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Calculation of the Rectangular Cross-Section Beams On the Side Buckling Taking into Account Creep

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Abstract. The article presents the derivation of the resolving equation for calculating the flat form of bending stability of the beams, taking into account creep. When deriving the basic equation, the application of the load with eccentricity, as well as the initial curvature of the beam is taken into account. The solution of the test problem for a wooden cantilever beam is presented. The value of the long-term critical load was introduced and it was shown that the load acting on the beam should not exceed it. A new criterion for determining the critical time is proposed.

1. Introduction

When designing rectangular beams in order to reduce material consumption, engineers strive to increase the ratio of cross-section height to width, which implies the need to check the structure for stability of a flat bending. A large number of works, including [1–3], are devoted to the calculation of prismatic beams on the stability of a flat form of deformation. However, in all these publications, the solution is performed in an elastic formulation. In this paper, we will consider the solution of this problem, taking into account creep.

When deriving the basic equation, we will use the criterion of initial imperfections as a criterion of stability. The initial irregularities are specified in the form of the initial deflection $v_0(x)$, the initial twist angle $\theta_0(x)$ and the eccentricity e. The critical time when using the specified criterion is determined conditionally by setting the maximum value of displacements or the maximum speed of their growth.

2. Derivation of resolving equations

The beam element after buckling is shown in figure 1. In the lateral buckling of a beam, a torque M_x arises in it. For the x axis of an ideal undeformed beam, we write the sum of moments:

$$M_x + dM_x + qdx\left(\frac{v + v_0 + \left(v + v_0 + d\left(v + v_0\right)\right)}{2} + e + a\left(\theta + \theta_0\right)\right) - M_x = 0. \qquad (1)$$

After discarding values of a higher order of smallness, we get:

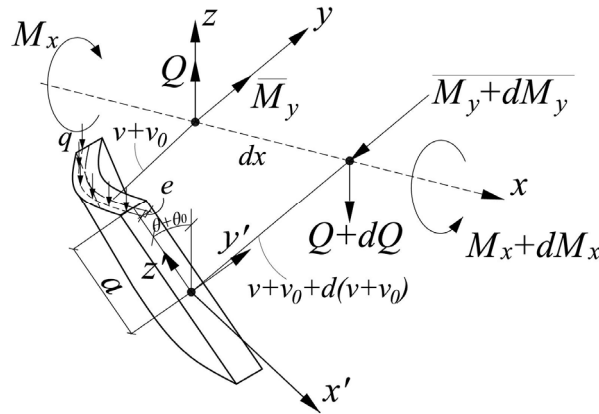$$\frac{dM_x}{dx} = -q\left(v + v_0 + e + a\left(\theta + \theta_0\right)\right). \qquad (2)$$

Figure 1. Beam element after deformation

Torque acting relative to the axis x', can be calculated by the formula:

$$M_{x'} = M_x - Q(v + v_0) + M_y \left(\frac{dv}{dx} + \frac{dv_0}{dx} \right). \tag{3}$$

For a beam experiencing bending in two planes, the total linear deformations based on the flat section hypothesis can be determined by the formula:

$$\varepsilon_x = -y \frac{d^2 v}{dx^2} - z \frac{d^2 w}{dx^2}. \tag{4}$$

On the other hand, the total deformations ε_x represent the sum of the elastic deformations and creep deformations:

$$\varepsilon_x = \frac{\sigma_x}{E} + \varepsilon_x^*. \tag{5}$$

Substituting (4) into (5) and expressing stresses through deformations, we get:

$$\sigma_x = -E \left(y \frac{d^2 v}{dx^2} + z \frac{d^2 w}{dx^2} + \varepsilon_x^* \right). \tag{6}$$

Bending moments are calculated as follows:

$$M_y = -\int_A \sigma_x z \, dA ; M_{z'} = \int_A \sigma_x y \, dA. \tag{7}$$

Substituting (6) into (7), we get:

$$M_y = EI_y \frac{d^2 w}{dx^2} - M_y^* ; M_{z'} = -EI_z \frac{d^2 v}{dx^2} - M_z^*, \tag{8}$$

where $M_y^* = -E \int_A \varepsilon_x^* z \, dA, M_z^* = E \int_A \varepsilon_x^* y \, dA$.

The relation between the twisting angle and the torque with regard to creep has the form [4]:

$$M_{x'} = M_t = GI_t \frac{d\theta}{dx} - M_t^*,$$ (9)

where $M_t^* = G \int_A (-\gamma_{xy}^* z + \gamma_{xz}^* y) dA$, γ_{xy}^* and γ_{xz}^* – shear creep strains, I_t – moment of inertia during torsion.

