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Competing risk models in reliability systems, an exponential distribution model with Bayesian analysis approach

I Iskandar

Faculty of Manufacturing Engineering, Universiti Malaysia Pahang, Pekan 26600, Pahang, Malaysia

ismed_iskandar@yahoo.com

Abstract. The exponential distribution is the most widely used reliability analysis. This distribution is very suitable for representing the lengths of life of many cases and is available in a simple statistical form. The characteristic of this distribution is a constant hazard rate. The exponential distribution is the lower rank of the Weibull distributions. In this paper our effort is to introduce the basic notions that constitute an exponential competing risks model in reliability analysis using Bayesian analysis approach and presenting their analytic methods. The cases are limited to the models with independent causes of failure. A non-informative prior distribution is used in our analysis. This model describes the likelihood function and follows with the description of the posterior function and the estimations of the point, interval, hazard function, and reliability. The net probability of failure if only one specific risk is present, crude probability of failure due to a specific risk in the presence of other causes, and partial crude probabilities are also included.

1. Introduction

Reliability theory concerns the ability of a component or a system, either a life creature or an equipment, to be functioning during its expected length of life. The studies of reliability theory become more important because they are generally more expensive than measurement data in most quality control processes. The estimation of the parameters of the assumed failure models may be based on data collected over life tests or obtained from past engineering experience or from handbook data, if available. Analysis of data, either historical or from life tests, may use any of the classical parameter estimation methods such as maximum likelihood, matching moments, regression, etc. However, most of recent analysis so far has been limited to single risk-mode models. A system or component only considered as in function or fail only without considering the causes of its failure.

But in the real systems, the failure may be a phenomenon within a single item such as the tread wear, puncture, or defective side walls or from other causes of an automobile tire. Or it can happen in physically distinct components of a system such as the CPU, hard-drive, or monitor, of a computer system. This phenomenon is commonly called as competing risks. The competing risks theory describes how many causes of failure act together to affect the performance of a system.

Another problem in this area: the failure data together with their causes of failure are simply quantitatively inadequate, time consuming and expensive to perform the life tests, especially in engineering areas. For these cases, Bayesian analyses are more beneficial than classical one. The Bayesian estimation analyses allow us to combine past knowledge or experience with less data available (which reflect the competing risks phenomenon) in the form of an apriori distribution to make inferences



of the parameter of interest. Although Bayesian estimation has been used widely in reliability analysis, the application so far has been limited to single risk-mode models.

Competing risks models have been studied for many years but not extensively. The estimation methods, however, have been classical. By considering all possible causes of failure for our estimations will lead us to the possibility of data scarcity for each cause and make the classical estimation methods difficult to apply. As the result it will also lead us to the wrong conclusion about our system reliability. To address this problem, the author propose to develop the robust and more precised competing risk models of a complex reliability system using the Bayesian analyses approach that can address the problems and provide the scientists and engineers with a better reliability design models of products and maintenance systems. Three models have been developed as mentioned above. More models are still available to be explored and developed.

This paper is an extended works of the author on developing some competing risks model in reliability engineering. Three models by using Weibull and Gamma distributions as well as for attribute type of data have been published previously [1][2][3].

2. Literature Review

The theory of competing risks has been in use in bio-medicine for a long time. It began in 1760 with Daniel Bernoulli when he tried to determine mathematically what would happen to a population mortality structure at different ages if smallpox were eliminated from that population. Most of the currently accepted techniques were developed during the 19th century when the problem had been of great interest and importance to actuaries for over 100 years. Since that time scientists, mostly in Bio-medical areas but a few in engineering , have put their contributions for the basic understanding and theory of the competing risks and its application.

The methods used in analyzing survival data was discussed with competing risks by using SAS software [4]. A software was used to implement the appropriate nonparametric methods for estimating cumulative incidence functions and for testing differences between these functions in multiple groups. A model was proposed in which it was applied to accelerated degradation data from plastic substrate active matrix light-emitting diodes (AMOLEDs), along with sensitivity analysis [5]. The reliability of the model was estimated with competing risks analyses from linear degradation and failure time data. Competing risk data were analyzed in the presence of complete information of failure cause [6]. In their paper, the authors interest was to consider the occurrence of missing causes as well as interval censored failure time. The competing risk of death and premature assessment of neurological prognosis pose significant challenges was studied to measuring these surrogate endpoints after cardiac arrest in order a need for new therapies to improve outcomes after cardiac arrest [7]. Competing risk analysis was also applied for patients who die early are precluded from developing on the incidence of ventilator-associated pneumonia (VAP) [8]. Interval censored failure time data with competing risks and a new estimator was analyzed for the cumulative incidence function [9]. It was derived using an approximate likelihood and a test statistic to compare two samples and then obtained by extending Sun's test statistic. Small sample properties of the proposed methods are examined by conducting simulations and a cohort dataset from AIDS patients is analyzed as a real example.

