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Dynamic reanalysis of spring-mass systems using sensitivity derivative

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Abstract. Studies on Structural Dynamic Modification (SDM) of beams are very important area of research work to both fields of Mechanical and Civil Engineering. Dynamic reanalysis of structures is about to find modified dynamic characteristics i.e. natural frequency with the modification of structure parameters. It plays an important role in smooth operation of structural analysis. Design of such structures is important to resist dynamic forces occurred due to natural hazardous. Modification in dynamic characteristics of a complex structure is highly expensive and time taking process. Hence the modified dynamic characteristics are evaluated using reanalysis methods. Dynamic reanalysis for structures can be done by using Taylor series in which sensitivity derivatives are substituted for finding the modified frequencies. In the present work, the dynamic reanalysis is applied for linear spring-mass systems. The results of direct method have been compared with reanalysis method. These comparisons prove preciseness of reanalysis method which is based on sensitivity derivatives and it can be considered as a powerful frame work for Eigen value analysis for modified spring-mass systems.

1. Introduction

Many structures are existing around us and majority of them were contrived by the vibrations. The consequences of vibrations are all over, particularly in engineering design. All the designs in upcoming days should be predicted and protected from the vibration effects. In order to reduce these problems there is a need to vary the structural behavior which is so called SDM problem and the modified dynamic characteristics are evaluated using reanalysis methods.

Reanalysis methods based on continuously varying cross-section beams used Rayleigh-Ritz method [1] calculating natural frequencies. Free vibration problem of tapered beam used Lagrange multiplier formalism [2] for finding a solution based on Timoshenko theory. An algorithm to find how much the system is modified [3] in physical spaces with a continuous damped beam system. Various methods in SDM which can be defined by dynamic behavior [4] of a system and improved by forecasting the modified parameter for lumped masses. A new method in which substructure

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techniques is combined with the reanalysis by applying boundary conditions[5] arising from finite element modeling of free vibration problems in large structural systems.

The present work is intended to find modified dynamic characteristics i.e. natural frequency with the modification of stiffness parameters of spring mass system in which dynamic reanalysis was used. Initially we derive the first and second order sensitivity derivatives of eigenvalues and eigenvectors. Dynamic reanalysis can be done by using Taylor series to find the modified natural frequencies of spring mass systems with the help of the sensitivity derivatives and are compared with the results of direct method.

2. Theoretical Study

Generalized vibration equation of a system can be written as

$$[M]\{\ddot{X}\} + [K]\{X\} = [F] \tag{1}$$

The solution can be obtained by solving the eigenvalue problem which can be written as

$$[M]\{\ddot{X}\} + [K]\{X\} = 0 \text{ where,}$$

λ - eigenvalues
$\{X\}$ - eigenvectors
$[M]$ - Mass Matrix
$[K]$ - stiffness matrix

$$\{[K] - \lambda [M]\}\{X\}=0 \tag{2}$$

The eigenvectors should satisfy the orthogonality conditions i.e.,

$$X_i^T [K] X_j = 0 \qquad X_j^T [M] X_i = 0$$

$$X_i^T [K] X_i = [K] \quad X_i^T [M] X_i = [M] \tag{3}$$

The first and second order eigen sensitive derivatives are obtained by differentiating equation (2) with respect to design parameter p and by applying orthogonality conditions of eigenvectors. These derivatives can be expressed as

$$\frac{\partial\lambda}{\partial P} = \frac{X^T\left[\frac{\partial K}{\partial P} - \lambda\frac{\partial M}{\partial P}\right]X}{X^T M X} \tag{4}$$

$$\frac{\partial^2\lambda}{\partial P^2} = \frac{X^T\left[\frac{\partial^2 K}{\partial P^2} - \lambda\frac{\partial^2 M}{\partial P^2} - 2\frac{\partial\lambda\partial M}{\partial P\,\partial P}\right]X + 2X^T\left[\frac{\partial K}{\partial P} - \lambda\frac{\partial M}{\partial P} - M\frac{\partial\lambda}{\partial P}\right]\frac{\partial X}{\partial P}}{X^T M X} \tag{5}$$

