of the model in [6] it is proposed to include into the set of input parameters, variables characterizing resistance of medium to the fluid flow: $\omega_i = \frac{\mu_i}{k_i h_i}$, where ω_i - coefficient of filtration resistance, μ_i - viscosity of fluid, k_i - permeability, h_i - fluid-saturated thickness of the block i . After that, we replace this set of blocks with a graph. Vertices of this graph are the centers of blocks, and arcs are lines connecting adjacent vertices. By adjacent vertices, we usually mean centers of adjacent blocks. Adjacent blocks are blocks that have a common boundary. The length of each arc is an average coefficient of flow resistance for two adjacent blocks. R_{ij} - the distance between blocks i and j , is the minimum total length of arcs

connecting them. It means that for each pair of vertices; we needed to solve the shortest path problem [7].

After selecting blocks containing wells, we need to determine the profile of a horizontal wellbore. The mathematical definition of this problem is the following. After we selected blocks for a well and SPU (that the well connects to), we can define azimuth of a wellbore horizontal part as an azimuth of the vector directed from the SPU ($X_{SPU}; Y_{SPU}$) to the well (X_{well}, Y_{well}) location. We take as the initial point ($X_{in}; Y_{in}$) (formation entry) and as the terminal point ($X_{out}; Y_{out}$) (well bottom) of the horizontal part of the well. We now consider only X and Y point coordinates; Z-coordinate to be optimized further. Let us define the depth of the horizontal part of the wellbore as 0 at the formation entry point. and $L = \sqrt{(X_{in} - X_{out})^2 + (Y_{in} - Y_{out})^2}$ at well bottom point. We then define a set of points ($X_i; Y_i$) for every $i = \overline{1, m}$ as:

$$X_i = X_{in} + X_{out} \frac{l_i}{L}, \quad (7)$$

$$Y_i = Y_{in} + Y_{out} \frac{l_i}{L}, \quad (8)$$

where l_i is the measured depth at point i :

$$l_i = L \cdot \frac{i - 1}{m - 1}, i = \overline{1, m}. \quad (9)$$

At every point ($X_i; Y_i$) defined above we can get the limiting Z_i^{top} and Z_i^{bot} values from reservoir top and bottom surfaces, respectively, and get a Z-value Z_i between these limiting surfaces. We then take a desired 3D-property (like remaining recoverable fluid-in-place, permeability, and some other), and we can calculate its value D at any ($X_i; Y_i; Z_i$), but since the number of 3D-grid blocks between top and bottom surfaces is limited, there could be only a limited number of Z-values at every layer i . We will sample Z_i at 3D-property cell centers. The problem is to maximize the total sum of values at planned wellbore points, but to keep the inclination angle increase/decrease value not greater than a defined angle I (here we only vary inclination (vertical angle), but not azimuth). Thus, the following problem to be solved:

$$\sum_{i=1}^m D(X_i; Y_i; Z_i) \rightarrow \max_{Z_i}, \quad (10)$$

$$Z_i^{bot} \leq Z_i \leq Z_i^{top}, \quad i = \overline{1, m}, \quad (11)$$

$$\left| \arctan \left(\frac{Z_{i+1} - Z_i}{\sqrt{(X_{i+1} - X_i)^2 + (Y_{i+1} - Y_i)^2}} \right) \right| \leq I, \quad i = \overline{1, m - 1}, \quad (12)$$

The problem (10)-(12) can be solved using standard dynamic programming and a Bellman's Principle of Optimality [8], since we can split it to subproblems at layer i .