Numerous scientists have contributed and provided the philosophical basis for Bayesian analysis in reliability systems and use it in many areas of application and research. They are not to be mentioned here because so many of them doing research in this area and because their Bayesian method of analysis were for single risk-mode model only. There were a limited number of scientists who studied and analysed the competing risks problems by using a Bayesian analyses approach in order to estimate the parameters of interest. Their models were mostly for applications in a specific condition but not for the basic understanding of general model applications. Bayesian analysis of system failure data was used under a competing-failure framework when the failure causes have not been exactly identified but narrowed down to a subset of all potential causes of failure [10]. The model of recurrent events used Bayesian reference-analysis with competing risks, where the sampling distribution of the observations due to some failure types is modeled through a proportional intensity homogeneous Poisson process [11]. Individuals are expected to experience repeated events along with the fixed covariates. The goal was to estimate the cause-specific intensity functions.

For competing risks models where the causes of failures are being considered, the life test data scarcity for each cause is inevitable. In many situations, especially in engineering areas, these life test data are time consuming to obtain and expensive to perform the life tests. For these cases, Bayesian analyses are more beneficial than classical one. To deal with this situation, some basic notions have been developed on a general competing risks model in reliability system using a Bayesian analysis approach [12]. Also three papers that deal with this matter using Weibull and Gamma as posterior functions and Uniform distribution as prior distribution for both posteriors have been published [2][3]. Another paper was for attribute type of life test data with Multi-nomial distribution model as the posterior and Beta distribution function as its prior [1].

3. Exponential Distribution Model

This section describes the likelihood function and follows with the description of the posterior function. A non-informative prior is used in our analysis. It is followed by the estimation of the failure rate and hazard function. The net, crude, and partial crude probabilities are also included.

The individual p.d.f. of the length of life t_i is given by

$$g_i(t_i|\lambda_i) = \lambda_i e^{-\lambda_i t_i}, \quad (1)$$

for $i = 1, \dots, k$, $\lambda_i > 0$, and $t_i \geq 0$. The parameter λ_i is known as the failure rate.

3.1. The Likelihood Function

Let us consider the case in which all lengths of life and associated causes of failure are known. The individual likelihood function is given by

$$L_i(\lambda_i) \propto \lambda_i^{n_i} \exp(-\lambda_i T) \quad (2)$$

and the statistic

$$T = \sum_{i=1}^k \sum_{j=1}^{n_i} t_{ij} \quad (3)$$

is the sufficient for the estimation of λ_i . The t_{ij} are the failure times for $i = 1, 2, \dots, k$ and $j = 1, 2, \dots, n_i$.

3.2. The Posterior Function

The individual posterior function is given by

$$g_i(\lambda_i|t_{ij}) = \frac{\lambda_i^{n_i} \exp(-\lambda_i T) f_i(\lambda_i)}{\int_0^{\infty} \lambda_i^{n_i} \exp(-\lambda_i T) f_i(\lambda_i) d\lambda_i}, \quad (4)$$

where $f_i(\lambda_i)$ is the individual prior distribution.

3.3. Non-informative Prior Distribution

Taking the logarithm of the individual likelihood function in equation (4.33) above, we will obtain

$$\ln L_i(\lambda_i) = s \ln \lambda_i - \lambda_i T + c, \quad (5)$$

where c is a constant.