Now exploiting the eigenvectors for first order sensitivity derivation and the equation follows as

$$\frac{\partial X_i}{\partial P} = \frac{[-X_j^T]\left[\frac{\partial K}{\partial P} - \lambda\frac{\partial M}{\partial P}\right][X_j]}{X_j^T[K-\lambda M]X_j}[X_i] \tag{6}$$

For reanalysis using Taylor series and the equation follows as

$$\lambda_m = \lambda + \sum_{i=1}^{n} \Delta P_i \frac{\partial\lambda}{\partial P_i} + \sum_{i=1}^{n} \frac{\Delta P_i^2}{2!}\frac{\partial^2\lambda}{\partial P_i^2} + \cdots \tag{7}$$

where, λ_m - Modified natural frequency Pi - change in parameter

IOP Conf. Series: Materials Science and Engineering **402** (2018) 012030 doi:10.1088/1757-899X/402/1/012030

3. Eigen Value Reanalysis of Spring-Mass Systems

Eigenvalue reanalysis can be applied to various case studies of spring-mass systems with the following procedure.

- Write the equation of motions of the spring-mass system.
- Formulate the eigenvalue problem of the spring-mass system using equation (2).
- Eigenvalues and eigenvectors of the system are to be evaluated.
- Determine the corresponding eigenvalue sensitivity derivatives using equations (4) and (5).
- Determine the modified eigenvalues with the modification of stiffness of springs using reanalysis equation (7).
- Compare the reanalysis results by the results of direct method.

3.1 Case Study.

STEP1: The equation of motions of system in the figure-1 follows as

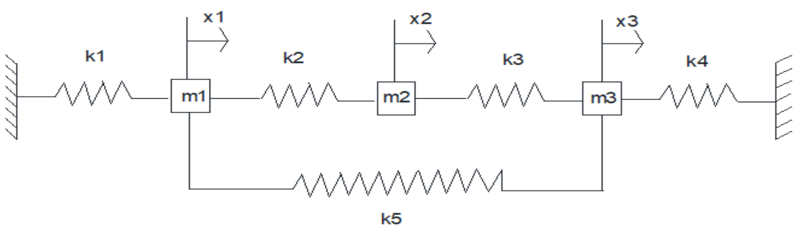

Figure 1. Spring-mass system

$$m_1\ddot{x}_1 + (k_1 + k_2 + k_5)x_1 - k_2x_2 - k_5x_3 = 0 \qquad \text{for } m_1$$

$$m_2\ddot{x}_2 + [k_2 + k_3]x_2 - k_3x_3 - k_2x_1 = 0 \qquad \text{for } m_2$$

$$m_3\ddot{x}_3 + [k_3 + k_4 + k_5]x_3 - k_3x_2 - k_5x_1 = 0 \qquad \text{for } m_3$$

STEP2: Eigen value equation for the above equations follows as

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 + k_5 & -k_2 & -k_5 \\ -k_2 & k_2 + k_3 & k_3 \\ -k_5 & k_3 & k_3 + k_4 + k_5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$[[K]-\lambda[M]][X] = 0$$

Where ,

$$K = \begin{bmatrix} k_1 + k_2 + k_5 & -k_2 & -k_5 \\ -k_2 & k_2 + k_3 & k_3 \\ -k_5 & k_3 & k_3 + k_4 + k_5 \end{bmatrix}$$

$$M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$

with assumed input values

m_1 = 1kg m_2= 2kg m_3 = 3kg

K_1 = 2000N/m K_2 = 3000N/m K_3 = 500N/m K_4= 1000N/m K_5=2500N/m

IOP Conf. Series: Materials Science and Engineering **402** (2018) 012030 doi:10.1088/1757-899X/402/1/012030

STEP 3: The eigenvalue and eigenvectors are given in the following table 1.

Table 1: Eigenvalue(rad/s) and eigenvectors of case-study

EIGEN VALUE	EIGENVECTORS
λ_1 = 436.91	X_1 = -0.3343
λ_2 =1706.1	X_2 = -0.4573
λ_3 =8440.3	X_3 = -0.33958

In the above table only the eigenvalues and corresponding eigenvectors of the given system are considered. For present case first eigenvalue is taken.

