

PAPER • OPEN ACCESS

Detection of Cracks in a Cantilever Beam Using Signal Processing and Strain Energy Based Model

To cite this article: P Mehta *et al* 2017 *IOP Conf. Ser.: Mater. Sci. Eng.* **234** 012008

View the [article online](#) for updates and enhancements.

You may also like

- [Ultrasound excited thermography: an efficient tool for the characterization of vertical cracks](#)
A Mendioroz, R Celorrio and A Salazar
- [Indentation-based methods to assess fracture toughness for thin coatings](#)
Jinju Chen
- [Identification of multiple cracks in 2D elasticity by means of the reciprocity principle and cluster analysis](#)
Efim I Shifrin and Alexander V Kaptsov



ECS
The
Electrochemical
Society
Advancing solid state &
electrochemical science & technology

DISCOVER
how sustainability
intersects with
electrochemistry & solid
state science research

Detection of Cracks in a Cantilever Beam Using Signal Processing and Strain Energy Based Model

P Mehta¹, A Kureshi², S Lad³, N Patel^{4*} and D Sharma⁵

^{1,2,3,4}Mechanical Department, School of Technology, Pandit Deendayal Petroleum University, Gandhinagar, Gujarat, India, 382007

⁵Mechanical Engineering Department, Faculty of Technology and Engineering, The M. S. University of Baroda, Gujarat, India

* nirav.npp@gmail.com

Abstract. Structure health monitoring is one of the most important aspects in an industry, as structures should work safely during their service life. Cracks are the most common damage that initiates a breakdown phase and hence timely and accurate detection of these cracks is imperative. In this article, a vibration based non-destructive technique is presented to detect one or multiple edge cracks in beam like structures. This model is based on variation in mode shapes and natural frequencies that provide accuracy in results as well as ease in practical applications. The crack location is identified using mode shapes of damaged beam wherein an appropriate signal processing technique is implemented by using which the noise in the signal can be reduced. Along with this, the crack severity is also determined using a strain energy based mathematical model. The model presented in this study is capable of detecting an arbitrary number of cracks in cantilever or simply supported configuration. The results obtained using the proposed method is also validated by considering few case studies.

1. Introduction

For any structure to work safely during its service life, it is necessary to assess its health on a regular basis. Hence, the continuous monitoring of the variation in its static or dynamic behavior is imperative. Mostly, these changes are directly influenced by the reduction of stiffness of the structure and in most cases, this reduction is caused by crack like defects. The presence of cracks in structure may affect the dynamic behavior like change in mode shapes, natural frequency, stiffness and damping ratio. The vibration based non-destructive technique (NDT) works upon this measured response of mode shapes and frequency. The use of vibration based technique has been explored by many researchers [1–6] for the detection of single and multiple cracks in various structures. Various articles implemented different approaches to model the cracks in beams. Some of the authors [7–11] replaced the crack by rotational spring while others [12–14] replaced by reduction in strain energy caused by a crack. Each method has its own specific advantages, although it is very difficult to satisfy all demands due to different types of changes in structures with various loading conditions. The spring based model has been applied to various types of beams like, stepped cantilever beam [7], geometrically segmented beams [8], simply supported beams [9], cantilever beam [10], multi-span beams [11] etc. The strain energy based model [12–14] has been used for crack detection in a cantilever beam of variable cross section. The basic idea of this model is to account for the energy absorbed by the crack and hence the crack can be substituted by any energy absorbing media. The strain energy based model works on the



energy parameter itself and hence it is better suited to model cracked structures. Now, the variation in frequency and mode shapes are used in the crack detection. This variation can be seen as a discontinuity. The sensitivity of wavelet transform to discontinuities caused by the shift of deflection function and its capability to increase the resolution of the singularity signal has led to considerable developments in utilizing wavelet-based damage detection methodologies. Wang and Deng [14] were the first to address this possibility of determining the crack features of location and depth of damage in beams. The wavelet transform for crack detection has been further utilized by many researchers [15–19].

The aim of this paper is to establish an accurate method to detect and locate cracks in cantilever beam using wavelet transform. The methodology to detect crack severity is also presented along with the location of crack by using a strain energy based model. The model can be used to detect one or more cracks in a cantilever beam. The model is also validated by considering few case studies.

