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Gears or rotors - three approaches to design of working units of hydraulic machines

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Abstract. The gerotor pump has many advantages over other hydraulic pumps, such as high reliability, compactness, lower noise in running conditions etc. Considering the increasingly stringent regulations in the field of environmental protection, one of the main goals in the industrial applications of gerotors is to improve their design by using light materials and optimize their performance. This paper will give an overview of modern research that focuses on improving the performance of gerotor pumps. A joint publication of three university centers, namely of Wroclaw, Terrassa and Kragujevac, presents their original approaches to the problem of designing gerotor and orbital systems. The presentations have been prepared in separate parts, but according to similar rules. The first part of the paper focuses on meshing between internal and external gear of gerotor. Analysis for both cases, moved axis and fixed axis, will be provided in accordance to their working characteristics and their application. The second part of the paper provides the design of a gerotor pump strikes a balance between volumetric efficiency, manufacturability and mechanical efficiency. The third part of the paper defines the methodology for selecting the best combination of geometric form parameters of the trochoidal gearing in relation to the reduction of contact stresses and improving other characteristics of the pump. All three parts provide manufacturing and application review in the fluid power machines. Paper concludes with presented theoretical and experimental review of the authors and their practice achievements.

1. Introduction

In hydraulic, pneumatic, mechanical and also internal combustion engines, gerotor or orbital systems are often used. Those words refer to systems composed of two elements, i.e. a rotor and a ring, which are internally engaged and in which the difference between the mating elements equals one. If both elements work at fixed axes and the rotor drives the ring, then the system is called a gerotor system. If, however, the rotor moves within a fixed ring by circular or orbital motion, then such a system is called an orbital system. Two fundamental problems related to gerotor and orbital systems need solving, namely: establishing a uniform terminology for this machine design area and formulating principles for the design of gerotor and orbital systems.

Among the authors who deal with the issues, there are two basic approaches to solving those problems. According to the first one, two co-operating elements are treated as gears so the problem is



located in the field of gears [1, 2, 3]. According to the second one, those elements are treated as cams with a special outline so the problem is located in the field of the theory of machines and mechanisms [4, 5, 6]. In both cases, different terminology is used.

This paper presents research approaches to the described gearing of three university centres, namely of Wrocław, Terrassa and Kragujevac. The methods presentation has been prepared in separate parts, but according to similar rules including geometry, kinematics, hydraulics, contact stress and technology. In the summary, all approach methods are compared.

2. Designing of gerotor and orbital gear systems for hydraulic machines according to FPRG WROCLAW

2.1 Geometry

In the gerotor design process, two basic groups of curves, i.e. epicycloids and hypocycloids, are used. These types of tooting are used because of its very good meshing characteristics, resistance to shock load, low noise, etc. In the following text of this chapter much closer description of gerotor profile types will be presented.

By treating the rotor and the ring as gears, parameters characteristic for gears were adopted for their description. In Figure 1 are shown uncorrected and corrected gear profiles.

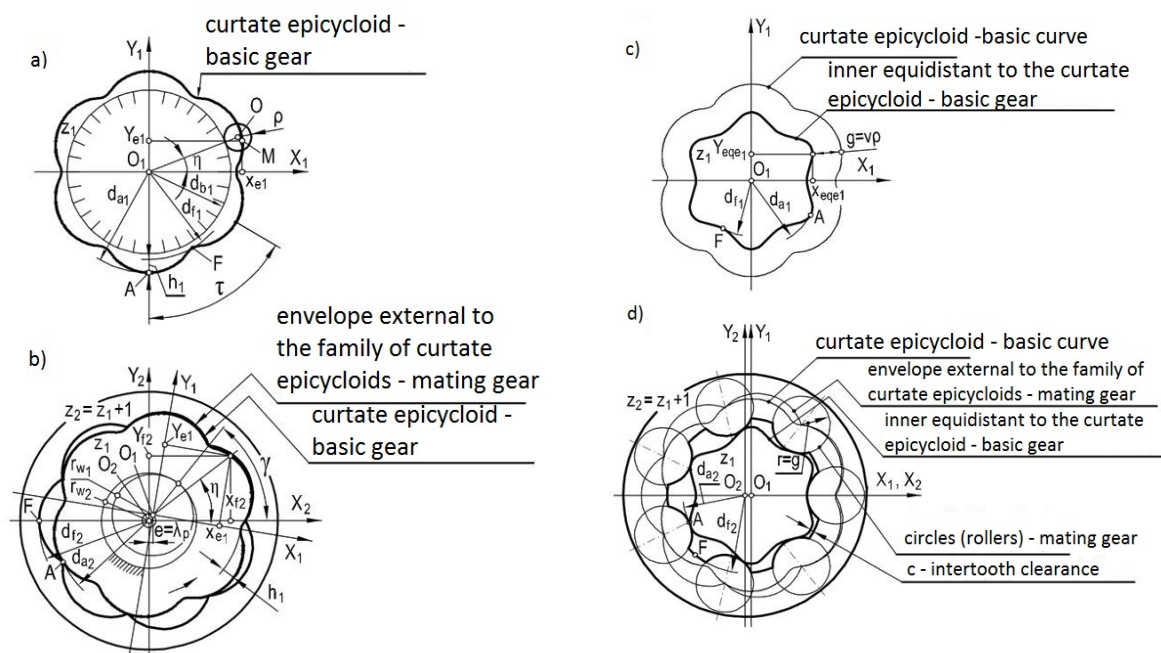


Figure 1. Designing epicycloid gears: a, b) uncorrected gears: internal and external; c, d) corrected gears: external and internal.

