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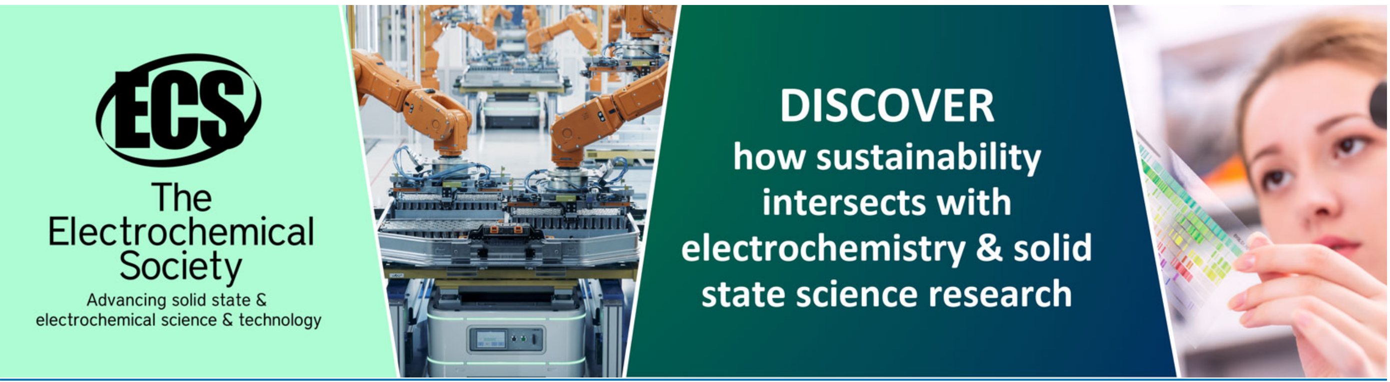

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Simulation Analysis of Resonant Frequency of Mix Sucker Rod with absorber

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Abstract. Considering the influence of sucker rod absorber (SRA) on longitudinal vibration of sucker rod string (SRS), a new model of mix sucker rod with absorber (MSRWA) in the form of numerical integration is built. In the detail, based on the dynamical theory of continuous systems, the wave equation of instantaneous motion of MSRWA is deduced. The dynamic response is solved out with the mode superposition method. The affecting factors of natural frequency are analyzed, such as the length, diameter, density and elastic modulus of SRS. Then three dimensional curves of natural frequency are obtained. The results are very important for the design and application of SRS.

1. Introduction

The sucker rod pumping system (SRPS) is an important method of the mechanical recovery methods in the domestic and overseas oilfields [1-3]. In order to reduce energy consumption and boost its crude production, the optimal design of SRPS had been done since the nineteen sixties [4, 5]. The resonant frequency of SRPS is foundation of optimal design of SRPS, so the resonant frequency of SRPS has been studied. Two steps in estimating natural frequencies and damping ratios are given by Lu X [6], the first step is from measurement records to the power spectral density function, the second step is from the power spectrum to system parameter estimates. Dong S gave the conclusion that there will be resonance phenomena, when the excited frequency and even multiple of fundamental frequency are equal [7]. Peng Y analyzes the twisting vibration of rotary sucker rod string, a formula of calculating every harmonic vibration inherent frequency is presented [8]. The inherent frequency of longitudinal vibration of single stage rod string is analyzed and calculated from the point of view of vibration theory by Zhu B [9]. The 3-D vibration of sucker rod string in the reciprocating motion is proposed by Chen J, such as horizontal vibration, longitudinal vibration and torsional vibration. The sucker rod string is considered the elastic body whose quality is distributed by using the elastic body vibration theory. The dynamic models for the vibrations of sucker rod string in three directions are established [10]. Although the resonant Frequency of SRPS has been studied for years, the most of aforementioned studies have not been studied sufficiently. In the detail, the formula of natural frequency is not accurate [11], when the free vibration of SRS is researched. When the SRA is installed between polished rod and SRS, the influence of SRA on the SRS's vibration is usually ignored. In the period of oil field development, the cost of oil production has been the focus of the global attention at present [12-13]. However, with application of SRA, the vibration of SRS will be absorbed and the stress of SRS and the peak of dynamic load will be reduced, at the same time, the motor load and power consumption will decrease. Therefore, the resonant frequency of longitudinal