The definition of a mathematical model for optimal placement of seabed production units (SPU) begins with the replacement of deposit domain by a set of equal blocks. We can install SPU in any of them. Let m be the total number of SPUs. Other initial parameters are: M the number of SPU, and K the maximum allowable number of wells in SPU. We state that $KM \geq s$, where s is the number of wells (number of bottom holes). Let C_i be the cost of construction of SPU in the block i , $i = 1, 2, \dots, m$; w_{ij} is the construction cost of well connecting the center of block i to bottom hole j , $j = 1, 2, \dots, s$; R_{ij} is the distance between block j with a bottom hole in it and the center of block i ; R is the maximum allowable distance between the bottom hole

and SPU. Let g_{ij} be coefficients such that if $R_{ij} \leq R$, then $g_{ij} = w_{ij}$; if $R_{ij} > R$, then $g_{ij} = W$, where, $W \gg \max_{i,j} \{w_{ij}\}$.

Now problem solution can be defined as two groups of variables $\{x_{ij}\}$ and $\{y_i\}$, where $x_{ij} = 1$, if the bottom hole j is connected to the SPU located in the block i , and $x_{ij} = 0$, otherwise; $y_i = 1$, if there is a SPU in block i , otherwise $y_i = 0$.

Hence to solve the problem, we need to identify sets $\{y_i\}$, $\{x_{ij}\}$ such that:

$$\sum_{i=1}^m \sum_{j=1}^s g_{ij} x_{ij} + \sum_{i=1}^m C_i y_i \rightarrow \min_{x,y}, \quad (13)$$

$$\sum_{i=1}^m y_i = M, \quad (14)$$

$$\sum_{j=1}^n x_{ij} \leq K y_i, \quad i = \overline{1, m}, \quad (15)$$

$$\sum_{i=1}^m x_{ij} = 1, \quad j = \overline{1, s}, \quad (16)$$

$$x_{ij} \in \{0, 1\}, \quad y_i \in \{0, 1\}, \quad i = \overline{1, m}, \quad j = \overline{1, s}. \quad (17)$$

The problem of central gathering node (CGN) placement is defined by the mathematical model (13)-(17), in which s equals the number of SPU, $M = 1$, $K = s$, and inequality constraints (15) are replaced by equalities. In addition, g_{ij} is the cost of building of a gas flow-line between SPU j and the CGN, if CGN is placed into the block i , and C_i is the cost of building the CGN in the block i , $i = 1, 2, \dots, m$, $j = 1, 2, \dots, s$. To solve the problem, we need to inspect all acceptable options for the CGN placement. As the best, we choose one for which the total cost of building gas flow-lines and CGN will be minimal.

3. Method for solving problems (1)-(5) and (13)-(17)

The specifics of problem (1)-(5) are: First, we can represent the set of desired solution variables in the form of a square matrix; Second, if by some consideration values are assigned to the elements of the main diagonal of this matrix, then the original problem is converted to the classical transport problem with the cost criterion (T-problem)[9]. The proposed solution method uses these features. Below we present a description of this method applied for the problem (1)-(5).

Let consider some assigned of variables $\{x_{ii}\}$ satisfying constraints (2) and (5), i.e. $\{x_{ii}\}$ is a set of the one of several admissible well placement. Consider the sets A , B and C : $A = 1, 2, \dots, n$, $B = \{i : x_{ii} = 1\}$, $C = \{i : x_{ii} = 0\}$, i.e.

$$A = B \cup C, \quad B \cap C = \emptyset, \quad |A| = n, \quad |B| = s, \quad |C| = n - s \quad (18)$$

Then, for a given set $\{x_{ii}\}$, it follows from (2) that $x_{ij} = 0$ for $i \in C$ & $j \in A$. Taking into account (18), problem (1) - (5) will be:

$$\sum_{i \in B} \sum_{j \in C} c_{ij} x_{ij} \rightarrow \min_x, \quad (19)$$

$$\sum_{j \in C} x_{ij} = \binom{n}{s} - 1, \quad i \in B, \quad (20)$$

$$\sum_{i \in B} x_{ij} = 1, \quad j \in C, \quad (21)$$

$$x_{ij} \in \{0, 1\}, \quad i \in B, \quad j \in C. \quad (22)$$

Let's analyze the problem (19) - (22). First, the right-hand side parts of constraints (20) and (21) are positive integers. Therefore, all the support plans for this problem will be integer-valued [10]. Second, from (22) we obtain that $x_{ij} \geq 0$, and taking into account (21) we get the "automatic" fulfillment of $x_{ij} \leq 1$. Therefore, conditions (22) can be replaced by restrictions on the sign of the unknown variables:

$$x_{ij} \geq 0, \quad i \in B, \quad j \in C. \quad (23)$$

Third, $(n - s)$ - the sum of the right-hand sides of the constraints (20) equals to the sum of the right-hand side parts of constraints (21). Therefore, with a fixed set $\{x_{ii}\}$ satisfying constraints (2) and (5), problem (1) - (5) turns into model (19) - (21), (23), which is a classical transport problem. Therefore, one of the known methods, for example, the method of potentials [9], can solve this problem. Thus, any admissible set $\{x_{ii}\}$ uniquely defines some valid solution for the original problem (1) - (5). Therefore, the method for solving problem (1) - (5) is reduced to generating sets $\{x_{ii}\}$ satisfying constraints (2) and (5), solving T-problem (19) - (21), (23) for each and choosing the best in terms of criteria (1)..

Let us estimate z - the lower bound for the reasonable number of test sets $\{x_{ii}\}$ in the Monte-Carlo optimal solution search. By this, we mean that conducting $\#z$ tests we get an admissible result with a preset probability P . Lower bound for z we can obtain from the following considerations. Let N be the number of all admissible sets of $\{x_{ii}\}$, and $D(L)$ be best solutions subset of size L for the problem (1) - (5) ($|D(L)| = L$). We assume that if at least one of the test sets $\{x_{ii}\} \in D(L)$, then problem (1) - (5) is solved. Let Q be the probability that the test solution of the problem (1) - (5) belongs to the set $D(L)$. It is clear that $Q = L/N$. Let P be the predefined probability that at least one of the $\#z$ test sets belong to the set $D(L)$. For a known value of N and given values of P and L , an estimation of $\#z$ we can get as the solution of the equation:

$$P = 1 - (1 - L/N)^z$$

which gives:

$$z = \ln(1 - P) / \ln(1 - L/N)$$

We need to emphasize that the use of any enhanced random search algorithm or the application of additional information in the process of sets $\{x_{ii}\}$ generation could significantly reduce this estimation.

For the case of a large number of numerical computations, it is better to use high-performance computing systems. If the total amount of computation is relatively small, then workstation with general-purpose graphics accelerators (NVIDIA, AMD) or many-core computing systems based on Intel Xeon Phi, Sunway 26010, and some other processors will be a good option. From our point of view, the Sunway TaihuLight supernode is the best choice for conducting optimization with the usage of hydrodynamic simulators.

Since the total amount of memory required for the generation of the input sets and subsequent transport problem solution is not large, the calculations for one dataset can be entirely performed on the CUDA core or the fast PCE core in Scratch Pad Memory of the SW26010 processor [11]. This approach can significantly speed up calculations. The parallelization of the above algorithms is obvious.

The application of this method for the solution of the problem (13) - (17) requires preliminary transformation. By adding $s^* = KM - s$ fictitious wells to the model (13) - (17) inequality constraints (17) can be converted into equality constraints. For the model including fictitious wells coefficients should be set to zero, i.e. $c_{ij} = 0$. Then the above method can be applied in this case, only instead of the sets $\{x_{ii}\}$, it requires to generate sets $\{y_i\}$ satisfying constraints (14) and (17). The set $\{y_i\}$ represents one of the possible SPU (or CGN) placements.

4. Example

We apply the described algorithm to solve a test problem. We have created a sample (artificial) model similar to models of deepwater gas fields. We have divided this model into square blocks $1 \times 1 \text{ km}^2$. The criterion we've used for (6) to calculate c_{ij} in (1) was a composite criterion $V_j = \overline{G}_j^{-0.5} \times \overline{kH}_j^{0.4} \times \overline{P}_j^{0.1}$, where \overline{G}_j is a normalized gas-in-place for the j -th cell, \overline{kH}_j is a normalized kH property (cell permeability multiplied by cell height), and \overline{P}_j is a normalized pressure. The map of the composite criterion for this test field is shown on the Figure 1.