2. Methodology to identify Crack

Methods that detect damages directly from mode shapes are not accurate since inherent noise present in the signal can lead to incorrect detection and determination of damages also surrounding environment at times can introduce frequency changes in the structure greater than that introduced by the damage. Hence, suitable signal processing needs to be done on the obtained mode shape to achieve accuracy. Discontinuities are introduced in the mode shape of a damaged beam due to the reduction in strain energy of the beam. The wavelet transform is a signal processing technique which is accurate in identifying the discontinuities in the signal and hence can be effectively used onto the mode shapes of a damaged beam in order to locate damages. The wavelet transform signal consists of noticeable peaks wherein the signal has a discontinuity. A crack is described using two variables: location x_n and depth a_n for n^{th} crack, respectively as shown in figure 1. In the present study, the case of multiple cracks is considered that leads to the identification of any number of cracks.

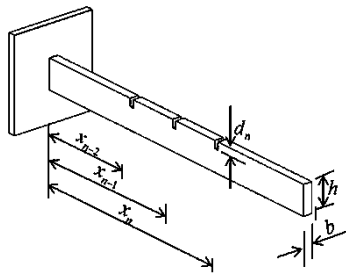


Figure 1. Beam Schematic

A mathematical model that describes the relationship between natural frequency variation and damage characteristic is presented in this article. The natural frequency of a beam can be derived considering the transverse vibration Euler-Bernoulli beam theory (beam is of constant cross section and rigidity). After using the displacement function as a method of separation of variables and putting it to the differential equation of a beam (Euler-Bernoulli beam theory), the following equation can be derived:

$$EI\Phi^{iv}(x) - \rho A\omega^2\Phi(x) = 0. \quad (1)$$

The solution of equation (1) is given as:

$$\Phi_i(x) = C_1\sin(\lambda x) + C_2\cos(\lambda x) + C_3\sinh(\lambda x) + C_4\cosh(\lambda x), \quad (2)$$

where, $\lambda^4 = \frac{\rho A\omega^2}{EI}$, (A is cross sectional area, I is moment of inertia, Φ_i is the displacement function for i^{th} harmonic and $C_{1,2,3,4}$ are constants).

Applying the boundary conditions of a cantilever beam to the equation (2), the following equation of mode shape can be derived:

$$\Phi_i(x) = \cos(\lambda_i x) - \cosh(\lambda_i x) - \frac{\cos(\lambda_i L) + \cosh(\lambda_i L)}{\sin(\lambda_i L) + \sinh(\lambda_i L)} (\sin(\lambda_i x) - \sinh(\lambda_i x)). \quad (3)$$

The mode shapes presented in equation (3) can directly influence the strain energy. The strain energy (W_i) and natural frequency ($f_{h,i}$) of healthy beam can be written in terms of the mode shapes as below [12]:

$$W_i = \frac{1}{2} \int_0^L EI [\Phi''_i(x_1)]^2 dx; \quad f_{h,i} = \sqrt{\frac{1}{2} \int_0^L EI [\Phi''_i(x_1)]^2 dx} \quad (4-5)$$

where, W_i and $f_{h,i}$ are the strain energy and natural frequency of the healthy beam in the i^{th} harmonic respectively. It can be observed from equations (4) and (5) that strain energy and frequency both are in direct proportional to the square of the second derivative of the mode shape. Similarly, in the case of the damaged beam, the strain energy from the section is reduced because of damage which results in the further reduction of natural frequency. Strain energy stored in a small section dx can be represented by equation (6):

$$dW_i(x_n) = \frac{1}{2} EI [\Phi''_i(x_n)]^2 dx. \quad (6)$$

It can also be interpreted from the equation (6) that a certain vibration energy change depends on the position of the damage on the beam i.e. the longitudinal location x_n , being influenced by the second derivative of the mode shape at the location while the depth of the crack only amplifies the response. As frequency is proportional to strain energy, change in frequency would be proportional to change in strain energy. Δf_i is the difference between $f_{h,i}$ (frequency of healthy beam) and $f_{c,i}$ (frequency of a cracked beam) in i^{th} mode of vibration ($\Delta f_i = f_{h,i} - f_{c,i}$). In the case of a cantilever beam having more than one crack, the total resultant change in modal strain energy can be considered as the summation of the individual change in modal strain energy caused by every single crack [20]:

$$\frac{\Delta f_i}{f_{h,i}} = \sum_{j=1}^n p(\bar{d}_i) [\Phi''_i(x_j)]^2. \quad (7)$$

