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Modeling of dynamic mass coupled system with Runge-Kutta fourth order

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Abstract. This paper shows the mathematical modeling process of a mechanical system of masses coupled by two springs and a shock absorber. The process of capture of movement of the mass system coupled with springs was done with software Tracker video analysis and modeling tool. The motion capture of the coupled masses A and B was made with a duration of 20 seconds. A comparison of the results of the movement of the two masses will be established as a first instance, by adjusting trajectories of curves in time using Matlab software and as a second instance a numerical solution of the Mass-Spring model will be established using the Runge Kutta method of 4 order. With this last method, it is expected to achieve a better precision modeling of the initial value problem. Finally, a comparison of the two mathematical models will be made analyzing.

1. Introduction

Studies in different fields of engineering have shown that mathematical modeling is of great help for the development and control of mechanical and industrial processes that have the basis of production the analysis of movement of objects by its geometric configuration in terms of weight, shape, location space and climatic conditions. Some authors as Qi, C. et al., doing the mathematical modeling to the modeling and identification of nonlinear distributed parameters dynamics of the micro-cantilever [1], Chen, Z. et al., use the mathematical modeling to perform the mathematical model and simulation for motion of underwater vehicle [2], Wang, D. et al., use the modeling in ordinary differential equation (ODE) to modeling, analyzing and controlling long distribution feeder [3], Rath, G. et al., make a direct numerical solution of stiff ODE system in optimal control [4], Lobao, W. J. A. et al., perform a genetic programming and automatic differentiation algorithms applied to the solution of ordinary and partial differential equation [5], Korolova, O. et al., show a performance of Runge-Kutta family numerical solvers for calculation of transient processes in AC machines [6], Krisko, V. et al., use mathematical modeling to nonlinear dynamics of contact interaction of MEMS beam elements accounting the Euler-Bernoulli hypothesis in a temperature field [7], Kern, R. and Gehring N. show a tracking control for a long pneumatic transmission line [8].

The Numerical analyzes and simulations through computer software such as Matlab [9 - 12] are important in the mathematical process modeling since through their calculation tools they allow to optimize time in execution of iterative cycles for solving differential equations problems and in this case in particularly help to solve the initial value problem [13 - 15].

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There are many methods to analyze mathematical models and adjust trajectories over time, taking into account the analysis of the initial value problem [13 - 15]. Some of these methods for curve fitting or trajectory adjustment are:

Methods focused on linear regressions. Some authors as Kavitha, S. et al., [16] make a comparative analysis on linear regression to use the correct model for better prediction and accuracy in business consumer interest.

Methods focused on Polynomials and multiple linear regressions to analyze higher-order polynomials. Hirose, H. et al., in [17] use the multiple linear regression to predict the continuous value of target variable using the values of explanation variables in prediction accuracy of the prices for auctions of used cars is drastically improved. Xuang, F. et al., [18] implement a contact temperature prediction of high voltage switchgear based on multiple linear regression model to analyze and process a monitoring point data.

Methods based on Fourier analysis involving fitting periodic functions to data through Fast Fourier transform (FFT). Kong, W. et al., [19] propose a novel grouping scheme of the basic functions within the framework of the fast Fourier transform (FFT) to create a block-sparse structure for the near-matrix on account of the efficient analysis of multiscale problems. Su, T. et al., [20] applied this method to present a power harmonic detection method in renewable power based in Nuttall double-window all-phase FFT algorithm as a basis for the study of harmonics.

Interpolation, estimating intermediate values between precise data points with newton's interpolating polynomial method. Laih, Chi-Sum. Et al., [21] use this method to the design of a single-key-lock mechanism to reduce the average computation time. On the other hand with LaGrange interpolating polynomials method. Van Beeumen, R. et al., [22] write about linearization of Lagrange and Hermite interpolating matrix polynomials as a tool to study the classical approach to investigating the polynomial eigenvalue problem.

Spline interpolation. In this method, Liang, Y. et al., [23] present a class of algebraic trigonometric interpolation splines and applications as an efficient new model for geometric design.

Curve fitting with sinusoidal functions. Pattanadech, N. et al., [24] in this method present a fast curve algorithm for parameter evaluation in lighting impulse test technique for the evaluation of the base curve of lightning impulse voltage and current.

In addition, methods with ordinary differential equations treatment ODEs like Euler's method and Runge-Kutta fourth order (RK4). Wang, X., [25] use Euler's method to find the strong convergence rates of the linear implicit Euler. Simons, T., [26] use RK4 method for the numerical integration of the radial Schrödinger equation. Haut, T. et al., [27] use the method to solve wave propagation problems via the direct construction of an approximate time-evolution operator

In this work, the procedure performed for the validation of a real coupled spring mass model with ODEs is presented. It is expected to show if our model describes the real behavior of the system. For this case it is highlighted that the constants that appear in our solution are related to the initial conditions and the parameters that characterize the system under study.

Additionally, this paper presents the RK4 method as an aid to solve the initial value problem through numerical and computational analysis.

The methodological development of the presented work will be given in the following way: In the second section the mathematical modeling realized to the two coupled masses will be shown with second order differential equations and the modeling through software Matlab, in the third section we present the modeling done with Rung-Kutta of fourth order and its fitting application with Matlab, in the fourth section results are presented. To finalize, the modeling conclusions and references will be presented.

To compare the different series of time obtained in this experiment we will use the MSE (Mean-Squared Error), because it can describe the degree of similarity between two set of time series data.[28].

The MSE between the time series data will be:

$$MSE(x,y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - y_i)^2$$

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2. Mathematical model

To start with the modeling process and determine the position of the two coupled masses, laboratory assembly was done see Figure 1, in which two masses are presented coupled to two springs and as a damping element water is used inside a glass container .