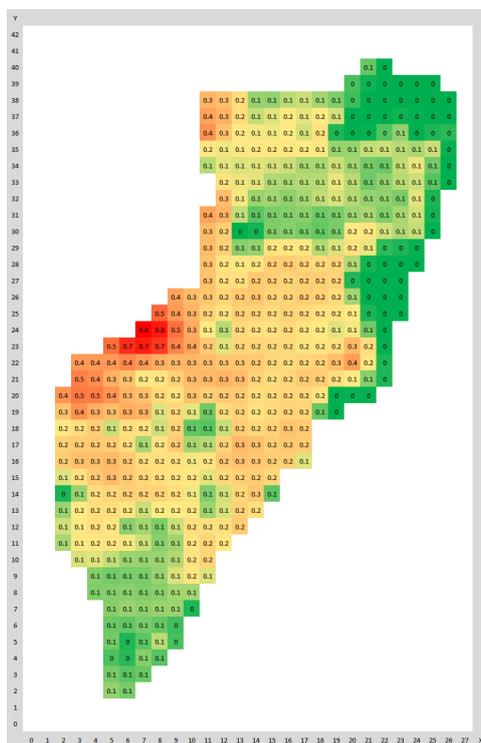


Figure 1. A representation of the used test field model: color scale represents the composite criterion value from the lowest (green) to the highest (red).

This model contains 492 blocks. Dividing total (recoverable) reserved to average produced gas estimate, we've decided to place 12 wells on this gasfield. We have tested different values of the parameter γ in equation (6). It is rational to select (as described in [6]) γ in the range

from 0.05 to 0.25. We have selected 3 γ values: 0.05, 0.15 and 0.25. We have decided to pick well positions for $\gamma = 0.15$, as this case looks more realistic in terms of engineering criteria [4], however a question of selecting γ depends on the specific field. After running the iterative solver [12], we got the following results (Figure 2, (a)). The wells were located in blocks numbered 48, 63, 118, 125, 182, 209, 236, 259, 311, 362, 400 and 454. The resulting zones corresponded to cells are shown at Figure 2, (b). We count the number of times that a j -th block appears in the (local) optimal well placement (Figure 2, (c)). The distribution of (local) optima, i.e. the values of the objective function vs. the theoretical lognormal distribution (with an estimated mean (ξ) and standard deviation (σ)) is shown at Figure 2, (d).