Figure 1a,b presents the principles of designing a system of uncorrected epicycloid gears. In Figure 1 the following parameters are shown:

z_1 - number of teeth identical to the number of the epicycloid's or hypocycloid's arcs present in the entire closed cycloidal curve,

m - a module which in accordance with the principle of constructing cycloidal curves is equal to the diameter of the moving wheel ρ , i.e. $m = 2\rho$

λ - tooth depth factor ($\lambda = OM/\rho$),

η - the angle of the cycloidal curve,

v - correction coefficient of the cycloidal profile,

g - shift of the profile ($g = v\rho$),
 b - tooth width.

As shown further in this work, using those parameters, the geometry and kinematics of cycloidal gears can be described, and also hydraulic parameters of machines, such as delivery (absorption) and the pulsation of delivery (absorption) can be determined. It shows the versatility of the adopted parameters system and its usefulness in the entire designing process.

The system of parametric equations describing the family of wheels with an epicycloid outline in the X_2Y_2 system has the form:

$$\begin{aligned} x_{fe2} &= \frac{m}{2} \left[\lambda \cos \gamma + (z_1 + 1) \cos \frac{z_1 \eta - \gamma}{z_1} - \lambda \cos \frac{(z_1 + 1) z_1 \eta - \gamma}{z_1} \right] \\ y_{fe2} &= \frac{m}{2} \left[\lambda \sin \gamma + (z_1 + 1) \sin \frac{z_1 \eta - \gamma}{z_1} - \lambda \sin \frac{(z_1 + 1) z_1 \eta - \gamma}{z_1} \right] \end{aligned} \quad (1)$$

Equations (1) should be supplemented with a condition of the envelope which connects angle of rotation γ of the basic wheel relative to the collaborating wheel with angle of the epicycloid η [3]:

$$\sin z_1 \eta - \lambda \sin \left[z_1 \eta + \frac{z_1 (\eta - \gamma)}{z_1} \right] + \sin \frac{z_1 (\eta - \gamma) - \gamma}{z_1} = 0 \quad (2)$$

Using equations (1) and (2) together, it is possible to determine the profile of the collaborating internal gear. The characteristic diameters of the collaborating gear are derived from formulas (3):

$$d_{a2} = 2(z_1 \rho + \rho - \lambda \rho + \lambda \rho) = m(z_1 + 1) \quad (3a)$$

$$d_{f2} = 2(z_1 \rho + \rho + \lambda \rho + \lambda \rho) = m(z_1 + 1 + 2\lambda) \quad (3b)$$

Analysing Figure 1b, it is noted that:

- formulas for determining the outline of an internal gear are difficult to use directly,
- the teeth of both gears have wrong proportions between the head and the foot, and the contact stresses may be too high,
- manufacturing of an internal gear of a complicated outline is difficult and expensive.

In such a situation, the gears and gear systems are corrected. First, the basic external gear is corrected. As shown in Figure 1c, the correction consists in the formation of the internal equidistant relative to the curtate epicycloid, and the shift of the equidistant equals $g = v\rho$. The system of parametric equations describing the equidistant takes the form:

$$\begin{aligned} x_{eqe1} &= \frac{m}{2} \left[(z_1 + 1) \cos \eta - \lambda \cos(z_1 + 1) \eta - v \frac{\cos \eta - \lambda \cos(z_1 + 1) \eta}{\sqrt{1 - 2\lambda \cos z_1 \eta + \lambda^2}} \right] \\ y_{eqe1} &= \frac{m}{2} \left[(z_1 + 1) \sin \eta - \lambda \sin(z_1 + 1) \eta - v \frac{\sin \eta - \lambda \sin(z_1 + 1) \eta}{\sqrt{1 - 2\lambda \cos z_1 \eta + \lambda^2}} \right] \end{aligned} \quad (4)$$

The formulas for calculating the outside diameter and the root diameter in the corrected gear are:

$$d_{a1} = 2(z_1 \rho + \rho + \lambda \rho - g) = m(z_1 + 1 + \lambda - v) \quad (5a)$$

$$d_{f1} = 2(z_1 \rho + \rho - \lambda \rho - g) = m(z_1 + 1 - \lambda - v) \quad (5b)$$

Next, the collaborating internal gear is corrected. As shown in Figure 1d, circles of a radius equal to the equidistant shift of $r = g$ are drawn from the vertices of the envelope, and then those circles are

connected to each other by arcs of the r_{f2} radius circle, which rotates from the centre of the collaborating gear O_2 . The characteristic diameters of the collaborating gear after correction are derived from the formulas:

$$d_{a2} = 2(z_1\rho + \rho - \lambda\rho - g + \lambda\rho) = m(z_1 + 1 - \nu) \quad (6a)$$

$$d_{f2} = 2(z_1\rho + \rho + \lambda\rho - g + \lambda\rho) = m(z_1 + 1 + 2\lambda - \nu) \quad (6b)$$

The detailed derivation of the presented equations for corrected gerotor profile could be found in the literature [3, 13-15].

