IOP Conference Series: Materials Science and Engineering

PAPER • **OPEN ACCESS**

Kernel-Spline Estimation of Additive Nonparametric Regression Model

To cite this article: Rahmat Hidayat *et al* 2019 *IOP Conf. Ser.: Mater. Sci. Eng.* **546** 052028

IOP Conf. Series: Materials Science and Engineering **546** (2019) 052028 doi:10.1088/1757-899X/546/5/052028

Kernel-Spline Estimation of Additive Nonparametric Regression Model

Rahmat Hidayat[1,2], I Nyoman Budiantara[2*], Bambang Widjanarko Otok[2], Vita Ratnasari[2]

[1]Department of Mathematics, Cokroaminoto Palopo University, Palopo, Indonesia.
[2]Department of Statistics, Faculty of Mathematics, Computation and Data Science, Institut Teknologi Sepuluh Nopember, Surabaya, East Java, Indonesia.

*Corresponding author: nyomanbudiantara65@gmail.com

Abstract. In this paper, we model the open unemployment rate with the Kernel-Spline model. We investigate and compare the performance of model Kernel-Spline by varying the Kernel function. The performance model has been compared with five Kernel function i.e. Kernel functions Uniform, Epanechnikov, Quartic, Gaussian, and Triweight. For these models, we conducted a comparison based on actual data sets, the unemployment rate in East Java. The best model was chosen based on the Generalized Cross Validation value and the coefficient of determination criteria. The empirical results obtained have shown that Spline-Kernel model by using the Gaussian Kernel better than other models.

1. Introduction

Labor is a capital for the movement of the wheel of development. The condition of the number and composition of the workforce will continue to change along with the ongoing demographic process. Unemployment is a labor problem faced by developing countries like Indonesia. The unemployment rate in Indonesia in 2016 was 6.56 percent, from 34 provinces in Indonesia, East Java is one of the densely populated provinces and is experiencing growth every year. The amount of unemployment rate has broad social implications because those who do not work have no income. The higher the level of the unemployment rate, the greater the potential for social insecurity caused, for example crime. Conversely, the lower the unemployment rate, the more stable social conditions in the community. It is very appropriate if the government often makes this indicator a measure of the success of the development. Based on Indonesia Statistic data, the unemployment rate in East Java in 2016 reached 4.21 percent. The high or low unemployment rate is influenced by many factors, including the level of education, economic growth, and population density.

Regression analysis is one of the analyzes in statistics used to investigate the pattern of functional relationships between two or more variables. In estimating the regression curve there are three approaches, namely parametric regression, nonparametric regression and semiparametric regression [1]. Early identification of data patterns can be done by utilizing past experience or using a scatter plot. Often in practice, the form of a pattern of functional relationships between response variables and predictor variables is unknown. In this condition, the parametric regression model is less suitable to be used, so the nonparametric regression model is used [2,3]. Nonparametric regression has high flexibility, where data is expected to search for its own regression curve estimation form without being influenced by the subjectivity of the researcher [2]. Some models in nonparametric regression that are widely used by researchers are Spline, Kernel, and Fourier Series.

Spline has the advantage that this model tends to look for data estimates wherever the data pattern moves. This advantage occurs because in the Spline there are continuous polynomial cut components that can provide better flexibility and allow them to adjust effectively to local characteristics [4,5,6]. Unlike the case with Spline, the Kernel estimator has a good ability to model the data with random patterns [7]. In addition, the Kernel estimator has a flexible nature, the mathematical form is easy, and can reach a relatively fast convergence level. In terms of computation, the kernel method is easier to do and easy to implement [8].

In nonparametric regression involving many predictors, predictors are often found with Spline characteristics and there are predictors with Kernel characteristics. The estimation of the right curve model is an estimate that matches the data pattern, so that this study refers to the use of Kernel and Spline mixed estimator models.