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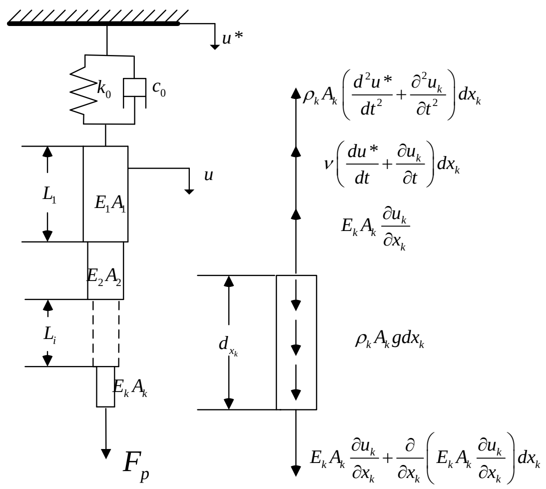
vibration of MSRWA is necessary to analyzed, which lays the foundation for the design and application of SRA.

2. Dynamic simulate model of MSRWA

The number stages of SRS assemblage are supposed to be k, and the polished rod displacement is u*. The relative displacement between polished rod and SRS top is u0. Considering the influence of SRA on SRS's longitudinal vibration, a mechanical model of MSRWA is established, as shown in Fig.1.



(a) (b)

Figure 1. The mechanical model of MSRWA

Based on the mechanical model of longitudinal vibration, a mathematical model is built, as follows

$$\begin{cases} \dfrac{\partial^2 u_k}{\partial t^2} - a_k^2 \dfrac{\partial^2 u_k}{\partial x_k^2} + c_k \dfrac{\partial u_k}{\partial t} = -\dfrac{d^2 u^*}{dt^2} - c_k \dfrac{du^*}{dt} + g \\ E_1 A_1 \dfrac{\partial u_1(0)}{\partial x_1} = k_0 u_0 + c_0 \dot{u}_0 \\ E_k A_k \dfrac{\partial u_k(L_d)}{\partial x_k} = F_p \end{cases} \tag{1}$$

Where,

$$\begin{cases} a_k = \sqrt{\dfrac{E_k}{\rho_k}} \\ c_k = \left[\dfrac{\left(0.20 + 0.39 \dfrac{D_k}{D_t}\right) + \dfrac{2.197 \times 10^4}{25}}{\left(\dfrac{D_{ck}}{D_t} - 0.381\right)^{2.57} \dfrac{D_{ck}{}^2 - D_k{}^2}{l_{0k} D_k}} \right] \dfrac{12\pi\mu m_k}{\rho_k A_k} \left(\dfrac{D_k}{D_t - D_k} \right) \end{cases} \tag{2}$$

Where, a_k is the velocity of sound in SRS (m/s); c_k is the friction coefficient (Pa.s); F_p is the load of pump plunger (N); ω is the exaction frequency (rad/s)

3. Free vibration

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Ignoring the influence of the damping of SRA on the SRS's vibration, the equation of free vibration is obtained based on Eq.(1).

$$\begin{cases} \frac{\partial^2 u_1}{\partial t^2} - a_1^2 \frac{\partial^2 u_1}{\partial x_1^2} = 0 \\ \vdots \qquad \cdots \qquad \vdots \\ \frac{\partial^2 u_k}{\partial t^2} - a_k^2 \frac{\partial^2 u_k}{\partial x_k^2} = 0 \end{cases} \tag{3}$$

The equations of boundary and continuous conditions are given, as follows.
When k=1

$$E_1 A_1 \frac{\partial u_1}{\partial x_1}\Big|_{x=0} = k_0 u_0 \,, \qquad E_{rk} A_{rk} \frac{\partial u}{\partial x}\Big|_{x=L} = 0 \tag{4}$$

When k=2, 3, 4…

$$\begin{cases} E_1 A_1 \frac{\partial u_1}{\partial x_1}\Big|_{x_1=0} = k_0 u_0, \qquad u_{k-1}\Big|_{x_{k-1}=L_{k-1}} = u_k\Big|_{x_k=0} \\ E_{(k-1)} A_{(k-1)} \frac{\partial u_{k-1}}{\partial x_{k-1}}\Big|_{x_{k-1}=L_{k-1}} = E_k A_k \frac{\partial u_k}{\partial x_k}\Big|_{x_k=0} \,,\; E_k A_k \frac{\partial u_k}{\partial x}\Big|_{x_k=L_k} = 0 \end{cases} \tag{5}$$