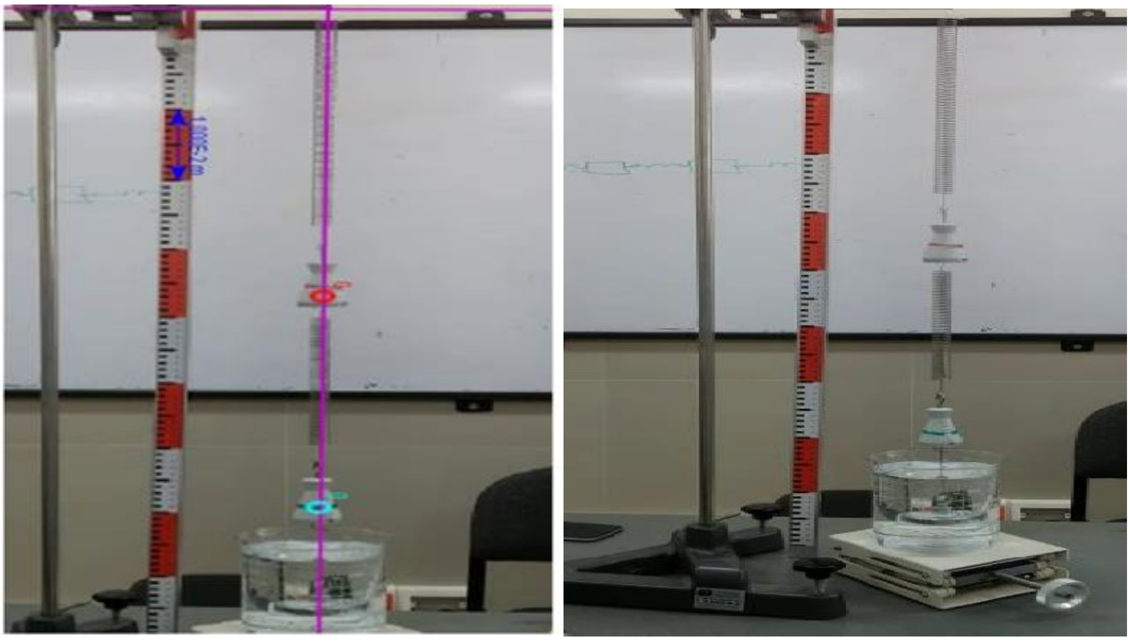

Figure 1. Laboratory assembly coupled masses

The real data of the system are shown in Table 1. The experimental analysis for the capture of the movement of the coupled system was done with the Tracker software (https://physlets.org/tracker/) and the capture of movement was made during 20 seconds in which the two masses were sliding. From the capture process, the displacement graphs of the masses were obtained, see Figure 2, and see Figure 3.

Table 1. Real data of the system.

Physical Data	Values
Mass 1 (m1)	0.2 (kg)
Mass 2 (m2)	0.325 (kg)
Elasticity (k1)	11.136 (N/m)
Elasticity (k2)	18.41 (N/m)
Damping (B)	0.08 (Ns/m)

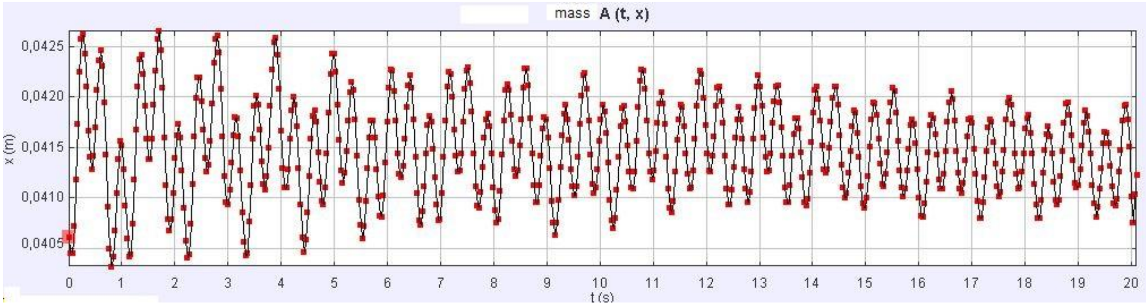

Figure 2. Displacement graphs mass A.

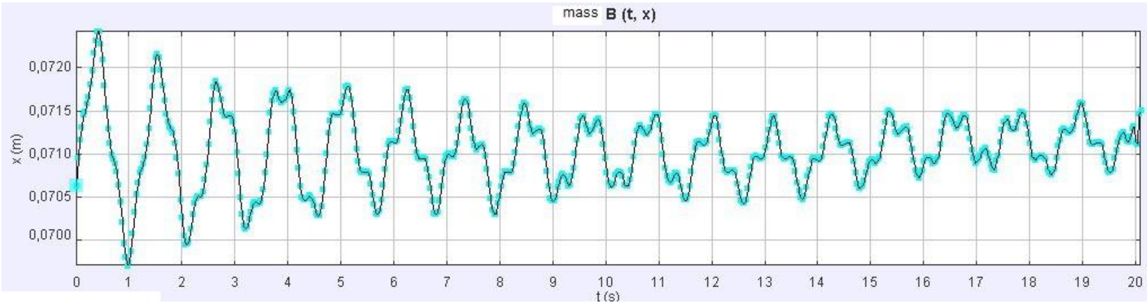

Figure 3. Displacement graphs mass B.

For the mathematical analysis, a diagram of the real situation is presented, see Figure 4.

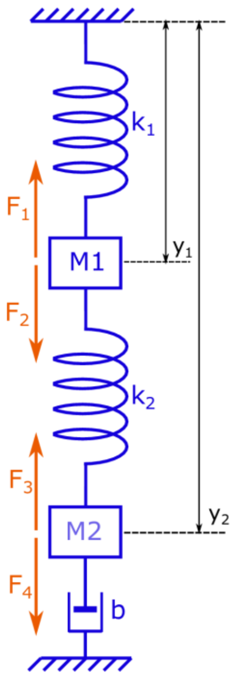

Figure 4. Analysis diagram free body.

Taking into account the second law of Newton, applied to the dynamic system of coupled masses. The mathematical model of the system is given in the following way:

$$m_1 * a_1 = F_1 - F_2 \tag{1}$$

$$m2 * a2 = F3 - F4 \tag{2}$$

Replacing by its derivatives in equations 1 and 2 it is observed that:

$$m_1 \frac{d^2 y_1}{dt^2} = -k_1 y_1 + k_2 \left(y_2 - y_1 \right) \tag{3}$$

$$m_2 \frac{d^2 y_2}{dt^2} = -k_2 \left(y_2 - y_1 \right) - b \frac{dy_2}{dt} \tag{4}$$

Reorganizing equations 3 and 4 we have:

$$m_1 \frac{d^2 y_1}{dt^2} + \left(k_1 + k_2 \right) y_1 - k_2 y_2 = 0 \tag{5}$$

$$m_2 \frac{d^2 y_2}{dt^2} + b \frac{dy_2}{dt} + k_2 y_2 - k_2 y_1 = 0 \tag{6}$$

If we make the following note in which:

$$\omega_{11}^2 = \frac{k_1}{m_1}, \omega_{22}^2 = \frac{k_2}{m_2}, \omega_{21}^2 = \frac{k_2}{m_1}, \lambda_b = \frac{b}{m_2} \tag{7}$$

