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Settlement and Bearing Capacity of the Pile in A Three-Layer Base Taking into Account the Elastic-Visco-Plastic Properties of Soils

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Abstract. The article presents the formulation and analytical solution of the problem of determining the settlement and bearing capacity of an incompressible pile in a three-layer soil base, taking into account the elastic and elastic-visco-plastic properties of surrounding and underlying soils. It is shown that the force applied to the pile head is distributed and redistributed between the lateral surface of the shell of pile and its tip in time, and that the stress under the tip of the pile increases in time and may approach the critical load. Formulas for determining the settlement and bearing capacity of the pile in time are obtained depending on the physical and mechanical properties of the surrounding and underlying soils, the length and diameter of the pile by the analytical method.

1. Introduction

It is known that for choosing the number and pitch of piles in the pile-slab foundation, the important indicators are the length and diameter of the pile, as well as its settlement and bearing capacity in the engineering geological conditions. Obviously, these parameters essentially depend on the physical and mechanical properties of the soils of the multilayer soil base [1,2], including their elastic, elastic-plastic and elastic-visco-plastic properties.

In the present work, the problem of quantifying the settlement and bearing capacity of a pile in a three-layer foundation with regard to linear and elastic-visco-plastic properties [3-5] is posed and solved by an analytical method. It is shown that in this case the force ($N = const$) applied to the pile top is distributed between its lateral surface and the lower tip in time, and that it can more and more be transmitted to the pile tip and this can lead to the development of progressive behaviour of the pile settlement process.

2. Formulation of the problem, initial conditions

As a calculation for solving the problem, let us take a geomechanical model in the form of a thick-walled soil cylinder of finite size ($L = l_1 + l_2 + l_3, 2b$), containing a pile of length $l = l_1 + l_2$ and diameter $2a$ assuming that the pile impact radius is limited to diameter $2b$, where the soil settlement is zero (figure 1).
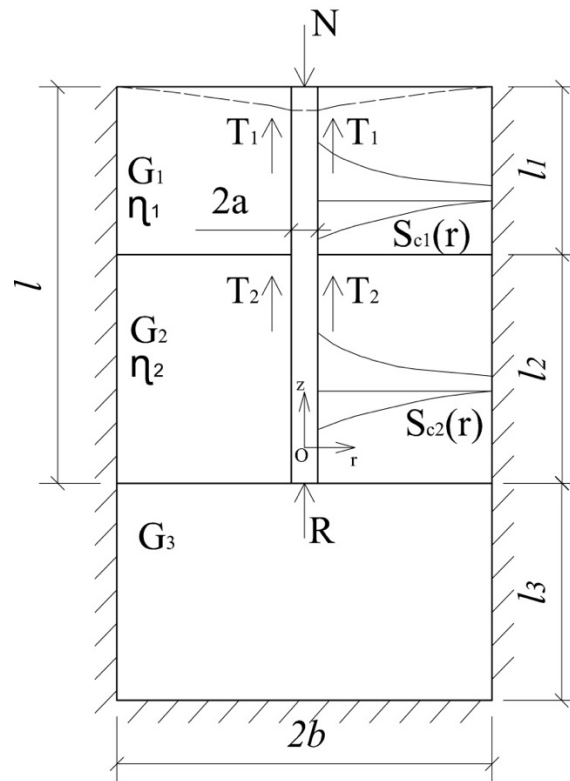
Figure 1. Calculation scheme of the pile interacted with three-layer soil base

From the design scheme (figure 1) it follows that an equilibrium condition holds that, in the first approximation if $\tau_{a1} = \tau_{a2} = \tau_a$, has the form:

$$N = T_1 + T_2 + R \text{ or } \sigma_N = \tau_a 2l_1 / a + \tau_a 2l_2 / a + \sigma_R , \tag{2.1}$$

$$\text{where } \tau_a = (T_1 + T_2) / 2\pi al \tag{2.2}$$

$$\text{if } a \leq r \leq b \tag{2.3}$$

The settlement around the pile from the friction forces on its side surface can be determined through the angular deformation of the soil assuming that there is a telescopic movement of coaxial cylindrical soil layers around the pile [6], i.e. we have:

$$S_g = -\int \gamma(r)dr + c , \tag{2.4}$$

where $\gamma(r)$ in the linear formulation, it is determined by the expression:

$$\gamma(r) = \tau / G \tag{2.5}$$

3. Linear elastic solution
Integration of (2.4) with regard to (2.5) ranging from "a" to "b" gives the maximum settlement from two layers of the surrounding soil when $r = a$, and it is equal to the settlement of the pile itself, from the condition of no slip between the pile and soils, that is:

$$S_g = S_{g1} + S_{g2} = a\tau_a \ln\frac{b}{a}\left(\frac{1}{G_1} + \frac{1}{G_2}\right) \tag{3.1}$$

