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# Three-dimensional vibrations of wind turbines

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Abstract. In this paper authors consider three-dimensional vibrations of a wind energy farms (WEF) of tower type VTR 50 Arctic. Low-frequency vibration of a WEF composed of a tower and rotor is considered in the out of balance condition. The method of analysis is based on the theory of mechanical oscillations and the theory of elastic shafts. We consider the tower as a shaft and the rotor as a rigid body. Using the Lagrange's equations for elastic structures in combination with computer simulation provides for the novelty and efficiency of the suggested method. The unique nonlinear equation is deduced and solved using the Wolfram Mathematica.

Keywords: alternative energy production, wind energy farms, Lagrange equations, general vibration theory

#### **Review of literature**

The climate change due to intensive use of fossil fuels, such as oil, coal, gas, etc., is one of the fundamental problems that the humanity is facing. World countries are negotiating quotas of greenhouse gas emissions reduction, which is impossible without alternative energy sources. Renewable energy sources are one of the best solutions. They have been extensively designed and installed in already in the course of several decades, due to the prospective depletion of fossil fuel and the greenhouse gas emissions issues. Renewable energy sources are much safer environmentally and economically, and in the future can replace the traditional ones. The energy of wind is one of the most promising directions in the alternative energy production [1].

Manufacturing of wind energy farms (hereinafter, WEFs), transforming the energy of a wind flow into the kinetic energy of rotor spinning and, further on, into electric current has been already put in practice in many countries [2]. It is important that WEFs do not exhaust natural resources in the process of their exploitation and do not add up to climate change. Operation of a wind generator of 1 MWt capacity within 20 years, would provide for saving 92 barrels of oil thousands cubic meters of gas. However, a major issue of the low vibration and noise in the area nearby the wind farms makes their dissemination problematic.

This brings us to the actuality of this article dedicated to mathematical modeling of WEF vibration for the assessment of the level of it. The purpose of this paper was the development of a method for such modeling which would make sense yet at the design stage of a WEF.

#### Theoretical data for solving the problem set

The object of research is a WEF of tower type VTR 50 Arctic which has a horizontal axis of rotation of the wind wheel. The low-frequency vibration of a WEF with tower and rotor is under analysis. As long

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as the diameter of the WEF tower is much smaller than its height, while the displacements are relatively small, we can consider the tower as an elastic console beam [3-5]. The authors believe that in that case to consider bending oscillations in two planes is pretty well justified [6-8].

The method is based on the theory of elastic beams [9, 10] and the general vibration theory [11, 12]. The rotor is considered as a solid body. The method is based on the Lagrange equations in combination with computer mathematics which has presently reached a very high level and provides for achieving results of high accuracy in solving differential equations [13]. The problem is solved with the use of the system of the Wolfram Mathematica.

We consider the WEF as a model with rotating rotor in two planes (Fig.1, Fig.2).





A tower with rotating rotor in the XZ plane



Figure 2 A tower with rotating rotor in the YZ plane

Here we have a system with non-stationary connection. For the systems of the kind the Lagrange equations are the best option. Point C in the picture is the mass center of the rotor. It is displaced from the moving axis of rotation at a small distance  $\varepsilon$  in XZ plane, the excentricity.

### Solving of the WEF linear problem

The method is based on the Lagrange equations:

$$\left(\frac{\partial K}{\partial \dot{q}_i}\right)^{\!\!\!\!\perp} - \frac{\partial K}{\partial q_i} = -\frac{\partial U}{\partial q_i} + Q_i \tag{1}$$

Here  $K(\dot{q}_i, q_i, t)$  is the kinetic energy of the system of the system as a function of generalized coordinates  $q_i$ , generalized velocities  $\dot{q}_i$  and (in some cases) directly present time;  $U(q_i, t)$  is the potential energy;  $Q_i$  are the generalized forces. The latter are derived from the expression of virtual work.

$$\delta A = \sum_{i=1}^{n} Q_i \delta q_i = Q^T \delta q \tag{2}$$

Here we used the matrix form of presentation with columns of generalized coordinates and forces.