The correction of the epicycloidal gear system can be compared with the P-0 correction (without the axes distance change) in the classic involute mesh.

To sum up this part of the considerations, it can be stated that:

- the formulas for calculating the gear outlines are simple and easy to use in the design process,
- the teeth and gears are proportional,
- manufacturing of the gears is much simpler and cheaper compared to the uncorrected gears, particularly in the case of the collaborating internal gear.

Hypocycloidal gears and gear systems can be designed in a similar way to the one the epicycloidal gears and gear systems are designed. That design problem has been thoroughly explained in [13, 14, 15].

2.2 Kinematics

Principles of collaboration between epicycloidal gears in a system at fixed axes of rotation are shown in Figure 2a.

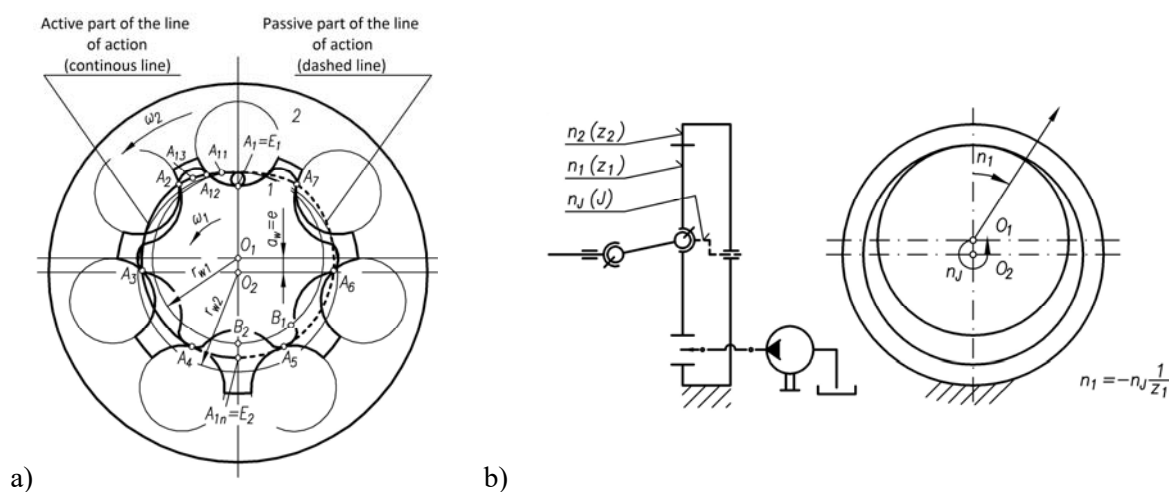


Figure 2. Collaboration (mating) of cycloidal gears; a) mating of the corrected epicycloidal gears with fixed axes O_1 , O_2 ; b) mating of the epicycloidal gears with moving axes, where O_1 rotates around O_2 .

The external gear drives the internal gear and all teeth of both gears are in constant contact with each other. The line of action has the shape of a loop (conchoid) which is divided into the active part carrying the load (continuous line) and the passive part not transferring the load (dashed line). The number of action is $\varepsilon = z_2/2$ and is much larger than in the involute gearing. That means that the teeth are much less loaded than in the involute gearing, and the mechanical cycloidal gear transmissions can transfer higher loads than involute gear transmissions. An extremely important feature of the collaboration is that between the gears, intertooth closed spaces are formed, which are necessary for transporting the working medium in hydraulic machines. The principles of the gears co-operation in the epicycloidal systems with moving axes are shown in Figure 2b. A case of a single hydraulic

gearing was considered. An internal gear of O_2 centre remains motionless, and on that gear, an external gear of O_1 centre rolls with the planetary motion. It is a kind of gear transmission, in which the motion of the external gear is hydraulically forced and the transmission equals $n_1=n_j/z_1$. That principle of collaboration was used in orbital hydraulic engines [16, 17] and in control units [17].

2.3 Hydraulics - delivery (absorption) and pulsation of the delivery (absorption) of hydraulic machines

Using the characteristic parameters of the teeth and the mesh adopted at the beginning of this work, it is possible to make formulas for calculating of the hydraulic parameters of the machines. This applies to the delivery, or, alternatively, the absorption, as well as pulsation of the delivery, or, alternatively, absorption of the delivery of machines with fixed axes (gerotor) and delivery, or alternatively, absorption of machines with moving axes (orbital).

The delivery (absorption) of gerotor machines is determined by the formula:

$$\frac{q_{st}}{\pi b m} = \frac{1}{4} \left[(z_1 + 1 + \lambda - \nu)^2 - \frac{z_1}{z_1 + 1} (z_1 + 1 - \nu)^2 + z_1 \lambda^2 \right] \quad (7)$$

and the pulsation of delivery (absorption) from the formula:

$$\delta = f(z, \lambda, \nu) \quad (8)$$

In the case of orbital machines, which, by principle, work only as engines, the absorption is determined by the formula:

$$q_{or} = z_2 q_{st} \quad (9)$$

2.4 Design and manufacturing solutions for gears of cycloidal hydraulic machines and mechanical transmissions

Using the presented method of designing cycloidal gear systems, their construction documentation can be developed and manufacturing can be started. Figure 3 presents selected methods of manufacturing cycloidal gears.

