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# The Normal Condition of Squeeze-film Damping Model for Torsion Micro-Resonators

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**Abstract:** The prediction of squeeze-film damping plays a significant role in the design of high Q MEMS devices. This paper presents an analytical solution for the effect of squeeze film damping on a rectangular torsion micro-mirrors whose torsion axis exists in different positions. We defined a variable  $l$  to represent the distance which torsion axis departs from its central axis. The double sine solution is derived from the linearized Reynolds equation to obtain the pressure distribution under the vibrating plate, and then calculate the analytical expression of the damping constant. Comparing the results with the finite element method(FEM), the accuracy of the model can be verified.

## 1. Introduction

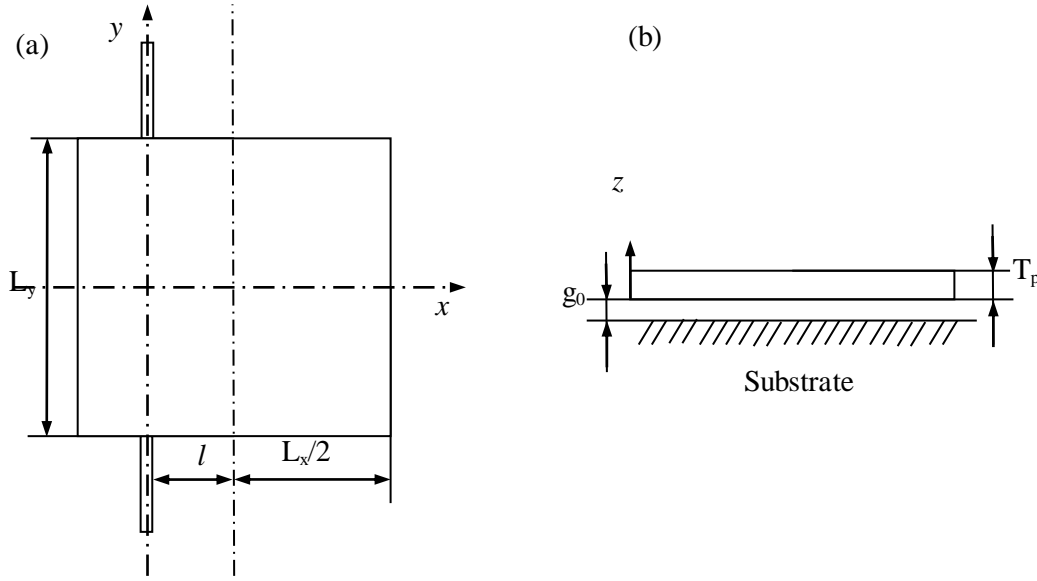
Squeeze-film damping, the main energy dissipation mechanism of MEMS resonators, plays an important role of device's dynamic response, which have been studied for several decades relating to the torsion micro-mirrors. The common tool named Reynolds equation is used to calculate the squeeze-film damping [1-2].

With the wide usage of the MEMS device, there is an urgent demand of analytical models describing the dynamic response. Langlois[3] proposed a general equation for the compressible flow gas film, this equation is the traditional Reynolds equation. Bao et al [4] proposed an analytical model for calculating the squeeze film damping of a rectangular torsion mirror at finite tilting angles. Pan et al [5] derived the Fourier series solution and the double sine series solution by solving a linearized Reynolds equation under the assumption of small displacements. However, in most of the previous studies, only the situation that torsion axis existing either in central axis or one side of the microplate have been studied, but lack of the normal situation, in which the torsion axis exists neither the central nor one side of the plate, but in the arbitrary position. In this paper we define  $l$  as the distance that torsion axis departs from the central axis, and the air pressure under the circular plate is approximated by the double series.

## 2. Problem formulation

A rigid rectangular plate size  $L_x \times L_y$  is shown in Figure 1.  $l$  is the distance which torsion axis departs





**Figure 1.** A schematic drawing of a rigid rectangular micro-plate. (a) Top view of perforated microplate. (b) Cross-sectional view

from its central axis,  $T_p$  is the thickness of plate, and  $g_0$  the thickness of the gap. In the condition of incompressible gas, the governing equation proposed by Bao et al [1] is

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{12\mu}{h_0^3} \frac{dh}{dt} \quad (1)$$

Where  $p(x, y, t)$  is the air pressure of squeeze-film gap,  $p(x, y, t) = p_a + \Delta p(x, y, t)$ ,  $p_a$  is the ambient pressure,  $\Delta p$  the pressure variation, and  $\mu$  the standard viscosity.  $h(t) = g_0 + x\theta e^{j\omega t}$ , in which  $\theta e^{j\omega t}$  is the angle of the torsion micro-plate.