2. Additive Regression Model

The relationship between response variables and predictor variables for multi-predictor nonparametric regression models:

$$\mathbf{y} = f\left(\mathbf{x}, \mathbf{z}\right) + \boldsymbol{\varepsilon} \tag{1}$$

where f is the regression curve. Assume that the form of the regression curve f is unknown and additive.

$$f\left(\mathbf{x}, \mathbf{z}\right) = \sum_{k=1}^{q} m_k(\mathbf{x}_k) + \sum_{j=1}^{p} t_j(\mathbf{z}_j) \tag{2}$$

The function m is approached using the Kernel function and t is approached using the Spline function. Form of the regression curve $t_j, j = 1, 2, \ldots, p$ unknown and assumed to be contained in the Sobolev space $W_2^m\left[a_j, b_j\right]$,

$$W_2^m\left[a_j, b_j\right] = \left\{ t_j : \int_{a_j}^{b_j} \left[t_j^{(m)}\left(z_j\right)\right]^2 dz_j < \infty \right\}.$$

3. Kernel and Spline Model

The estimation of the regression curve in equation (2) is solved by completing the m curve and the t curve partially then solving together is solved by penalized least square optimization.

3.1. Kernel Function

A function $K : R \rightarrow R$ called the Kernel function if the function is continuous, symmetrical, limited and

$$\int_{-\infty}^{\infty} K(x)dx = 1 \tag{3}$$

From this definition, if K is a non-negative function then K is also interpreted as a function of solid chance (density function) [9]. In general, K Kernels with bandwidth α is defined by:

$$K_\alpha\left(x_i\right) = \frac{1}{\alpha} K\left(\frac{x}{\alpha}\right); \ -\infty < x < \infty, \ \alpha > 0 \tag{4}$$

Some types of kernel functions are:

(1) *Uniform kernel*: $K\left(x\right) = \frac{1}{2} I\left(|x| \le 1\right)$

(2) *Epanechnikov kernel*: $K\left(x\right) = \frac{3}{4}\left(1 - x^2\right) I\left(|x| \le 1\right)$

IOP Conf. Series: Materials Science and Engineering **546** (2019) 052028 doi:10.1088/1757-899X/546/5/052028

(3) *Quartic kernel*: $K(x)=\frac{15}{16}\left(1-x^2\right)^2 I\left(|x|\leq 1\right)$

(4) *Triweight kernel*: $K(x)=\frac{35}{32}\left(1-x^2\right)^3 I\left(|x|\leq 1\right)$

(5) *Gaussian kernel*: $K(x)=\frac{1}{\sqrt{2\pi}}\exp\left(\frac{-x^2}{2}\right),\ -\infty<x<\infty$,

with I is indicator function [10,11].

Lemma 1:
If the Kernel component in equation (2) is given by:

$$m(\mathbf{x})=\sum_{k=1}^{q} m_k(\mathbf{x}_k)$$

and is approached with Nadayara-Watson Kernel then:

$$m(\mathbf{x})=\mathbf{A}_{\boldsymbol{\alpha}}\mathbf{y}$$

with

$$\mathbf{A}(\boldsymbol{\alpha})=\begin{pmatrix} n^{-1}\sum_{k=1}^{q}W_{\alpha_k 1}(x_{k1}) & n^{-1}\sum_{k=1}^{q}W_{\alpha_k 2}(x_{k1}) & \cdots & n^{-1}\sum_{k=1}^{q}W_{\alpha_k n}(x_{k1}) \\ n^{-1}\sum_{k=1}^{q}W_{\alpha_k 1}(x_{k2}) & n^{-1}\sum_{k=1}^{q}W_{\alpha_k 2}(x_{k2}) & \cdots & n^{-1}\sum_{k=1}^{q}W_{\alpha_k n}(x_{k2}) \\ \vdots & \vdots & \ddots & \vdots \\ n^{-1}\sum_{k=1}^{q}W_{\alpha_k 1}(x_{kn}) & n^{-1}\sum_{k=1}^{q}W_{\alpha_k 2}(x_{kn}) & \cdots & n^{-1}\sum_{k=1}^{q}W_{\alpha_k n}(x_{kn}) \end{pmatrix},\ \mathbf{y}=\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} \text{ and}$$

$$W_{\alpha_k i}(x_{ki})=\frac{K_{\alpha_k}(x_k-x_{ki})}{n^{-1}\sum_{i=1}^{n}K_{\alpha_k}(x_k-x_{ki})},\ i=1,2,...,n$$

Proof:
If the Kernel component $m(\mathbf{x})$ approached with the Nadayara-Watson Kernel then:

$$m_\alpha=n^{-1}\sum_{i=1}^{n}W_{\alpha_k i}(x_k)y_i \tag{5}$$

The parameter α_k is the bandwidth parameter for the *k*-predictor variable, and the function $W_{\alpha_k i}(x_k)$ is the weighting function given by:

$$W_{\alpha_k i}(x_k)=\frac{K_{\alpha_k}(x_k-x_{ki})}{n^{-1}\sum_{i=1}^{n}K_{\alpha_k}(x_k-x_{ki})}, \tag{6}$$

the quantity $K_{\alpha_k}(x_k-x_{ki})$ given by:

$K_{\alpha_k}(x_k-x_{ki})=\frac{1}{\alpha_k}K\left(\frac{x_k-x_{ki}}{\alpha_k}\right)$ and K is Kernel function.