The settlement of the pile at its lower end can be determined on the basis of the well-known formula [7] on the extrusion of a rigid round stamp into an elastic space in the form:

$$S_R = \frac{\pi a(1-v_3)\sigma_R}{4b_3}K(l), \tag{3.2}$$

where σ_R - the stress under the pile tip, G_3 - shear modulus of the third layer, $K(l)$ - coefficient taking into account the influence of the depth of the stamp [8].

Comparing (3.1) and (3.2), we obtain the connection between τ_a and σ_R:

$$\tau_a = \frac{\pi(1-v_3)\sigma_R K(l)}{4G_3 a \ln(b/a)} \cdot \frac{G_1 \cdot G_2}{G_1 + G_2} \tag{3.3}$$

From the equilibrium condition (2.1) it follows that:

$$\sigma_N = \sigma_R + \tau_a 2l/a, \tag{3.4}$$

where $\sigma_R = R/\pi a^2$, $\sigma_N = N/\pi a^2$, $l = l_1 + l_2$.

Substituting the value τ_a from (3.3) here we get:

$$\sigma_N = \sigma_R \frac{\pi(1-v_3)\sigma_R K(l)}{4G_3 a \ln(b/a)} \cdot \frac{G_1 \cdot G_2}{G_1 + G_2} \cdot \frac{2l}{a} + 1 \tag{3.5}$$

$$\text{or } \sigma_R = \sigma_N/A, \tag{3.6}$$

where $A = \dfrac{\pi(1-v_3)\sigma_R K(l)}{4G_3 a \ln(b/a)} \cdot \dfrac{G_1 \cdot G_2}{G_1 + G_2} \cdot \dfrac{2l}{a} + 1$.

Taking into account (3.4) and (3.6), we have:

$$\tau_a = (\sigma_N - \sigma_N/A)\frac{a}{2l} = \sigma_N (A-1)/A \cdot \frac{a}{2l} \tag{3.7}$$

To determine the settlement from the action of the force *N*, use (3.2) and (3.6), i.e. we get:

$$S_c = \frac{\pi a(1-v_3)K(l)}{4G_3} \cdot \frac{\sigma_N}{A} \tag{3.8}$$

Comparing σ_R (3.6) with the initial critical load under a round stamp [6] we can determine the degree of approximation to the limiting state of soils under the tip of the pile:

$$K_p = \sigma_R/\sigma_R^* < 1, \tag{3.9}$$

$$\sigma_R^* = \bar{\gamma} \cdot d + \frac{2\bar{\gamma} \cdot d \cdot \sin\varphi_3 + 2c_3 \cdot \cos\varphi_3}{1 - 2v_3}, \tag{3.10}$$

where is $\bar{\gamma}$ the average specific weight of soils within l_1 and l_2, d is the depth of the bottom of the pile; φ_3, c_3 - strength parameters of layer 3.

Note that formula (3.10) differs from the Puzyrevsky formula [5], which is designed to determine the initial critical load under the action of the load applied on the surface of a soil along a width $2a$ (plane strain problem).

4. The solution in the elastic-viscous-plastic formulation

In this case, we will take an elastic-visco-plastic model of the Bingham-Shvedov-Maslov type [9] of the following form as a calculation for describing the stress-strain state of the surrounding soil:

$$\dot{\gamma} = \frac{\tau - \tau_i^*}{\eta_i} + \frac{\dot{\tau}}{G} \tag{4.1}$$

where $\tau_i^* = \bar{\sigma}_i \cdot tg\varphi_i + c_i \ (i = 1, 2)$ (4.2)

$\bar{\sigma}_i, \varphi_i, c_i$ - the average value of the normal stress on the pile surface, friction angle and cohesion in surrounding soils, i.e. in layers (1) and (2), accordingly.