The system becomes:

$$\frac{d^2 y_1}{dt^2} + \left(\omega_{11}^2 + \omega_{21}^2 \right) y_1 - \omega_{21}^2 y_2 = 0 \tag{8}$$

$$\frac{d^2 y_2}{dt^2} + \lambda_b \frac{dy_2}{dt} + \omega_{22}^2 y_2 - \omega_{22}^2 y_1 = 0 \tag{9}$$

Making a change of variables $x_1 = y_1 \;\; x_2 = y_2 \;\; \dot{x}_1 = v_1 = \dot{y}_1 \;\; \dot{x}_2 = v_2 = \dot{y}_2$ and replacing in equation 5 and 6 we have that:

$$\dot{v}_1 = -\left(\omega_{11}^2 + \omega_{21}^2 \right) x_1 - \omega_{21}^2 x_2 \tag{10}$$

$$\dot{v}_2 = -\lambda_b v_2 - \omega_{22}^2 x_2 - \omega_{22}^2 x_1 \tag{11}$$

Taking into account equations 7 and 8, the system can be written matrix as follows:

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$$\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{x}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\left(\omega_{11}^2+\omega_{21}^2\right) & 0 & \omega_{21}^2 & 0 \\ 0 & 0 & 0 & 1 \\ \omega_{22}^2 & 0 & -\omega_{22}^2 & -\lambda_b \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} \tag{12}$$

The system can be described as follows in equation 13:

$$\dot{x} = A_x \tag{13}$$

Where matrix A is the deterministic matrix of the system and x is the solution vector. The general solution can be written as follows:

$$x(t) = \sum_{i=1}^{n} C_i e^{\lambda it} V_{0i} \tag{14}$$

Where λi are the proper values of A and $0i$ the eigenvectors of A.

To determine λi we must determine the roots of the n degree equation resulting from performing the following operation:

$$det(A-l)=0 \tag{15}$$

Where l it is an identity matrix and to determine the eigenvalues it must be solved:

$$AV_0 = \lambda V_0 \tag{16}$$

Now taking into account our dynamic coupled mass system:

$$det(A-l)=\begin{bmatrix} -\lambda & 1 & 0 & 0 \\ -\left(\omega_{11}^2+\omega_{21}^2\right) & -\lambda & \omega_{21}^2 & 0 \\ 0 & 0 & -\lambda & 1 \\ \omega_{22}^2 & 0 & -\omega_{22}^2 & -\lambda_b-\lambda \end{bmatrix}=0 \tag{17}$$

By developing the following fourth degree polynomial is obtained:

$$\lambda^4 + \lambda_b\lambda^3 + \left(\omega_{11}^2+\omega_{21}^2+\omega_{22}^2\right)\lambda^2 + \lambda_b\left(\omega_{11}^2+\omega_{21}^2\right)\lambda + \omega_{11}^2\omega_{22}^2 = 0 \tag{18}$$

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Our System is described as: however, in order to determine the solution of our system we have to write it in canonical form. For this we make the following change of variable as it is presented:

$$X = PY \tag{19}$$

Where Y will be a system equivalent to X and P is a transformation matrix as follows:

$$X' = PY'$$
$$AX = PY'$$
$$APY = PY'$$
$$Y' = p^{-1}APY = JY$$

Where J is the canonical matrix of Jordan [29 - 30]. For our case with complex values of $\alpha_j \pm \beta_{ji}$

$$J = P^{-1}AP\begin{pmatrix} B_1 & \cdots & \\ \vdots & \ddots & \vdots \\ & \cdots & B_j \end{pmatrix} = o \text{, where } b_j = \begin{bmatrix} \alpha_j & \beta_j \\ -\beta_j & \alpha_j \end{bmatrix} \tag{20}$$

The eigenvectors V_j which corresponds to the j-esimo eigenvalue are also complex and the vector V_{2j} (pairs) is the complex conjugate V_{2j-1} (odd). Then P will be formed by the columns of eigenvectors such that:

$$P_{2j-1} = Re\left(V_{2j-1}\right) \tag{21}$$

And:

$$P_{2j} = lm\left(V_{2j-1}\right) \tag{22}$$

Then, the general solution of $X' = AX$ will be $X = PY(t)$ where:

$$Y(t) = \begin{pmatrix} a_1e^{\alpha_1 t}cos(\beta_1 t) + b_1e^{\alpha_1 t}sin(\beta_1 t) \\ -a_1e^{\alpha_1 t}sen(\beta_1 t) + b_1e^{\alpha_1 t}cos(\beta_1 t) \\ a_je^{\alpha_j t}cos(\beta_j t) + b_1e^{\alpha_j t}sin(\beta_j t) \\ -a_je^{\alpha_j t}sen(\beta_j t) + b_1e^{\alpha_j t}cos(\beta_j t) \\ \end{pmatrix} \tag{23}$$

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Taking into account the physical values of the system in Table 1. The initial values and values calculated in Table 2 can be observed.

Table 2. Real data of the system.

	Initials Value		Calculated Value
$x_1(0)$	-0.0099 (m)	λ_b	0.2462
$v_1(0)$	0.041 (m/s)	ω_{11}^2	36.05
$x_2(0)$	0.015 (m)	ω_{21}^2	64.45
$v_2(0)$	0.053 (m/s)	ω_{22}^2	39.66

Now our system is reflected in the following way:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{x}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -(147.73) & 0 & 92.05 & 0 \\ 0 & 0 & 0 & 1 \\ 56.6462 & 0 & -56.6462 & -0.2462 \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} \tag{24}$$

Now our system with complex eigenvalues is shown as follows:

$$D = \begin{bmatrix} -0.0287 & + & 13.6948i \\ -0.0287 & + & -13.6948i \\ -0.0944 & + & 4.0998i \\ -0.0944 & + & -4.0998i \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} \tag{25}$$

And with complex eigenvectors is shown as follows:

$$V_0 = \begin{bmatrix} -0.0001-0.0668i & -0.0001+0.0668i & -0.0023-0.1362i & -0.0023+0.1362i \\ 0.9154+0.0000i & 0.9154+0.0000i & 0.5587+0.0033i & 0.5587+0.0033i \\ -0.0005+0.0289i & -0.0005-0.0289i & -0.0045-0.1938i & -0.0045+0.1938i \\ -0.3959-0.0078i & -0.3959+0.0078i & 0.7948+0.0000i & 0.7948+0.0000i \end{bmatrix} \tag{26}$$

The P matrix will be in this way:

$$P = \begin{bmatrix} -0.0001 & -0.0668 & -0.0023 & -0.1362 \\ 0.9154 & 0 & 0.5587 & 0.0033 \\ -0.0005 & 0.0289 & -0.0045 & -0.1938 \\ -0.3959 & -0.0078 & 0.7948 & 0 \end{bmatrix} \tag{27}$$

