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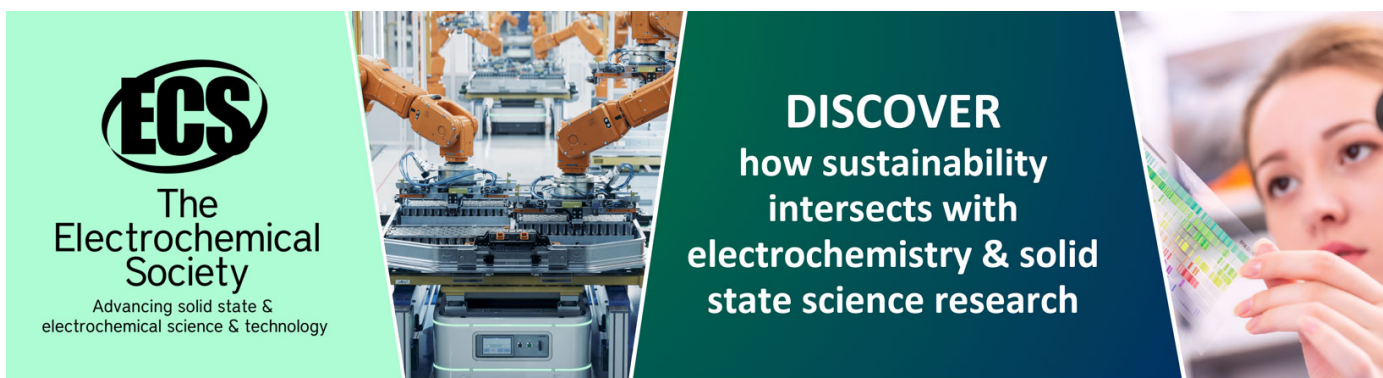
Heat transfer original studies –Theory and applications

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Heat transfer original studies –Theory and applications

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Abstract. Thermal stresses, thermal fatigue, materials' constants variation with the temperature, residual stresses and micro-structure of the parts after the solidification process are heat transfer phenomenon related problems. The calculation instrument of a given age of science and technology has a great influence on the mathematical solutions and the according applications in engineering. The paper presents a synthesis, including the authors' contributions in the heat transfer problems, which may be regarded in a broader context of the distinct branches of engineering that are the fields of expertise of the authors. The first step is to analyse the theoretical aspects to be mastered in order to wisely apply them in concrete engineering problems. In this way, there are presented the basic equations used by the authors in their analytical studies and the background of the numerical methods used in engineering, i.e. the finite difference method and the finite element method. In the following case studies are presented the influences of the heat transfer in an internal combustion engine related phenomena, the connections with the casting manufacturing process and some interesting aspects regarding the connections between the melt temperature with the residual stresses. To conclude, the distinct fields of expertise of the authors are useful to present the various facets of the heat transfer problem which is one of their long run concerns in research.

1. Introduction

Temperature gradients and the according fields of temperatures have a major influence in engineering, some of the most important issues being: thermal stresses, thermal fatigue, influence onto the materials' constants, residual stresses and micro-structure of the parts after the solidification process. Being an important problem, several solutions of the heat transfer phenomenon were conceived over the time, their application in engineering depending on the currently used calculation instrument at that age of technological development.

In this way, the mathematical equations, which govern the heat (and mass) transfer phenomenon were solved using either analytical methods for simple problems, or by the use of the approximation methods which are discretizing the equations.

2. Mathematical aspects of the heat transfer

Several levels of the solutions regarding the heat transfer problems may be noticed. Before the solutions in various engineering applications, there must be considered some basic aspects of the heat transfer phenomenon, i.e. the mathematical equations used in the subsequent studies. These equations may be used for simple analytical models of the heat engines, [1].



2.1. Equations in heat transfer models

The conductive heat transfer equation is also designated as Fourier equation. Its form in a homogeneous and isotropic material subjected to conductive heat transfer is equation (1):

$$\frac{\partial T}{\partial t} = a \cdot \nabla^2 T + \frac{q_v}{c_p \cdot \rho}, \quad (1)$$

where the significance of the according parameters is: T temperature in $[K]$, t time in $[s]$, a thermal diffusivity $\left[\frac{m^2}{s}\right]$, q_v specific power of the inner heat source in $\left[\frac{W}{m^3}\right]$, c_p is specific heat capacity in $\left[\frac{J}{kg \cdot K}\right]$, ρ density in $\left[\frac{kg}{m^3}\right]$. The thermal diffusivity may be expressed as $a = \frac{\lambda}{c_p \cdot \rho}$ and it expresses the thermal inertia of the body, where λ is the thermal conductivity in $\left[\frac{W}{m \cdot K}\right]$.

The convective heat transfer is based on several laws of physics which also include hydrodynamic phenomena.

