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The dynamic stability analysis of telescopic booms of the crane based on the energy method

S Liu[1*], J Liu[1], K Zhang[1] and L Meng[1]

[1] School of Mechanical Engineering, Shenyang Jianzhu University, Shenyang 110168, China

[*] E-mail: liushiming_1983@163.com

Abstract. According to the energy method, the dynamic stability of a crane's telescopic boom under the periodic load is studied, which is known as parametric resonance, based on the design code for a crane (GB/T3811-2008), the stability analysis model of the box telescopic boom can be regarded as variable-section stepped columns. The parametric vibration equation of n-stepped columns which expressed as Mathieu Equation is deduced by the Hamilton Principle, then the Critical frequency equation of dynamic instability regions of the telescopic booms are derived, finally, the effects of damping to the dynamic stability of telescopic boom are discussed. The results show that~~,~~ the dynamic instability regions are reduced when the damping coefficient is increased and the effect on the second dynamic instability region is more obvious, that means damping improves the structure's dynamic stability.

1. Introduction

The telescopic boom is the main load-carrying part of the truck crane and is the most important working part. When the lifting quality rises or falls off the ground during the braking process or suddenly unloads, the inertia of the hoisting crane and the flexibility of the structure can cause vibration of the telescopic boom, especially when the telescopic boom is in a fully extended state and at minimum amplitude, the telescopic boom structure may suffer from dynamic instability. Based on the crane design specification (GB/T3811-2008), the calculation model of telescopic boom static instability is a variable cross-section stepped column, and its instability critical force can be solved by the energy method, the exact finite element method, etc [1-3] . Therefore, in the study of the dynamic stability of the crane telescopic boom, the telescopic boom is also equivalent to a variable cross-section stepped column model, which is in line with the actual engineering requirements. To solve the dynamic stability under cyclic loading, the method of the former Soviet Union scholar~~s~~, Bolotin is often used to determine the dynamic stability and dynamic instability regions of the structure [4]. Sun Qiang [5] used the Mathieu-Hill equation to study the dynamic stability of isometric straight bars under cyclic loading. Li Xiaodong [6] determined the dynamic instability region of the buckling restrained supporting bar under cyclic loading according to the Bolotin method. Ratko Pavlovic [7] studied the dynamic stability of an isometric straight rod under the combined effect of axial and bending moments by using the differential equation method; MA De Rosa [8] studied the dynamic stability of the continuously variable beam with the DQM method. However, the above studies on the dynamic stability under axial cyclic loading mainly focus on single components with continuous section changes. The dynamic stability of the stepped columns with discontinuous changes in the cross-section and the multiple structures composed of multiple individual components are investigated. In [9-11], the finite element method was used to study the dynamic stability of the crane's telescopic boom under cyclic loading. Although this method can effectively solve the dynamic stability problem of the telescopic boom under cyclic loading, it has the disadvantage of a complicated solution which is not conducive to the practical application of engineering. Therefore, in this paper, the energy method combined with the Hamilton principle is used to establish the parametric vibration equation of the telescopic boom which is expressed in the form of the Mathieu equation. The critical frequency equation of the boundary of the dynamic instability region of the telescopic arm is deduced, and the influence of the damping on the unstable region of the dynamics is discussed. The proposed method provides a basis for design calculations and can guide the practical application of engineering.

2. Establishment of parametric vibration equation of telescopic boom

The dynamic stability analysis model of the telescopic boom of a truck crane can be equivalent to a multi sectioned column, the mechanical models are shown in figure. 1. Assuming l_i as the total length of i sections telescopic boom. L is the total length of the telescopic boom namely when $i=n$, $L=l_n$. m_i is the mass per unit length of section i, E is the elastic modulus of the material ,J_i is the inertia moment of section i, the axial resonant force $P_0+P_t\cos\theta t$ is applied at the top of the column, where P_0, P_t are the amplitudes of the resonant force respectively, θ is the resonant circular frequency.