And the canonical Jordan form [28 - 30] will be in this way:

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$$J = P^{-1}AP = \begin{bmatrix} -0.0287 & 13.6948 & 0 & 0 \\ -13.6948 & -0.0287 & 0 & 0 \\ 0 & 0 & -0.0944 & 4.0998 \\ 0 & 0 & -4.0998 & -0.0944 \end{bmatrix} \tag{28}$$

And the $Y(t)$ matrix will be:

$$Y(t) = \begin{bmatrix} e^{-0.0287t}\left(a_1 cos(13.6948t) + b_1 sin(13.6948t)\right) \\ e^{-0.0287t}\left(-a_1 sin(13.6948t) + b_1 cos(13.6948t)\right) \\ e^{-0.0944t}\left(a_2 cos(4.0998t) + b_2 sin(4.0998t)\right) \\ e^{-0.0944t}\left(-a_2 sin(4.0998t) + b_2 cos(4.0998t)\right) \end{bmatrix} \tag{29}$$

$$Y(0) = \begin{bmatrix} a_1 \\ b_1 \\ a_2 \\ b_2 \end{bmatrix} = P^{-1}X(0) = \begin{bmatrix} -0.01 \\ -0.008 \\ -0.0091 \\ 0.0048 \end{bmatrix} \tag{30}$$

The mathematical model of the mass, spring and damper system will be as follows:

$$\begin{aligned} x_1(t) = & e^{-0.0287t}\left(0.0005cos(13.70t) - 0.0007sin(13.70t)\right) + \\ & e^{-0.09447t}\left(-0.0006cos(4.09980t) - 0.0012sin(4.0998t)\right) \end{aligned} \tag{31}$$

$$\begin{aligned} v_1(t) = & e^{-0.0287t}\left(-0.0091cos(13.70t) - 0.00074sin(13.70t)\right) + \\ & e^{-0.09447t}\left(-0.0051cos(4.09980t) - 0.0027sin(4.0998t)\right) \end{aligned} \tag{32}$$

$$\begin{aligned} x_2(t) = & e^{-0.0287t}\left(0.0040cos(13.70t) + 0.0003sin(13.70t)\right) + \\ & e^{-0.09447t}\left(-0.0009cos(4.09980t) - 0.0018sin(4.0998t)\right) \end{aligned} \tag{33}$$

$$\begin{aligned} v_2(t) = & e^{-0.0287t}\left(-0.0091cos(13.70t) - 0.00074sin(13.70t)\right) + \\ & e^{-0.09447t}\left(-0.0072cos(4.09980t) - 0.0038sin(4.0998t)\right) \end{aligned} \tag{34}$$

2.1. Fitting and modelling in MATLAB

In this part of work, we presented the general model of fitting curve [31 - 34] using MATLAB software.

General mode movement of mass A:

$$x_1(x) = \exp(-a1.*x).*(a2*\cos(a3*x)+a4*\sin(a3*x)) + \exp(-a5.*x).* \\ (a6*\cos(a7*x)+a8*\sin(a7*x)) + a9 \tag{35}$$

Coefficients (with 95 % confidence bounds):

$$\begin{aligned} a1 &= 0.02774(0.02533, 0.03015) \\ a2 &= -0.003153(-0.003328, -0.002977) \\ a3 &= 17.29(17.28, 17.59) \\ a4 &= -0.006619(-0.006795, -0.006442) \\ a5 &= 0.09154(0.08601, 0.09707) \\ a6 &= -0.003745(-0.003971, -0.003519) \\ a7 &= 5.429(5.423, 5.434) \\ a8 &= 0.005036(0.004809, 0.005264) \\ a9 &= 0.4149(0.4148, 0.005264) \end{aligned} \tag{36}$$

Now we can see the adjustment of the movement of mass A. See Figure 5.

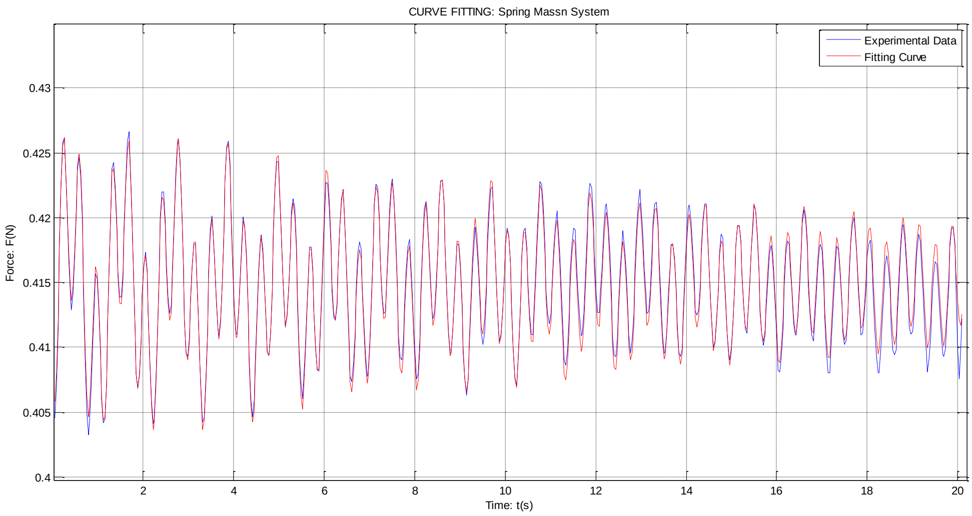

Figure 5. Adjustment of mass movement A.

General mode movement of mass B:

$$x_2(x) = a1*\exp(-a2*x).*\cos(a3*x+a4) + a5\exp(-a6*x).*\cos(a7*x+a8) + a9 \tag{37}$$

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Coefficients (with 95 % confidence bounds):

$$\begin{aligned} a1 &= -0.002426(-0.002682, -0.00217) \\ a2 &= -0.03434(0.02334, -0.04533) \\ a3 &= 17.28(17.27, 17.29) \\ a4 &= 2.084(1.979, 2.188) \\ a5 &= 00.01071(0.01039, 0.01103) \\ a6 &= -0.08843(0.08403, 0.09283) \\ a7 &= 5.426(5.421, 5.43) \\ a8 &= -2.168(-2.197, -2.139) \\ a9 &= 0.7104(0.7103, 0.7105) \end{aligned} \tag{38}$$

Now we can see the adjustment of the movement of mass B. See Figure 6.