If equation (6) processed for $i=1,2,3,...,n$ then obtained:

$$\begin{pmatrix} m_{\alpha_k}(x_{k1}) \\ m_{\alpha_k}(x_{k2}) \\ \vdots \\ m_{\alpha_k}(x_{kn}) \end{pmatrix} = \begin{pmatrix} n^{-1}W_{\alpha_k 1}(x_{k1}) & n^{-1}W_{\alpha_k 2}(x_{k1}) & \cdots & n^{-1}W_{\alpha_k n}(x_{k1}) \\ n^{-1}W_{\alpha_k 1}(x_{k2}) & n^{-1}W_{\alpha_k 2}(x_{k2}) & \cdots & n^{-1}W_{\alpha_k n}(x_{k2}) \\ \vdots & \vdots & \ddots & \vdots \\ n^{-1}W_{\alpha_k 1}(x_{kn}) & n^{-1}W_{\alpha_k 2}(x_{kn}) & \cdots & n^{-1}W_{\alpha_k n}(x_{kn}) \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

with matrix notation can be written to be

$$m_{\alpha_k}(\mathbf{x}_k) = \mathbf{A}_k(\alpha_k)\mathbf{y} \tag{7}$$

Based on equation (7), the Kernel component $m(\mathbf{x})$ will become

$$\begin{aligned} \mathbf{m}(\mathbf{x}) &= \sum_{k=1}^{q}\left(\mathbf{A}_k(\alpha_k)\mathbf{y}\right) \\ &= \mathbf{A}_1(\alpha_1)\mathbf{y} + \mathbf{A}_2(\alpha_2)\mathbf{y} + ... + \mathbf{A}_q(\alpha_q)\mathbf{y} \\ &= \left(\mathbf{A}_1(\alpha_1) + \mathbf{A}_2(\alpha_2) + ... + \mathbf{A}_q(\alpha_q)\right)\mathbf{y} \\ &= \mathbf{A}_{\boldsymbol{\alpha}}\mathbf{y} \end{aligned} \tag{8}$$

3.2. Spline Function

Polynomial pieces play an important role in approximation and statistic theory. Polynomial pieces have a flexible and effective nature to deal with the local nature of a function or data [2]. One important type of polynomial cut is the polynomial spline. The regression curve $\mathbf{t}_j$ in equation (2) is stated as:

$$t_j(\mathbf{z}_j) = \sum_{v=1}^{m}\alpha_{jv}\phi_{jv}(z_{ji}) + \sum_{j=1,l=1}^{n}\beta_i\theta_j\psi_j(z_{ji}, z_{jl}) \tag{9}$$

Nonparametric regression curves $t(\mathbf{z}) = \sum_{j=1}^{p} t_j(z_{ji})$ are estimated using the PLS method by minimizing:

$$n^{-1}\left(y_i - \sum_{j=1}^{p} t_j(z_{ji})\right)^2 + \lambda\sum_{j=1}^{p}\theta_j^{-1}\int_{a_j}^{b_j}\left(t_j^{(m)}(z_j)\right)^2 dz_j, \tag{10}$$

with $\lambda_j = \lambda/\theta_j$ parameter control the balance between the goodness of fit and the size of the smoothness of the function. To get the Spline estimator in the multi-predictor nonparametric regression model, the extension method was used [3]:

$$\mathbf{t}(\mathbf{z}) = \sum_{j=1}^{p} t_{j_{\lambda,\theta_j}}(z_{ji}) = \sum_{j=1}^{p}\sum_{v=1}^{m}\omega_{jv}\phi_{jv}(z_{ji}) + \sum_{i=1,l=1}^{n}\varsigma_i\sum_{j=1}^{p}\theta_j\psi_j(z_{ji}, z_{jl}) \tag{11}$$

or can be written in vector and matrix form:

$$\mathbf{t}(\mathbf{z}) = \sum_{j=1}^{p}\mathbf{t}_{j_{\lambda,\theta j}} = \mathbf{R}\boldsymbol{\omega} + \mathbf{W}\boldsymbol{\varsigma} \tag{12}$$

with:

$\mathbf{R} = (\mathbf{R}_1, ..., \mathbf{R}_p)$, $\mathbf{R}_j = \left\{\phi_{jv}(z_{ji})\right\}_{i=1,v=1}^{n,m}$,

$$\boldsymbol{\omega} = \left(\omega_{11},\ldots,\omega_{1m_1},\ldots,\omega_{p1},\ldots,\omega_{pm_p}\right)'$$

$$\mathbf{W} = \theta_1\mathbf{W}_1 + \theta_2\mathbf{W}_2 + \cdots + \theta_p\mathbf{W}_p, \mathbf{W}_j = \left\{\psi_j\left(z_i, z_l\right)\right\}_{i=1,l=1}^{n,n}$$

$$\boldsymbol{\varsigma} = \left(\varsigma_1,\varsigma_2,\ldots,\varsigma_n\right)'$$

After obtaining each form of the estimator for Kernel and Spline components, the Kernel-Spline estimator is then presented.

4. Kernel-Spline Model

The Kernel-Spline model is a combination of the Spline and Kernel regression models. This model is used if the pattern of relationships between a set of predictor variables on the response variable, there are those whose patterns change at certain sub-intervals and there are patterns that are difficult to describe. Suppose the form of a regression curve as in equation (2), $\boldsymbol{\varepsilon}$ is a random error that is mutually independent with a zero mean and variance σ^2. The m function is a Kernel component and the function t is a Spline component. The Kernel-Spline estimator in the non-parametric regression model is obtained by minimizing penalized least square:

$$n^{-1}\sum_{i=1}^{n}\left(y_i - \sum_{k=1}^{q} m_k(x_{ki}) - \sum_{j=1}^{p} t_j(z_{ji})\right)^2 + \lambda_1\int_{a_1}^{b_1}\left(t_1^{(m)}\left(z_1\right)\right)^2 dz_1 + \cdots + \lambda_p\int_{a_p}^{b_p}\left(t_p^{(m)}\left(z_p\right)\right)^2 dz_p$$

$$n^{-1}\sum_{i=1}^{n}\left(y_i - \sum_{k=1}^{q} m_k(x_{ki}) - \sum_{j=1}^{p} t_j(z_{ji})\right)^2 + \sum_{j=1}^{p}\lambda_j\int_{a_j}^{b_j}\left(t_j^{(m)}\left(z_j\right)\right)^2 dz_j \tag{13}$$

Completion optimization (13) gives results:

$$n^{-1}\left(\mathbf{y} - \mathbf{R}\boldsymbol{\omega} - \mathbf{W}\boldsymbol{\varsigma} - \mathbf{A}_{\boldsymbol{\alpha}}\mathbf{y}\right)'\left(\mathbf{y} - \mathbf{R}\boldsymbol{\omega} - \mathbf{W}\boldsymbol{\varsigma} - \mathbf{A}_{\boldsymbol{\alpha}}\mathbf{y}\right) + \lambda\boldsymbol{\varsigma}'\mathbf{W}\boldsymbol{\varsigma} \tag{14}$$

The optimization solution (13) can be obtained by using the partial derivative (14) respectively towards $\boldsymbol{\omega}$ and $\boldsymbol{\varsigma}$, then equal the result to zero obtained:

$$-\left(\mathbf{I} - \mathbf{A}_{\boldsymbol{\alpha}}\right)\mathbf{y} + \mathbf{R}\hat{\boldsymbol{\omega}} + \left(\mathbf{W} + n\lambda\mathbf{I}\right)\hat{\boldsymbol{\varsigma}} = \mathbf{0} \tag{15}$$

$$-\mathbf{R}'\left(\mathbf{I} - \mathbf{A}_{\boldsymbol{\alpha}}\right)\mathbf{y} + \mathbf{R}'\mathbf{R}\hat{\boldsymbol{\omega}} + \mathbf{R}'\mathbf{W}\hat{\boldsymbol{\varsigma}} = \mathbf{0} \tag{16}$$

by completing equations (15) and (16) simultaneously obtained:

$$\hat{\boldsymbol{\omega}} = \left(\mathbf{R}'\mathbf{M}^{-1}\mathbf{R}\right)^{-1}\mathbf{R}'\mathbf{M}^{-1}\left(\mathbf{I} - \mathbf{A}_{\boldsymbol{\alpha}}\right)\mathbf{y} \tag{17}$$

$$\hat{\boldsymbol{\varsigma}} = \mathbf{M}^{-1}\left(\mathbf{I} - \mathbf{R}\left(\mathbf{R}'\mathbf{M}^{-1}\mathbf{R}\right)^{-1}\mathbf{R}'\mathbf{M}^{-1}\right)\left(\mathbf{I} - \mathbf{A}_{\boldsymbol{\alpha}}\right)\mathbf{y}. \tag{18}$$

From equations (8), (17) and (18) an estimator Kernel-Spline is obtained:

$$\begin{aligned}\hat{\mathbf{f}} &= \mathbf{R}\hat{\boldsymbol{\omega}} + \mathbf{W}\hat{\boldsymbol{\varsigma}} + \mathbf{A}_{\boldsymbol{\alpha}}\mathbf{y} \\ &= \left\{\mathbf{R}\left(\mathbf{R}'\mathbf{M}^{-1}\mathbf{R}\right)^{-1}\mathbf{R}'\mathbf{M}^{-1}\left(\mathbf{I} - \mathbf{A}_{\boldsymbol{\alpha}}\right) + \mathbf{W}\mathbf{M}^{-1}\left(\mathbf{I} - \mathbf{R}\left(\mathbf{R}'\mathbf{M}^{-1}\mathbf{R}\right)^{-1}\mathbf{R}'\mathbf{M}^{-1}\right)\left(\mathbf{I} - \mathbf{A}_{\boldsymbol{\alpha}}\right) + \mathbf{A}_{\boldsymbol{\alpha}}\right\}\mathbf{y} \\ &= \mathbf{F}_{\lambda,\boldsymbol{\theta},\boldsymbol{\alpha}}\mathbf{y}\end{aligned} \tag{19}$$

IOP Conf. Series: Materials Science and Engineering **546** (2019) 052028 doi:10.1088/1757-899X/546/5/052028

with:

$$\hat{\boldsymbol{\omega}} = \left(\mathbf{R}'\mathbf{M}^{-1}\mathbf{R}\right)^{-1}\mathbf{R}'\mathbf{M}^{-1}\left(\mathbf{I}-\mathbf{A}_\alpha\right)\mathbf{y}$$

$$\hat{\boldsymbol{\varsigma}} = \mathbf{M}^{-1}\left(\mathbf{I}-\mathbf{R}\left(\mathbf{R}'\mathbf{M}^{-1}\mathbf{R}\right)^{-1}\mathbf{R}'\mathbf{M}^{-1}\right)\left(\mathbf{I}-\mathbf{A}_\alpha\right)\mathbf{y}.$$

$$\hat{\mathbf{m}}(\mathbf{z}) = \mathbf{A}_\alpha\mathbf{y}$$

5. Data Application

The Kernel-Spline nonparametric regression model will be applied to the open unemployment rate (y) data with the observation unit consisting of 38 districts/cities in the East Java province. The predictor variables used are the average length of school (LS) and population growth rate (PG). Mathematically the regression model can be written as follows:

$$y = f(LS) + g(PG) \tag{20}$$

Data exploration based on complete descriptive statistics can be seen in Table 1. Based on Table 1, it is known that 38 districts/cities in East Java province in 2016, the average unemployment rate is 4,358 percent with an average length of school of 7,470 years and an average the average population growth rate is 0.6032 people/km^2.

Table 1. Descriptive Statistics Response Variables and Predictor Variables

Variable	Mean	StDev	Minimum	Maximum	Range
UR	4.358	1.729	0.970	8.460	7.490
LS	7.470	1.704	3.790	11.090	7.300
PG	0.6032	0.3319	0.0600	1.6000	1.5400

Visually, this can be seen from the scatter plot presented in Figure 1 and Figure 2.