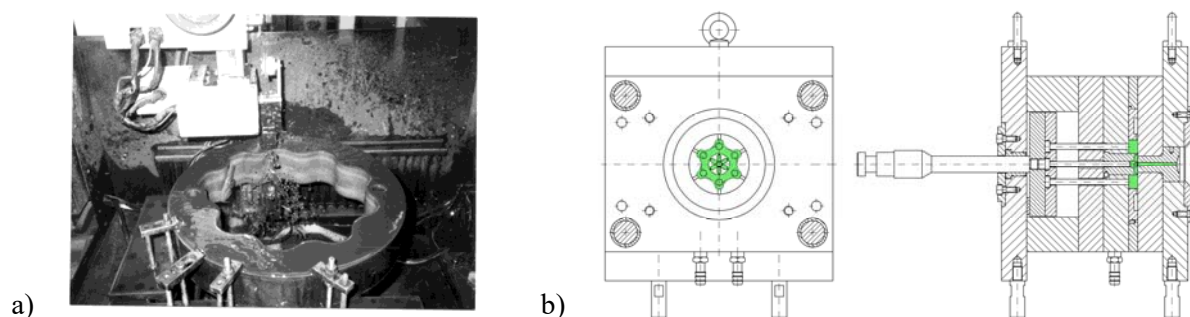


Figure 3. Manufacturing of the cycloidal gears: a) spark erosion of the internal hypocycloidal gear, b) manufacturing of the internal epicycloid gear by injection moulding.

Individual gears, or small series, can be made by spark erosion [3] (Figure 3a), and the plastic cycloidal gears can be made by injection moulding [18, 19] (Figure 3b). The technological data, tools and instruments needed for the manufacturing have been developed by scientists of the FPRG WROCLAW. As a result of the manufacturing process, various cycloidal gear systems have been made and used in the construction of hydraulic machines [3, 17] and mechanical transmissions [21].

3. Designing of gerotor for hydraulic machines according to IAFARG & LABSON TERRASSA

The design of a gerotor pump from scratch is drawn on existing research work and the own authors' experience and know-how. As a first step, the conceptual stage with the dimensional constraints lead to obtain a complete gear set by means of four basic parameters and simple formulae. The design process, a step forward, the most important characteristics of the pump such as theoretical performance indexes and the porting are presented.

The available gerotor gear sets by means of sheet commercial literature shows a great number of standard sets and each application has to be adapted to fit one of them. However, the selection of the design parameters that has led to the geometry of these standard sets is not clear, at least in the available information sheets. Despite of being the cost effective solution, it is advisable to choose the right selection of the design parameters that allows reaching a balance between contact stress, volumetric characteristics and flow irregularity. The methodology presented in this section is specifically applied to circular-tooth profiles (conventional-toothed gerotor).

3.1 Dimensional constraints parameters in a new-born gerotor pump

The characterization of the gerotor pump will be based on the volumetric capacity c_v in [cm³/rev]. Then, the dimensional constraints will set up the corrected epicycloidal gear thickness H in [mm] and the external diameter of the external gear D_c in [mm]. The gear thickness is proportional to the volumetric capacity. However, the main influence of H is related with the filling capability of the chambers, and subsequently, on the volumetric performance and efficiency. The eccentricity e in [mm] of the gear set, which is the most difficult geometrical parameter to predict, can be calculated as,

$$e^2 - \left(\frac{D_c}{4} - \frac{wc}{2} \right) \cdot e + \frac{39.8 \cdot c_v}{H} = 0 \quad (10)$$

where wc in [mm] is the wall wide of the external gear (see Figure 4). The value of wc can be estimated by using the following expression,

$$wc = 0.075 \cdot D_c \quad (11)$$

being just an estimation because it is not taking into account the number of teeth of the external gear, working pressure, contact stress and gear material. Now, the arc radius of the external gear tooth g in [mm] can be calculated as:

$$g = \frac{z_2 \cdot e}{\lambda} - 0.995 \cdot (0.5 \cdot D_c - wc) + 2 \cdot e \quad (12)$$

where z_2 is the number of teeth of the external gear, z_1 is the number of teeth of the internal gear ($z_2 - z_1 = 1$) and λ is the tooth depth factor, with recommended values going from 0.6 (larger g) to 0.8 (shorter g) from Stryczek et al. [18].

Then, by knowing c_v , H and D_c from the specifications of the application, and by choosing z_2 the four basic parameters are defined and the designer can use the previous formulae, from equation (1) to (12), to determine the profiles of the gear set.

3.2 Theoretical performance indexes and porting

The theoretical torque T in [N·m/bar], theoretical power N in [W/bar·rpm], the tip velocity of the internal gear V_t in [m/s·rpm] and the theoretical radial load RL in [N/bar] can be estimated by using the following expressions:

$$T = 0.016 \cdot c_v \quad (13)$$

$$N = 0.0016 \cdot c_v \quad (14)$$

$$V_t = 0.0001 \cdot \left(\frac{D_c}{2} - wc - e \right) \quad (15)$$

$$RL = 0.1 \cdot H \cdot (D_c - 2 \cdot wc - 3 \cdot e) \quad (16)$$

RL value is a reference to the load on the shaft and its bearings, and V_t value must be taking into account regarding to the volumetric efficiency since the increase of V_t has a negative effect on the filling capability of the chambers.