The continuity equation is equation (2):

$$\frac{\partial \rho}{\partial t} + \nabla(\rho \cdot \bar{v}) = 0, \quad (2)$$

where $\bar{v}$ is the velocity.

The Navier-Stocks equation is equation (3):

$$\rho \cdot \frac{\partial \bar{v}}{\partial t} + \rho \cdot \bar{v} \cdot \nabla \bar{v} = g \cdot \rho \cdot (1 - \beta \cdot \Delta T) - \nabla p + \eta \cdot \nabla^2 \bar{v} + \frac{\eta}{3} \cdot \nabla(\nabla \bar{v}), \quad (3)$$

where g is the gravitational acceleration in $\left[\frac{m}{s^2}\right]$, β is the coefficient of thermal expansion in $\left[\frac{1}{s}\right]$,

p is the pressure in $[Pa]$ and η is the dynamic viscosity in $\left[\frac{m^2}{s}\right]$.

To define the energy equation, firstly we define the material or substantial derivative. In this way let us consider a tensor field, $f(x_i, t)$. Its substantial derivative is equation (4):

$$\frac{D f(x_i, t)}{Dt} = \frac{\partial f}{\partial t} + \sum_{k=1}^3 \left(v_k \cdot \frac{\partial f}{\partial x_k} \right) = \frac{\partial f}{\partial t} + \bar{v} \cdot \nabla f \quad (4)$$

and it is the time rate of change of the $f(x_i, t)$ physical quantity of a fluid parcel in a flow velocity field variation of $\bar{v}$. The general form of the energy equation is equation (5):

$$\rho \cdot \frac{D h}{dt} - \frac{D p}{dt} = \lambda \cdot \nabla^2 T + \ell_d, \quad (5)$$

where h is the enthalpy and ℓ_d is the work spent to deform a volume element.

The diffusivity equation of the 'A' component may be expressed as equation (6):

$$\frac{DC_A}{dt} + c_A \cdot \nabla \bar{v} = DC_A \cdot \nabla^2 c_A, \quad (6)$$

where DC_A is the diffusion coefficient of the 'A' component and c_A is its concentration.

Newton's law may be considered a particular form of the Fourier equation and it has the equation (7) form:

$$\frac{dQ}{dt} = -h \cdot A \cdot [T_f(t) - T_w(t)], \quad (7)$$

where Q is the convective transfer thermal energy flow in $[J]$, h is the heat transfer coefficient in $\left[\frac{W}{m^2 \cdot K}\right]$, A is the area of the convective heat transfer process in $[m^2]$, $T_f(t)$ is the temperature of the fluid in $[K]$ and $T_w(t)$ is the temperature of the wall in $[K]$.

2.2. Finite difference method solution

The solution of the differential equations using the finite difference method is done by replacing the differential operators with finite difference operators. A detailed presentation of the finite difference method applied in conductive heat transfer is given in [2]. In this way, let us consider an unknown function, u . By expanding $u_{i+1,j}$ in a Taylor series next to the (i, j) node it finally results equation (8):

$$\underbrace{\left(\frac{\partial u}{\partial x}\right)_{i,j}}_{\text{differential operator}} = \underbrace{\frac{u_{i+1,j} - u_{i,j}}{\Delta x}}_{\text{finite difference operator}} + \underbrace{0[\Delta x]}_{\text{truncation error}} \quad (8)$$

which defines a right or forward finite difference operator, that is a first order approximation.

Similarly, there are defined equation (9) and equation (10):

$$\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{u_{i,j} - u_{i-1,j}}{\Delta x} + 0[\Delta x] \quad (9)$$

which is a left or backward finite difference operator, that is also a first order approximation, and:

$$\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{u_{i+1,j} - u_{i-1,j}}{2 \cdot \Delta x} + 0[\Delta x^2] \quad (10)$$

which is a central difference operator, in this case it is a second order approximation.

A similar procedure is followed in order to deduce the second order partial derivatives, i.e.

$$u_{i+1,j} + u_{i-1,j} = 2u_{i,j} + \left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} (\Delta x)^2 + \left(\frac{\partial^4 u}{\partial x^4}\right)_{i,j} \frac{(\Delta x)^4}{12} + \dots$$

and it results equation (11):

$$\left(\frac{\partial^2 u}{\partial x^2} \right)_{i,j} = \frac{u_{i+1,j} - 2 \cdot u_{i,j} + u_{i-1,j}}{(\Delta x)^2} + 0[\Delta x^2]. \quad (11)$$

We also deduce equation (12):

$$\left(\frac{\partial^2 u}{\partial x \partial y} \right)_{i,j} = \frac{u_{i+1,j+1} - u_{i+1,j-1} - u_{i-1,j+1} + u_{i-1,j-1}}{4 \cdot \Delta x \cdot \Delta y} + 0[\Delta x^2, \Delta y^2] \quad (12)$$

And equation (13):

$$\left(\frac{\partial^2 u}{\partial y^2} \right)_{i,j} = \frac{u_{i,j+1} - 2 \cdot u_{i,j} + u_{i,j-1}}{(\Delta y)^2} + 0[\Delta y^2]. \quad (13)$$

Let us consider a simple example regarding the usefulness of these differential operators, i.e. the one-dimensional equation of the conductive heat transfer, the a thermal diffusivity being constant, equation (14):

$$\frac{\partial T}{\partial t} = a \cdot \frac{\partial^2 T}{\partial x^2}, \quad (14)$$

for which we use an explicit scheme and an implicit scheme.