$p(\bar{d}_i)$ reflects the influence of the depth of the crack while the term $\Phi''_i(x_i)$ reflects the influence of the location of the crack. This relation is valid for different types of configuration as well as dimensions of the beam considering suitable beam theories. The value of $p(\bar{d}_i)$ can be written as below [5]:

$$p(\bar{d}_i) = 6\pi(1-\nu^2) \left(\frac{0.6272(\bar{d}_i)^2 - 1.04533(\bar{d}_i)^3 + 4.5948(\bar{d}_i)^4 - 9.9736(\bar{d}_i)^5 + 20.2948(\bar{d}_i)^6 - 33.0341(\bar{d}_i)^7 + 47.163(\bar{d}_i)^8 - 40.7556(\bar{d}_i)^9 + 19.6(\bar{d}_i)^{10}}{(\bar{d}_i)^6 - 33.0341(\bar{d}_i)^7 + 47.163(\bar{d}_i)^8 - 40.7556(\bar{d}_i)^9 + 19.6(\bar{d}_i)^{10}} \right), \quad (8)$$

where, $\bar{d}_i = d_i/h$ is a dimensionless damage depth and the functions $p(\bar{d}_i)$ is stress intensity factor. Using the values of equation (8), one can write equation (7) for different mode shapes:

$$\left\{ \frac{\Delta f_1}{f_{h,1}}, \frac{\Delta f_2}{f_{h,2}}, \dots, \frac{\Delta f_n}{f_{h,n}} \right\} = \sum_{j=1}^n p(\bar{d}_j) [\Phi''_1(x_j)]^2, \sum_{j=1}^n p(\bar{d}_j) [\Phi''_2(x_j)]^2 \dots \sum_{j=1}^n p(\bar{d}_j) [\Phi''_n(x_j)]^2. \quad (9)$$

Solving the equation (9), one can obtain the values for $\{d_1, d_2 \dots d_n\}$. Overall process to detect location and depth can be summarized in figure 2.

3. Validation Study

The methodology to detect one or multiple cracks in a cantilever beam is presented in this article. The model can be validated by considering a beam with various configurations of cracks. The cantilever beam with Young's Modulus (E) = 70.09 GPa, density (ρ) = 2700 kg/m³, length (L) = 0.24 m, height (h) = 0.0191 m, width (b) = 0.0064 m and moment of Inertia (I) = 3716 mm⁴ is considered in present study. The mode shapes of the damaged beam can be obtained by experimentation or by using finite element analysis based commercial software package. In the present study, a cantilever beam has been used with two edge cracks at normalized location 0.25 and 0.5 with normalized depths 0.35 and 0.45 respectively. The finite element analysis model is presented in figure 3.