The following non-dimensional variables are introduced for convenience:

$$P(x, y, t) = \frac{\Delta p(x, y, t)}{p_a}, H(t) = h_0 e^{j\omega t}, h_0 = \frac{\theta_0}{g_0} \quad (2)$$

Substituting equation (2) into equation (1), and linearizing the outcome around  $p_a$  and  $g_0$  which leads to

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = \alpha^2 \cdot x \frac{dH}{dt} \quad (3)$$

Where  $\alpha^2 = \frac{12\mu}{p_a g_0^2}$ ,  $l$  is the distance departing from the central axis, and the boundary condition can be changed to

$$P\left(x, -\frac{L_y}{2}, t\right) = P\left(x, \frac{L_y}{2}, t\right) = P\left(-\frac{L_x}{2} + l, y, t\right) = P\left(\frac{L_x}{2} + l, y, t\right) = 0 \quad (4)$$

The expression of  $P(x, y, t)$  is able to expressed as a double sine series:

$$P(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin\left[\frac{m\pi}{L_x} \left(x + \frac{L_x}{2} - l\right)\right] \sin\left[\frac{n\pi}{L_y} \left(y + \frac{L_y}{2}\right)\right] \cdot e^{j\omega t} \quad (5)$$

Where  $a_{mn}$  is the complex amplitude need determination. Obviously,  $P(x, y, t)$  satisfies the boundary condition (equation (4)). Substituting equation (5) and  $H(t) = h_0 e^{j\omega t}$  into equation (3) the following expression can be derived.

$$-\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[ \left( \frac{m\pi}{L_x} \right)^2 + \left( \frac{n\pi}{L_y} \right)^2 \right] a_{mn} \sin \left[ \frac{m\pi}{L_x} \left( x + \frac{L_x}{2} - l \right) \right] \sin \left[ \frac{n\pi}{L_y} \left( y + \frac{L_y}{2} \right) \right] = j\omega \alpha^2 h_0 x \quad (6)$$

According to the orthogonality of trigonometric series, we can lead to

$$\begin{aligned} a_{mn} &= ja_{mn}^I \\ &= \frac{8h_0}{mn\pi^2} \left\{ L_x - [1 + (-1)^{m+1}] \cdot \left( \frac{L_x}{2} + l \right) \right\} \frac{j\omega \alpha^2}{\left[ \left( \frac{m\pi}{L_x} \right)^2 + \left( \frac{n\pi}{L_y} \right)^2 \right]} \end{aligned} \quad (7)$$

Where  $a_{mn}^I$  is the imaginary parts of  $a_{mn}$ , and the expression of  $a_{mn}^I$  is :

$$a_{mn}^I = \frac{8h_0}{mn\pi^2} \left\{ L_x - [1 + (-1)^{m+1}] \cdot \left( \frac{L_x}{2} + l \right) \right\} \frac{\omega \alpha^2}{\left( \frac{m\pi}{L_x} \right)^2 + \left( \frac{n\pi}{L_y} \right)^2} \quad (8)$$

$$m = 1, 2, 3, \dots; n = 1, 3, 5, \dots$$

The torque which caused by the pressure of the squeeze film can be derived as

$$T_{squeeze} = \int_{-\frac{L_x}{2}}^{\frac{L_x}{2}+l} \int_{-\frac{L_y}{2}}^{\frac{L_y}{2}} \Delta p \cdot x dx dy = jT_{damping} \cdot e^{j\omega t} \quad (9)$$

The total damping torque acting on the microplate are

$$\begin{aligned} T_{damping} &= T_{squeeze}^I = p_a \cdot a_{mn}^I \cdot \int_{-\frac{L_x}{2}}^{\frac{L_x}{2}+l} \int_{-\frac{L_y}{2}}^{\frac{L_y}{2}} x \cdot \sin \left[ \frac{m\pi}{L_x} \left( x + \frac{L_x}{2} - l \right) \right] \sin \left[ \frac{n\pi}{L_y} \left( y + \frac{L_y}{2} \right) \right] dx dy \\ &= -p_a h_0 \cdot \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{16L_x L_y}{m^2 n^2 \pi^4} \left\{ L_x - [1 + (-1)^{m+1}] \cdot \left( \frac{L_x}{2} + l \right) \right\}^2 \frac{\omega \alpha^2}{\left( \frac{m\pi}{L_x} \right)^2 + \left( \frac{n\pi}{L_y} \right)^2} \end{aligned} \quad (10)$$