Since the derivation of non-informative prior is mathematically very close associated with the Fisher's information [13] that is given by

$$J(\hat{\lambda}_i) = \left(\frac{n_i}{\hat{\lambda}_i^2} \right)_{\lambda_i = \hat{\lambda}_i}, \quad (6)$$

a non-informative prior distribution for λ_i can be obtained as follows

$$\begin{aligned} f_i(\lambda_i) &\propto J^{1/2}(\lambda_i) \\ &\propto \frac{1}{\lambda_i}, \quad \lambda_i > 0. \end{aligned} \quad (7)$$

3.4. Failure Rate Estimation

The individual posterior function in equation (4.35) becomes

$$g_i(\lambda_i | t_{ij}) = \frac{T^{n_i}}{\Gamma(n_i)} \lambda_i^{n_i-1} \exp(-\lambda_i T), \quad 0 < \lambda_i < \infty. \quad (8)$$

This is a *gamma distribution* with the shape parameter n_i and the scale parameter T .

3.5. Point Estimation

The individual posterior mean is given by

$$E_i(\lambda_i | t_{ij}) = h_i(t) = \frac{n_i}{T}, \quad (9)$$

and the individual posterior variance is as follows

$$Var_i(\lambda_i | t_{ij}) = \frac{n_i}{T^2}. \quad (10)$$

This posterior mean is also called as the hazard function or the failure rate.

The hazard function for the system is as follows

$$h(t) = \sum_{i=1}^k \frac{n_i}{T}, \quad (11)$$

We know that all functions above are constant over time.

3.6. Interval Estimation

The $100(1-\gamma)\%$ two-sided Bayes confidence interval (TBCI) equations for each individual posterior mean are given by

$$\lambda_{i,L} = \frac{\chi_{\gamma/2}^2(2n_i)}{2T}, \quad (12)$$

and

$$\lambda_{i,U} = \frac{\chi^2_{1-\gamma_i/2}(2n_i)}{2T}, \quad (13)$$

where $\lambda_{i,L}$ and $\lambda_{i,U}$ are the lower and the upper bounds of the interval, respectively.

3.7. Reliability Estimation

The reliability of the system is given by

$$R_{\text{sys}}(t|\lambda_i) = \exp\left\{-\left(t\right)\sum_{i=1}^k \frac{n_i}{T}\right\}. \quad (14)$$

3.8. The Net, Crude, and Partial Crude Probabilities

The net probability of failure, i.e. probability of failure within a particular time period if the specific risk is the only risk present, is given by

$$q_{i,\text{net}} = 1 - \exp\left\{-\left(t\right)\sum_{i=1}^k \frac{n_i}{T}\right\}. \quad (15)$$

The crude probability of failure, i.e. the probability refers to the failure within a particular time interval from a specific cause in the presence of all other risks, is given by.

$$Q_{i,\text{crude}} = \frac{n_i}{\sum_{i=1}^k n_i} \left\{1 - \exp\left(-\left(t\right)\sum_{i=1}^k \frac{n_i}{T}\right)\right\}. \quad (16)$$

and the partial crude probability of failure, i.e. the probability of failure from a specific cause when another risk (or risks) is eliminated from the population, is given by

$$Q_{i,1,\text{prt.crude}} = \frac{n_i}{\sum_{i=1}^k n_i - n_1} \exp\left\{-\frac{t}{T}\left(\sum_{i=1}^k n_i - n_1\right)\right\}. \quad (17)$$

4. Conclusions

There are some models that are open to be developed in this area, depending on the problems of interest. In this paper we select our investigation by using the exponential distribution as a posterior function and non-informative distribution as its prior. This distribution is widely used in reliability analysis for a constant hazard rate and is one of some other common failure models as well as Weibull, exponential, Normal, Log-normal, and Inverse Gaussian models. It should be noted that the choice of the time to failure distribution functions depend on the choice of its reliability measures that is meaningful and useful to the problem of interest. Different pattern of operating life of a device may required to have a specific probability of successfully performing its required function.

The estimations of this exponential model for its individual posterior function (the individual mean time to failure / MTTF) and variance are developed and followed by the estimations of the hazard and the reliability functions. Based on these estimations, others such as the estimations for reliable or design life for a specific reliability and the probability of survival between a specific interval of time can also be calculated. One important part of competing risks analysis is its ability to elaborate the net probability if only one risk is present and the crude / partial crude probabilities if one or more risk is eliminated from the system while other risks remain.

The author realizes that for all descriptions above of this model need a real or simulated data to test and prove the validity of the model. But the limitation of time to obtain or to conduct a simulation for such a data has put this matter for further investigation without eliminating the importance of this model. Another important measures of this analysis is to include the dependent risks in our model for further research.

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