First we use an explicit scheme and we discretize the left member with the forward operator in the temporal space, equation (15):

$$\left(\frac{\partial T}{\partial t} \right)_i^n = \frac{T_i^{n+1} - T_i^n}{\Delta t} \quad (15)$$

and the right member with the second order central finite difference, equation (16):

$$\left(\frac{\partial^2 T}{\partial x^2} \right)_i^n = \frac{T_{i+1}^n - 2 \cdot T_i^n + T_{i-1}^n}{(\Delta x)^2}. \quad (16)$$

It results:

$$\frac{\partial^2 T}{\partial t^2} - a \cdot \frac{\partial^2 T}{\partial x^2} = \frac{T_i^{n+1} - T_i^n}{\Delta t} - \frac{T_{i+1}^n - 2 \cdot T_i^n + T_{i-1}^n}{(\Delta x)^2} + \underbrace{\left[- \left(\frac{\partial^2 T}{\partial t^2} \right)_i^n \frac{\Delta t}{2} + a \cdot \left(\frac{\partial^4 T}{\partial t^4} \right)_i^n \frac{(\Delta x)^2}{12} + \dots \right]}_{\text{Truncation error}} = 0$$

where the superscript is the time step and the subscript is the index of the grid's node. Moreover, for $\Delta t \rightarrow 0$ and $\Delta x \rightarrow 0$ the truncation error trends to become zero.

The (14) equation is parabolic, therefore we may apply a time-marching (advance in time) numerical method, which may be noticed in the previous relation if we disregard the truncation error, i.e. equation (17):

$$T_i^{n+1} = T_i^n + a \cdot \frac{\Delta t}{(\Delta x)^2} \cdot (T_{i+1}^n - 2 \cdot T_i^n + T_{i-1}^n). \quad (17)$$

To conclude the example regarding the explicit schemes, the temperatures in node i at time $n+1$ is explicitly computed using the temperatures at time n .

An implicit scheme is defined if in the (16) finite difference operator the T_{i+1}^n , T_i^n , T_{i-1}^n temperatures are replaced by weighted values of the temperatures at the n and $n+1$ time steps, i.e. equation (18):

$$\left(\frac{\partial^2 T}{\partial x^2} \right)_i^n = \frac{\theta \cdot (T_{i+1}^{n+1} - 2 \cdot T_i^{n+1} + T_{i-1}^{n+1}) + (1-\theta) \cdot (T_{i+1}^n - 2 \cdot T_i^n + T_{i-1}^n)}{(\Delta x)^2}. \quad (18)$$

For the Crank-Nicholson scheme $\theta = \frac{1}{2}$ and we get:

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = a \cdot \frac{\frac{1}{2}(T_{i+1}^{n+1} + T_{i+1}^n) + \frac{1}{2}(-2 \cdot T_i^{n+1} - 2 \cdot T_i^n) + \frac{1}{2}(T_{i-1}^{n+1} + T_{i-1}^n)}{(\Delta x)^2},$$

which leads to equation (19):

$$\underbrace{\frac{a \cdot \Delta t}{2 \cdot (\Delta x)^2}}_A \cdot T_{i-1}^{n+1} - \underbrace{\left[1 + \frac{a \cdot \Delta t}{(\Delta x)^2} \right]}_B \cdot T_i^{n+1} + \underbrace{\frac{a \cdot \Delta t}{2 \cdot (\Delta x)^2}}_A \cdot T_{i+1}^{n+1} = \underbrace{-T_i^n - \frac{a \cdot \Delta t}{2 \cdot (\Delta x)^2} (T_{i+1}^n - 2 \cdot T_i^n + T_{i-1}^n)}_{(D_i^n)}. \quad (19)$$

