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To cite this article: Haojie Li *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **631** 052022

IOP Conf. Series: Materials Science and Engineering **631** (2019) 052022 doi:10.1088/1757-899X/631/5/052022

One Step Anti-Windup Design for Nonlinear Systems: An Indirect LMI-Based Approach

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Abstract. This paper proposes an indirect linear matrix inequality (LMI) approach to design one step anti-windup dynamic output feedback compensator for a class of nonlinear systems subject to actuator saturation. When the system state satisfies certain conditions, the nonlinear system can be reconstructed accurately by T-S fuzzy model, based on Lyapunov stability analysis and a result about matrix inequality, we show that the feasibility of an LMI guarantees the solvability of the corresponding one step anti-windup compensator. Once the solvability issue is determined, then we give an algorithm of one step anti-windup nonlinear dynamic output feedback compensator. The effectiveness of the proposed method is illustrated with numerical example.

1. Introduction

Actuator saturation is unavoidable in almost all real systems. In some cases, it may lead to system instability. In the literature, several methods now exist to handle saturation effects, but the most popular and effective one remains the anti-windup (AW) approach[1]. LMI-based synthesis of anti-windup compensators has been proposed recently to synthesize either static[2] or dynamic[3] anti-windup compensators; an overview of these results can be found in the survey[4]. Most of these works deal with a two step method in which the controller and the AW strategy are designed separately[5]. An alternative solution called one step method, which designs the controller and the AW compensator simultaneously, has been proposed[6].

Using T-S fuzzy model, the nonlinear system can be reconstructed accurately, then LMI-based relaxed nonquadratic stabilization conditions for nonlinear systems can be presented[7]. For technological or economic reasons, the state variables are not all measured in most real-world applications[8].

The note is organized as follows. The problem stated in Section 2. The main result is stated in Section 3. Section 4 gives example. Finally, Section 5 gives Some concluding remarks.

Notations. For a real symmetric matrix M, $M>0$ and $M<0$, means the matrix is positive definite and negative definite respectively. For two symmetric matrices, A and B, $A>B$ means that $A-B>0$, M^{T} denotes the transpose of the matrix M. $M(a)$ denotes the ath row of matrix M, and $M(a)^{\mathrm{T}}=(M(a))^{\mathrm{T}}$. Here, $\mathrm{He}M=M+M^{\mathrm{T}}$ for real square matrix M, $*$ stands for the symmetric blocks. I_m denotes the $m\times m$ dimension identity matrix. $\mathrm{diag}(A,B)$ is a block-diagonal matrix.

2. Problem statement

Consider the autonomous continuous time-invariant nonlinear system with input saturation

$$\dot{x}(t) = f(x(t))x(t) + g(x(t))\mathrm{sat}(u(t)), y = h(x(t))x(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^p$ are the plant state vector, the control input and the output of the system, respectively. Continuous time-invariants matrices function $f(x) \in \mathbb{R}^{n\times n}, g(x) \in \mathbb{R}^{n\times m}$, $h(x) \in \mathbb{R}^{p\times n}$. Saturation function $\mathrm{sat}(u_i(t)) = \mathrm{sign}(u_i(t))\min\{u_{0i}, |u_i(t)|\}, i = 1,2,\cdots,m,$ where u_{0i} denotes the symmetric amplitude bound relative to control input.

Consider exact T–S fuzzy model,

Rules R_i : if $z_1(t)$ is F_{i1}, $z_2(t)$ is F_{i2}, $z_j(t)$ is F_{ij}, and $z_s(t)$ is F_{is}, then

$$\dot{x}(t) = A_i x(t) + B_i \mathrm{sat}(u(t)), y = C_i x(t), i = 1,2,\cdots,q, j = 1,2,\cdots,s \tag{2}$$

with F_{ij} representing the fuzzy set *j* of each rules R_i, and matrices $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^{n\times m}$, $C_i \in \mathbb{R}^{p\times n}$.

Assumption 1. (A_i, B_i) is stabilizable and (A_i, C_i) is detectable, $i = 1,2,\cdots q$.

