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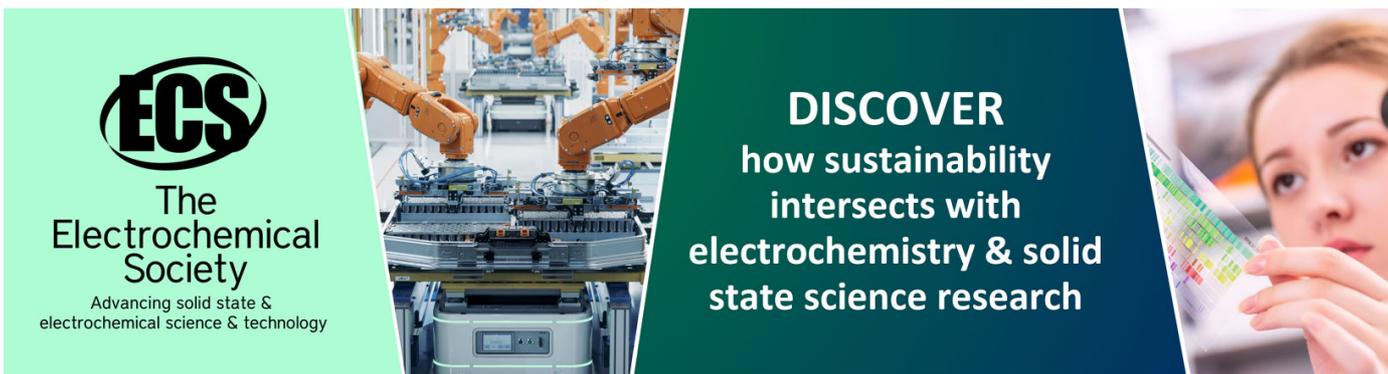
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On the connection of the generalized cascade-probability function with the Boltzmann equation

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Abstract. It was revealed that the generalized cascade-probability function (GCPF) can be obtained from the three-dimensional equation of the cascade process (of the Boltzmann type). It was established that this function, depending on the number of interactions, first increases, reaches a maximum, and then decreases. With the increase in H , the maximum of GCPF is shifted to the right, and its value decreases.

1. Introduction

When modeling radiation-physical processes, there is often a problem of choosing a theoretical calculation method, developing physical and mathematical models of research. At present, the most well-known and widely used theoretical methods for calculating the passage of particles through matter in physics are the Monte Carlo method, the Boltzmann kinetic method, and so on [1,2].

The analytical-numerical method of calculation, called cascade-probabilistic (CP), was developed in the process of many years of work in the field of applied mathematics [3], elementary particle physics, cosmic ray physics [4,5], radiation physics of materials [6–9], microelectronics [10, 11] and positron physics [12], and also on the basis of analysis and generalization of these studies. Its validity has been verified on a large number of specific problems from various fields of nuclear physics and solid state physics [6]. In the framework of the cascade-probabilistic method [12], we also succeeded in obtaining an analytical expression and calculating the threshold energy of displacement for various elements. The history of the CP-method is connected with the name of the great mathematician of the last century, S.D. Poisson, in particular with the distribution of random variables, named in his honor [13]. The Poisson distribution for certain values of the parameters is the limiting transition of the CP-functions, which are the basis of the CP-method [6].

2. Physical model

Let us consider the passage of particles of one kind through the material. Let a particle (nucleon, electron, positron, primary-knocked out atom, etc.) be generated at a certain variable depth x at the zenith and azimuth angles γ and φ to the chosen direction (relative to the perpendicular to the sample surface or with respect to the zenith). The most realistic description of the process is the three-dimensional model of particle distribution in matter, since the trajectory of particle motion is twisted strongly in space.