By denoting with A the coefficient of T_{i-1}^{n+1} and T_{i+1}^{n+1} , with B the coefficient of T_i^{n+1} and with (D_i^n) the right member of the previous relation, it results the following system of equations, equation (20):

$$\begin{bmatrix} -B & A & 0 & 0 & \dots & \dots & \dots & \dots & 0 \\ A & -B & A & 0 & \dots & \dots & \dots & \dots & 0 \\ 0 & A & -B & A & \dots & \dots & \dots & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & & & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & & & & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & \dots & \dots & \dots & A & -B & A \\ 0 & 0 & \dots & \dots & \dots & \dots & 0 & A & -B \end{bmatrix} \begin{bmatrix} T_2^{n+1} \\ T_3^{n+1} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ T_{i_{\max}-1}^{n+1} \end{bmatrix} = \begin{bmatrix} (D_2^n) \\ (D_3^n) \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ (D_{i_{\max}-1}^n) \end{bmatrix}. \quad (20)$$

The previous system of equations has a tri-diagonal matrix, therefore the algorithm of Thomas is preferred, which is an exact method. Firstly, the system is transformed to become bi-diagonal using

the Gaussian elimination and the last stage is to use substitutions to finally compute the unknowns, i.e. the temperatures at the new time step.

2.3. Finite element method solution

Let us consider the temperature as a general function which is variable in space (coordinates) and time, for generality reasons being denoted as φ . A general field problem in a transient state may be expressed by equations defined on the Ω domain which have the form of equation (21):

$$\frac{\partial}{\partial x} \left(k_x \frac{\partial \varphi}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial \varphi}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial \varphi}{\partial z} \right) = f(x, y, z) + k_t \cdot \dot{\varphi} + k_{tt} \cdot \ddot{\varphi}. \quad (21)$$

The k_t and k_{tt} parameters may be functions of space and time. If $k_{tt} = 0$ and $k_t \neq 0$ the previous equation is parabolic and if $k_{tt} \neq 0$, the equation is hyperbolic.

The boundary conditions may be:

- Dirichlet conditions applied on the S_1 boundary, equation (22):

$$\varphi = \Phi(x, y, z, t), \text{ for } t > 0 \quad (22)$$

- Neumann conditions applied on the S_2 boundary, equation (23):

$$k_x \frac{\partial \varphi}{\partial x} \cdot n_x + k_y \frac{\partial \varphi}{\partial y} \cdot n_y + k_z \frac{\partial \varphi}{\partial z} \cdot n_z + q(x, y, z, t) + h(x, y, z, t) \cdot \varphi = 0, \text{ for } t > 0 \quad (23)$$

where the entire boundary of the Ω domain is $\Gamma = S_1 + S_2$.

The initial conditions are equation (24):

$$\varphi = \varphi_0(x, y, z), \quad \dot{\varphi} = \zeta(x, y, z), \text{ for } t < 0. \quad (24)$$

The elemental equations may be deduced using one of the following methods:

- by applying the Galerkin method directly to the differential equation that governs the phenomenon;
- by applying a variational principle, if one may be found for the physical problem under investigation; this principal may minimize either the energy, or a functional related to the energy.

In this case there is no classical variational principle, therefore the Galerkin method must be applied. In this way we consider the N_i interpolation functions, in an element being the field variable, equation (25):

$$\varphi^{(e)} = \sum_{i=1}^{NODEL} N_i(x, y, z) \varphi_i(t) = [N] \{\varphi\}^{(e)}. \quad (25)$$

According to the Galerkin method, for each $i = 1, 2, \dots, NODEL$ there must be fulfilled the condition given by equation (26):

$$\int_{\Omega^{(e)}} N_i \left[\frac{\partial}{\partial x} \left(k_x \frac{\partial \varphi^{(e)}}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial \varphi^{(e)}}{\partial y} \right) + \frac{\partial}{\partial z} \left(k_z \frac{\partial \varphi^{(e)}}{\partial z} \right) - f - k_t \dot{\varphi}^{(e)} - k_u \ddot{\varphi}^{(e)} \right] d\Omega^{(e)} = 0. \quad (26)$$

In this way is deduced the elemental equation (27):

$$[K]^{(e)} \cdot \{\varphi\}^{(e)} + [K_t]^{(e)} \cdot \left\{ \dot{\varphi} \right\}^{(e)} + [K_u]^{(e)} \cdot \left\{ \ddot{\varphi} \right\}^{(e)} + [K_{S_2}]^{(e)} \cdot \{\varphi\}^{(e)} + \{R_1(t)\}^{(e)} = \{0\}, \quad (27)$$

which is used to create a system of equations that is solved in order to compute all the $\{\varphi\}$ nodal unknowns resulted from the domain discretization. The results, i.e. the temperatures in the nodes of the grid, may be further on used to compute the thermal stresses.

3. Discussion and applications

Solutions of the heat transfer problems require computer based instruments. The most common investigation instruments nowadays are using the finite element method, [3], the according solvers being included in commercial software applications.

Fields of temperatures which result from the aforementioned heat transfer studies, are influencing many phenomena, such as: thermal stresses, thermal processes in internal combustion engines, casting process, residual stresses. In this section are presented some of the phenomena related to the heat transfer problems.