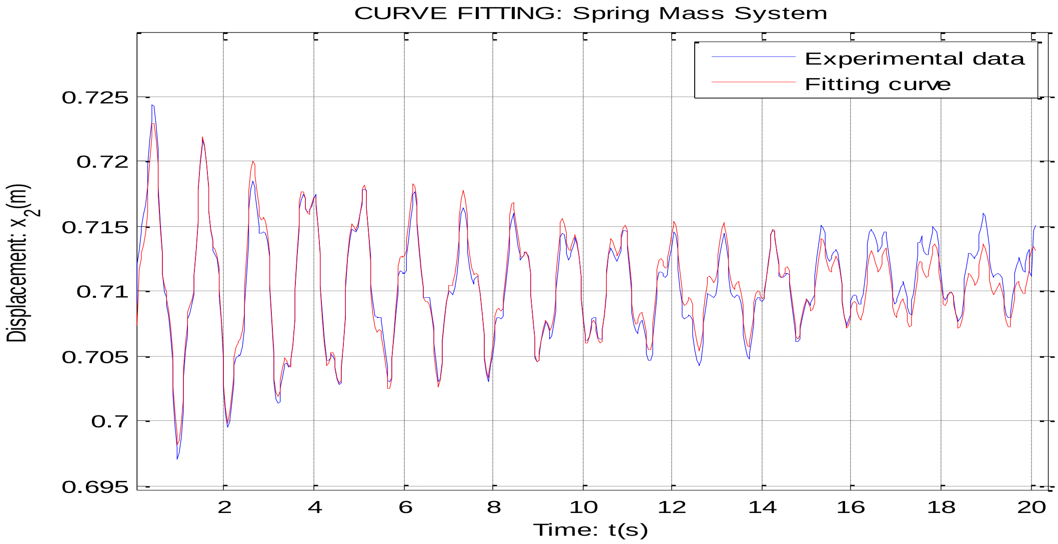

Figure 6. Adjustment of mass movement B.

Taking into account the calculation made for the general mode of movement of the mass B. It should be taken into account that in order to calculate the $\lambda_b = a2$, the general mode of the equation must be as shown in the equation:

$$FIT(x) = a1.*\exp(-a2.*x).*\cos(a3*x + a4) + a5 \tag{39}$$

Coefficients (with 95 % confidence bounds):

$$\begin{aligned} a1 &= 0.0106(0.008578, 0.01262) \\ a2 &= 0.2436(0.1817, 0.311) \\ a3 &= 17.08(16.94, 17.23) \\ a4 &= -0.3941(-0.9898, 0.2016) \\ a5 &= 0.7149(0.7144, 0.7154) \end{aligned} \tag{40}$$

Knowing that, $\lambda_b = \dfrac{b}{m2}$, we have to:

$$v_1(t) = e^{-0.0287t}\left(-0.0091\cos(13.70t) - 0.0074 sen(13.70t)\right) + e^{-0.00944t}\left(-0.0006\cos(4.098t) - 0.0012 sen(4.0998t)\right) \tag{41}$$

$$b = \lambda_b m_2 = 0.2463 * 0.325 = 80 + 10^{-3} \tag{42}$$

Now we can see the damped motion adjustment of mass B. See Figure 7.

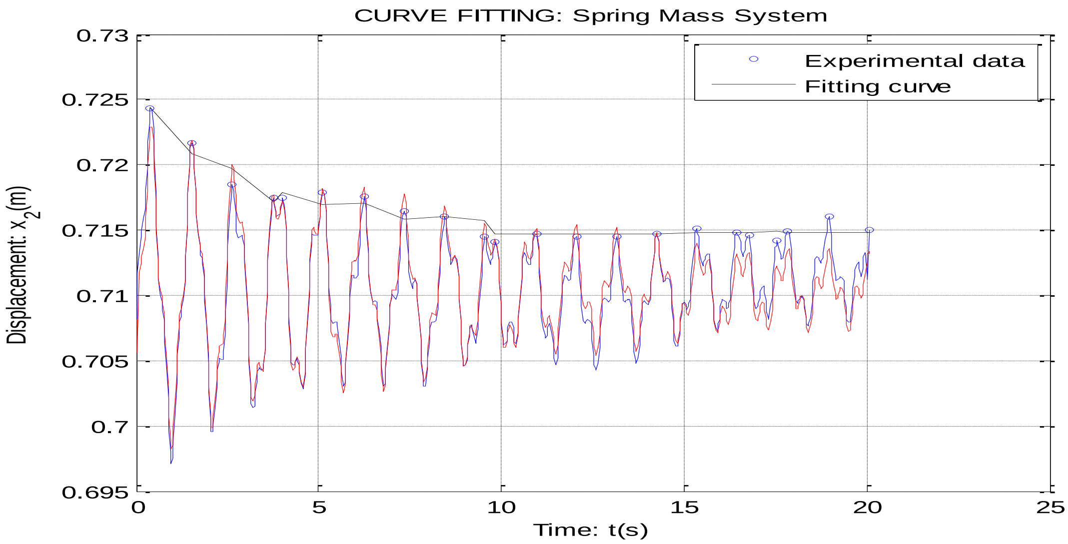

Figure 7. Damped motion adjustment of mass B.

2.2. Numerical Solution with, Runge Kutta 4 order

It is clarified that the 4-order Runge-Kutta Method [35 - 37] achieves the precision of the Taylor series, without having to determine the higher order derivatives. This is done to determine the solution of the initial value problem in the following way:

$$\frac{dy}{dt} = f(t, y), y(t_0) = y_0 \tag{43}$$

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Through the general form:

$$w_{i+1} = w_i + \phi h \tag{44}$$

Where $\phi = a_1 k_1 + a_2 k_2 + ... + a_n k_n$

a_i Are constants.

$k_i = f\left(t_i + p_{i-1}h, y_i + q_{i-1,1}k_1 h + q_{i-1,2}k_2 h + ... + q_{i-1,i-1}k_{i-1}h\right)$

p_i Y q_i are constants.