Then the system (1) can be refactored as

$$\dot{x}(t) = \sum_{i=1}^{q} \mu_i(t)(A_i(t) + B_i \mathrm{sat}(u(t))), y(t) = \sum_{i=1}^{q} \mu_i(t) C_i x(t) \tag{3}$$

An important issue related to fuzzy control systems is that the nonlinear T–S representation (2) usually has a specific bounded domain of validity Eq. (1). One way to represent such domain of validity is by a polyhedral set $\Omega(x) = \left\{ x \in \mathbb{R}^n \middle| |L_i x| \le d_i, i = 1,2,\cdots,l \right\}$, with $L_i \in \mathbb{R}^{1\times n}$, $d_i > 0$. consider the set

$$\Xi = \left\{ \mu(t) = (\mu_1(t), \mu_2(t), \cdots, \mu_q(t))^{\mathrm{T}} \middle| \mu_i(t) \ge 0, \sum_{i=1}^{q} \mu_i(t) = 1 \right\}$$

where $\mu_i(t) = \alpha_i(t) \Big/ \sum_{j=1}^{q} \alpha_j(t)\ (i = 1,2,\cdots,q)$ and functions $\alpha_i(t) = \prod_{j=1}^{s} F_{ij}(z_j(t))$, $F_{ij}(z_j(t))$ are the grades of member functions corresponding to the fuzzy terms F_{ij}.

We introduce an n_c order dynamic output stabilizing compensator

$$\dot{x}_c(t) = \sum_{i=1}^{q} \mu_i(t)(A_{ci} x_c(t) + B_{ci} y(t) - E_{ci} \psi(u(t))), u(t) = \sum_{i=1}^{q} \mu_i(t)(C_{ci} x_c(t) + D_{ci} y(t)) \tag{4}$$

where $x_c(t) \in \mathbb{R}^{n_c}$, $\psi(u): \mathbb{R}^m \to \mathbb{R}^m$ is the dead-zone nonlinearity, given by $\psi(u) = u - \mathrm{sat}(u)$ .where $A_{ci}, B_{ci}, C_{ci}, D_{ci}$ are supposed to be time-invariant matrices of appropriate dimensions. E_{ci} is an anti-windup gain matrix. From Eq. (3) and Eq. (4), we can represent the closed loop system by:

$$\dot{\xi} = \sum_{i=1}^{q}\sum_{j=1}^{q}\sum_{k=1}^{q} \mu_i(t)\mu_j(t)\mu_k(t)(A_{ijk}\xi - E_{ij}\psi(u)), u = \sum_{i=1}^{q}\sum_{j=1}^{q} \mu_i(t)\mu_j(t) K_{ij}\xi \tag{5}$$

with $\xi = \begin{pmatrix} x \\ x_c \end{pmatrix}$, $A_{ijk} = \begin{bmatrix} A_i + B_i D_{cj} C_k & B_i C_{cj} \\ B_{cj} C_k & A_{cj} \end{bmatrix}, E_{ij} = \begin{bmatrix} B_i \\ E_{cj} \end{bmatrix}, K_{ij} = \begin{bmatrix} D_{ci} C_j & C_{ci} \end{bmatrix}$.

Problem. Determine the matrices $A_{ci}, B_{ci}, C_{ci}, D_{ci}, E_{ci}$ and give a region of local stability for the closed loop system (5). And then optimize the attraction domain estimation.

3. Main Results

3.1. Dynamic output feedback stabilization

We define polyhedral set $\Sigma=\bigcap_{i=1}^{q}\bigcap_{j=1}^{q}\left\{\xi(t)\left|\left|(K_{ij}(a)-G_{ij}(a))\xi(t)\right|\le u_{0a}, a=1,2,\cdots,m\right.\right\}$.

Lemma 1. Consider the function $\psi(u)$ defined as before. If $\xi(t)\in\Sigma$ then the relation

$$\psi^{\mathrm{T}}(u(t))T(G_{ij}\xi(t)-\psi(u(t)))\ge 0, i,j=1,2,\cdots,q \tag{6}$$

is verified for any matrix $T\in\mathbb{R}^{m\times m}$ diagonal and positive definite.