3.1. Case study no. 1 – heat transfer in mechanical engineering

Internal combustion engines are a complex type of heat engines. The study of the stresses in the cylinder block, as well as the stresses in the mobile equipment must consider the field of temperatures in running conditions. In this case, beside the general strength problem regarding the thermal stresses, there should be also considered the variation of the material constants with respect to the temperature, figure 1, [4], and the local strength problems, such as the restrained dilation produced by the steel bolts in the aluminium alloy block of cylinders.

Thermal stresses in the cylinder block are computed with respect to the field of temperatures, therefore the first stage is to solve a conductive heat problem. The results of a finite element study which outputs the nodal temperatures is presented in figure 2, [5]. The problem was solved using the COSMOS/M commercial application. The results were verified using the measurements with an infrared non-contact thermometer. The temperatures were used to correct the strains which were measured in running conditions using three-gases-rosettes.

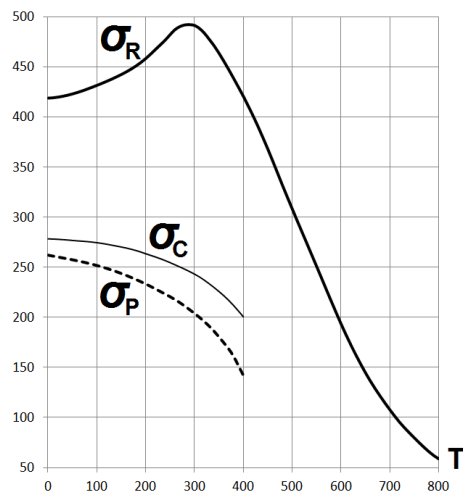


Figure 1. Example regarding the variation of the material constants with respect to the temperature.

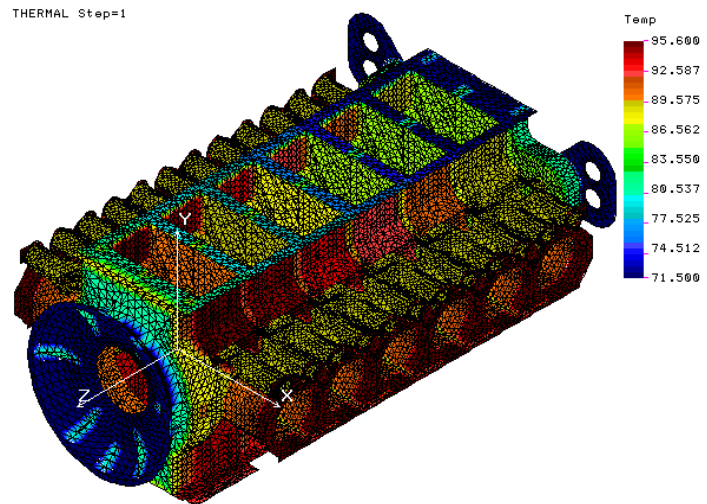


Figure 2. Field of temperatures resulted from the conductive heat transfers study based on the finite element method.

A more profound approach is to solve heat transfer problems in internal combustion engines starting from the combustion process with the according influences, predictions of heat release rate aspects, which may be also related with the pollution actual concerns, [6 - 9].

3.2. Case study no. 2 – heat transfer phenomena during the casting process

In terms to calculate the casting process and to optimize the final casting many parameters as: initial temperature of the mould, shape and size of the mould, mould wall thickness, material of the mould, pouring temperature, time of pouring, composition of the metal and solidification, the time must be considered, [10, 11].

To establish the casting process, the key step is the determination of the heat transfer coefficients, which are usually available in theory, as correlations involving the Reynolds number of the flow and the relative geometric orientation, of the radiating surfaces of the mould.

Considering the specific equations and parameters involved, is possible to carry out, relatively accurate calculations, of the solidification rate, mould characteristics and the heat transfer coefficient.

The heat transfer coefficient at the bottom surface of a splat rapidly solidified on a cold substrate is self-consistently and quantitatively investigated by P.S. Wei and al. [12].

Using and solving one-dimensional unsteady heat conduction equations and accounting for distinct properties between phases and splat and substrate P.S. Wei and al., show that the figures are time dependent heat transfer coefficient can be divided into five regimes: liquid splat-solid substrate, liquid splat-liquid substrate, nucleation of splat, solid splat-solid substrate, and solid splat-liquid substrate. The results obtained by Wei and al. show that the heat transfer during rapid solidification increases with increasing the contact heat transfer coefficient, Stefan number, density ratio, initial temperature, kinetics coefficients, melting temperatures, and solid conductivities of the splat and substrate, and decreasing nucleation temperature and specific heat ratio. The results obtained by Kh. Abdel-Aziz and al., [13], studying A356 alloy reinforced with micro or nano-alumina particles, show the influence of the mould wall thickness, which affect the rate of solidification, as if the mould wall thickness increases, then the cooling rate also increases and during the solidification process the grains become finer.