Now we are looking to solve the initial value problem in this way:

$$\frac{dy}{dx} = f\left(t, y\right), y\left(t_0\right) = y_0 \tag{45}$$

The Runge-Kutta method of order 4 [35 - 37] is used based on iterations to approach the solution of $y\left(t\right)$ through w , using a step size h in this form:

$$w\left(i+1\right) = w\left(i\right) + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)h \tag{46}$$

Where:

$k_1 = f\left(t_i, y_i\right)$

$k_2 = f\left(t_i + \frac{1}{2}h, y_i + \frac{1}{2}k_1 h\right)$

$k_3 = f\left(t_i + \frac{1}{2}h, y_i + \frac{1}{2}k_2 h\right)$

Taking into account the aforementioned equations, the systems of differential equations are represented as follows:

$$\left\{\begin{matrix} \dfrac{dy}{dt} = f_1\left(t, y_1, y_2, ..., y_n\right) \\ \dfrac{dy}{dt} = f_2\left(t, y_1, y_2, ..., y_n\right) \\ . \\ . \\ . \\ \dfrac{dy}{dt} = f_n\left(t, y_1, y_2, ..., y_n\right) \end{matrix}\right\} \tag{47}$$

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Now to find the solution of n differential equations, is required n initial values $y_1(t_0) = y_{0,1}, y_2(t_0) = y_{0,2}, ..., y_n(t_0) = y_{0,n}$

It is clarified that the solution of the $j-esima$ equation, is found iteratively in the following way:

$$w(i+1, j) = w(i, j) + \frac{1}{6}\left(k_{ij} + 2k_{2j} + 2k_{3j} + k_{4j}\right)h \tag{48}$$

Where:

$$k_{ij} = f_j\left(t_i, y_{i,1}, ..., y_{i,n}\right)$$
$$k_{2j} = f_j\left(t_i + \frac{1}{2}h, y_{i,1} + \frac{1}{2}k_{1,1}h, ..., y_{i,n} + \frac{1}{2}k_{1,n}h\right)$$
$$k_{3j} = f_j\left(t_i + \frac{1}{2}h, y_{i,1} + \frac{1}{2}k_{2,1}h, ..., y_{i,n} + \frac{1}{2}k_{2,n}h\right)$$
$$k_{4j} = f_j\left(t_i + h, y_{i,1} + k_{3,1}h, ..., y_{i,n} + k_{3,n}h\right)$$

And $j = 1, 2, 3, ..., n$

2.3. Numerical Solution with, Runge Kutta 4 order

In this section we present the mathematical model development of the mass coupled with spring and shock absorber with the Runge-Kutta method of 4 order [38 - 39]. It should be clarified that the method works with first-order differential equations and therefore the system of equations must be brought to such a level.

Taking into account the equation (1), (2), (3) and (4), the system can be written as follows:

$$\left\{\begin{array}{l} \dot{x}_1 = v_1 \\ \dot{v}_1 = -\left(w_{11}^2 + w_{21}^2\right)x_1 + w_{21}^2 x_2 \\ \dot{x}_2 = v_1 \\ \dot{v}_2 = w_{22}^2 x_1 - w_{22}^2 x_2 - \lambda_b v_2 \end{array}\right\} \tag{49}$$

The system with values replaced will be:

$$\left\{\begin{array}{l} \dot{x}_1 = v_1 \\ \dot{v}_1 = -147.73x_1 + 92.05x_2 \\ \dot{x}_2 = v_1 \\ \dot{v}_2 = 56.6462x_1 - 56.6462x_2 - 0.2462v_2 \end{array}\right\} \tag{50}$$

$$\begin{bmatrix} x_1(0) \\ v_1(0) \\ x_2(0) \\ v_2(0) \end{bmatrix} = \begin{bmatrix} 0.0099 \\ 0.041 \\ 0.0015 \\ 0.053 \end{bmatrix} \tag{51}$$

The analytical solution will look like this:

$$x_1(t) = e^{-0.0287t}\left(0.0005\cos(13.70t) - 0.0007sen(13.70t)\right) + e^{-0.00944t}\left(-0.0006\cos(4.0998t) - 0.0012sen(4.0998t)\right) \tag{52}$$

$$v_1(t) = e^{-0.0287t}\left(-0.0091\cos(13.70t) - 0.0074sen(13.70t)\right) + e^{-0.00944t}\left(-0.0051\cos(4.0998t) + 0.0027sen(4.0998t)\right) \tag{53}$$

$$x_2(t) = e^{-0.0287t}\left(0.0002\cos(13.70t) + 0.0003sen(13.70t)\right) + e^{-0.00944t}\left(-0.0009\cos(4.0998t) - 0.0018sen(4.0998t)\right) \tag{54}$$

$$v_2(t) = e^{-0.0287t}\left(0.040\cos(13.70t) + 0.0003sen(13.70t)\right) + e^{-0.00944t}\left(-0.0072\cos(4.0998t) + 0.0038sen(4.0998t)\right) \tag{55}$$

Finally, after executing the coding in MATLAB software with RK4 application we have the mass A position and velocity of mass A. See Figure 8 and Figure 9. And the mass B position and velocity of mass B. See Figure 10 and Figure 11.

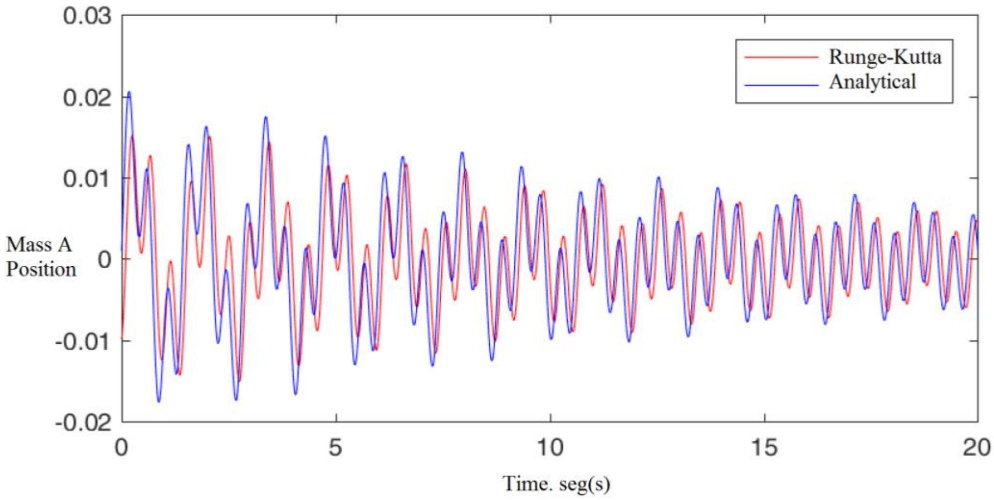

Figure 8. Mass A position with RK4.