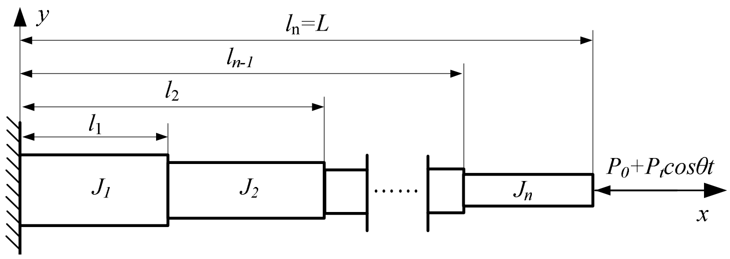

Figure 1. The mechanical model of dynamic stability for multi sectioned telescopic boom.

Assuming the lateral vibration displacement curve of the telescopic arm is $V(x,t)=f(t)(1-\cos\frac{\pi x}{2L})$ which conforms to the boundary conditions of figure.1. Then the kinetic energy of the telescopic boom $T(t)$ is obtained by using the energy principle

$$\begin{aligned} T(t) &= \frac{1}{2}\int_0^{l_1} m_1(\frac{\partial V}{\partial t})^2 dx + \frac{1}{2}\int_{l_1}^{l_2} m_2(\frac{\partial V}{\partial t})^2 dx + \cdots + \frac{1}{2}\int_{l_{n-1}}^{l_n} m_n(\frac{\partial V}{\partial t})^2 dx \\ &= \dot{f}^2(t)\sum_{i=1}^{n}\frac{m_i L}{4}(\frac{l_i-l_{i-1}}{L} - \frac{\sin(\frac{2\pi l_i}{L})-\sin(\frac{2\pi l_{i-1}}{L})}{2\pi}) \end{aligned} \tag{1}$$

The potential energy of the telescopic boom is expressed as $U(t)=U_1(t)+U_2(t)$, where $U_1(t)$ is the bending strain energy ,$U_2(t)$ is the external potential energy.

$$\begin{aligned} U_1(t) &= \frac{1}{2}\int_0^{l_1} EJ_1(\frac{\partial^2 V}{\partial x^2})^2 dx + \frac{1}{2}\int_{l_1}^{l_2} EJ_2(\frac{\partial^2 V}{\partial x^2})^2 dx + \cdots + \frac{1}{2}\int_{l_{n-1}}^{l_n} EJ_n(\frac{\partial^2 V}{\partial x^2})^2 dx \\ &= \frac{\pi^4 f^2(t)}{4L^3}\sum_{i=1}^{n} EJ_i(\frac{l_i-l_{i-1}}{L} - \frac{\sin(\frac{2\pi l_i}{L})-\sin(\frac{2\pi l_{i-1}}{L})}{2\pi}) \end{aligned} \tag{2}$$

$$U_2(t) = -\frac{1}{2}\int_0^L (P_0+P_t\cos\theta t)(\frac{\partial V}{\partial x})^2 dx = -\frac{\pi^2(P_0+P_t\cos\theta t)}{4L}f^2(t) \tag{3}$$

The damping of the telescopic boom in vibration is equivalent to linear viscous damping, then, the energy consumed by the damping $D(t)$ is gained.

$$\begin{aligned} D(t) &= \frac{1}{2}\int_0^{l_1} C_1(\frac{\partial V}{\partial t})^2 dx + \frac{1}{2}\int_{l_1}^{l_2} C_2(\frac{\partial V}{\partial t})^2 dx + \cdots + \frac{1}{2}\int_{l_{n-1}}^{l_n} C_n(\frac{\partial V}{\partial t})^2 dx \\ &= \dot{f}^2(t)\sum_{i=1}^{n}\frac{C_i L}{4}(\frac{l_i - l_{i-1}}{L} - \frac{\sin(\frac{2\pi l_i}{L}) - \sin(\frac{2\pi l_{i-1}}{L})}{2\pi}) \end{aligned} \tag{4}$$

where: C_i is the equivalent damping coefficient of section i; EJ_i is the flexural rigidity of section i. According to the Hamilton Principle,

$$\begin{cases} L = T(t) - U(t) \\ \frac{d}{dt}(\frac{\partial L}{\partial \dot{f}}) - \frac{\partial L}{\partial f} + \frac{\partial D}{\partial \dot{f}} = 0 \end{cases} \tag{5}$$