Lemma 2(Schur complement)[9]. Let symmetric matrix $A=\begin{pmatrix}A_{11} & A_{12}\\ * & A_{22}\end{pmatrix}$, then, $A>0$ if and only if $A_{11}>0,\Delta>0$ or $A_{22}>0,\bar{\Delta}>0$, where $>$ can be changed as $\ge$, $<$ or $\le$, with Schur complement $\Delta=A_{22}-A_{12}{}^{\mathrm{T}}A_{11}{}^{-1}A_{12}$, $\bar{\Delta}=A_{11}-A_{12}A_{22}{}^{-1}A_{12}{}^{\mathrm{T}}$.

Suppose P is a positive definite matrix, N_{ij} is a matrix of appropriate dimensions such that

$$\begin{bmatrix}P & K_{ij}(a)^{\mathrm{T}}-PN_{ij}(a)^{\mathrm{T}}\\ * & u_{0a}^2\end{bmatrix}\ge 0, i,j=1,2,\cdots,q, a=1,2,\cdots,m \tag{7}$$

$$\begin{bmatrix}P_{11} & L_i^{\mathrm{T}}\\ * & d_i^2\end{bmatrix}\ge 0, i=1,2,\cdots,l \tag{8}$$

We shall partition P, Q, and N as $P=\begin{pmatrix}P_{11} & P_{12}\\ * & P_{22}\end{pmatrix}$, $Q=\begin{pmatrix}Q_{11} & Q_{12}\\ * & Q_{22}\end{pmatrix}$, $N_{ij}=\begin{pmatrix}N_{ij}{}^{(1)} & N_{ij}{}^{(2)}\end{pmatrix}$, with $P_{11},Q_{11}\in\mathbb{R}^{n\times n}$, $P_{22},Q_{22}\in\mathbb{R}^{n_c\times n_c}$, $P_{12},Q_{12}\in\mathbb{R}^{n\times n_c}$ $N_{ij}{}^{(1)}\in\mathbb{R}^{m\times n}$, $N_{ij}{}^{(2)}\in\mathbb{R}^{m\times n_c}$ .let $G_{ij}=N_{ij}P,$ we have $u_{0a}^2-(K_{ij}(a)-G_{ij}(a))P^{-1}(K_{ij}(a)-G_{ij}(a))^{\mathrm{T}}\ge 0$, let $\Lambda=\left\{\xi(t)\left|\xi^{\mathrm{T}}(t)P\xi(t)\le 1\right.\right\}$, If $\xi(t)\in\Lambda$, based on Holder inequality, we have $\left|(K_{ij}(a)-G_{ij}(a))/u_{0a}\right|\le 1$, If (8) are satisfied, then $P_{11}-(L_i^{\mathrm{T}}/d_i)(L_i/d_i)\ge 0$, when $\xi(t)\in\Lambda$, we have $x^{\mathrm{T}}(L_i^{\mathrm{T}}/d_i)(L_i/d_i)x\le x^{\mathrm{T}}P_{11}x=\begin{pmatrix}x^{\mathrm{T}} & 0\end{pmatrix}P\begin{pmatrix}x^{\mathrm{T}} & 0\end{pmatrix}^{\mathrm{T}}\le 1$, which implies $x(t)\in\Omega(x)$. Define Lyapunov function $V=\xi^{\mathrm{T}}P\xi$, then

$$\dot{V}=\sum_{i=1}^{q}\sum_{j=1}^{q}\sum_{k=1}^{q}\mu_i(t)\mu_j(t)\mu_k(t)(\xi^{\mathrm{T}}\mathrm{He}(PA_{ijk})\xi-2\xi^{\mathrm{T}}PE_{ij}\psi(u))\le\sum_{i=1}^{q}\sum_{j=1}^{q}\sum_{k=1}^{q}\mu_i(t)\mu_j(t)\mu_k(t)\eta^{\mathrm{T}}\Psi_{ijk}\eta$$