The porting located at the housing on the casing is an important factor in the gerotor pump volumetric performance and efficiency and a proper porting design requires experience and knowledge by the pump designer. As a first step in the design process, the theoretical porting can be defined based on the contact points P where a chamber is comprised between two successive contact points and the end inlet port (fa), the beginning outlet port (ii) and the beginning inlet port (ia). The external porting radius G in [mm] and the internal porting radius $RiRi$ in [mm] can be calculated as:

$$G = 0.5 \cdot D_c - wc \quad (17)$$

$$RiRi = 0.5 \cdot D_c - wc - 3 \cdot e \quad (18)$$

and the upper limit, the porting angle, and the lower limit, the porting length, can be simply obtained by calculating the corresponding contact points (P_{ia} , P_{fa} , P_{ii} and P_{fi}) by means of the line of contact [7]. The theoretical porting does not accomplish the minimum requisites, specially providing the best chamber filling as possible, and a former porting is then proposed in [5].

3.3 GeroLAB software

The specifications of the application configure three basic parameters c_v , H and D_c , and the fourth one by choosing z_2 . These parameters can be introduced in specialized software named GeroLAB that will calculate and provide the designer with the technical drawings in CAD format (Figure 5), volumetric characteristics, contact stress, teeth clearance and former porting [22].

GEROLAB software is an integrated package system based on modules and written by using an open source code. Basically, the methodology consists of three phases, each of which is related to a basic module: design, volumetric characteristics and contact stress [23]. By consecutively following each phase, the software is capable of combining the design of a gear set for a gerotor pump. Then, as an integrated package, it is completed with two new modules. The minimum clearance module is the forth module that takes into account real effects regarding the geometry paths of the manufactured tolerances and clearances. The fifth module integrates the new approach of modelling relief grooves located at the porting, to make the transition from the theoretical to the former porting [24].

3.4 New gerotor technology: GeroMAG concept and the magnet-sleeve-sealed with polymer composite concept

GeroMAG concept is an innovative variable flow pump, sealed, compact and non shaft-drive with magnetic-driving outer rotor [25]. Up to date in the standard gerotor pump technology, the rotational movement was transmitted from the internal gear (driving) to the external gear (driven) by means of an exterior shaft-drive motor. The external gear takes the role of primary pump gear conducting the rotational movement as a driving outer rotor in the GeroMAG concept, without exterior shaft-drive, using electro-magnetic driving on the external gear through pole pieces (see Figure 5a).

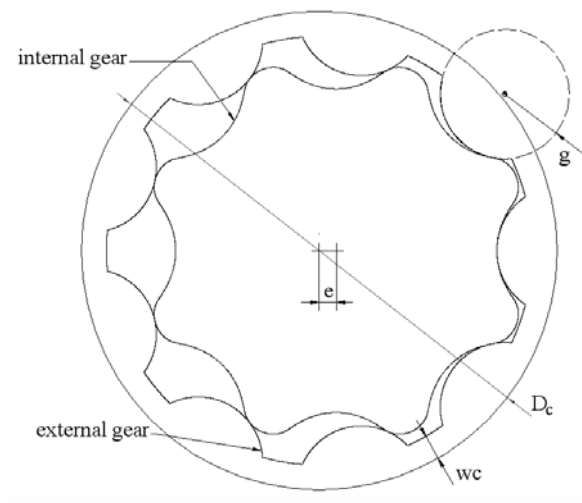


Figure 4. The basic geometrical parameters of the corrected epicycloidal gear.

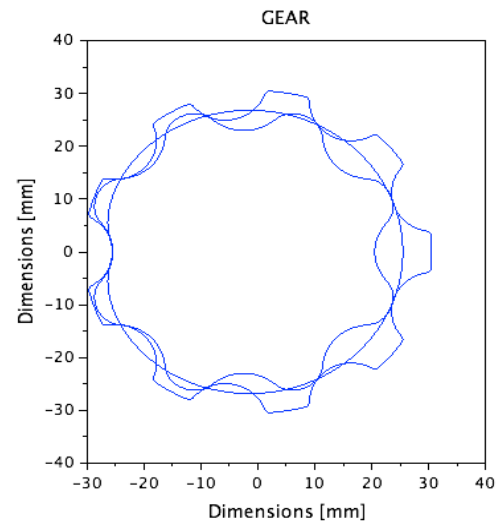


Figure 5. Gear set and line of contact by using GeroLAB [22].