K.C. Bala et al. underline, during the heat transfer analysis, at the casting on grey cast iron mould of aluminium alloy (LM14), if the thickness of the mould increases, the solidification takes place at a faster solidification rate. This is due to the cooling effect of the mould, which is also dependent on the heat transfer coefficient and the heat content of the molten metal, [14].

The grain size and mechanical properties of cast metal are studied by M. M. Pariona and al., [15], by analysing the geometrical characteristics of the mould and the thermo-physical properties of the metal and the mould. Numerical simulations made on pure iron solidification in industrial AI 50/60 AFS using the finite element technique and the ANSYS software program, show that in the cooling in the sand system was slower than in the mullite system.

In order to have more accurate temperature measurements using thermocouples during the experimental tests, as D. Aichbhaumik and al. present in their research on A356 alloy, [16], for permanent moulds, the length of the thermocouple, must be situated perpendicular to the isotherms and the thermocouples in sand moulds should be positioned parallel to the isotherms.

According to [17], Burton et al. proposed an equation for effective distribution coefficient, K_{ef} . This can be considered as a function of the solidification speed, V_L , equation (28):

$$K_{ef} = \frac{K_{eq}}{K_{eq} + (1 - K_{eq}) \cdot e^{\left(\frac{V_L \cdot \delta}{\Delta L}\right)}} \quad (28)$$

where K_{eq} is the equilibrium partition coefficient, δ diffusion layer thickness of the segregated solute ahead the solid/liquid interface and ΔL is the liquid solute diffusivity.

Studying the variation of dendrite spacing's with alloys composition, solidification rate and temperature gradient in the liquid, for Al-4,5wt pct Cu and Al-15 wt pct Cu alloys, J. Quaresma and all. find analytical expressions have been developed describing thermal gradients (G), dendrites growth model rate (V) and show that the relationship between G and V is actually linear. To obtain required mechanical properties from the casting, this expression provides an insight into the programming of solidification process, [18].

Studying the polypropylene casting A. Sporrer and al., [19], show that the elevated mould cavity surface temperatures can limit degradation of the surface finish. Moreover, the final morphology of structural PP foams studied can be varied over a huge bandwidth, affecting the mechanical properties like modulus, strength, and toughness of the final piece. The thicker the skin layers, the higher the examined mechanical values.

Solidification speed depends on the casting parameters. In terms to obtain the required structure and technical properties, at the end of the casting process, and to optimize the costs of the final product, can calculate and establish the details for the entire casting process.

3.3. Case study no. 3 – influence of the melt temperature onto the residual stresses

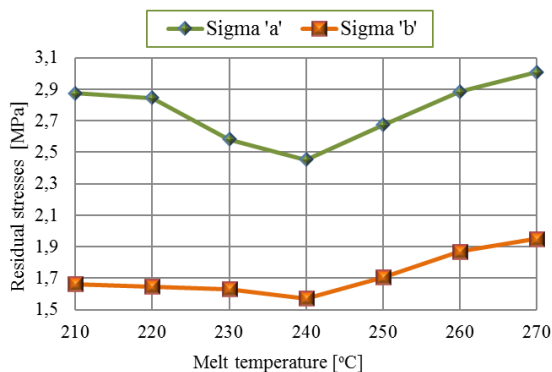
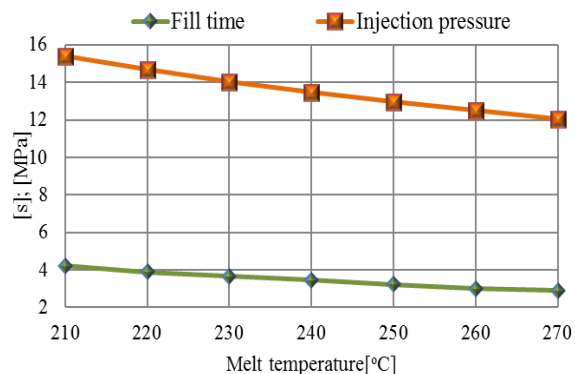
Many of the actual parts are made of moulded polymeric materials, because they have great advantages in comparison with the classic materials. The quality of the melt that depends on the temperature is hard to control, and it has a direct influence on the quality of the final parts. Numerical simulations have an important role in the injection moulding process, being useful methods to approach most of the moulding problems.

The authors developed several simulation studies in order to find the best parameters of the moulding injection process using a plate-wise specimen, which lead to a minimum level of the residual stresses and to obtain information regarding: the polymer melt filling patterns; weld line and air trap locations; required injection pressure and clamp tonnage; fibre orientation; cycle time; the final parts' shape and the deformation and the mechanical properties of moulded parts. Commercial software, such as Autodesk MoldFlow, simulate the moulding process in the free surface approximation. An analyst may finally obtain a quantitative description in which polymers' properties can be adjusted. MoldFlow Plastics simulates the filling and packing phases of the injection moulding.