Where $\eta=\begin{pmatrix}\xi\\ \psi(u)\end{pmatrix}$, $\Psi_{ijk}=\begin{pmatrix}\mathrm{He}(PA_{ijk}) & -PE_{ij}+G_{ij}{}^{\mathrm{T}}T\\ * & -2T\end{pmatrix}$, let $Q=P^{-1}$, $T_1=T^{-1}$, pre- and post-multiplying Ψ_{ijk} by $\mathrm{diag}(Q,T_1)$, and we have

$$\begin{pmatrix}\mathrm{He}(A_{ijk}Q) & -E_{ij}T_1+N_{ij}{}^{\mathrm{T}}\\ * & -2T_1\end{pmatrix}<0, i,j,k=1,2,\cdots,q \tag{9}$$

3.2. Algorithm of one step anti-windup nonlinear output dynamic compensator

Lemma 3[10]. Given a symmetric matrix $\Gamma\in\mathbb{R}^{n\times n}$ and matrices $X\in\mathbb{R}^{i\times n}$, $Y\in\mathbb{R}^{j\times n}$. There exists a matrix Θ of compatible dimension such that $\Gamma+X^{\mathrm{T}}\Theta^{\mathrm{T}}Y+Y^{\mathrm{T}}\Theta X<0$, if and only if $N_X^{\mathrm{T}}\Gamma N_X<0,$

$N_Y^{\mathrm{T}}\Gamma N_Y < 0,$ where N_X and N_Y are any matrices whose columns form bases of the null spaces of X and Y respectively.

Let $\tilde{A}_i = \begin{pmatrix} A_i & 0 \\ 0 & 0 \end{pmatrix}$, $\tilde{B}_i = \begin{pmatrix} 0 & B_i \\ I & 0 \end{pmatrix}$, $\Theta_j = \begin{pmatrix} A_{cj} & B_{cj} \\ C_{cj} & D_{cj} \end{pmatrix}$, $\tilde{C}_k = \begin{pmatrix} 0 & I \\ C_k & 0 \end{pmatrix}$, $\bar{B}_i = \begin{pmatrix} \tilde{B}_i & 0 \\ 0 & 0 \end{pmatrix}^{\mathrm{T}}$, $\bar{\Theta}_j = \begin{pmatrix} \Theta_j & 0 \\ 0 & 0 \end{pmatrix}$,

$G_{Qij} = \begin{pmatrix} \mathrm{He}(\tilde{A}_i Q) & -\begin{pmatrix} B_i T_1 \\ E_{cj} T_1 \end{pmatrix} + N_{ij}{}^{\mathrm{T}} \\ * & -2T_1 \end{pmatrix}$, $\bar{C}_{Qk} = \begin{pmatrix} \tilde{C}_k Q & 0 \\ 0 & 0 \end{pmatrix}$, then Eq. (9) can be expressed as

$$G_{Qij} + \mathrm{He}(\bar{B}_i^{\mathrm{T}} \bar{\Theta}_j \bar{C}_{Qk}) < 0 \tag{10}$$

By Lemma 3, the inequality (10) is equivalent to

$$N_{\bar{B}_i}^{\mathrm{T}} G_{Qij} N_{\bar{B}_i} < 0,\ N_{\bar{C}_{Qk}}^{\mathrm{T}} G_{Qij} N_{\bar{C}_{Qk}} < 0, \tag{11}$$

Let $\bar{C}_k = \begin{pmatrix} \tilde{C}_k & 0 \\ 0 & 0 \end{pmatrix}$, $\bar{Q} = \begin{pmatrix} Q & 0 \\ 0 & I \end{pmatrix}$, then

$$N_{\bar{C}_{Qk}}^{\mathrm{T}} G_{Qij} N_{\bar{C}_{Qk}} = N_{\bar{C}_k}^{\mathrm{T}} \bar{Q}^{-1} G_{Qij} \bar{Q}^{-1} N_{\bar{C}_k} = N_{\bar{C}_k}^{\mathrm{T}} G_{Q^{-1}ij} N_{\bar{C}_k} = N_{\bar{C}_k}^{\mathrm{T}} G_{Pij} N_{\bar{C}_k},$$