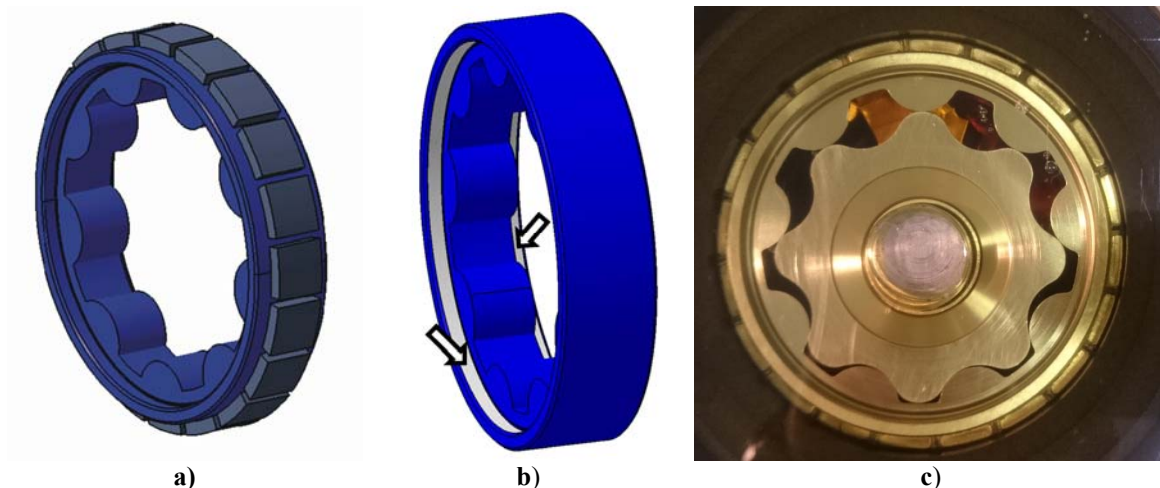


Figure 5. a) Pole pieces fixed on the external gear, in grey; b) guiding internal surface, in grey indicated with arrows; c) GeroMAG prototype with an in house manufactured transparent polymer pump cover.

In addition, the coaxially of each gear in its own rotation centre, as well as the eccentricity has to be guaranteed. A guiding internal surface is designed to guide the rotational movement of the outer gear. The guide is performed by the path of the inner surface in both sides of the external gear teeth faces (see Figure 5b).

This novel configuration accomplishes a standard volumetric flow rate at low rotational speed with satisfactory volumetric efficiency (see Figure 5c). GeroMAG concept is believed to lead to a cleaner and noiseless hydraulic technology to be part of a wider part of sectors. Moreover, if the configuration of the magnetic-driven outer rotor is also transferred to the internal gear, becoming likewise a magnetic-driven inner rotor, then the rotational movement could be performed with zero inter-teeth contact stress by controlling the electrical torque of both rotors.

Another example of new concept in gerotor technology is the magnet-sleeve-sealed mini gerotor pump [20]. This novel variable flow pump configuration, despite of the difficulties occurred and several disappointed fallouts obtained, such as frictional performance of the gear units, accomplishes a

standard volumetric flow rate at low rotational speed with satisfactory volumetric efficiency. Experimental results prove feasibility as well as proof of concept.

4. Designing of gerotor for hydraulic machines according to Kragujevac

4.1 Mathematical description of gerotor toothing profile

Analysis takes into account the case where exterior gear of the trochoidal pump has one tooth more than the interior gear. The interior gear profile is described by peritrochoid equidistant, while the exterior one is described by the circular arc with radius r_c . The basic geometrical relations during unmodified and modified peritrochoid gearing generation are illustrated by Figure 6a. Peritrochoid is presented by the point D , fixed in a circle plane of radius r_a , which is rolling inward side on the outside of an immovable circle of the radius r_i . The equations of the peritrochoid are defined in the coordinate system of the trochoid $O_t x_t y_t$, according to Figure 6a [26].

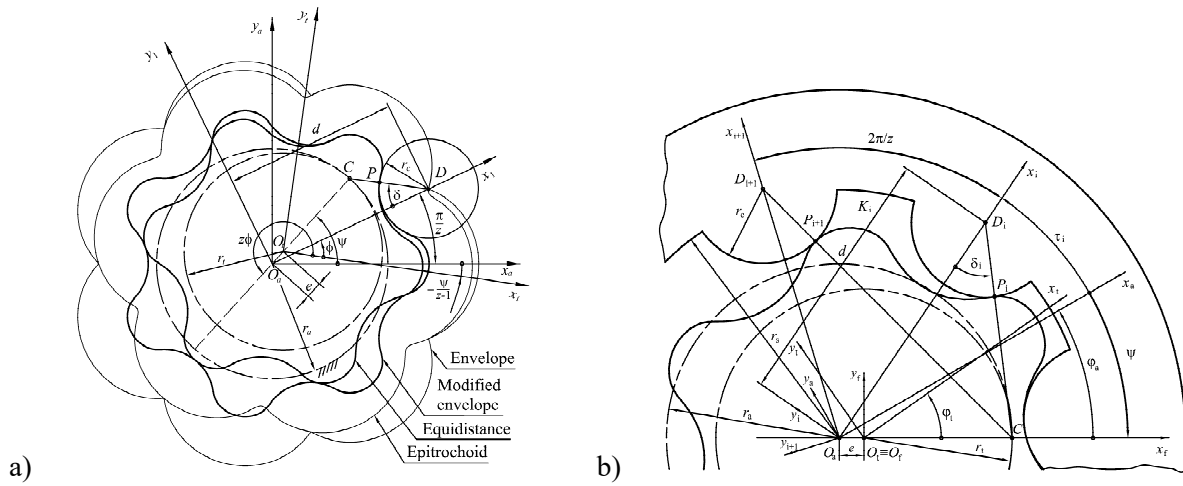


Figure 6. Gerotor gearing: a) generation of the trochoidal gearing; b) Schematic presentation of the gerotor pump gear pair with fixed shaft axes.