Table 1. Values of residual stresses resulted from the numerical analysis.

Melt temperature [°C]	Residual stress near the injection point “a” [MPa]	Residual stress near the plate edge “b” [MPa]	Fill time [s]	Maximum injection pressure [MPa]
210	2.873	1.664	4.228	15.39
220	2.843	1.647	3.892	14.69
230	2.581	1.631	3.670	14.02
240	2.453	1.571	3.448	13.45
250	2.691	1.706	3.222	12.95
260	2.883	1.870	2.998	12.49
270	3.008	1.951	2.884	12.03

Studies were performed on different polymeric flat parts, for different values of melt temperatures for polystyrene (PP) between 210°C and 270°C and different sizes of the mesh (between 8 mm and 2 mm) to determinate the optimum injection process. The flat plate polymer test specimens have 200x200x5mm dimensions. The authors computed the residual stresses in two critical points, for 7 melting temperature values. The first one is near the point of injection and the second one is near the plate edge, in the part depth of 3.3 mm, as you can see in table 1 and diagrams presented in figure 3 and figure 4. It can be seen in the table 1, that for the melting temperature of 240°C we have minimal residual stresses, [20, 21].

**Figure 3.** Variation of the residual stresses vs. the melt temperature.**Figure 4.** Variation of the fill time and of the injection pressure vs. the melt temperature.

The flat plate was injected from PP and the parameters of the injection process was: mould surface temperature 38°C, melt temperature, 240°C, fill time 3.448 s, cooling time 20 s, maximum injection pressure inside the mould 13.45 MPa, as it can be noticed in figure 5.

FEM technology help us to save time, money and raw material, improve product quality and management and reduce the time-to-market parameter. Numerical simulations were done in order to find the proper conditions for the moulding process.

Using numerical simulation, we can obtain: the maximum value of the injection pressure which is necessary to fill all the area of the part, as it can be noticed in figure 6; and the clamp force, that can be compared with the clamp force limit for the injection moulding machine to be used, figure 7.

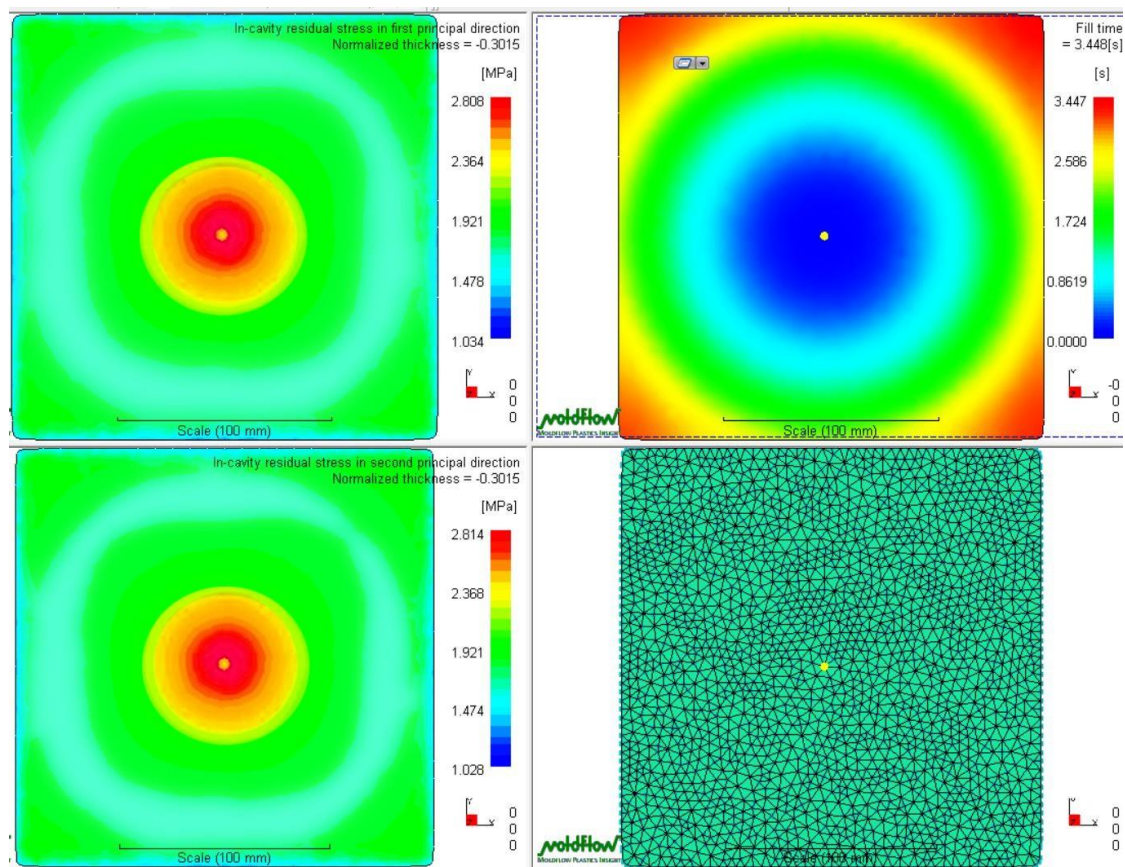


Figure 5. Simulation of the residual stress distribution for mesh 5 mm.