Substitute equation (1) - (4) into equation (5)

$$f''(t) + 2\beta f'(t) + \Omega^2(1 - 2\mu\cos\theta t)f(t) = 0 \tag{6}$$

Equation (6) is the famous Mathieu equation, assuming the dimensionless coefficient $\alpha_i = l_i / L (i = 0,1,2,\cdots n)$, $\lambda_i = J_i / J_1 (i = 1,2,\cdots n)$, $\alpha_0 = 0$, then the symbols in equation(6) can be further expressed as follows

$$\Omega = \omega\sqrt{1 - \frac{P_0}{\rho}} \tag{7}$$

$$\mu = \frac{P_t}{2(\rho - P_0)} \tag{8}$$

$$\omega^2 = \frac{\frac{\pi^4 EJ_1}{(2L)^4}\sum_{i=1}^{n}\lambda_i((\alpha_i + \frac{\sin(\pi\alpha_i)}{\pi}) - (\alpha_{i-1} + \frac{\sin(\pi\alpha_{i-1})}{\pi}))}{\sum_{i=1}^{n} m_i((3\alpha_i - \frac{8\sin(\pi\alpha_i/2)}{\pi} + \frac{\sin(\pi\alpha_i)}{\pi}) - (3\alpha_{i-1} - \frac{8\sin(\pi\alpha_{i-1}/2)}{\pi} + \frac{\sin(\pi\alpha_{i-1})}{\pi}))} \tag{9}$$

$$\rho = \frac{\pi^2 EJ_1}{(2L)^2}\sum_{i=1}^{n}\lambda_i((\alpha_i + \frac{\sin(\pi\alpha_i)}{\pi}) - (\alpha_{i-1} + \frac{\sin(\pi\alpha_{i-1})}{\pi})) \tag{10}$$

$$\beta = \frac{\sum_{i=1}^{n}\frac{C_i L}{4}(\alpha_i - \alpha_{i-1} - \frac{\sin(2\pi\alpha_i) - \sin(2\pi\alpha_{i-1})}{2\pi})}{2\sum_{i=1}^{n}\frac{m_i L}{4}(\alpha_i - \alpha_{i-1} - \frac{\sin(2\pi\alpha_i) - \sin(2\pi\alpha_{i-1})}{2\pi})} \tag{11}$$

where: Ω is the natural frequency of the lateral free vibration of the telescopic arm under the axial resonance force amplitude P_0, and the unit is Hz, μ is the excitation coefficient, ω is the natural frequency of lateral free vibration of the stepped column without any load, and the unit is Hz, ρ is the static destabilizing critical load of the stepped column, the unit is N, β is the dimensionless damping coefficient.

Equations (6) - (11) are the parametric vibration equations of n sections crane telescopic boom.

3. Determination of dynamic stability and instability regions of telescopic boom

The equation (6) is a Mathieu equation, we judge it must have periodic solutions of T and $2T$ based on the properties of the mentioned equation. In order to get the critical frequency equation of the dynamic stability region, the Fourier series expansion is used to get solutions of period $2T$ and T, and they can be expressed respectively as follows:

$$f(t) = \sum_{n=1,3,5\cdots}^{\infty} a_n \sin\frac{n\theta t}{2} + b_n \cos\frac{n\theta t}{2} \tag{12}$$

$$f(t) = b_0 + \sum_{n=2,4,6\cdots}^{\infty} a_n \sin\frac{n\theta t}{2} + b_n \cos\frac{n\theta t}{2} \tag{13}$$