Equating (9) to (3), we get:

$$GI_t \frac{d\theta}{dx} - M_t^* = M_x - Q(v + v_0) + M_y \left(\frac{dv}{dx} + \frac{dv_0}{dx} \right).$$ (10)

Next we differentiate equality (10) with respect to *x*:

$$GI_t \frac{d^2\theta}{dx^2} - \frac{dM_t^*}{dx} = \frac{dM_x}{dx} - \frac{dQ}{dx}(v + v_0) - Q \frac{d(v + v_0)}{dx} + \frac{dM_y}{dx} \frac{d(v + v_0)}{dx} + M_y \frac{d^2(v + v_0)}{dx^2}.$$ (11)

Further we substitute (2) in (11) and take into account that $\frac{dQ}{dx} = -q$ and $\frac{dM_y}{dx} = Q$. After simplifications, equality (11) takes the form

$$GI_t \frac{d^2\theta}{dx^2} - \frac{dM_t^*}{dx} = -q(e + a(\theta + \theta_0)) + M_y \frac{d^2v}{dx^2} + M_y \frac{d^2v_0}{dx^2}.$$ (12)

We express from (8) the value $\frac{d^2v}{dx^2}$:

$$\frac{d^2v}{dx^2} = -\frac{M_{z'} + M_z^*}{EI_z}.$$ (13)

Considering that $M_{z'} = M_y (\theta + \theta_0)$, we rewrite equality (13) in the form:

$$\frac{d^2v}{dx^2} = -\frac{M_y (\theta + \theta_0)}{EI_z} - \frac{M_z^*}{EI_z}.$$ (14)

Substituting (14) into (12), we obtain the main resolving equation:

$$GI_t \frac{d^2\theta}{dx^2} + \left(\frac{M_y^2}{EI_z} + qa \right)\theta = \frac{dM_t^*}{dx} - q(e + a\theta_0) + M_y \frac{d^2v_0}{dx^2} - \frac{M_y M_z^*}{EI_z} - \frac{M_y^2 \theta_0}{EI_z}.$$ (15)

3. Methods

For the calculation, the grid is introduced in time t and the x coordinate. The cross section is divided into segments Δy and Δz. At the first stage, the solution of equation (15) is performed by the method of finite differences at $t = 0$, $\varepsilon_x^* = 0$, $\gamma_{xz}^* = \gamma_{xy}^* = 0, M_t^* = 0, M_y^* = 0, M_z^* = 0$.

After determination of the twist angle $\theta(x)$ from equation (15), we calculate the relative angle of twist $\vartheta = \dfrac{d\theta}{dx}$ for each cross section with step Δx. Then, for each section, the tangential stresses are calculated by solving the differential equation obtained in [4]:

$$\nabla^2 \Phi(y,z) + 2G\vartheta - G\left(\frac{\partial \gamma_{xz}^*}{\partial y} - \frac{\partial \gamma_{xy}^*}{\partial z}\right) = 0, \tag{16}$$

where Φ is the stress function, given by the formulas:

$$\tau_{xy} = \frac{\partial \Phi}{\partial z}; \tau_{xz} = -\frac{\partial \Phi}{\partial y}. \tag{17}$$

The boundary conditions on the contour of the cross section have the form: $\Phi = 0$.

Curvature changes are also determined. The value $\dfrac{d^2 v}{dx^2}$ is determined by the formula (14), and the values $\dfrac{d^2 w}{dx^2}$ can be calculated by the formula:

$$\frac{d^2 w}{dx^2} = \frac{M_y + M_y^*}{EI_y}. \tag{18}$$

Knowing the changes in curvatures, it is possible to determine the normal stresses by the formula (6). Further, the magnitudes of the normal and tangential stresses are determined by the rate of growth of creep deformations. Then, using the Euler method, the strains $\gamma_{xz}^*, \gamma_{xy}^*, \varepsilon_x^*$ are found at time $t + \Delta t$, as well as the values M_t^*, M_y^*, M_z^*. Then the process is repeated for the next step.

4. Results and discussions

In order to control the reliability of the resulting equation and the developed technique, a test problem was solved for a cantilever wooden beam, to which the force F is applied with the eccentricity e (figure 2).