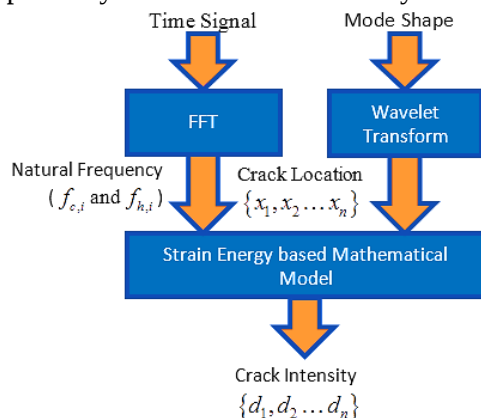


Figure 2. Flow chart of crack detection method

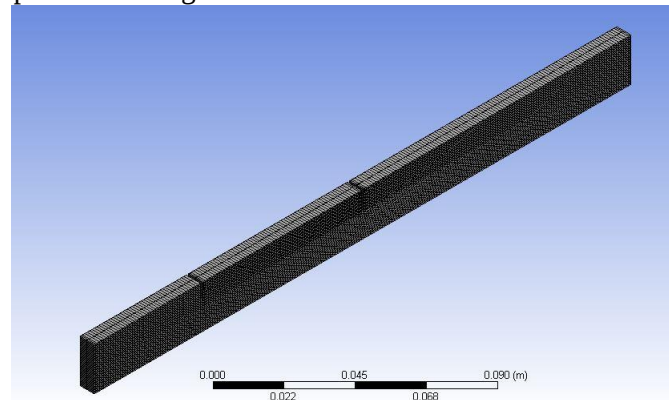


Figure 3. FEA model of a cantilever beam

Applying proper boundary conditions in FEA model (figure 3), the mode shape or displacement can be obtained for any type of damaged beam. Such displacement plot is presented in figure 4. Figure 5 shows the wavelet transform of the second mode shape of the damaged beam. It can be seen clearly that the wavelet transform picked up a disturbance in the mode shape at the normalized location of the crack: $x_1 = 0.24$ and $x_2 = 0.49$, which is then compared to the actual location. Similarly, the results are also validated for different locations of crack (table 1). It is evident that when the location of the crack is near the fixed end, the error in locating the crack increases since near the fixed end the displacement of the beam would be less and subsequently, the discontinuity in the mode shape would also be less. Additionally, the severity of the crack can also be identified in the present model using equation (9). The different cases of crack depth (at the same location as table 1) have been considered. The detected location from table 1 and the natural frequency are inserted in the mathematical model to obtain the crack severity. The comparison is listed in table 2.

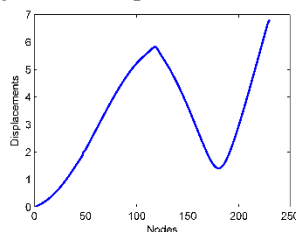


Figure 4. Second Mode Shape of the damaged beam

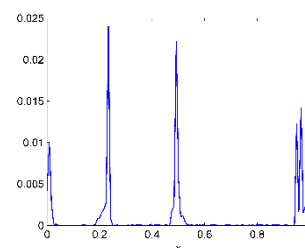


Figure 5. Wavelet transform of the mode shape

Table 1. Crack Location

Sr No	Actual Normalized Location		Detected Normalized Location		Error	
	Crack 1	Crack 2	Crack 1	Crack 2	Crack 1	Crack 2
1	0.25	0.5	0.24	0.49	-4	-2

2	0.2	0.5	0.2	0.49	0	-2
3	0.25	0.5	0.24	0.48	-4	-4
4	0.1	0.5	0.11	0.5	9	0
5	0.25	0.5	0.24	0.49	-4	-2
6	0.15	0.5	0.15	0.51	0	2

Table 2. Crack Severity

Sr No	Natural Frequency		Normalized Actual Crack Depth		Normalized Detected Crack Depth		Error	
	Mode 1	Mode 2	Crack 1	Crack 2	Crack 1	Crack 2	Crack 1	Crack 2
1	237.7	1415.4	0.35	0.45	0.35	0.43	0.89	-3.89
2	232.5	1590.7	0.45	0.25	0.46	0.26	3.13	3.76
3	208.02	1309.9	0.45	0.45	0.44	0.42	-2.23	-6.67
4	231.9	1636.9	0.46	0.15	0.44	0.14	-3.04	-3.46
5	240.29	1393.8	0.3	0.45	0.3	0.41	0	-8
6	233.07	1521.1	0.43	0.35	0.43	0.33	-0.46	-4.85

The mean error (table 2) is 3.8% which is quite accurate for such Non-Destructive Technique. It is evident that the measurements were consistent up to $\bar{d} = 0.5$. The vibration coupling (at larger depths) resulted in a crowded spectrum and a complex time signal that influence the accuracy of natural frequency. Moreover at depths smaller than $\bar{d} = 0.1$, the difference from the natural frequency of the un-cracked beam was not measurable. Therefore, the results will be limited to $0.1 \leq \bar{d} \leq 0.6$ for present case [5].

4. Conclusion

A quick, accurate and reliable vibration based non-destructive technique to detect cracks is presented in this study. The method can be applied to identify any number of cracks in a cantilever beam. The methodology can also be extended to the various configuration of beams with a change in mode shapes. The method presented in this study is also validated by considering various cases of cracks locations and crack depths. The method can be used by any designer who wants to detect cracks in beam like structures.