$$\bar{\sigma}_i = \gamma_i l_i / 2 \tag{4.3}$$

$$c_i = c_{c,i} + c_{w,i} + c_{\eta,i} \tag{4.4}$$

where $c_{c,i}, c_{w,i}, c_{\eta,i}$ - cohesions due to: structural strength, water-colloid bonds and viscous resistance of the soil, respectively, and:

$$c_{\eta i} = \dot{\gamma} \cdot \eta_i \tag{4.5}$$

It should be noted that the first two terms in expression (4.4) were introduced by N.N. Maslov in the 60s of the last century [4]. However, they could not take into account the influence of the shear rate on the ultimate shear resistance, which is significant for the kinematic shear $\left(\dot{\gamma} = const \right)$. From the condition of equality of soil movement on a radius, it follows that the angular deformations of soils in the layers are also equal $\dot{\gamma}_1 = \dot{\gamma}_2 = \dot{\gamma}$. In this case, the contact shear stress in layers 1 and 2 is also identical, i.e. $\tau_{u_1} = \tau_{u_2} = \tau_a = (T_1 + T_2) / 2\pi a (l_1 + l_2)$.

However, the limiting resistance to shear of soils in each layer must be considered separately according to (4.2).

The rate of Settlement of the surrounding pile of soil from the action of tangential stresses τ_a (2.3) according to (2.4) can be determined by the formula:

$$\dot{S}_g = -\int \dot{\gamma}(r) dr + C , \tag{4.6}$$

where $\dot{\gamma}$ - the rate of the angular deformation of the soils, may be obtained by the formula (4.1). Than we have:

$$\dot{S}_g = -\int \left\{ \frac{\tau(r)-\tau_1^*}{\eta_1} + \frac{\dot{\tau}(r)}{G_1} + \frac{\tau(r)-\tau_2^*}{\eta_2} + \frac{\dot{\tau}(r)}{G_2} \right\} dr + C, \tag{4.7}$$

where C determined from the condition $S_g(r=b)=0$.

If the indefinite integral (4.7) is replaced by a definite one and integrated within the limits from "a" to "b", we obtain the maximum speed of the soil settlement at, i.e. we get:

$$\dot{S}_r = \left\{ a \cdot \dot{\tau}_a \left(\frac{1}{G_1} + \frac{1}{G_2} \right) + a\tau_a \left(\frac{1}{\eta_1} + \frac{1}{\eta_2} \right) \right\} \ln(b/a) - \left(\tau_1^* \cdot \frac{1}{\eta_1} + \frac{\tau_2^*}{\eta_2} \right)(b-a), \tag{4.8}$$

where $\tau_1^* = \overline{\sigma}_1 \cdot tg\varphi_1 + c_{c1} + c_{w1} + \dot{\gamma}\eta_1$, $\tau_2^* = \overline{\sigma}_2 \cdot tg\varphi_2 + c_{c2} + c_{w2} + \dot{\gamma}\eta_2$ (4.9)

(3.2) leads to:

$$\dot{S}_c = \frac{\pi a(1-\nu_3) \cdot \dot{\sigma}_R}{4G_3} \cdot K(l) \tag{4.10}$$

From the condition of the lack of slippage follows the equality of the settlement of the pile S_c and the surrounding soil S_g at $r=a$. So we have:

$$\dot{\tau}_a + \tau_a \frac{G_1 \cdot G_2}{G_1+G_2} \frac{\eta_1+\eta_2}{\eta_1\eta_2} - \left(\frac{\tau_1^*}{\eta_1} + \frac{\tau_2^*}{\eta_2} \right) \cdot \frac{b-a}{a\ln(b/a)} \cdot \frac{G_1 \cdot G_2}{G_1+G_2} = \frac{\pi a(1-\nu_3)\dot{\sigma}_R K(l)}{4G_3} \cdot$$
$$\cdot \frac{G_1 \cdot G_2}{G_1+G_2} \cdot \frac{1}{a\ln(b/a)} \tag{4.11}$$

From (2.1) at $\dot{N}=0$ follows that:

$$\dot{\sigma}_R = -\dot{\tau}_a \frac{2l}{a} \tag{4.12}$$

Substituting this value $\dot{\sigma}_R$ into (4.11) after some rearrangement, we get:

$$\dot{\tau}_a \left(1 + \frac{\pi a(1-\nu_3)}{4G_3} K(l) \cdot \frac{G_1 \cdot G_2}{G_1+G_2} \cdot \frac{2l}{a} \right) + \tau_a \frac{G_1 \cdot G_2}{G_1+G_2} \cdot \frac{\eta_1+\eta_2}{\eta_1 \cdot \eta_2} = \left(\frac{\tau_1^*}{\eta_1} + \frac{\tau_2^*}{\eta_2} \right) \frac{b-a}{a\ln(b/a)} \cdot$$
$$\cdot \frac{G_1 \cdot G_2}{G_1+G_2} \tag{4.13}$$