The  $\delta$  symbol in (2) and further on indicates variation, i.e., infinitesimal increment of the value which we specify on our own. The Lagrange equations (1), which are an absolute tool in analytical mechanics, could not be understood outside the frames of variational analysis [9,10,14].

The kinetic energy of the tower looks like the following expression:

$$2K_t = \int_0^L \rho \dot{u}^2 dz \tag{3}$$

The potential energy of the tower is:

$$2U_t = \int_0^L a u''^2 dz \tag{4}$$

We consider the vibration in two planes, therefore the deflection includes two components  $u_x(z,t)$ ,  $u_y(z,t)$ . Generalized coordinates for the vibrating beam (the WEF tower) could be introduced through the approximation of the deflections:

$$u_{x}(z,t) = \varphi_{1}q_{1}(t)$$

$$u_{y}(z,t) = \varphi_{2}q_{2}(t)$$

$$u(z,t) = \sqrt{u_{x}^{2}(z,t) + u_{y}^{2}(z,t)}$$
(5)

Here  $\varphi_1, \varphi_2$  are coordinate functions. We shall specify them but we must bear in mind the accurate boundary conditions in the rigid support of the beam:  $\varphi_{1,2}(0) = \varphi_{1,2}'(0) = 0$ . We suppose that  $\varphi_1 = \varphi_2 = \varphi = z^2$ .

Then the kinetic energy and the potential energy of the tower are:

$$K_{t} = \frac{1}{2} \int_{0}^{L} \rho \dot{u}^{2} dz = \frac{1}{2} \int_{0}^{L} \rho \left( \dot{u}_{x}^{2} + \dot{u}_{y}^{2} \right) dz = \frac{1}{2} M \cdot (\dot{q}_{1}^{2} + \dot{q}_{2}^{2}), M = \int_{0}^{L} \rho z^{4} dz$$
(6)

$$U_{t} = \frac{1}{2} \int_{0}^{L} a u''^{2} dz = \frac{1}{2} \int_{0}^{L} a \left( u_{x}''^{2} + u_{y}''^{2} \right) dz = \frac{1}{2} C \left( q_{1}^{2} + q_{2}^{2} \right), C = 4 \int_{0}^{L} a dz$$

Here *M* is the matrix of inertia and *C* is the matrix of stiffness of the tower.

Let's assume that the body of the rotor impacts only the kinetic energy of the rotor. First of all, the angles of the small rotation of the rotor:

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$$\psi_{y}(t) = u'_{x}(L,t) + \Omega t$$

$$\psi_{x}(t) = u'_{y}(L,t)$$
(7)

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According to the König's theorem the kinetic energy of the rotor [14]:

$$2K_r = mv_c^2 + J_x \dot{\psi}_x^2 + J_y \dot{\psi}_y^2 + 2J_{xy} \dot{\psi}_x \dot{\psi}_y$$
(8)

(Where  $J_x$ ,  $J_y$ ,  $J_{xy}$  are the inertia moments in relation to the axes going through the mass center, *m* is the mass of the rotor).

We should write the speed of the mass center:

$$x_{c} = u_{x}(L,t) + \varepsilon \cos \psi_{y}, y_{c} = u_{y}(L,t), z_{c} = L - \varepsilon \sin \psi_{y};$$

$$u_{x}(L,t) = L^{2}q_{1}, u_{y}(L,t) = L^{2}q_{2};$$

$$v_{c}^{2} = \dot{x}_{c}^{2} + \dot{y}_{c}^{2} + \dot{z}_{c}^{2} = L^{4}(\dot{q}_{1}^{2} + \dot{q}_{2}^{2}) + \varepsilon^{2}\dot{\psi}_{y}^{2} - 2L^{2}\varepsilon\dot{q}_{1}\dot{\psi}_{y}\sin\psi_{y}$$
(9)