Figure 6.a illustrates that during the relative moving of kinematic circles, point D generates peritrochoid, while point P generates equidistant. The angle specified as δ is an angle between the normal $n-n$ and radius vector of the point D , and can be defined as leaning angle.

Since at the trochoidal gearing is characterized with simultaneous meshing of all the teeth, it is necessary to defines general equations of the profile points coordinates applicable to all the teeth. The internal gear profile (Figure 6b) is defined by a position vector of the contact point P_i in the coordinate system of the trochoid $O_t x_t y_t$ using the following equations:

$$\begin{aligned} x_{P_i}^{(t)} &= e \left\{ z_2 \lambda_c \cos \left[\tau_i + \frac{\psi}{z_2 - 1} \right] - \cos \frac{z_2 \psi}{z_2 - 1} - c \cos \left[\tau_i + \frac{\psi}{z_2 - 1} + \delta_i \right] \right\} \\ y_{P_i}^{(t)} &= \left\{ z_2 \lambda_c \sin \left[\tau_i + \frac{\psi}{z_2 - 1} \right] - \sin \frac{z_2 \psi}{z_2 - 1} - c \sin \left[\tau_i + \frac{\psi}{z_2 - 1} + \delta_i \right] \right\} \end{aligned} \quad (19)$$

where $r_c = ce$, $d = \lambda_c e z_2$, i is the ordinal number of the tooth, $\tau_i = \frac{\pi(2i-1)}{z_2}$,

$$\delta_i = \arctan \frac{\sin(\tau_i - \psi)}{\lambda_c - \cos(\tau_i - \psi)} \text{ and } \psi = (z_2 - 1) \left(\phi - \frac{\pi}{z_2} \right).$$

The derived equations can be applied to different kinematic schemes of trochoidal pumps. They enable modelling of the gear pair meshing during the rotation, as well as determination of necessary parameters for any teeth pair at an arbitrary moment. In pumps with fixed shaft axes where the drive shaft is fixed to the internal gear (Figure 6b), the following expressions define the relations between the referent angle ψ and the shaft rotation angles $\varphi_a = -\psi$ and $d\varphi_i = \frac{z_2}{z_2 - 1} d\varphi_a$.

To unify the various geometric design of working units of hydraulic machines, parameters used in the chapter 2 can be related to variables used in this chapter as follows: $r_b = r_i d / r_a$, $\rho = \lambda_c e$, $g = r_c$, $\lambda = 1/\lambda_c$, $v = c/\lambda_c$ and $\eta = \phi$.

4.2 Gerotor functional characteristics

In order to obtain functional dependency which would provide projecting of the pump gear pair, based on the given starting data, a mathematical model of trochoidal gearing pump capacity characteristics has been developed. First to be considered was distribution of working fluid and definition of characteristic phases in the pump operating cycle, followed by description of the methods for defining working capacity and actual rate of delivery of a pump. Based on the analysis of results, relevant values were identified which have influence on pulsation of the rate of delivery and uneven flow.

Concerning of conditions which reduce contact forces creation, and thus reduce wear, analyzed were forces and moments which influence the gear pair of the trochoidal rotational pump. Starting with the specific conditions in which the load is transmitted simultaneously at a number of contact points, considered were fluid thrust force which affect the sides of the gear teeth, contact forces and contact stress [27, 28, 29]. To check analytical methods and determine error which occurs when applying above noted postulations, the finite element method was applied (Figure 7).

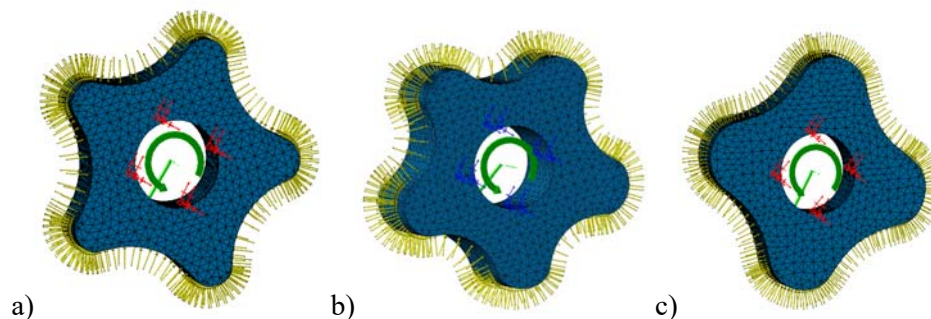


Figure 7. Models of the discrete gears simulation with forces defined and visualization of reaction support: a) $z_2=6$, $\lambda_c=1.575$ b) $z_2=6$, $\lambda_c=1.375$ c) $z_2=5$, $\lambda_c=1.85$.

Maximum contact stresses, forces and moments were analysed using analytical and numerical methods, taking into consideration the pressure variation in the pump chambers due to the fluid flow [26]. It should be pointed out that in the described investigations the theoretical trochoidal profile was considered, with presumptions on ideally precise geometrical characteristics. Starting from the fact that the real profiles are made with inevitable technological gaps, it is considered modelling of real meshing profiles [30].