And finally:

$$\dot{\tau}_a + \tau_a \cdot P = Q, \tag{4.14}$$

where $P = B_2 / B_1$, $Q = B_3 / B_1$, $B_1 = 1 + \frac{\pi a(1-\nu_3)}{4G_3} K(l) \cdot \frac{G_1 \cdot G_2}{G_1+G_2} \cdot \frac{2l}{a}$, $B_2 = \frac{G_1 \cdot G_2}{G_1+G_2} \cdot \frac{\eta_1+\eta_2}{\eta_1 \cdot \eta_2}$,

$$B_3 = \left(\frac{\tau_1^*}{\eta_1} + \frac{\tau_2^*}{\eta_2} \right) \frac{b-a}{a\ln(b/a)} \cdot \frac{G_1 \cdot G_2}{G_1+G_2} \tag{4.15}$$

Solution (4.14) is known and looks like [4,5]:

$$\tau_a(t) = e^{-\int P \cdot dt} \left\{ \int Q \cdot e^{\int P dt} dt + C \right\} \tag{4.16}$$

If $Q = const$, we get:

$$\tau_a(t) = \frac{Q}{P} + C \cdot e^{-Pt} \tag{4.17}$$

Taking into account, that $\tau_a(t=0) = \frac{a}{2l} \sigma_N \frac{A-1}{A}$, we get:

$$C = \frac{u}{2l} \cdot \sigma_N \frac{A-1}{A} - \frac{Q}{P} \tag{4.18}$$

And finally:

$$\tau_a(t) = \frac{Q}{P}\left(1 - e^{-Pt}\right) + \frac{a}{2l} \sigma_N \frac{A-1}{A} \cdot e^{-Pt} \tag{4.19}$$

If $t \to \infty$:

$$\tau_a(\infty) = \frac{Q}{P} = \frac{B_3 / B_1}{B_2 / B_1} = \frac{B_3}{B_2}, \tag{4.20}$$

where B_3 and B_2 determined from (4.15).

Substituting the τ_a value from (4.19) into (4.12), we get:

$$\overset{\Box}{\sigma_R} = -\frac{2l}{a} \cdot \left\{ Q \cdot e^{-Pt} - \frac{a}{2l} \sigma_N \frac{A-1}{A} P e^{-Pt} \right\} \tag{4.21}$$

Integration of (4.21) gives:

$$\sigma_N(t) = -\frac{2l}{a}\left\{ -\frac{Q}{P} e^{-Pt} + \frac{a}{2l} \sigma_N \frac{A-1}{A} P \cdot e^{-Pt} \right\} + C_2 \tag{4.22}$$

As $\sigma_n(0) = \frac{\sigma_N}{A}$, we have:

$$C_2 = \frac{2l}{a}\frac{Q}{P} - \sigma_N \frac{A-1}{A} + \frac{\sigma_N}{A} = \frac{2l}{a}\frac{Q}{P} + \sigma_N \tag{4.23}$$

And finally:

$$\sigma_R(t) = \sigma_N - \frac{2a}{l}\frac{Q}{P}\left(1 - e^{-Pt}\right) \tag{4.24}$$

If $t \to \infty$:

$$\sigma_R(\infty) = \sigma_N - \frac{2a}{l}\frac{Q}{P} \tag{4.25}$$

Comparing this value with the initial critical load according to (3.10), one can determine the degree of approximation $\sigma_R(\infty)$ to the beginning of the formation of limiting equilibrium in the underlying layer of the cylinder (figure 1).