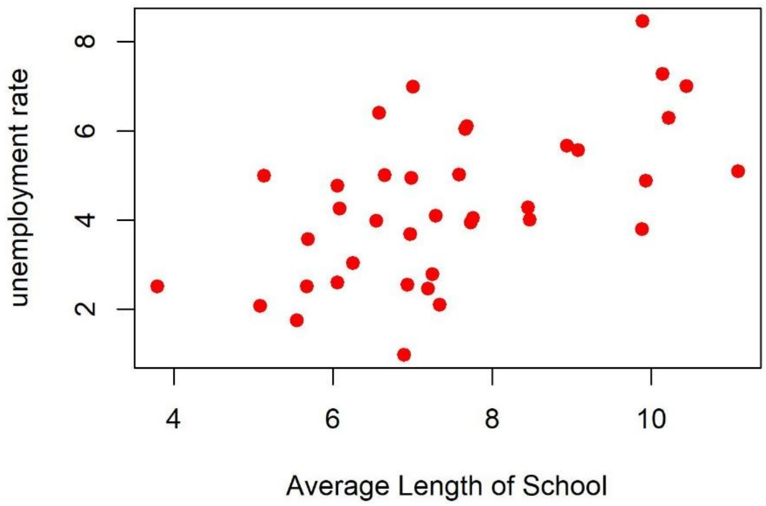

Figure 1. Scatter plot unemployment rate data with LS

Figure 1 shows that the pattern of the relationship between the response variable (unemployment rate) and predictor variable (average length of school) has an unknown pattern, so it is modeled as nonparametric. The pattern of the relationship between the unemployment rate and the average length of school tends to change behavior at points 7 and 9. In the average of 4-7 years of schooling, the unemployment rate tends to increase. While the average length of school at intervals of 7-9 years, the unemployment rate tends to decline. Furthermore, the average length of school at the 9-12 years interval, the unemployment rate has increased again. So that in theory, it can be approached with the Spline function.

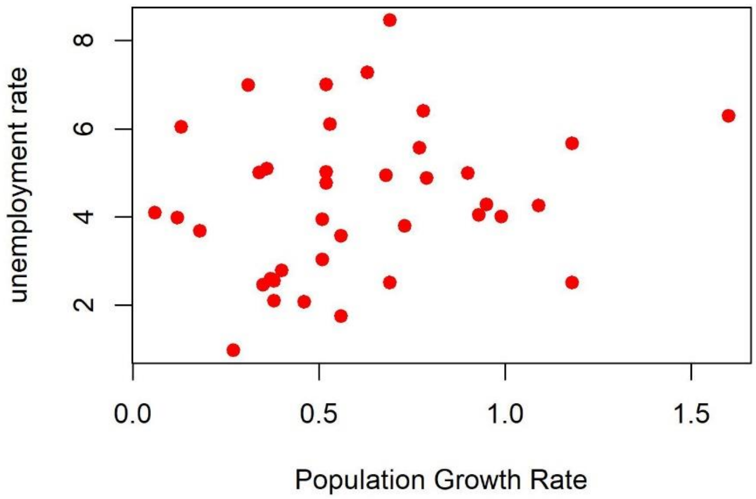

Figure 2. Scatter plot unemployment rate data with PG

The pattern of the relationship between the response variable (unemployment rate) and the predictor variable (population growth) seen in Figure 2 tends not to follow a certain pattern, so it is modeled as a nonparametric Kernel.

From the scatter plot in Figure 1 and Figure 2, it can be seen that LS variables and PG tend not to have a certain pattern. So that the appropriate model to explain the relationship between predictor variables and the response variable in East Java unemployment data is the multi-predictors nonparametric regression model. Based on the Kernel-Spline model obtained in equation (19), then variations on the Kernel function are carried out to see which Kernel functions are more suitable. The results are shown in the following table:

Table 2. GCV values and R-squared Kernel-Spline models with various Kernel functions

Kernel Function	GCV	R-squared
Uniform	0.053298413	0.74891
Epanechnikov	0.006724193	0.82348
Quartic	0.003897512	0.82987
Gaussian	0.001908752	0.88762*
Triweight	0.002001231	0.82546

Based on the table above, the Kernel-Spline model with the Kernel Gaussian function gives the best results, namely the minimum GCV of 0.001908752 and the highest R-squared value of 0.88762. It can be concluded that the best model for modeling the unemployment rate in East Java with the average predictor of school length and population growth rate is the Kernel-Spline model with the Gaussian kernel function.