The angles of the small rotation:

$$\psi_{y}(t) = 2Lq_{1} + \Omega t$$

$$\psi_{x}(t) = 2Lq_{2}$$
(10)

For the kinetic energy under the selected approximation, we will have:

$$2K_{r} = (mL^{4} + 4L^{2}m\epsilon^{2} - 4L^{3}m\epsilon \cdot \sin(2Lq_{1} + \Omega t) + 4L^{2}J_{y}) \cdot \dot{q}_{1}^{2} + (mL^{4} + 4L^{2}J_{x}) \cdot \dot{q}_{2}^{2} + (4Lm\epsilon^{2}\Omega - 2L^{2}m\epsilon\Omega \cdot \sin(2Lq_{1} + \Omega t) + 4J_{y}L\Omega) \cdot \dot{q}_{1} + (11) + m\epsilon^{2}\Omega^{2} + J_{y}\Omega^{2} + 4J_{xy}L\dot{q}_{2}(2L\dot{q}_{1} + \Omega)$$

The kinetic energy of the system is the following:

$$K = \frac{1}{2} ((mL^{4} + 4L^{2}m\epsilon^{2} - 4L^{3}m\epsilon \cdot \sin(2Lq_{1} + \Omega t) + 4L^{2}J_{y} + M) \cdot \dot{q}_{1}^{2} + (mL^{4} + 4L^{2}J_{x} + M) \cdot \dot{q}_{2}^{2} + (4Lm\epsilon^{2}\Omega - 2L^{2}m\epsilon\Omega \cdot \sin(2Lq_{1} + \Omega t) + 4J_{y}L\Omega) \cdot \dot{q}_{1} + (12) + m\epsilon^{2}\Omega^{2} + J_{y}\Omega^{2} + 4J_{xy}L\dot{q}_{2}(2L\dot{q}_{1} + \Omega))$$

The potential energy of the system is summed up from the energy of deformation of the tower  $\Pi_t$  and the energy of the gravitation field:

$$U = U_t - mg\varepsilon\sin\psi_y = \frac{1}{2}C(q_1^2 + q_2^2) - mg\varepsilon\sin(2Lq_1 + \Omega t)$$
(13)

The expressions for the kinetic (12) and potential energies (13) we can put into the Lagrange equations. After deriving the relevant derivatives, we come to ODE (14):

$$\begin{cases} -2mg\varepsilon L \cdot \cos(2Lq_{1} + \Omega t) - m\varepsilon L^{2}\Omega^{2} \cdot \cos(2Lq_{1} + \Omega t) + cq_{1} - -4m\varepsilon L^{3}\Omega \cdot \cos(2Lq_{1} + \Omega t) \cdot \dot{q}_{1} - 4m\varepsilon L^{4} \cdot \cos(2Lq_{1} + \Omega t) \cdot \dot{q}_{1}^{2} + +4I_{y}L^{2}\ddot{q}_{1} + 4m\varepsilon^{2}L^{2}\ddot{q}_{1} + mL^{4}\ddot{q}_{1} + M\ddot{q}_{1} - -4m\varepsilon L^{3} \cdot \sin(2Lq_{1} + \Omega t)\ddot{q}_{1} + 4J_{xy}L^{2}\ddot{q}_{2} = 0 \end{cases}$$
(14)

Under the following conditions, the ODE (14) could be solved with the means of computer mathematics:  $I_x = 8.86 \cdot 10^3 kg \cdot m^2$ ,  $I_y = 28.1 \cdot 10^3 kg \cdot m^2$ ,  $I_{xy} = 88.6 kg \cdot m^2$ ,  $\varepsilon = 50mm$ , m = 580 kg,

 $\Omega = 1,25 \text{ s}^{-1}$ . The material of the tower is steel which has the density  $7800 kg/m^3$  and the Young's modulus 200 *GPa*. The tower concludes two parts: cylindrical and conical. The height of the tower L = 24.3 m, the cylindrical part's depth h = 1,8m, the section is a ring with the outside radius R(x) = 2,2m and the thickness 0,012 m. The conical part's depth h = 22,5 m, the section is a ring with the outside radius  $R(x) = (-1,296/l \cdot x + 1,896) m$  and the thickness 0,01m.