In the balance equation for the particle flow at the depth of x we will have a decrease in the particles due to their absorption, on the other hand, formation due to generation. In this case, the equation of the cascade process (of the Boltzmann type) for the stationary case can be written in the form of:

$$\frac{dN(E, \theta, \varphi, x)}{dx} = -\frac{N(E, \theta, \varphi, x)}{\lambda(E)\cos\theta} + \frac{1}{2\pi} \int_E^{E_m} \int_{\theta_1}^{\theta_1'} \int_{\varphi_1}^{\varphi_1'} N(E', \theta', \varphi', x) \frac{n(E', E, \theta', \theta)}{\lambda(E')\cos\theta'} dE' d\theta' d\varphi', \quad (1)$$

with the initial condition $N(E, \theta, \varphi, x) = N(E, 0)$.

Here, $N(E, \theta, \varphi, x)$ is the particle flow at the depth of x ; $dN(E, \theta, \varphi, x)/dx$ is the change in the radiation flow in an infinitesimally small dx layer, $\lambda(E)$ and γ are the interaction range and the emission angle of the particle relative to the vertical after the interaction, $n(E', E, \theta', \theta)$ is the normalized differential energy spectrum of the secondary particles in an elementary act, E is energy.

3. Results and discussion

Let us find the solution of equation (1) in a general form. After the Laplace transform, we have:

$$N(E, \theta, \varphi, p) = \frac{1}{p + \frac{1}{\lambda\cos\theta}} \frac{1}{2\pi} \int_E^{E_m} \int_{\theta_1}^{\theta_1'} \int_{\varphi_1}^{\varphi_1'} N(E', \theta', \varphi', p) \frac{n_p(E', E, \theta', \theta, \varphi)}{\lambda(E')\cos\theta'} dE' d\theta' d\varphi' + \frac{N(E, \theta, \varphi, 0)}{p + \frac{1}{\lambda\cos\theta}}. \quad (2)$$

where p is the transformation parameter.

In the general case, the $N(E, \theta, \varphi, p)$ value can be represented as follows:

$$N(E, \theta, \varphi, p) = \sum_{n=0}^{\infty} N_n(E, \theta, \varphi, p), \quad (3)$$

where

$$N_n(E, \theta, \varphi, p) = \int_E^{E_m} \int_{\theta_1}^{\theta_1'} \int_{\varphi_1}^{\varphi_1'} \dots \int_{E_{n-1}}^{E_{n-1}'} \int_{\theta_{n-1}}^{\theta_{n-1}'} \int_{\varphi_{n-1}}^{\varphi_{n-1}'} N(O, E_n, \theta_n, \varphi_n) \frac{n_p(E_n, E_{n-1}, \theta_n, \theta_{n-1}, \varphi_n, \varphi_{n-1})}{2\pi} \dots * \quad (4)$$

$$* \frac{n_p(E_1, E, \theta_1, \theta, \varphi_1, \varphi)}{2\pi} \prod_{i=0}^n \frac{1}{p + \frac{1}{\lambda(E_i)\cos\theta_i}} \frac{dE_n d\theta_n d\varphi_n}{\lambda(E_n)\cos\varphi_n} \dots \frac{dE_1 d\theta_1 d\varphi_1}{\lambda(E_1)\cos\varphi_1}.$$

Making the inverse Laplace transform, we find the original. After the appropriate calculations, we obtain:

$$N(E, \theta, \varphi, x) = \sum_{n=0}^n Y_n(E, \theta, \varphi, x), \quad (5)$$

$$J_n(E, \theta, \varphi, x) = \int_E^{E_m} \int_{\theta_1}^{\theta_1'} \int_{\varphi_1}^{\varphi_1'} \dots \int_{E_{n-1}}^{E_{n-1}'} \int_{\theta_{n-1}}^{\theta_{n-1}'} \int_{\varphi_{n-1}}^{\varphi_{n-1}'} N(E_1, \theta_1, \varphi_1, 0) \frac{n_p(E_n, E_{n-1}, \theta_n, \theta_{n-1}, \varphi_n, \varphi_{n-1})}{2\pi} \dots \times \quad (6)$$