STEP 4: In the below table 2 are the eigenvalue sensitivity derivatives with respect to k1, k2, k3, k4 and k5.

Table 2: Sensitivity derivatives of case-study

System parameter	Sensitive derivation ($\partial\lambda_1/\partial K_i$)
K1	0.1118
K2	0.0151
K3	0.0038
K4	0.1567
K5	0.0038

Depending on the order of sensitivity derivatives i.e. K4, K1, K2, K3 and K5 the parameters are modified.

STEP 5: Reanalysis equation of the system is,

$$\lambda_m = \lambda + \sum_{i=1}^{n} \Delta P_i \frac{\partial \lambda}{\partial P_i} + \sum_{i=1}^{n} \frac{\Delta P_i^2}{2!} \frac{\partial^2 \lambda}{\partial P_i^2}$$

Table 3: Modified Eigenvalues of case study

Modified parameter	Increase % of parameter	Frequencies using reanalysis equation
K4	5	21.0891
	10	21.2740
	15	21.4573
	20	21.6391
	25	21.8193
	30	21.9560
	35	22.1186
	40	22.2778
(K1and K4)	5	21.3524
	10	21.7931
	15	22.2249
	20	22.6486
	25	23.0645
	30	23.4730
	35	23.8745

IOP Conf. Series: Materials Science and Engineering **402** (2018) 012030 doi:10.1088/1757-899X/402/1/012030

	40	24.2694
(k4,K1and K2)	5	21.3546
	10	21.7974
	15	22.2313
	20	22.6569
	25	23.0747
	30	23.4851
	35	23.8884
	40	24.2850
(k4,K1,K2, K3 and K5)	5	21.4187
	10	21.9227
	15	22.4154
	20	22.8976
	25	23.3697
	30	23.8325
	35	24.2865
	40	24.7322

ΔP_i is change in parameter and refers to parameters k1, k2, k3, k4 and k5. By varying the stiffness values, the modified eigenvalues of the system are shown in the above table 3.

4. Results and Conclusion

4.1 Results

Comparing reanalysis results with direct results are given in the following table 4.

Table 4: Comparing results of Case Study

Modified parameter	Modification (%)	Reanalysis (rad/s)	Direct(rad/s)	Error(%)
K4	5	21.0891	21.0878	0.00061
	10	21.2740	21.2691	0.02303
	15	21.4573	21.4465	0.05035
	20	21.6391	21.6200	0.08834
	25	21.8193	21.7898	0.13538
	30	21.9560	21.9560	0.19174
	35	22.1186	22.1186	0.25679
	40	22.2778	22.2778	0.33037
(K1 and K4)	5	21.3524	21.3494	0.01405
	10	21.7931	21.7813	0.03417
	15	22.2249	22.1990	0.11667
	20	22.6486	22.6035	0.19952
	25	23.0645	22.9954	0.30049
	30	23.4730	22.3755	0.41710
	35	23.8745	23.7443	0.54834
	40	24.2694	24.1025	0.69245
(k4,K1 and K2)	5	21.3546	21.3519	0.01264
	10	21.7974	21.7867	0.04911
	15	22.2313	22.2079	0.10536
	20	22.6569	22.6164	0.17909
	25	23.0747	23.0129	0.26854
	30	23.4851	23.3981	0.37182

	35	23.8884	23.7728	0.48627
	40	24.2850	24.1374	0.61149
(k4,K1,K2 , K3 and K5)	5	21.4187	21.4157	0
	10	21.9227	21.7557	0.7676
	15	22.4154	22.0872	1.4859
	20	22.8976	22.4136	2.15940
	25	23.3697	22.7351	2.79127
	30	23.8325	23.0519	3.38627
	35	24.2865	23.3642	3.94749
	40	24.7322	23.6722	4.47782

The maximum error between direct and reanalysis method is 4.47782 and the minimum error is 0 in last case. It shows that reanalysis method coincides with the direct method. The results obtained using reanalysis method and direct method are shown in graphs for comparing the (%) of error in frequencies for case study. The graphs are plotted from the results of above tables 4 of case study.