The boundary conditions for this problem are written as:

$$\text{at } x = 0 : \theta = 0;$$

$$\text{at } x = l : M_t = Pe = GI_\kappa \frac{d\theta}{dx} - M_t^*. \tag{19}$$

As the creep law, we used Maxwell-Thomson equation, which has the form:

$$\frac{\partial \varepsilon_x^*}{\partial t} = \frac{1}{nE}\left[\left(1 - \frac{E_\infty}{E}\right)\sigma_x - E_\infty \varepsilon_x^*\right];$$

$$\frac{\partial \gamma^*}{\partial t} = \frac{1}{nG}\left[\left(1 - \frac{G_\infty}{G}\right)\tau - G_\infty \gamma^*\right],$$

(20)

where E is the instantaneous modulus of elasticity of the material ($E = 1.48 \cdot 10^4$ MPa), E_∞ is the long modulus of deformation ($E_\infty = (0.6 \div 0.75) E = 10^4$ MPa), n is the relaxation time ($n = 10 \div 25$ days, is usually taken 18 days), G and G_∞ are respectively the instantaneous and long-term shear modulus.

In the calculations we assumed $G = 500$ MPa, and $G_\infty = 0.675G = 338$ MPa.
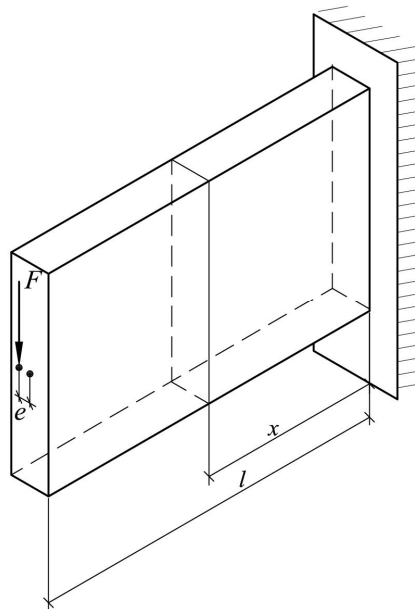

Figure 2. Design scheme

For an ideal elastic beam, the loss of stability occurs at the following critical force [5]:

$$F_{cr} = \frac{4.01}{l^2}\sqrt{GI_t EI_z}.$$

(21)

The value of long-term critical load was introduced by the formula:

$$F_\infty = \frac{4.01}{l^2}\sqrt{G_\infty I_t E_\infty I_z}.$$

(22)

The behaviour of the beam was investigated at $F < F_\infty$, $F = F_\infty$ and $F > F_\infty$. The initial data was taken as follows: $b = 10$ mm, $h = 100$ mm, $l = 1$ m, $e = 0.1$ mm. Long-term critical load for the beam is $F_\infty = 117.8$ N. Figure 3 shows the graphs of the twist angle maximum value variation in time for three values of the load ($F = 100$ N $< F_\infty$, $F = 117.8$ N $= F_\infty$ and $F = 125$ N $> F_\infty$).
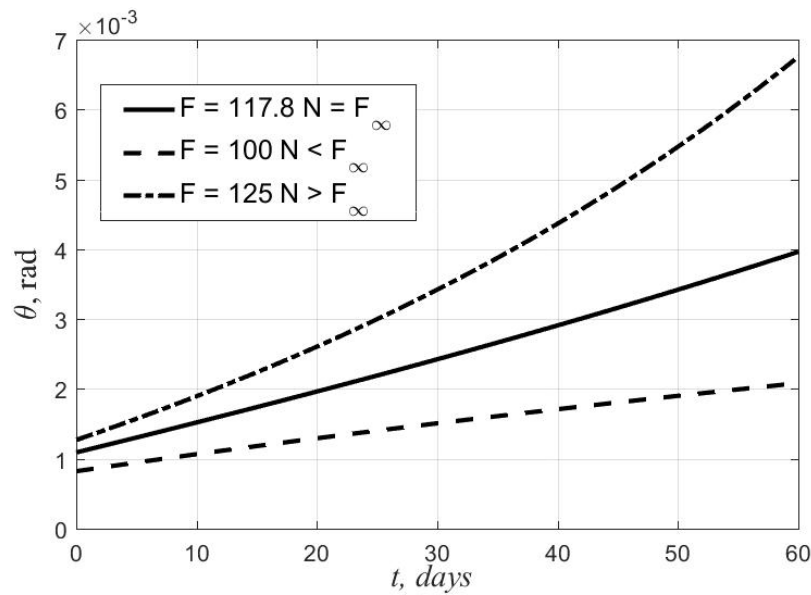
Figure 3. Graphs of the maximum twist angle growth for various values of the force F

From figure 3, it can be seen that, if a load is less than a long-term critical, the growth rate of displacements decays with time. When $F = F_\infty$, the displacements grow at a constant speed, and when $F > F_\infty$, the displacements growth rate increases with time. A similar character of displacement growth curves was obtained for compressed rods in the paper [6]. The results obtained by us confirm the validity of the technique.