Where  $T_{squeeze}^I$  is the imaginary part of  $T_{squeeze}$ . The corresponding damping constant  $C_\theta$  owing to the pressure of the squeeze film is given by

$$C_\theta = \frac{-T_{damping}}{\theta_0 \cdot \omega} = \frac{p_a}{g_0} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{16L_x L_y}{m^2 n^2 \pi^4} \left\{ L_x - [1 + (-1)^{m+1}] \cdot \left( \frac{L_x}{2} + l \right) \right\}^2 \frac{\alpha^2}{\left( \frac{m\pi}{L_x} \right)^2 + \left( \frac{n\pi}{L_y} \right)^2} \quad (11)$$

$$m = 1, 2, 3, \dots; n = 1, 3, 5, \dots$$

### 3. Validation

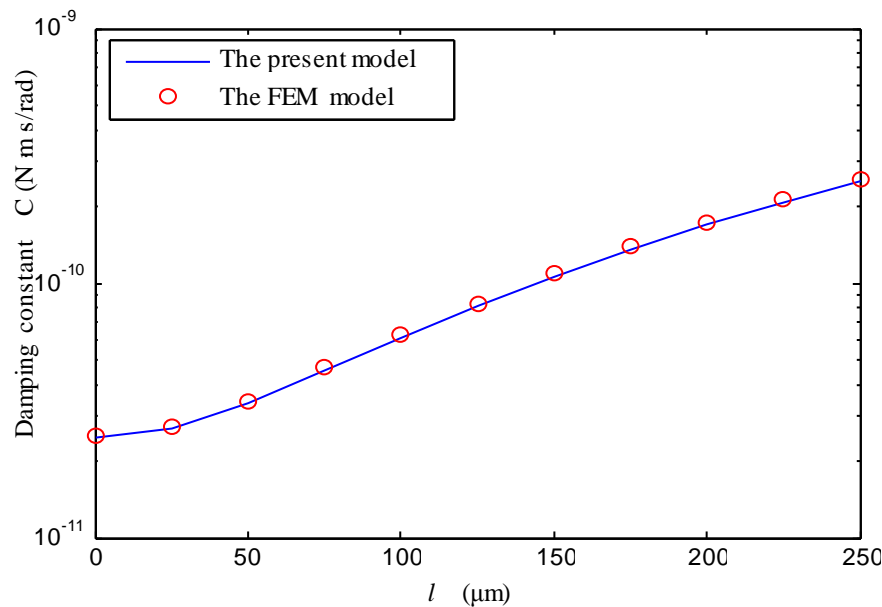
In this section, using the present model to calculate the squeeze film damping in a different values of  $l$ , we will compare the present results with FEM results. In ANSYS, FLUID136 element is used for modelling the squeeze-film. The parameters and dimensions of the rectangular micro-plate are listed in Table 1. Figure 2 shows the comparison of damping constants as a function of distance ( $l$ ) that torsion axis departing from the central axis obtained by the present model (equation (11)).

We varied the value of  $l$  from 0 to 250  $\mu\text{m}$ , finding that the present results are in good agreement with the FEM results in a wide range of distance. We also find that the damping constant increased as the

increasing of the distance that torsion axis departing from the central axis, and the damping constant also increased as the micro-mirror became more and more large.

**Table 1.** The parameters and dimensions of the torsion micro-mirror

Parameters	Values	Description
$L_x$	500 $\mu\text{m}$	Length of the plate
$L_y$	500 $\mu\text{m}$	Width of the plate
$T_p$	10 $\mu\text{m}$	Thickness of the plate
$g_0$	5 $\mu\text{m}$	Gap spacing
$\rho$	2330 $\text{Kg}/\text{m}^3$	Density
$\mu_0$	$1.85 \times 10^{-5} \text{ N} \cdot \text{s}/\text{m}^2$	Viscosity
$f$	5000 Hz	Frequency



**Figure 2.** The comparison of damping constants with present model and the FEM model as  $L_y = 500 \mu\text{m}$

#### 4. Conclusion

In this paper, an analytical model for squeeze-film damping in a rigid rectangular torsion microplate is presented. The pressure distribution of the gas under the plate calculated by the double sine series, and therefore we get the expressions of damping constant. Comparing them with the FEM results, we find that the analytical model fits the FEM model well, and as the increase of the distance which torsion axis departing from the central axis, the damping constants increased in the same time.

#### Acknowledgement

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