Finally, to determine the time-varying settlement of the pile, it is necessary to substitute in (3.2) instead of σ_R put $\sigma_R(t)$ to (4.24), i.e. receive:
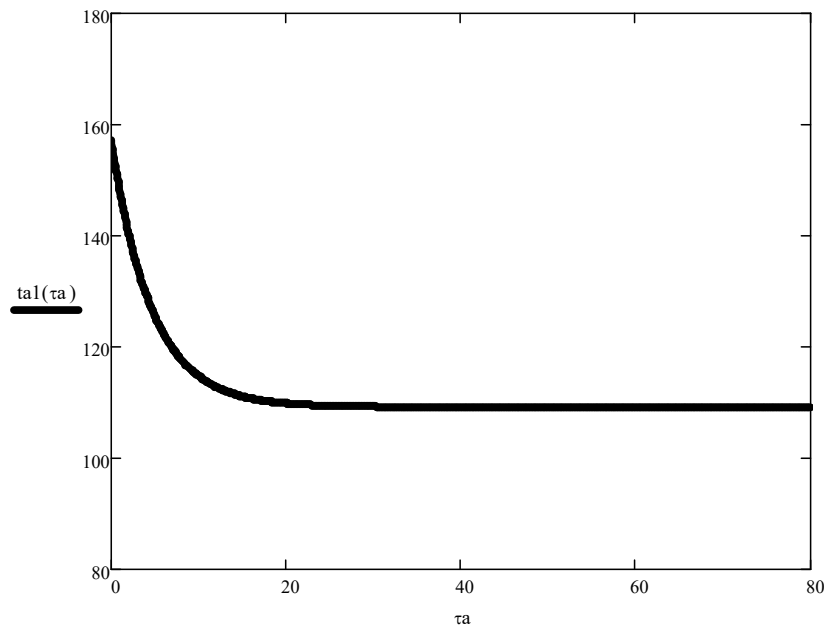
$$S_c(t) = \frac{\pi a (1 - v_3)}{4 G_3} K(l) \cdot \sigma_R(t), \qquad (4.26)$$

where $\sigma_R(t)$ can be determined by (4.24).

The results can be illustrated by a test calculation with the following initial data:

a=0.5 m, b=1.5 m, v =0.3 m, l=15 m, τ_1^*=60 kPa, τ_2^*=100 kPa, η_1=10000 Pa·s, η_2=100000000 Pa·s, G_1=10000 kPa, G_2=50000 kPa, G_3=70000 kPa, $K(l)$=0.8; N_1=10000 kN.

Using the MathCAD software, we have got the resulting curves of τ_a(t), σ_R(t) and $S_c(t)$, using (4.19), (4.24) and (4.26).

Figure 2. Shear stress on the lateral surface of the pile vs. time curve

5. Results and discussions

1. The force applied to the pile top in a linear formulation is distributed between the lateral surface and the lower end of the pile in proportion to the ratio of the stiffnesses of the surrounding and underlying soils $(G_1 : G_2 : G_3)$, as well as to the ratio of the geometric parameters of the pile $2l / a \ln(b / a)$.

2. In the case of an elastic-visco-plastic formulation, the distribution specified in clause 1 may be complex and change over time. This essentially depends on the deformation and strength properties of surrounding and underlying soils, including viscous shear resistance $\left(C_\eta = \overset{\Box}{\gamma} \eta \right)$.

3. It is shown that the stresses under the tip of the pile σ_R and on the side surface of the pile τ_a are redistributed in time, with the initial value of σ_R increasing in time, and τ_a, conversely, decreasing.