The vibration model functions are defined, as follows.

$$\begin{cases} \Phi_1(x_1) = B_1 \sin(b_1 x_1) + D_1 \cos(b_1 x_1) \quad 0 \le x_1 \le L_1 \\ \Phi_2(x_2) = B_2 \sin(b_2 x_2) + D_2 \cos(b_2 x_2) \quad 0 \le x_2 \le L_2 \\ \Phi_i(x_i) = B_i \sin(b_i x_i) + D_i \cos(b_i x_i) \quad 0 \le x_i \le L_i \end{cases} \tag{6}$$

Combining Eqs. (4) and (5), the transcendental equation of natural frequency is given, as follows

$$B_k \cos(b_k L_k) - D_k \sin(b_k L_k) = 0 \tag{7}$$

Where,

$$\begin{cases} D_1 = B_1 \frac{E_1 A_1 b_1}{k_0} \\ B_2 = B_1 \frac{E_1 A_1 b_1}{E_2 A_2 b_2}\left[\cos(b_1 L_1) - \frac{E_1 A_1 b_1}{k_0}\sin(b_1 L_1)\right] \\ D_2 = B_1\left[\sin(b_1 L_1) + \frac{E_1 A_1 b_1}{k_0}\cos(b_1 L_1)\right] \\ B_i = B_1 \frac{E_{i-1} A_{i-1} b_{i-1}}{E_i A_i b_i}\left[\cos(b_{i-1} L_{i-1}) - \frac{D_{i-1}}{B_1}\sin(b_{i-1} L_{i-1})\right] \\ D_i = B_1\left[\frac{B_{i-1}}{B_1}\sin(b_{i-1} L_{i-1}) + \frac{D_{i-1}}{B_1}\cos(b_{i-1} L_{i-1})\right] \end{cases} \tag{8}$$

Then the formula of regular function is obtained, as follow.

$$\begin{cases} \varphi_1(x_1) = B_1\left[\sin(b_1x_1) + \dfrac{E_1A_1b_1}{k_0}\cos(b_1x_1)\right] & 0 \le x_1 \le L_1 \\ \varphi_2(x_2) = B_1\left[\dfrac{E_1A_1b_1}{E_2A_2b_2}\cos(b_1L_1)\sin(b_2x_2) - \dfrac{E_1^2A_1^2b_1^2}{E_2A_2b_2k_0}\sin(b_1L_1)\sin(b_2x_2)\right. & \\ \left. +\sin(b_1L_1)\cos(b_2x_2) + \dfrac{E_1A_1b_1}{k_0}\cos(b_1L_1)\cos(b_2x_2)\right] & 0 \le x_2 \le L_2 \\ \varphi_i(x_i) = B_1\left[\dfrac{E_{i-1}A_{i-1}b_{i-1}}{E_iA_ib_i}\cos(b_{i-1}L_{i-1})\sin(b_ix_i) - \dfrac{E_{i-1}A_{i-1}b_{i-1}D_{i-1}}{E_iA_ib_iB_1}\sin(b_{i-1}L_{i-1})\sin(b_ix_i)\right. & \\ \left. +\dfrac{B_{i-1}D_i}{B_1^2}\sin(b_{i-1}L_{i-1})\cos(b_ix_i) + \dfrac{D_{i-1}D_i}{B_1^2}\cos(b_{i-1}L_{i-1})\cos(b_ix_i)\right] & 0 \le x_i \le L_i \end{cases} \tag{9}$$

Where,

$$\begin{cases} B_1 = \sqrt{\dfrac{1}{\rho_1A_1G_1+\rho_2A_2G_2+\rho_iA_iG_i}} \\ G_1 = \left[\dfrac{L_1}{2} - \dfrac{1}{4b_1}\sin(2b_1L_1)\right] + \dfrac{D_1^2}{B_1^2}\left[\dfrac{L_1}{2} + \dfrac{1}{4b_1}\sin(2b_1L_1)\right] + \dfrac{D_1}{2b_1B_1}\left[1-\cos(2b_1L_1)\right] \\ G_2 = \dfrac{B_2^2}{B_1^2}\left[\dfrac{L_2}{2} - \dfrac{1}{4b_2}\sin(2b_2L_2)\right] + \dfrac{D_2^2}{B_1^2}\left[\dfrac{L_2}{2} + \dfrac{1}{4b_2}\sin(2b_2L_2)\right] + \dfrac{B_2D_2}{2b_2B_1^2}\left[1-\cos(2b_2L_2)\right] \\ G_i = \dfrac{B_i^2}{B_1^2}\left[\dfrac{L_i}{2} - \dfrac{1}{4b_i}\sin(2b_iL_i)\right] + \dfrac{D_i^2}{B_1^2}\left[\dfrac{L_i}{2} + \dfrac{1}{4b_i}\sin(2b_iL_i)\right] + \dfrac{B_iD_i}{2b_iB_1^2}\left[1-\cos(2b_iL_i)\right] \end{cases} \tag{10}$$