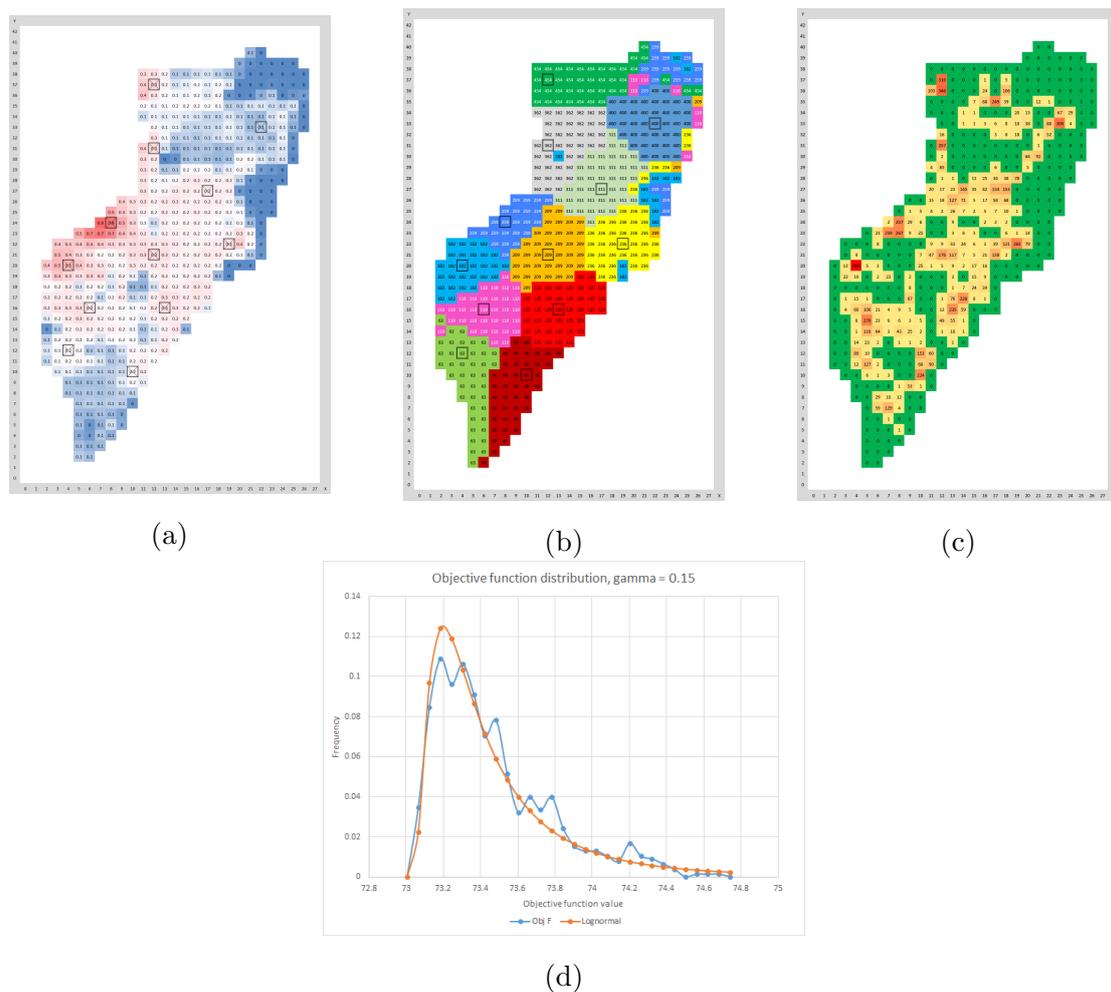


Figure 2. For $\gamma = 0.15$ (a) wells (crossed bold square block) optimal placement, color represents criterion V_j values from smallest (blue) to largest (red); (b) zones for wells (crossed block) distribution, color represents different zones; (c) the number of times that the block j has appeared in the (local) optimal solution, color represents values from smallest (green) to largest (red); (d) cumulative local optima distribution: solution vs. theoretical lognormal distribution with an estimated mean ξ and standard deviation σ .

For this location of wells, we then solve the problem of placing several SPU (13)-(17) having M set to 3 in (14). By doing this, we can drop the term $\sum_{i=1}^m C_i y_i$ in (13) since it turns into

a constant and can be eliminated (since the cost values C_i are assumed the same for all i). Nevertheless, we use the constraint (15) in a way to connect any number of wells to an SPU. We also assume that the cost of a length unit of a well is equal for all wells. Using these assumptions, g_{ij} in (13) morphs into a simple distance from block i to j . The complexity of this problem for 492 cells, 3 SPUs and 12 wells is $C_{492}^3 \times 12 \times 3 = 710221680$ cases, so it could be solved directly.

We have solved this problem for the test field model having wells placed into blocks defined above (Figure 2). The solution is shown at Figure 3, (a). The optimal result was unique, and SPUs were placed into blocks with numbers 118, 209, and 379. Note that tow first SPUs were placed in blocks with wells, while the thirds one between wells. We further solved the problem of placing a CGN for there three SPUs using the same approach. The solution is shown at Figure 3, (b). This position of the CGN guarantees optimal sum of distances; again, CGN was located at block number 209, where a well and an SPU were already placed.