Substituting equation (12) and equation (13) into equation (6), in order to make the establishment of equations, each coefficient of sin ($n\theta t/2$) and cos ($n\theta t/2$) must be equal to zero. The linear equations in terms of $\{a_1,b_1,a_3,b_3, \cdots a_{2k-1},b_{2k-1}\}^T$ and $\{b_0,a_2,b_2,a_4,b_4,\cdots a_{2k},b_{2k}\}^T$ can be derived, the necessary and sufficient condition for a system of linear equations to have a nonzero solution is that the determinant of its coefficient matrix must be zero. Therefore, let the mentioned determinant of coefficient be equal to zero, then this infinite determinant is convergent and it can be expressed as follows.

The corresponding critical frequency equation for the $2T$ cycle is as follows:

$$\begin{vmatrix}
1+\mu-\frac{\theta^2}{4\Omega^2} & -\frac{\theta\beta}{\Omega^2} & -\mu & 0 & 0 & 0 & \vdots & \vdots \\
\frac{\theta\beta}{\Omega^2} & 1-\mu-\frac{\theta^2}{4\Omega^2} & 0 & -\mu & 0 & 0 & \vdots & \vdots \\
-\mu & 0 & 1-\frac{9\theta^2}{4\Omega^2} & -\frac{3\theta\beta}{\Omega^2} & -\mu & 0 & \vdots & \vdots \\
0 & -\mu & \frac{3\theta\beta}{\Omega^2} & 1-\frac{9\theta^2}{4\Omega^2} & 0 & -\mu & \vdots & \vdots \\
0 & 0 & -\mu & 0 & 1-\frac{25\theta^2}{4\Omega^2} & -\frac{5\theta\beta}{\Omega^2} & \vdots & \vdots \\
0 & 0 & 0 & -\mu & \frac{5\theta\beta}{\Omega^2} & 1-\frac{25\theta^2}{4\Omega^2} & \vdots & \vdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{vmatrix} = 0 \tag{14}$$

where: n=1, 3, 5, 7,⋯2k-1.

The corresponding critical frequency equation for the T cycle is as follows:

$$\begin{vmatrix}
1 & 0 & -\mu & & & & & \vdots \\
0 & 1-\frac{\theta^2}{\Omega^2} & -\frac{2\theta\beta}{\Omega^2} & -\mu & 0 & & & \vdots \\
-2\mu & \frac{2\theta\beta}{\Omega^2} & 1-\frac{\theta^2}{\Omega^2} & 0 & -\mu & 0 & & \vdots \\
0 & -\mu & 0 & 1-\frac{4\theta^2}{\Omega^2} & -\frac{4\theta\beta}{\Omega^2} & -\mu & 0 & \vdots \\
0 & 0 & -\mu & \frac{4\theta\beta}{\Omega^2} & 1-\frac{4\theta^2}{\Omega^2} & 0 & -\mu & \vdots \\
0 & 0 & 0 & -\mu & 0 & 1-\frac{9\theta^2}{\Omega^2} & -\frac{9\theta\beta}{\Omega^2} & \vdots \\
0 & 0 & 0 & 0 & -\mu & \frac{9\theta\beta}{\Omega^2} & 1-\frac{9\theta^2}{\Omega^2} & \vdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{vmatrix} = 0 \tag{15}$$

where: n = 2, 4, 6, 8,⋯2k.

Equation (14) and equation (15) are the critical frequency equations of the boundary of the dynamic unstable region of the structure. The critical frequency refers to the external load frequency corresponding to the boundary of the dynamic unstable region. However, the first and second areas of dynamic stability are some of the most dangerous in practical engineering.