4. The settlement of the pile in time develops slowly. Its speed depends on the elastic, viscous (G, η) and plastic (c, φ) properties of the surrounding and underlying soils, as well as ratios of the pile radius and the containing cylinder (a / b).
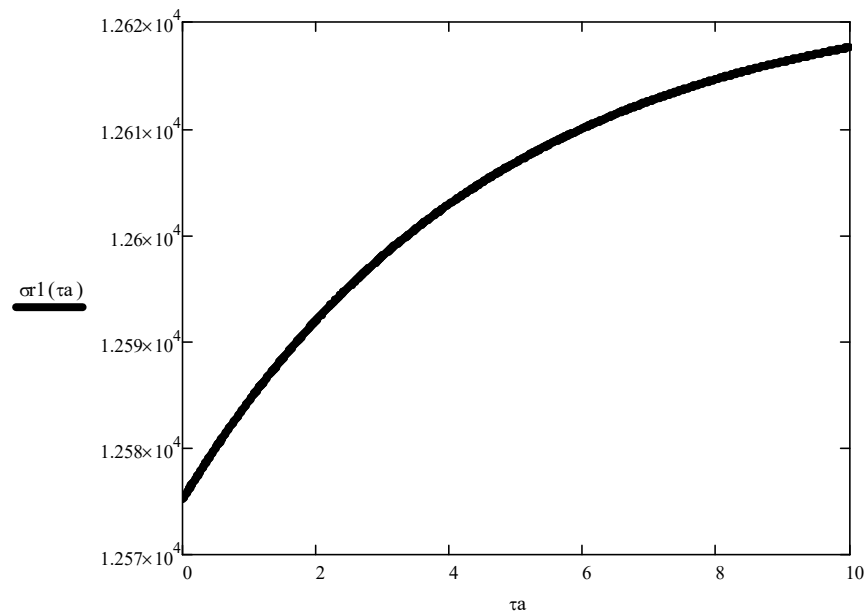
Figure 3. Vertical stress under the pile tip vs. time curve
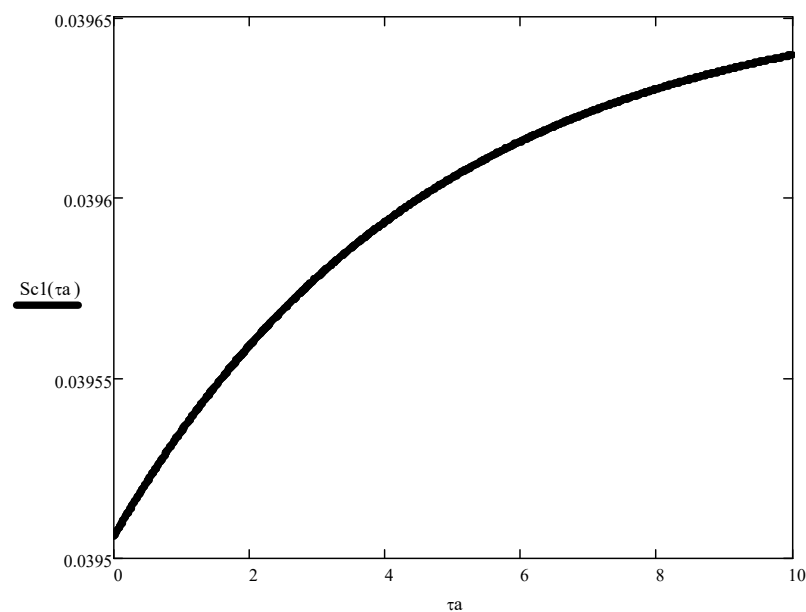
Figure 4. Settlement of the pile vs. time curve

6. Conclusions

The problem is completely solved. The pile settlement and pressure under its tip are determined, as well as the degree of pile approach to the limit state.

The results obtained in this work tasks in a closed form can be used when choosing the diameter, length and pitch of the piles in the pile-slab foundation [10].

7. Acknowledgements

References
[1] Z. G. Ter-Martirosyan, A.Z. Ter-Martirosyan, V.V. Sidorov, Initial and critical pressure under circle footing and under a tip of a boring circle pile, Natural and technical sciences, vol. 11-12(78), pp. 372-376, 2014 (in Russian)
[2] Z. G. Ter-Martirosyan, Nguen Z.N. The interaction of long piles with a two-layer elastic-creeping base, Bulletin of the civil engineers, vol. 1(10), pp. 52-55, 2007 (in Russian)
[3] S. S. Vyalov. Rheological bases of soil mechanics: textbook for construction universities, High school, 1978 (in Russian)
[4] N. N. Maslov. Bases of soil mechanics and engineering geology, High school, 1968 (in Russian)
[5] Z. G. Ter-Martirosyan. Rheological parameters of soils and structures bases calculations, Stroyizdat, 1990 (in Russian)
[6] Z. G. Ter-Martirosyan, V. Sidorov. Settlement and bearing capacity of the circular foundation, MATEC Web of Conf., vol. 196, 03019, 2018
[7] Z. G. Ter-Martirosyan, A.Z. Ter-Martirosyan. Experimental and theoretical foundations of the transformation of weak water-saturated clay soils in the surface and deep compaction, Engineering geology, vol.4, pp. 16-25, 2015 (in Russian)
[8] Z. G. Ter-Martirosyan, A.Z. Ter-Martirosyan, A.V. Manukyan, G.O. Anzhelo. The interaction of piles-drains with the surrounding compacted clay soil and a grillage, taking into account the time factor, Civil construction, vol.11, pp. 26-29, 2017 (in Russian)
[9] Z. A. R. Rzhanitsyn. The theory of creep, Stroyizdat, 1968. (in Russian)
[10] SP 24.13330.2011 Pile foundations. Actual redaction of SNiP 2.02.01-85.