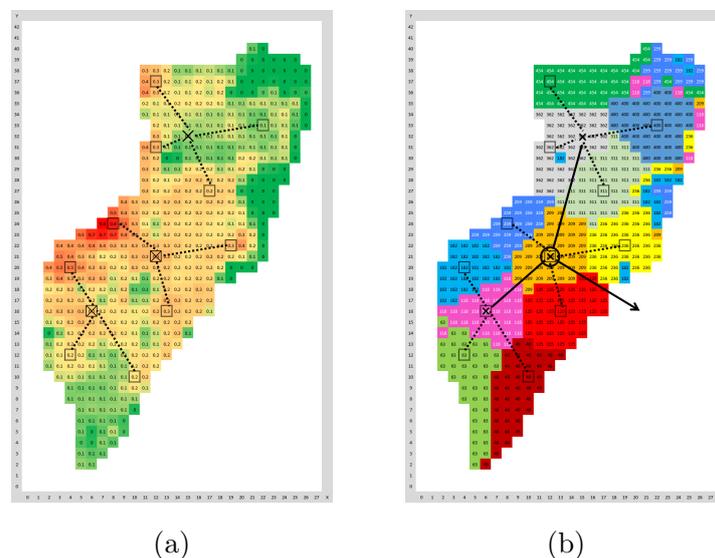


Figure 3. For $\gamma = 0.15$ (a) SPU (crossed block) optimal placement; well (bold block) connections to SPUs are shown as dashed lines; color represents criterion V_j values from smallest (green) to largest (red), (b) zones for wells (bold block) distribution, SPUs (crossed blocks) and CGU (bold round); color represents different zones.

We also solve the problem of optimal well-placement inside the formation (10)-(12) for the found set of locations of wells and SPUs. As for the desired $3D$ -property, we have selected kH above gas-water contact. The maximal inclination variation angle I is set to 2 per depth step. An example of the optimal position for the well in block number 259, presented in Figure 4.

5. Conclusion

The presented models and optimization algorithms allow taking into account the experience and knowledge of specialists in the design of oil and gas deposits development projects and using the capabilities of mathematical tools to support selected decisions. The primary purpose of models and algorithms is a creation of the preliminary set of optional projects for the development and reservoir surface facility construction for oil and gas deposits, from which subsequent choose of the best option needs to be conducted.

The advantages of the proposed approach for solving the problems of production technological elements placement include: First, a significant reduction in the number of calls to hydrodynamic

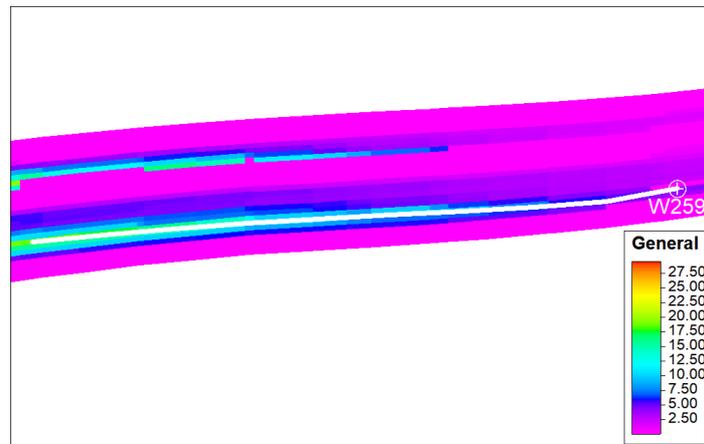


Figure 4. Optimal placement of the well 259 inside the formation with kH set as the target 3D-property. The well is shown in white color.

simulators; Second, the possibility of using linear programming methods; Third, the proposed algorithms are well suited for different parallelization technologies. All this allows us to proceed for solving optimization problems of high dimensions, which is specific for the production project design for oil and gas fields.

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