The critical frequency equation of the first dynamic unstable region is obtained by substitute n=1 into equation (14), that also means keeping the first order solution of the determinant of $2T$ cycle

$$\begin{vmatrix} 1+\mu-\dfrac{\theta^2}{4\Omega^2} & -\dfrac{\theta\beta}{\Omega^2} \\ \dfrac{\theta\beta}{\Omega^2} & 1-\mu-\dfrac{\theta^2}{4\Omega^2} \end{vmatrix}=0 \tag{16}$$

By solving equation (16), we can find:

$$\theta_{1,2}=2\sqrt{\Omega^2-2\beta^2\pm\sqrt{4\beta^2\left(\beta^2-\Omega^2\right)+\mu^2\Omega^4}} \tag{17}$$

Using the same method, the critical frequency equation of the second dynamic unstable region is obtained by substitute n=2 into equation(15), that also means keeping the second order solution of the determinant of T cycle, then we can get:

$$\theta_{3,4}=\sqrt{\Omega^2\left(1-\mu^2\right)-2\beta^2\pm\sqrt{4\beta^4-2\beta^2\Omega^2\left(1-\mu^2\right)+\mu^4\Omega^4}} \tag{18}$$

Under the action of a given axial resonance force $P(t)=P_0+P_t\cos\theta t$, the boundary region composed of critical frequencies θ can be determined, the region of dynamic stability is limited by the critical frequency of different cycles, and the region of dynamic instability is determined by two critical frequencies of the same cycles. That is, the dynamic unstable region is separated from the stable region by the solution of the cycles T and $2T$, and the critical frequency between the two cycles is the dynamic stable region. By substitute equation (7), equation (8) and equation (11) into equation (17) and equation (18), then the dynamic stable region and unstable region of crane's telescopic boom under periodic load can be obtained.

4. Examples

Taking the telescopic boom of a typical five-section crane as an example, the dynamic stability calculation model is a cantilever stepped beam which is shown in figure 2, the total length of the stepped column L=30m, Young's modulus E=200GPa, Sectional moment of inertia $J_1=1.1786\times10^{2}\text{m}^4$, density 7800kg/m^3, $l_1=0.24L$, $l_2=0.43L$, $l_3=0.62L$, $l_4=0.81L$, $l_5=L$, $\lambda_1=1.0$, $\lambda_2=J_2/J_1=0.77$, $\lambda_3=J_3/J_1=0.59$, $\lambda_4=J_4/J_1=0.46$, $\lambda_5=J_5/J_1=0.35$, analyze the dynamic stability of the telescopic crane when the cyclic axial loads are $P(t)=P_0+P_t\cos\theta t$.

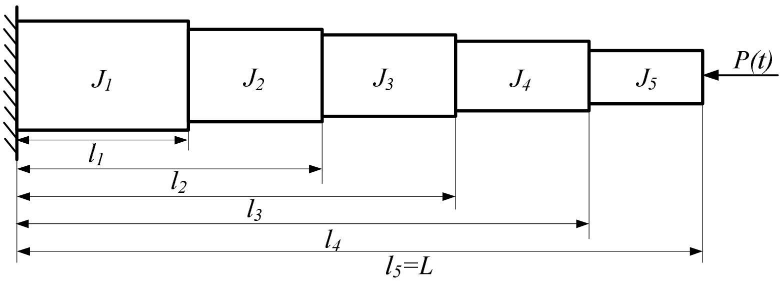

Figure 2. The calculation model of 5-section telescopic boom.

In order to analyze the influence of different values P_0 on the dynamic instability of the telescopic boom, the value of P_0 is chosen by 0 N, 0.5×10^6 N, 1.0×10^6 N, 1.5×10^6 N, 2.0×10^6 N respectively, assuming the damping coefficient β=0, the amplitude of the vibration force of the periodic load P_t is taken as the horizontal coordinate, and the critical vibration frequency $\theta/2\Omega$ is taken as the vertical coordinate. Drawing the first and second dynamic stability curves of a five-section telescopic arm under different axial resonant force amplitudes P_0 and P_t , which is shown in figure 3. The figures show that with the increase of P_0, the area of dynamic instability increases, when P_0 is equal to the static destabilizing critical load ρ, no irrespective of the value P_t is, the structure will still lose bearing capacity.