where $G_{Pij} = \begin{pmatrix} \mathrm{He}(P\tilde{A}_i) & -P\begin{pmatrix} B_i T_1 \\ E_{cj} T_1 \end{pmatrix} + PN_{ij}{}^{\mathrm{T}} \\ * & -2T_1 \end{pmatrix}$. When $(N_{B_i^{\mathrm{T}}} \neq 0)$, we have $N_{\bar{B}_i} = \begin{pmatrix} \begin{pmatrix} N_{B_i^{\mathrm{T}}} \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ 0 & I_{m\times m} \end{pmatrix}$,

then

$$\begin{cases} Q_{11} > 0, \\ N_{\bar{B}_i}^{\mathrm{T}} G_{Qij} N_{\bar{B}_i} = \begin{pmatrix} N_{B_i^{\mathrm{T}}}^{\mathrm{T}} \mathrm{He}(A_i Q_{11}) N_{B_i^{\mathrm{T}}} & -N_{B_i^{\mathrm{T}}}^{\mathrm{T}} B_i T_1 + N_{B_i^{\mathrm{T}}}^{\mathrm{T}} (N_{ij}{}^{(1)})^{\mathrm{T}} \\ * & -2T_1 \end{pmatrix} < 0. \end{cases} \tag{12}$$

When $(N_{C_k} \neq 0)$, we have $N_{\bar{C}_k} = \begin{pmatrix} \begin{pmatrix} N_{C_k} \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ 0 & I_{m\times m} \end{pmatrix}$, define $M_j = P_{12} E_{cj}$, $R_{ij} = P_{12}(N_{ij}{}^{(2)})^{\mathrm{T}}$, then

$$\begin{cases} P_{11} > 0, \\ N_{\bar{C}_k}^{\mathrm{T}} G_{Pij} N_{\bar{C}_k} = \begin{pmatrix} N_{C_k}^{\mathrm{T}} \mathrm{He}(P_{11} A_i) N_{C_k} & -N_{C_k}^{\mathrm{T}} P_{11} B_i T_1 - N_{C_k}^{\mathrm{T}} M_j T_1 + N_{C_k}^{\mathrm{T}} P_{11} (N_{ij}{}^{(1)})^{\mathrm{T}} + N_{C_k}^{\mathrm{T}} R_{ij} \\ * & -2T_1 \end{pmatrix} < 0. \end{cases} \tag{13}$$

If we choose the full column rank matrix P_{12}, then we can determine the matrices P and Q by the condition of $PQ = I$, and if

$$\begin{pmatrix} P_{11} & I \\ I & Q_{11} \end{pmatrix} > 0 \tag{14}$$

Based on the lemma 2, we have $P_{22} - P_{12}{}^{\mathrm{T}} P_{11}{}^{-1} P_{12} = -P_{12}{}^{\mathrm{T}} (P_{11} - P_{11} Q_{11} P_{11})^{-1} P_{12} > 0$, and then $P > 0, Q > 0$.

Hence, we have the algorithm of one step anti-windup nonlinear output dynamic compensator

Step 1. Solve the Eq. (12) to get the matrices Q_{11}, T_1, $N_{ij}^{(1)}, i, j = 1, 2, \cdots, q$;

Step 2. Solve the Eq. (8), Eq. (13) and Eq.(14) to get the matrices P_{11}, M_j, $R_{ij}, i, j = 1, 2, \cdots, q$;

Step 3. Choose full column rank matrix P_{12} and solve $PQ = I$ to get the matrices P, Q, and we have $E_{cj} = (P_{12}^{\mathrm{T}} P_{12})^{-1} P_{12}^{\mathrm{T}} M_j$, $N_{ij}^{(2)} = R_{ij}^{\mathrm{T}} P_{12} \left(P_{12}^{\mathrm{T}} P_{12}\right)^{-1}, i, j = 1, 2, \cdots, q$;

Step 4. Solve the following equations (Eq. (7) and Eq. (9)) to get the matrices $A_{cj}, B_{cj}, C_{cj}, D_{cj}$. $j = 1, 2, \cdots, q$.

$$\begin{cases} G_{Qij} + \mathrm{He}(\bar{B}_i^{\mathrm{T}} \bar{\Theta}_j \bar{C}_{Qk}) < 0, \\ \begin{pmatrix} P & -\left(D_{ci} C_j \quad C_{ci}\right)^{\mathrm{T}} + P\left(N_{ij}^{(1)} \quad N_{ij}^{(2)}\right)^{\mathrm{T}} \\ * & U_{0m} \end{pmatrix} \geq 0, \end{cases} \tag{15}$$

Where $U_{0m} = \mathrm{diag}(u_{01}^2, u_{02}^2, \cdots, u_{0m}^2)$.