4.3 Methodology for selection of the optimum teeth profile

Selection of the optimum profile should begin by defining the initial values of the designed gear pair. External load is one of the in advance given values, and it is defined through the working pressure and the pump volume. The geometric characteristics that remain constant and that are determined with the overall size of the gear pair are the following: the gear width b , the eccentricity e and the external gear root radius expressed by the meshing envelope parameter S_{fa} [27]. It is necessary to determine the

external gear teeth number z_2 and the values of the coefficient λ_c and c , which define the optimum shape of the teeth profile. The convex-convex contact is taken into consideration. Since the convex part of the external gear profile is a circular arc of the radius r_c , the problem comes down to the analysis of the contact at the point of the internal gear profile with the smallest curvature radius. In that case, the equivalent curvature radius has the lowest value for the given profile and it can be calculated using the equation:

$$(\rho_{ekv})_{\min} = \frac{r_c \rho_{c\min}}{r_c + \rho_{c\min}}. \quad (20)$$

Introducing the expression for $\rho_{c\min}$

$$\rho_{c\min} = e \left(z_2 \sqrt{\left(\frac{3}{z_2 + 1} \right)^3 (\lambda_c^2 - 1)(z_2 - 1) - c} \right) \quad (21)$$

into the equation (22), taking into account the equation (21), obtaining the first derivate of the equation (20) per the parameter λ_c and then setting it equal to zero, we get the condition for the existence of the maximum equivalent radius of the curvature $(\rho_{ekv})_{\min}$ dependent on λ_c , in the form of:

$$\frac{1}{(z_2 \lambda_c + 2 - S_{fa})^2} + \frac{\left(\frac{3}{z_2 + 1} \right)^3 (z_2 - 1) \lambda_c \left[\left(\frac{3}{z_2 + 1} \right)^3 (z_2 - 1) (\lambda_c^2 - 1) \right]^{\frac{1}{2}} - 1}{\left\{ z_2 \left[\left(\frac{3}{z_2 + 1} \right)^3 (z_2 - 1) (\lambda_c^2 - 1) \right]^{\frac{1}{2}} - z_2 \lambda_c - 2 + S_{fa} \right\}^2} = 0. \quad (22)$$

As this equation is *transcendental* and the parameter λ_c cannot be expressed explicitly, the solution is obtained by an iterative procedure. Thus, chosen value is further checked against the complex criteria. It has been shown that gear pairs can be generated based on minimizing the equivalent curvature of the profile of the tooth in contact, when the condition that the specific sliding of the meshing profiles are equal is fulfilled. This procedure has been applied to a commercial pump, and the obtained solution has been proposed as a better solution for the given conditions. The analysis results have shown that, for the same working conditions, significant reductions in maximum contact stresses can be achieved. For the same teeth number, when the value of the parameter λ_c is decreased, the maximum contact stress can be reduced by about 16.8%, while for the smaller number of teeth, with an increase in the value of the parameter λ_c , approximately 35% lower values are obtained compared to the commercial pump [27]. The results obtained in this paper have wide and practical application especially in design of gerotor pumps with better performance characteristics.

4.4 Experimental verification

After the analytical check-up of the theoretical models, an experimental verification of results was conducted. Measuring of flow rate and volumetric efficiency was conducted on three different models of gear pairs (Figure 8) with simulation of real conditions of pump exploitation, in the same pump housing. One of them was a commercial gear pair, shown in Figure 8,a and the other two gear pairs, whose profiles were derived from calculations and proposed as optimal are made and shown in Figure 8,b and Figure 8,c.

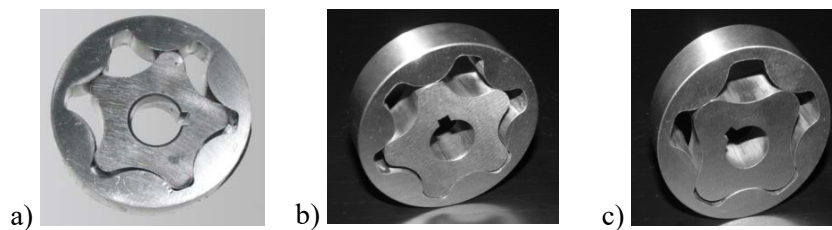


Figure 8. Models of three tested gear pairs: a) Commercial model
b) Optimal model, $z_2=6$; c) Optimal model, $z_2=5$.

5. Conclusions

Wrocław Fluid Power Research Group assumes that gerotor and orbital systems should be treated as gear systems. Following that assumption, the scientists of the group has developed the knowledge of the fundamentals of designing and manufacturing of cycloidal gears and gear systems constituting gerotor and orbital systems. It is complex knowledge which includes geometry, kinematics, strength, hydraulics and cycloidal gears technology. For the description of those issues, an integrated system of technical parameters has been used, which is similar to the one applied for the classic involute gears.

The research group IAFARG & LABSON from Terrassa considers the gerotor pump as a hydraulic machine of which pumping action is carried out by means of a trochoidal-envelope toothed gear set. The research group unifies several approaches to characterize performance indexes either an existing gerotor pump or a new-born gerotor unit. These approaches mainly are dynamic simulation by using bond graph technique, computational fluid dynamics numerical simulation and experimental work.

Contribution of the Kragujevac group research lays in the detailed analysis of the effects of the geometric parameters of the trochoidal gearing on the reduction of contact stresses performed using the analytical-numerical method and in the new possibilities it opens up for further studies. The developed mathematical model and the obtained results can be of use to the constructors of the gerotor pump and motors for choosing the best constructive solutions that reaches higher coefficient of efficiency.

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