$$\times \frac{n_p(E_1, E, \theta_1, \theta, \varphi_1, \varphi)}{2\pi} \psi_n(x, \lambda_0 \div \lambda_n) dE_n d\theta_n d\varphi_n \dots dE_1 d\theta_1 d\varphi_1.$$

where

$$\psi_n(x, \alpha_0 \div \alpha_n) = \sum_{j=0}^n \frac{\alpha_n \alpha_j^{n-1}}{\prod_{\substack{i=0 \\ i \neq j}}^n (\alpha_j - \alpha_i)} \exp\left(-\frac{x}{\alpha_j}\right). \quad (7)$$

$$\alpha_n = \lambda_n(E) \cos \theta_n; \quad \alpha_i = \lambda_i(E) \cos \theta_i; \quad \alpha_j = \lambda_j(E) \cos \theta_j.$$

As can be seen from the solution of (6), it includes a generalized cascade-probability function that takes into account the distribution of radiation in three-dimensional space. It has the sense of the probability that a particle generated at the depth of x' will reach the depth of x , after n number of collisions. Thus, GCPF can be obtained from the three-dimensional Boltzmann equation. Next, we developed an algorithm, compiled a program and performed calculations of this function on a computer. The results of the calculations are shown in Figure 1 as the dependence of $\psi_n(x, \alpha_0 \div \alpha_n)$ on n for different values of $H = (x - x')/\alpha_n$. It follows that this function, depending on the number of interactions, first increases, reaches a maximum, and then decreases (solid lines). With increasing H , the maximum of the generalized cascade-probability function shifts to the right and decreases, and its maximum value is at $n = (x - x')/\alpha_n$. It also follows from the Figure 1 that a slight change in H (by 1%) leads to a change in the CP function by more than 40% (dashed lines).

We also carried out a mathematical analysis of GCPF, which shows that the equation (7) satisfies all the properties of probability theory and can be obtained not only from the Boltzmann equation, but also from the recurrence relation:

$$\psi_i(h', h, \alpha_0) = \int_{h'}^h \psi_{i-1}(h', h'', \alpha_0) \psi_0(h'', h, \alpha_0) \frac{dh''}{\alpha_0}$$

$$\text{at } \psi_0(h', h, \alpha_0) = \exp\left(-\frac{h - h'}{\alpha_0}\right)$$

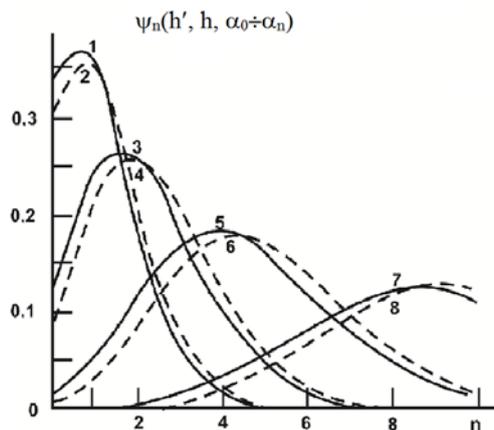


Figure 1. Dependence of the generalized CPF on the number of interactions n for different values of $H = (h - h') / \alpha_n = 1, 2, 4$ and 7 , respectively: 1, 3, 5, 7 – generalized CPF at $\alpha_n = 0.99\alpha_{n-1}$; 2, 4, 6, 8 – the simplest CPF [6].

4. Conclusions

1. The connection between the generalized cascade-probability function (GCPF) and the three-dimensional Boltzmann equation is established. It is shown that GCPF can be obtained from this equation by the Laplace transform method.
2. The calculation of GCPF on a computer was made. It is revealed that this function, depending on

the number of interactions, first increases, reaches a maximum, and then decreases. With increasing H , the maximum of the generalized cascade-probability function shifts to the right, and its value decreases, with the most probable number of interactions $n = (h - h') / \alpha_n$.

Acknowledgments

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