Remark 1: If $n \leq m$ or $n \leq p$, then step 1 to step 3 can be problematic in the following three situations.

Case 1: $\mathrm{rank}(B_i) = \mathrm{rank}(C_k) = n, i, k = 1, 2 \cdots q$, then we have $N_{B_i^{\mathrm{T}}} = N_{C_k} = 0, i, k = 1, 2, \cdots, q$, we will get $N_{ij}^{(1)}, N_{ij}^{(2)}, E_{cj}$ by step 4 instead of step 1 to step 3.

Case 2: $\mathrm{rank}(B_i) = n, i = 1, 2 \cdots q$, and exist k subject to $\mathrm{rank}(C_k) \neq n,$ then we have $N_{B_i^{\mathrm{T}}} = 0, i = 1, 2, \ \cdots, q$, define $S_{ij} = P_{11}(N_{ij}^{(1)})^{\mathrm{T}}$, solve Eq.(13) we can get S_{ij}, then we get $N_{ij}^{(1)}$.

Case 3: $\mathrm{rank}(C_k) = n, k = 1, 2 \cdots q$, and exist i subject to $\mathrm{rank}(B_i) \neq n,$ then we have $N_{C_k} = 0, k = 1, 2, \cdots, q$, we will get $N_{ij}^{(2)}, E_{cj}$ by step 4 instead of step 3.

Remark 2: It should be remarked that although Assumption 1 has not been explicitly used in the reasoning of the algorithm, but it is necessary for the Eq. (11).

3.3. Attraction domain estimation and optimization

The problem of maximizing $\left\{x \middle| x^{\mathrm{T}}(t) P_{11} x(t) \leq 1\right\}$ can be accomplished by solving the optimization problem: $\min\limits_{P_{11}, M_j, R_{ij}, i, j = 1, 2, \cdots q} \lambda, \text{subject to } \lambda I \geq P_{11}, \mathrm{LMIs}(8), (13), (14)$.

4. Numerical example

Consider the following chaos Rossler system with actuator saturation:

$$\dot{x}(t) = f(x(t))x(t) + B\mathrm{sat}(u(t)) + d, y(t) = Cx(t) \tag{16}$$

With $x(t) \in \mathbb{R}^3$, $u(t) \in \mathbb{R}^3$, $y(t) \in \mathbb{R}$, $f(x) = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0.2 & 0 \\ 0 & 0 & x_1 - 5 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$, $B = I_3, d = \begin{pmatrix} 0 \\ 0 \\ 0.2 \end{pmatrix}$,

when the initial value $x(0) = \begin{pmatrix} 0.8 & -1.5 & 0.5 \end{pmatrix}^{\mathrm{T}}$, it is easy to know $\Omega = \left\{x \middle| |Lx| \leq 10.5\right\}$ is a invariant set of the system (16), $L = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$, Consider T-S fuzzy model:

Rule 1: if x_1 is larger (tend to be 10.5), then $\dot{x} = A_1 x + B\mathrm{sat}(u) + d, y = Cx$;

Rule 2: if x_1 is lesser (tend to be -10.5), then $\dot{x} = A_2 x + B\mathrm{sat}(u) + d, y = Cx$.