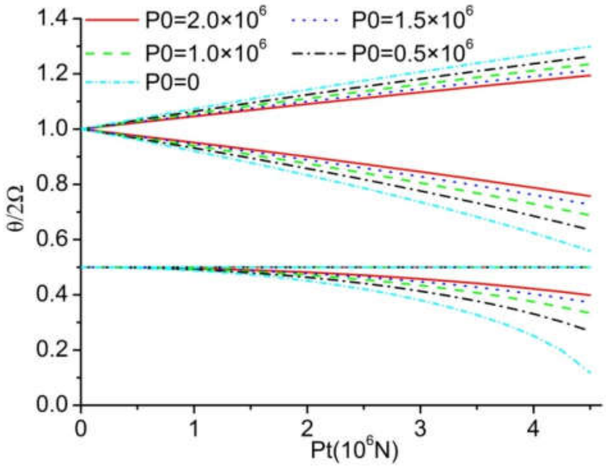

Figure 3. Dynamic curves of clamped 5-sections stepped column under different axial resonant forces.

In order to study the influence of damping on the dynamic unstable region, taking the 5-sections crane telescopic boom shown in figure 2 as an example, assume $P(t)=P_0+P_t\cos\theta t$, $P_0=1.0\times10^6$ N. Making P_t as the horizontal coordinate~~,~~ and the critical vibration frequency $\theta/2\Omega$ as the vertical coordinate, according to the energy method, the first and second dynamic stability region curves of a five-section telescopic arm under different damping coefficient β=0, 0.2, 0.4, 0.6 are drawn, which is shown in figure 4.

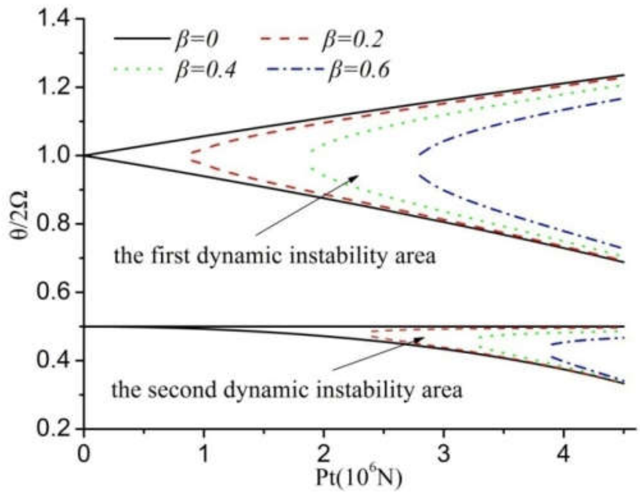

Figure 4. Parametric vibration curves of telescopic boom with different damping coefficients.

According to figure 4, it can be seen that as the damping coefficient gradually increases, the instability region of the telescopic boom decreases, the existence of damping makes the instability region smaller, and the influence on the second instability region is more obvious. At the same time, the existence of damping "cut~~s~~ offs" the unstable region connected with the ordinate~~,~~ which makes it impossible for the structure of the crane's telescopic boom to be unstable when the load amplitude P_t is small. The results show that the existence of damping reduces the possibility of dynamic instability of the crane telescopic boom, that is, the presence of damping is conducive to structural dynamic stability.

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5. Conclusions

In this paper, based on the energy method, the Hamiltonian principle is used to establish the parametric vibration equation of the crane telescopic arm expressed in the form of Mathieu equation. The critical frequency equation of the dynamic instability boundary of the telescopic boom is deduced, and the dynamic stability region and the dynamic instability region of the telescopic boom are determined, even more, the impact of damping on dynamic stability is discussed. The results show that the dynamic instability region increases with the increase of the amplitude of the axial resonance force P_0. When $P_0=\rho$, the structure loses the bearing capacity regardless of the value of P_t; With the gradual increase of the damping coefficient, the dynamic instability region decreased~~,~~ and the impact on the second dynamic instability area is more obvious.

Acknowledgments

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