The calculation was also carried out at $F = 125$ N for the different values of eccentricity e. The corresponding $\theta(t)$ curves are shown in figure 4. From the presented graphs it can be seen that, if the value of displacements or the rate of their growth is taken as a criterion for the loss of stability, the initial imperfections have a significant effect on the value of the critical time. Thus, the loads acting on the beam should not exceed the long-term critical force.
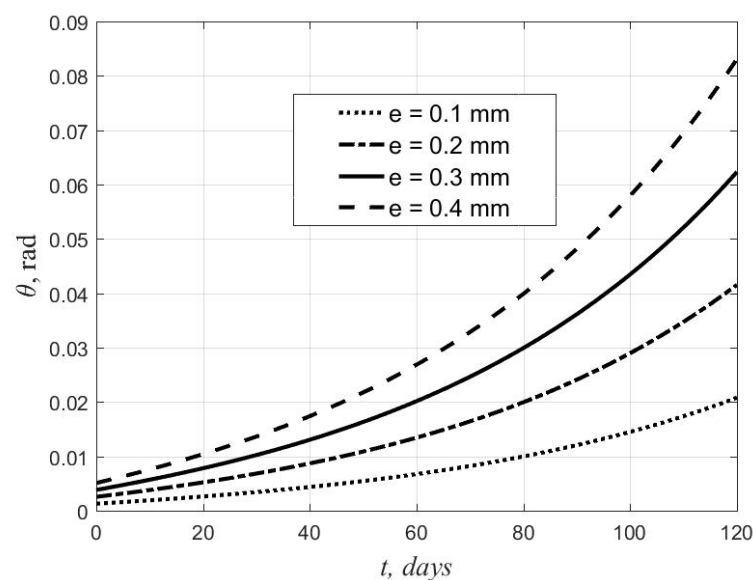

Figure 4. The growth of the twist angle maximum value for the different values of eccentricity e

A rather interesting picture is observed on the graphs of the normal stresses maximum values change in time. Up to a certain point in time, despite the increase in the twisting angle, the normal stresses decrease, but then they begin to increase. From figure 5 it can be seen that the higher the value of eccentricity e is, the earlier the moment comes from which normal stresses begin to rise. The time corresponding to the extremum point in the σ_{max} (t) graphs can be taken as critical. In case of using such a criterion for buckling critical time was 150 days for $e = 0.1$ mm, 112 days for $e = 0.2$ mm, 90 days for e = 0.3 mm and 76 days for $e = 0.4$ mm.
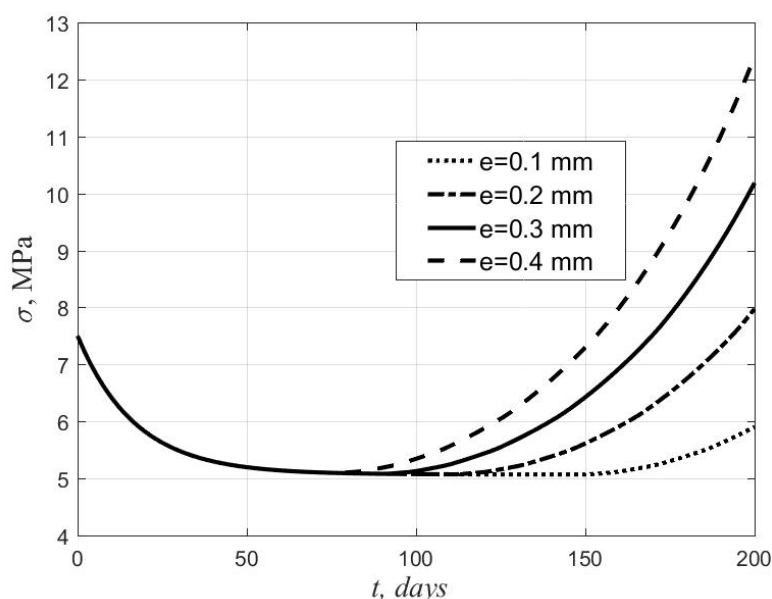

Figure 5. The change in time of the normal stress maximum values

The maximum shear stresses in the beam, as well as the twisting angle, only increase in time.

5. Conclusions

A universal resolving equation was obtained for calculating the flat bending stability of rectangular section beams, suitable for arbitrary creep law. Initial imperfections in the form of initial deflection, initial angle of twist and load eccentricity were taken into account. The phenomenon of lateral buckling of beams during creep was studied using the example of a cantilever wooden beam. The value of long-term critical load F_∞ was introduced and an analogy with the problems of compressed rods stability in creep was established.

It was also found that when $F > F_\infty$, the maximum value of the normal stress first decreases in time, and then from a certain moment the stress σ_x begins to increase. The time corresponding to the minimum stress point on the $\sigma_x(t)$ graphs can be taken as critical.

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