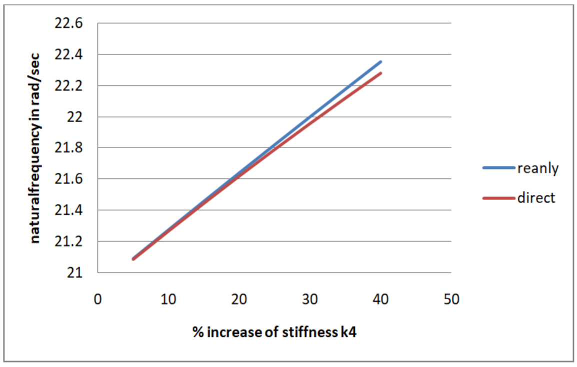

Figure 2. Varying parameter K4

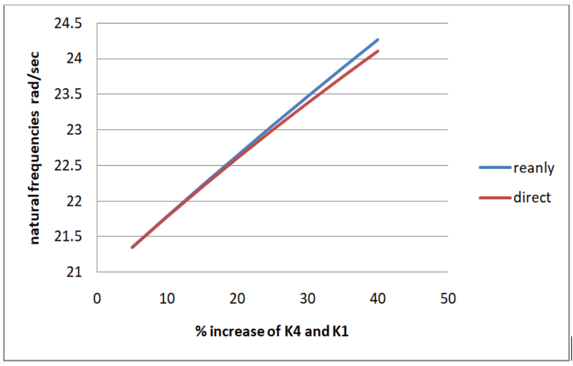

Figure 3. Varying parameter K4 and K1

As per the order of high sensitive parameter in figure 2, the % of stiffness is varied in parameter k4 and then in figure 3 the % of stiffness is varied in parameter K4 and K1.

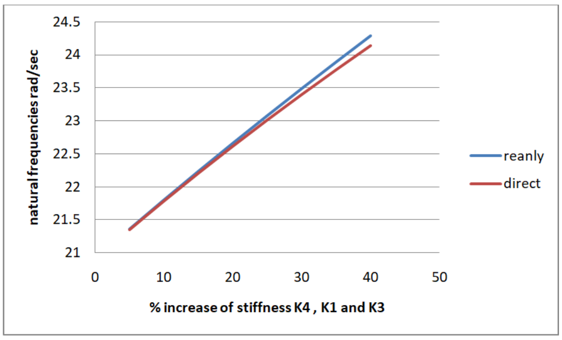

Figure 4. Varying parameter K4, K1 and K3

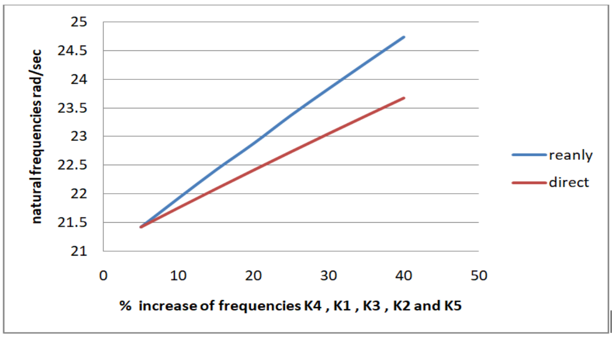

Figure 5. Varying parameter K4, K1, K3, K2 and K5

As per the order of high sensitive parameter in figure 4, the % of stiffness is varied in parameter k4, K1 and K3 and then in figure 5 the % of stiffness is varied in parameter K4, K1, K3, K2 and K5.

The results are plotted for reanalysis and direct methods which show that the values of reanalysis are close to direct method.

4.2 Conclusion

The following are the conclusions that can be drawn from the dynamic reanalysis of spring-mass systems:

- This method economize the time and cost to the scope of realizing the Eigen values not actually using direct method solution methods on spring-mass systems.
- The values of the frequencies obtained by dynamic reanalysis method are to be found very near to the values got by direct method. As shown in the graphs the error incurred in this method is less and inconsiderable which can be taken for practical situations also.
- The values of a particular parameter (K4, K3, K2, K1 and K5) of the system which are highly sensitive were identified and obtained required natural frequencies by using reanalysis method.

5. References

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