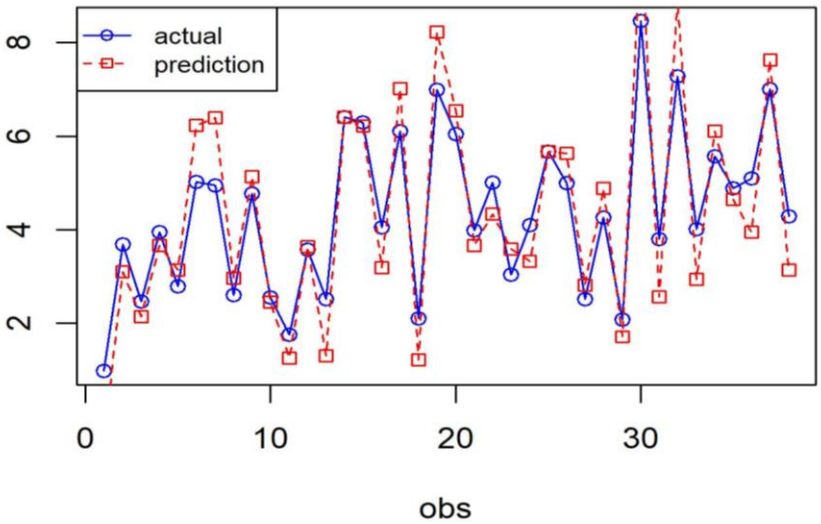

Figure 3. Unemployment rate actual and unemployment rate prediction

6. Conclusion

The estimation result of the Kernel-Spline model in nonparametric regression is $\hat{\mathbf{f}} = \mathbf{F}_{\lambda,\theta,\alpha}\mathbf{y}$. The best model for modeling unemployment rates in East Java with predictors of average school years and population growth rates is the Kernel-Spline model with Gaussian kernel functions. This can be seen based on the lowest GCV value with the largest R-squared value.

Acknowledgments

Thanks to Direktorat Jendral Sumber Daya IPTEK and DIKTI, Kemristekdikti, in collaboration with Lembaga Pengelola Dana Pendidikan (LPDP), which give the Beasiswa Unggulan Dosen Indonesia (BUDI) as a funder of this research.

References

[1] Budiantara I N, Ratna M, Zain I and Wibowo W 2012 Modeling the Percentage of Poor People in Indonesia Using Spline Nonparametric Regression Approach *Int. J. Basic and App. Sciences* **12** pp. 118-124
[2] Eubank R L 1999 *Spline Smoothing and Nonparametric Regression* (New York: CRC Press)
[3] Hidayat R, Budiantara I N, Otok B W, and Ratnasari V 2019 A Reproducing Kernel Hilbert Space Approach and Smoothing Parameters Selection in Spline-Kernel Regression *J. of Theo. and App. Inf. Tech.* **97** pp. 465-475
[4] Hidayat R, Budiantara I N, Otok B W, and Ratnasari V 2019 An Extended Model of Penalized Spline with The Addition of Kernel Functions in Non-parametric Regression Model *App. Math. and Inf. Sci.* **13** pp. 1-7
[5] Aydin D and Tuzemen M 2010 Estimation in Semi-parametric and Additive Regression using Smoothing and Regression Spline *Sec. Int. Conf. on Comp. Res. and Dev.* **2**, pp. 465-469
[6] Becher H, Kauermann G, Khomski P, and Kouyate B 2009 Using Penalized Splines to Model Age and Season of Birth Dependent Effects of Childhood Mortality Risk Factors in Rural Burkina Faso *Bio. J.* **51** pp. 110-122
[7] Lin X, Wang N, Welsh A H, and Carrol R J 2004 Equivalent Kernels of Smoothing Spline in Nonparametric Regression for Clustered/ Longitudinal Data *J. Biometrika* **91** pp. 177-193
[8] Cheng M Y, Paige R L, Sun S, and Yan K 2010 Variance Reduction for Kernel Estimator in Clustered/Longitudinal Data Analysis *J. of Stat. Planning and Inf.* **140** pp. 1389-1397
[9] Kayri M and Zirhhoglu G 2009 Kernel Smoothing Function and Choosing Bandwidth for

Nonparametric Regression Methods *Ozean J. of App. Sci.* **2** pp. 49-54
[10] Nadayara E A 1964 On Estimating Regression *Theo. of Prob. and Its App.* **9** pp. 141-142
[11] Watson G S 1964 Smooth Regression Analysis *Sankhya: The Ind. J. of Stat. A* **26** pp. 359-372.