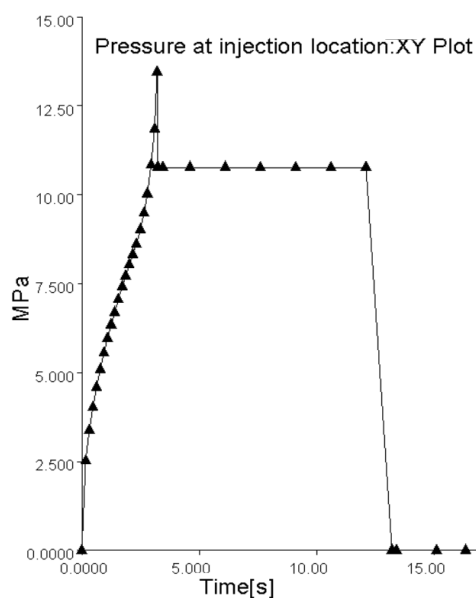


Figure 6. Injection pressure.

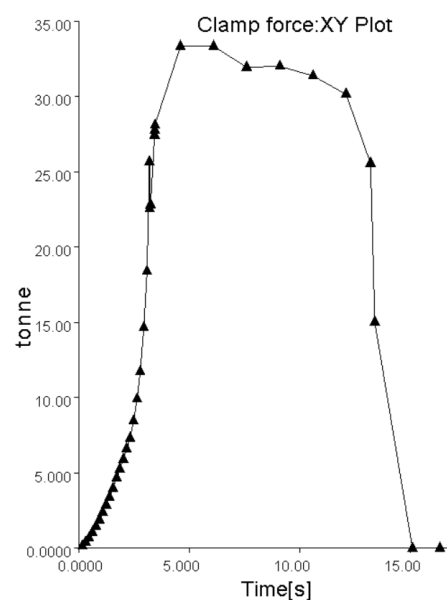


Figure 7. Clamp force.

The filling time of the injection mould is very important for the solidification of the melt subjected to transient heat transfer. If the injection process is too quickly, the polymer chains may break and thus

we can have a material degradation. For the numerical simulation using the temperature value of the 240°C, polymeric material flow is laminar, and with increasing the value of the injection pressure, viscosity is also increased to prevent turbulent flow.

Autodesk Moldflow analysis of the injection process allows viewing the evolution of the process without to manufacture the injection mould. The numerical simulation offers the possibility to obtain the injection moulding machine parameters and can solve the problems that may appear even from the design process, in this way having an increased quality of polymer parts [22].

High residual internal stresses in the injected parts may occur due to high pressures, temperature gradients of the molten material, and temperature gradients in the mould walls. As a result, they can cause defects in the injected parts causing contractions, bending and other defects which may affect the performance of the final products during operation. Also, the residual stresses may cause viscoelastic deformation of the product when the product is used for an extended period of time or has been exposed to relatively high temperature, [23, 24].

4. Conclusions

Regarding the finite difference method, one can notice that in comparison to the explicit schemes, the implicit schemes are creating a system of equations. Once solved, the T_i^{n+1} temperatures are known in any node of the grid. We notice that the implicit schemes require a higher volume of calculi than the explicit schemes.

Deducing the differential operators used for a heat transfer program in a general, abstract system of axes allows the analyst to work with an abstract domain, using a general approach. The method is in particular useful for a 3D general domain. The solver of the system of equations uses a triple sweep method, [25].

Solving engineering problems using the finite difference method requires to be known the set of mathematical relations and laws which are governing the phenomena. The stability and convergence of the method must be studied in order to assess the size of the grid, the data processing details (i.e. the sweep directions) and the error propagation. The implementation of the solutions based on the finite difference method leads to solvers which may be used to effectively solve a particular problem. These solvers may be used in a wider context, i.e. a hybrid model, [5]. The finite element method is more flexible, all the commercial software having modules which solve heat transfer problems. However, there may be found online solvers for simple problems, [26]. There may be cases when a particular heat transfer problem requires an original software instrument, in this case being necessary to firstly develop a library which must solve data handling problems, error minimising parallel data processing techniques and memory allocation solutions.

5. References

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