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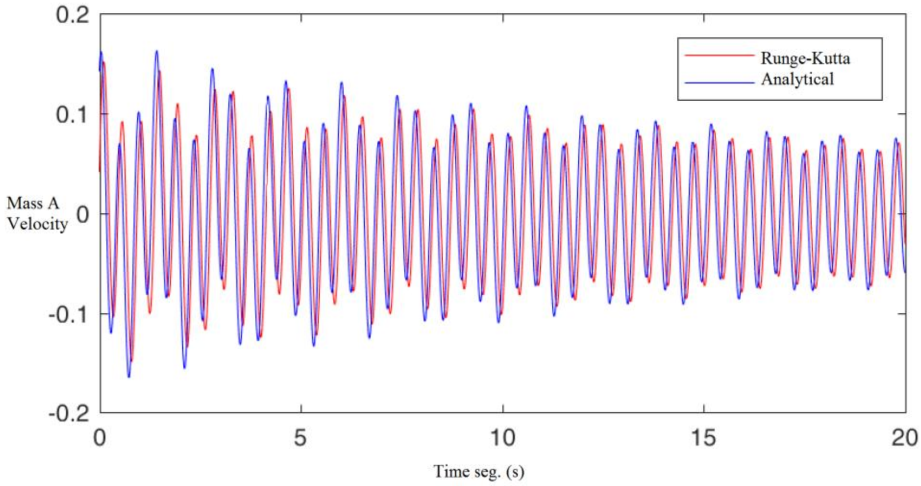

Figure 9. Mass A velocity with RK4

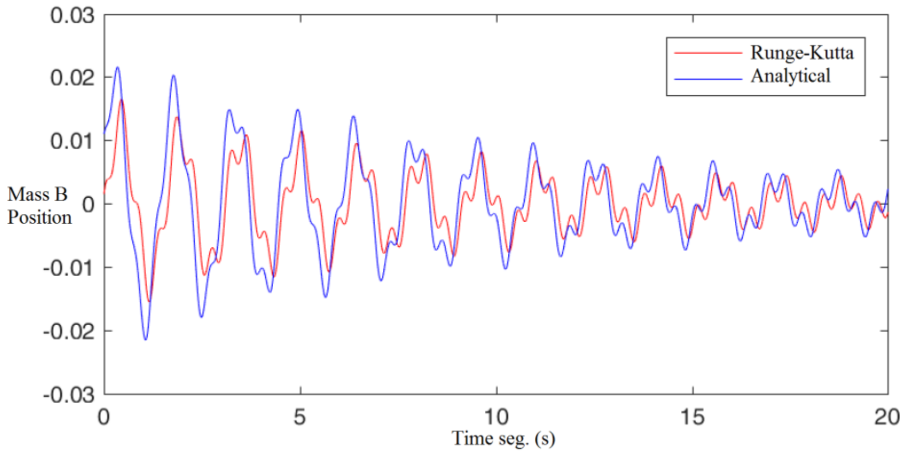

Figure 10. Mass B position with RK4.

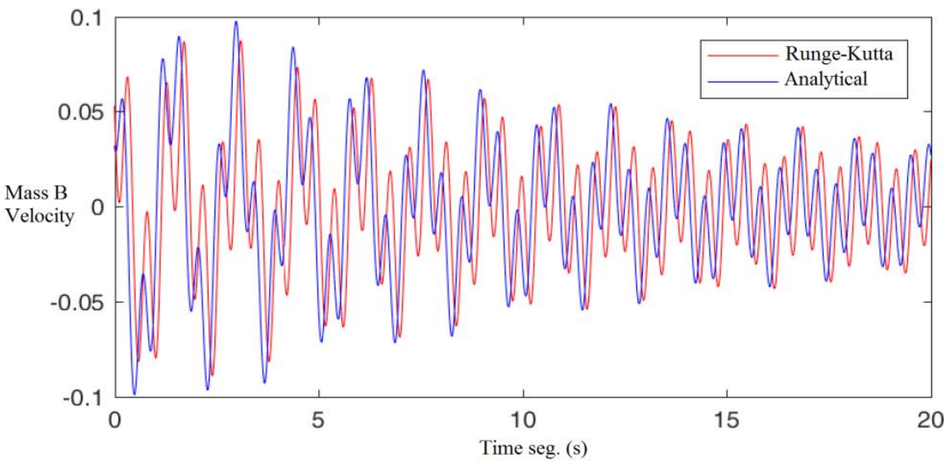

Figure 11. Mass B velocity with RK4

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3. Conclusions

Making a comparison of the two analysis methods to adjust curves in modeling of second order dynamical systems through differential equations. It can be concluded that the RK4 method performs a better approximation to the initial state values. See Figures 5 and 6, in comparison with Figures 8 and 10.

The MSE between the data taken from Tracker and the data calculated using the Runge-Kutta fourth order method obtained for mass A, is 5.4005e-005 and for mass B, is 2.7258e-005, which were too small compared with their initial amplitudes.

It is concluded that the methods of analysis for modeling of coupled mass dynamic systems were ideal and are consistent with the movement and displacement capture evidenced in the Tracker software.

References

[1] Qi C, Zhao X, Gao F and Li H 2014 Modeling and identification of nonlinear distributed parameter dynamics of the micro-cantilever *Proceedings of the 33rd Chinese Control Conference* 5924-5929

[2] Chen Z , Qi Q and Zhou Y 2013 Mathematical Model and Simulation for Motion of Underwater Vehicle 2013 *Sixth International Symposium on Computational Intelligence and Design* 293-296

[3] Wang D, Turitsyn K and Chertkov M 2012 DistFlow ODE: Modeling, analyzing and controlling long distribution feeder *IEEE 51st IEEE Conference on Decision and Control (CDC)* 5613-5618

[4] Rath G, Harker M and Zaev E 2017 Direct numerical solution of stiff ODE systems in optimal control *6th Mediterranean Conference on Embedded Computing (MECO)* 1-5

[5] Lobão W, Dias D and Pacheco M 2016 Genetic programming and automatic differentiation algorithms applied to the solution of ordinary and partial differential equations *IEEE Congress on Evolutionary Computation (CEC)* 5286-5292.

[6] Korolova O, Popp M, Mathis W and Ponick B 2017 Performance of Runge-Kutta family numerical solvers for calculation of transient processes in AC machines *20th International Conference on Electrical Machines and Systems (ICEMS)* 1-6

[7] Krysko V, Saltykova O, Krysko A and Papkova I 2017 Nonlinear dynamics of contact interaction of MEMS beam elements accounting the Euler-Bernoulli hypothesis in a temperature field *Dynamics of Systems, Mechanisms and Machines (Dynamics)* 1-6

[8] Kern R and Gehring N 2017 Tracking control for a long pneumatic transmission line *22nd International Conference on Methods and Models in Automation and Robotics (MMAR)* pp 180-185

[9] Guangyao L, Yugang D, Chengning Z and Liwen P 2014 Numerical simulation and analysis of contact thermal resistance on motor controller based on Matlab *IEEE Conference and Expo Transportation Electrification Asia-Pacific (ITEC Asia-Pacific)* 1-4

[10] Swetha G, Priya M, Gupta R and Pandey A 2017 Implementation of chaotic oscillator using Matlab *International Conference on Recent Innovations in Signal processing and Embedded Systems (RISE)* 97-101

[11] Nenov H, Dimitrov B and Marinov A. 2013 Algorithms for computational procedure acceleration for systems differential equations in matlab *36th International Convention on Information and Communication Technology, Electronics and Microelectronics (MIPRO)* 238-242.