References

- [1] Adams RD, Cawley P, Pye CJ, Stone BJ. A Finite Element Approach to Plate Vibration Problems. *J Mech Eng Sci.* 1965;7(1):5.
- [2] Cawley P. The Location of Defects in Structures From Measurements of Natural Frequencies. *J strain Anal.* 1978;14(2):49–57.
- [3] Dimarogonas AD, Papadopoulos CA. Vibration of cracked shafts in bending. *J Sound Vib.* 1983;91(4):583–93.
- [4] Shen MH, Pierre C. Free Vibrations of beams with single edge crack. *Jounral Sound Vib.* 1994;170(2):237–59.
- [5] Chondros TG, Dimarogonas A. D, Yao J. A Continuous Cracked Beam Vibration Theory. *J Sound Vib* [Internet]. 1998;215:17–34. Available from: [http:// www.sciencedirect.com /science/ article/ pii /S0022460X98916406](http://www.sciencedirect.com/science/article/pii/S0022460X98916406)
- [6] Dimarogonas AD. Vibration of cracked structures: A state of the art review. *Eng Fract Mech.* 1996;55(5):831–57.
- [7] Nandwana BP, Maiti SK. Detection of the Location and Size of a Crack in Stepped Cantilever Beams Based on Measurements of Natural Frequencies. *J Sound Vib* [Internet]. 1997;203(3):435–46. Available from: <http://www.sciencedirect.com/science/article/pii/S0022460X96908561>
- [8] Chaudhari TD, Maiti SK. A study of vibration of geometrically segmented beams with and

- without crack. *Int J Solids Struct* [Internet]. 2000;37(5):761–79. Available from: <http://www.sciencedirect.com/science/article/pii/S0020768399000542>
- [9] Patil DP, Maiti SK. Detection of multiple cracks using frequency measurements. *Eng Fract Mech*. 2003;70(12):1553–72.
- [10] Patil DP, Maiti SK. Experimental verification of a method of detection of multiple cracks in beams based on frequency measurements. *J Sound Vib*. 2005;281(1–2):439–51.
- [11] Sharma DS, Mungla MJ, Barad KH. Vibration-based non-destructive technique to detect crack in multi-span beam. *Nondestruct Test Eval* [Internet]. 2015;30(4):291–311. Available from: <http://www.tandfonline.com/doi/full/10.1080/10589759.2015.1029475>
- [12] Gillich G-R, Praisach Z-I. Modal identification and damage detection in beam-like structures using the power spectrum and time–frequency analysis. *Signal Processing* [Internet]. 2014;96:29–44. Available from: <http://www.sciencedirect.com/science/article/pii/S0165168413001722>
- [13] Yang XF, Swamidas ASJ, Seshadri R. Crack Identification in Vibrating Beams Using the Energy Method. *J Sound Vib* [Internet]. 2001;244(2):339–57. Available from: <http://linkinghub.elsevier.com/retrieve/pii/S0022460X00934989>
- [14] Wang Q, Deng XM. Damage detection with spatial wavelets. *Int J Solids Struct*. 1999;36:3443–68.
- [15] Douka E, Loutridis S, Trochidis A. Crack identification in beams using wavelet analysis. *Int J Solids Struct*. 2003;40(13–14):3557–69.
- [16] Chang CC, Chen LW. Detection of the location and size of cracks in the multiple cracked beam by spatial wavelet based approach. *Mech Syst Signal Process*. 2005;19(1):139–55.
- [17] Ovanesova A V., Suárez LE. Applications of wavelet transforms to damage detection in frame structures. *Eng Struct*. 2004;26(1):39–49.
- [18] Khatam H, Golafshani a. a., Beheshti-Aval SB, Noori M. Harmonic Class Loading for Damage Identification in Beams Using Wavelet Analysis. *Struct Heal Monit*. 2007;6(1):67–80.
- [19] Xiang J, Liang M. Wavelet-Based Detection of Beam Cracks Using Modal Shape and Frequency Measurements. *Comput Civ Infrastruct Eng*. 2012;27(6):439–54.
- [20] Kisa M. Free vibration analysis of a cantilever composite beam with multiple cracks. *Compos Sci Technol*. 2004;64(9):1391–402.