When $x \in \Omega$, the system (16) can be rewritten as

$$\dot{x}=\sum_{i=1}^{2}\mu_i(t)(A_i x+B\mathrm{sat}(u)+d), y=Cx \tag{17}$$

Choose $\mu_1(t)=0.5+{x_1}/{21}, \mu_2(t)=0.5-{x_1}/{21}$, then $A_1=\begin{pmatrix}0 & -1 & -1\\ 1 & 0.2 & 0\\ 0 & 0 & 5.5\end{pmatrix}, A_2=\begin{pmatrix}0 & -1 & -1\\ 1 & 0.2 & 0\\ 0 & 0 & -15.5\end{pmatrix}$

The three-order dynamic output stabilizing compensator is as follow:

$$\dot{x}_c(t)=\sum_{i=1}^{2}\mu_i(t)(A_{ci}x_c(t)+B_{ci}y(t)-E_{ci}\psi(u+d)),\ u(t)=\sum_{i=1}^{2}\mu_i(t)(C_{ci}x_c(t)+D_{ci}y(t))-d$$

Then we can get the closed loop system

$$\dot{\xi}=\sum_{i=1}^{2}\sum_{j=1}^{2}\mu_i(t)\mu_j(t)(A_{ij}\xi-E_j\psi(\bar{u})), \bar{u}=\sum_{i=1}^{2}\mu_i(t)K_i\xi \tag{18}$$

with $\xi=\begin{pmatrix}x\\ x_c\end{pmatrix}$, $A_{ij}=\begin{bmatrix}A_i+BD_{cj}C & BC_{cj}\\ B_{cj}C & A_{cj}\end{bmatrix}$, $E_j=\begin{bmatrix}B\\ E_{cj}\end{bmatrix}$, $K_i=\begin{bmatrix}D_{ci}C & C_{ci}\end{bmatrix}$, $\bar{u}=u+d$, control bounds $u_{0i}=1$. Combined with the algorithm of one step anti-windup nonlinear output dynamic compensator and correlative data, the feasible result with the help of LMI toolbox of MATLAB is obtained that:

$$A_{c1}=\begin{pmatrix}-3.6595 & 0.1305 & -0.2025\\ 0.0542 & -3.4178 & -0.0957\\ 0.1491 & 0.1041 & -3.6315\end{pmatrix}, A_{c2}=\begin{pmatrix}-2.4914 & 0.5086 & -0.0005\\ 0.4097 & -1.3436 & -0.0160\\ -0.1533 & -0.1019 & -2.2950\end{pmatrix}, B_{c1}=\begin{pmatrix}-0.2404\\ -0.0115\\ 0.8510\end{pmatrix},$$

$$B_{c2}=\begin{pmatrix}0.1971\\ -0.0497\\ -0.6215\end{pmatrix}, C_{c1}=\begin{pmatrix}-0.1061 & -0.9119 & -1.1973\\ -0.7482 & 0.1751 & -0.7724\\ 0.2040 & 0.1350 & -2.3517\end{pmatrix}, C_{c2}=\begin{pmatrix}0.3916 & -0.4783 & -0.5416\\ -0.1834 & 0.1598 & -0.2539\\ -1.1538 & -0.7445 & 0.7837\end{pmatrix},$$

$$D_{c1}=\begin{pmatrix}-4.2104\\ -3.3210\\ 0.7877\end{pmatrix}, D_{c2}=\begin{pmatrix}-1.1209\\ -0.9568\\ -4.7611\end{pmatrix}, E_{c1}=\begin{pmatrix}-1.2393 & 1.8701 & 0\\ 1.2393 & -1.8701 & 0\\ 0 & 0 & 0\end{pmatrix}, E_{c2}=\begin{pmatrix}-1.2393 & 1.8701 & 0\\ 1.2393 & -1.8701 & 0\\ 0 & 0 & 0\end{pmatrix},$$

$$P_{11}=\begin{pmatrix}3.8937 & 0.4592 & 0\\ 0.4592 & 5.6831 & 0\\ 0 & 0 & 3.9288\end{pmatrix}, Q_{11}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}, \lambda_{\max}=5.7941,$$

After optimization, we have $P_{11}=\begin{pmatrix}1.0008 & 0.0137 & 0\\ 0.0137 & 1.2475 & 0\\ 0 & 0 & 1.1374\end{pmatrix}$, $\lambda_{\max}=1.2483$.