[12] Molina A. and Lazcano G. 2018 Matlab programmed method for the optical flow estimation based on the integral image *4th International Conference on Control, Automation and Robotics (ICCAR)* 394-399

[13] Bai Y and Zhang M 2009 Visual Teaching of Numerical Analysis Based on MATLAB *First International Conference on Information Science and Engineering* 3552-3555

[14] Trimbitas R and Trimbitas M 2007 Runge-Kutta Methods and Inverse Hermite Interpolation

Ninth International Symposium on Symbolic and Numeric Algorithms for Scientific Computing 118-123
[15] Ghassemi A, Fazel S, Maghsoud I and Farshad S 2014 Comprehensive study on the power rating of a railway power conditioner using thyristor switched capacitor *IET Electrical Systems in Transportation* **4** 4 97-106
[16] Kavitha S, Varuna S and Ramya R 2016 A comparative analysis on linear regression and support vector regression *Online International Conference on Green Engineering and Technologies (IC-GET)* 1-5
[17] Hirose H, Soejima Y and Hirose 2012 K NNRMLR: A Combined Method of Nearest Neighbor Regression and Multiple Linear Regression *IIAI International Conference on Advanced Applied Informatics* 351-356
[18] Feng X , Zhou Y, Hua T, Zou Y and Xiao J 2017 Contact temperature prediction of high voltage switchgear based on multiple linear regression model *32nd Youth Academic Annual Conference of Chinese Association of Automation (YAC)* 277-280
[19] Kong W, Zhou H, Zheng K, Mu X and Hong W 2017 FFT-Based Method With Near-Matrix Compression *IEEE Transactions on Antennas and Propagation* **65** 11 5975-5983
[20] Su T, Yang M, Jin T and Flesch R 2018 Power harmonic and interharmonic detection method in renewable power based on Nuttall double-window all-phase FFT algorithm *IET Renewable Power Generation* **12** 8 953-961
[21] Laih C, Harn L and Lee J 1989 On the design of a single-key-lock mechanism based on Newton's interpolating polynomial *IEEE Transactions on Software Engineering* **15** 9 1135-1137
[22] Van Beeumen R, Michiels W and Meerbergen K 2015 Linearization of Lagrange and Hermite interpolating matrix polynomials *IMA Journal of Numerical* Analysis **35** 2 909-930
[23] Lian Y, Juncheng L and Guohua C 2010 A Class of Algebraic Trigonometric Interpolation Splines and Applications *International Conference on Computational and Information Sciences* 1174-1177
[24] Pattanadech N and Yutthagowith P 2015 Fast curve fitting algorithm for parameter evaluation in lightning impulse test technique *IEEE Transactions on Dielectrics and Electrical Insulation* **22** 5 2931-2936
[25] Wang X 2017 Strong convergence rates of the linear implicit Euler method for the finite element discretization of SPDEs with additive noise *IMA Journal of Numerical Analysis* **37** 2 965-984
[26] Simos T 2001 A fourth algebraic order exponentially-fitted Runge–Kutta method for the numerical solution of the Schrödinger equation *IMA Journal of Numerical Analysis* **21** 4 919-931
[27] Haut T, Babb T, Martinsson P and Wingate B 2016 A high-order time-parallel scheme for solving wave propagation problems via the direct construction of an approximate time-evolution operator *IMA Journal of Numerical Analysis* **36** 2 688-716
[28] Zhou W and Bovik A 2009 Mean squared error: Love it or leave it? A new look at signal fidelity measures *IEEE signal processing magazine* **26** 1 98-117
[29] Zhu L and Rivera L 2002 A note on the dynamic and static displacements from a point source in multilayered media *Geophysical Journal International* **148** 3 619-627
[30] Szyld D and Xue F 2014 Several properties of invariant pairs of nonlinear algebraic eigenvalue problems *IMA Journal of Numerical Analysis* **34** 3 921-954
[31] Yoshida H and Tanaka T 2007 Positive Controllability Test for Continuous-Time Linear Systems *IEEE Transactions on Automatic Control* **52** 9 1685-1689
[32] Pattanadech N and Yutthagowith P 2015 Fast curve fitting algorithm for parameter evaluation in lightning impulse test technique *IEEE Transactions on Dielectrics and Electrical Insulation* **22** 5 2931-2936
[33] Bakhshizadeh M 2017 The Application of Vector Fitting to Eigenvalue-Based Harmonic Stability Analysis *IEEE Journal of Emerging and Selected Topics in Power Electronics* **5** 41487-1498
[34] Ding T, Li C, Huang C, Yang Y, Li F and Blaabjerg F 2018 A Hierarchical Modeling for Reactive

Power Optimization With Joint Transmission and Distribution Networks by Curve Fitting *IEEE Systems Journal* **12** 3 2739-2748

[35] Batzelis E, Kampitsis G and Papathanassiou S 2017 Power Reserves Control for PV Systems With Real-Time MPP Estimation via Curve Fitting *IEEE Transactions on Sustainable Energy* **8** 3 1269-1280

[36] Haut T, Babb T, Martinsson P and Wingate B 2016 A high-order time-parallel scheme for solving wave propagation problems via the direct construction of an approximate time-evolution operator *IMA Journal of Numerical Analysis* **36** 2 688-716

[37] Nicolet A and Delince F 1996 Implicit Runge-Kutta methods for transient magnetic field computation *IEEE Transactions on Magnetics* **32** 3 1405-1408

[38] Darkner S, Pai A, Liptrot M and Sporring J 2018 Collocation for Diffeomorphic Deformations in Medical Image Registration *IEEE Transactions on Pattern Analysis and Machine Intelligence*, **40** 7 1570-1583

[39] Abdelgawad H and Sood V 2016 Performance analysis of boost converter by using different integration algorithms *IEEE Canadian Conference on Electrical and Computer Engineering (CCECE)* 1-5

[40] Talbi S, Kassmi K and Malek R 2016 Modeling and simulation of a solar oven box-type with thermal storage *International Renewable and Sustainable Energy Conference (IRSEC)* 348-352