The calculation is quite exemplify, because resisting forces is not taken into account. We can see the calculation result on Fig. 3: the deflection of the tower top u(L,t) under the above indicated parameters. As it was shown, the vibration amplitude is not big. With the increase of the eccentricity in 10 times, we will receive minor increase of the amplitude, which proved that the non-linearity is not manifesting.



Vibrations of the tower top with rotating rotor

#### Conclusion

In this paper, authors presented an actual method for calculations of vibrations of a WEF tower with moving rotors and brought a justification of it. The method is based on the Lagrange equations. An accurate calculation by all introduced formulas was made with the use of computer mathematics. Diagram of tower vibrations with rotating rotor was presented. The diagram showed that the level of vibrations of the tower top was quite low and did not constitute any danger, though the eccentricity of the rotating rotor was significant.

We conclude that value of eccentricity has not considerable influence on vibration level; therefore, authors intend to find out factors affect vibrations significantly.

#### References

- [1] V.V. Elistratov, I.A. Konstantinov, A.A. Panfilov. Dynamic analysis of the wind plant-foundationbase system. Saint-Petersburg, SPbGTU, P. 49 (1999)
- [2] Robert Gasch, Jochen Twele eds. Wind Power Plants. Fundamentals, Design, Construction and Operation, Springer (2012)
- [3] V.V. Eliseev, A.A. Moskalets, E.A. Oborin Deformation analysis of turbine blades based on complete one-dimensional model. Russ J Heavy Mach 5:35–38 (2015) (in Russian)
- [4] V.V. Eliseev, A.A. Moskalets, E.A. Oborin. Applying of Lagrange equations to calculation of turbine blade vibration. Handbook. An Eng J 8:21–24. doi:10.14489/hb.2015.08 (2015) (in Russian)
- [5] V.V. Eliseev, A.A. Moskalets, E.A. Oborin. One-dimensional models in turbine blades. Advances in mechanical engineering, Lecture notes in mechanical engineering, Springer International Publishing, pp. 93–104. doi:10.1007/978-3-319-29579-4 (2016)
- [6] T.V. Zinovieva, A.A. Moskalets. Modal Analysis of Turbine Blade as One-and Three-Dimensional Body //Advances in Mechanical Engineering. – Springer, Cham, – pp. 195-204 (2018)
- [7] V.V. Eliseev, A.A. Moskalets. Vibrations of Turbine Blades as Elastic Shells //Advances in Mechanical Engineering. Springer, Cham, 2018. pp. 53-60.
- [8] A.A. Moskalets, T.V. Zinovieva, A.K. Belyaev. Turbine blade vibration analysis using helicoidal shell model // Proceedings of Summer School-Conference «Advanced Problems in Mechanics 2018», St. Petersburg, pp. 181-190.
- [9] V.V. Eliseev. Mechanics of elastic bodies (Mekhanika uprugih tel). St. Petersburg State Polytechnic University Publishing House, St. Petersburg, 2003. P. 336 (in Russian)
- [10] V.V. Eliseev. Mechanics of deformable solid bodies //St. Petersburg State Polytechnical University Publishing House, St. Petersburg. – 2006 (in Russian)
- [11] W. Weaver Jr., S.P. Timoshenko & D.H. Young. Vibration problems in engineering. John Wiley & Sons (1990)
- [12] V.L. Biderman. Theory of mechanical vibrations. Vysshaya Shkola, Moscow (1980) (in Russian)
- [13] D.V. Kiryanov. Mathcad 14. BHV-Peterburg, St Petersburg 704 (2007) (in Russian)
- [14] Loytsyanskiy L.G., Lurye A.Y. Kurs teoreticheskoy mekhaniki. Tom 2 (Course of theoretical mechanics. Volume 2) (1983)