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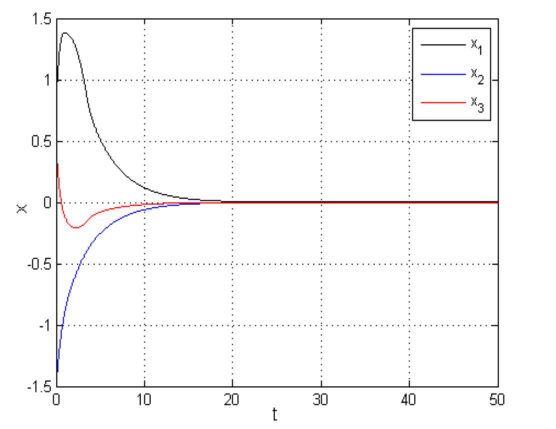

Figure 1. After introducing the compensator, the states diagram of system (18) are given in this figure

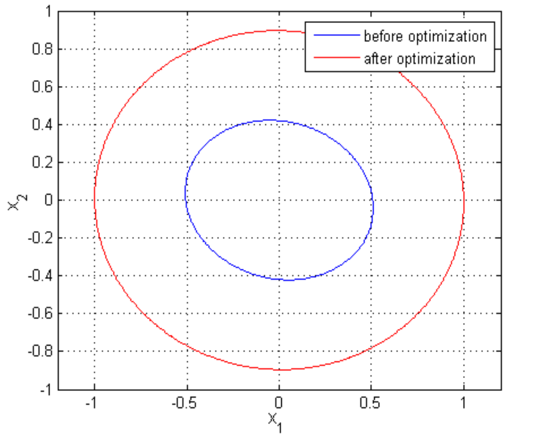

Figure 2. The projection of attraction domain estimation before and after optimization onto plane x_1-O-x_2

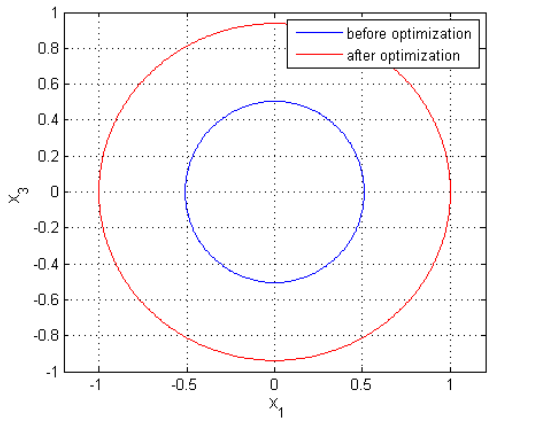

Figure 3. The projection of attraction domain estimation before and after optimization onto plane x_1-O-x_3

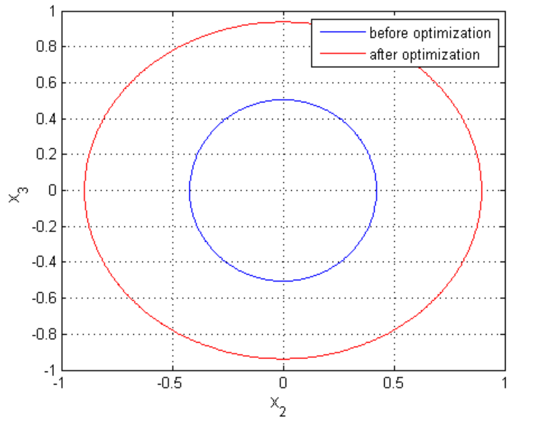

Figure 4. The projection of attraction domain estimation before and after optimization onto plane x_2-O-x_3

As we can see, the algorithm has played an important role in resistance to saturation, and the system state tends to 0 after 20 seconds. and the attraction domain expanded significantly after optimization, and the optimization algorithm has played a good optimization effect.

5. Conclusions

An indirect LMI-based approach to design one step anti-windup dynamic output feedback compensator for a class of nonlinear systems subject to actuator saturation has been proposed. In this approach, we can give an algorithm to solve this actuator saturation control problem. Unlike the traditional two step anti-windup scheme, the proposed algorithm gives one step anti-windup.

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