The numerical solutions of the time functions in regular coordinate system are solved out with the fourth order Runge-Kutta method. Then the numerical solutions of the time functions in Natural coordinate system are obtained with the coordinate transformation.

4. Characteristic analysis

The basic parameters are as follows: when the length of SRS L_d is 1000m, the length of first stage L_1 is 600m and the length of second stage L_2 is 400m. When the length of SRS L_d is 5000m, the length of first stage L_1 is 3000m and the length of second stage L_2 is 2000m. The Young's modulus of SRS E_1 as well as E_2 is 2.1×10^{11} Pa; the density of rod ρ1 as well as ρ_2 is 7850 kg/m^3; the diameter of first stage D_1 is 22mm and the diameter of second stage D_2 is 19mm.

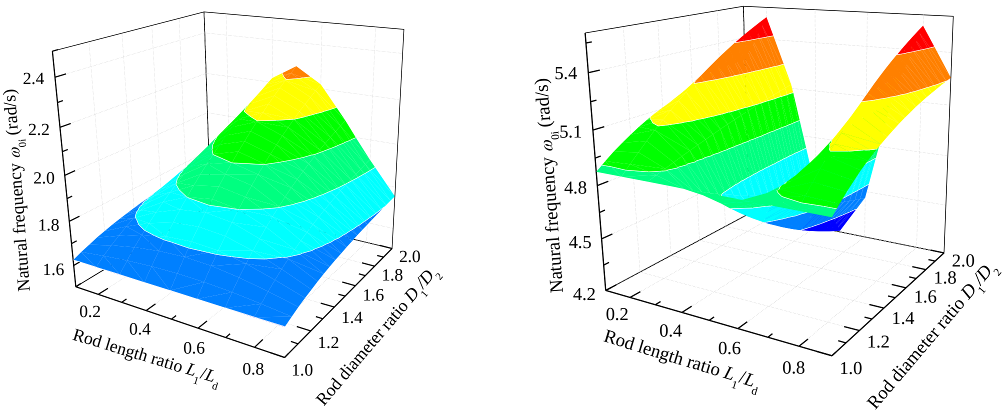
Table 1. The comparison of natural frequency

older	Theoretical	Current model		New model	
	rad/s	rad/s	Error %	rad/s	Error %
1	8.1	8.1	0	8.1	0
2	24.4	24.4	0	24.4	0
3	40.6	56.9	16.3	40.6	0
4	56.9	67.7	10.8	56.9	0
5	73.1	89.2	16.1	73.1	0
6	89.4	138.1	48.7	89.4	0

When L_d =1000m, L_1=L_2, two stages SRS is transformed into one stages SRS. Based on the current model, new model and the analytic expression of natural frequency, the six natural frequencies are obtained, in Tab.1. The table shows that there is a large error in results of current model, but the simulation results of the new model are found to be in good agreement with the simulation results of theoretical value. Therefore, the new model of natural frequency in the paper is advised to use in actual engineering.

When the elasticity modulus of SRS E_1=E_2=2.1×10^{11} Pa, the density of SRS ρ_1=ρ_2=7850kg/m^3, the length of SRS L_d=5000m, the diameter of SRS D_1=0.022m. The combination ratios of length and diameter are changed and three dimensional curves of natural frequency are obtained, in Fig.2.

As you can see in the above figures, the first natural frequency shows large changes when the diameter ratio of SRS is larger than 1.5 or the length ration of SRS is an interval value 0.3-0.7. The first natural frequency is increasing with the diameter ratio increasing. With the length ratio increasing, the first natural frequency is first increasing and then decreasing. With second natural frequency, the length ratio is major factor of natural frequency.

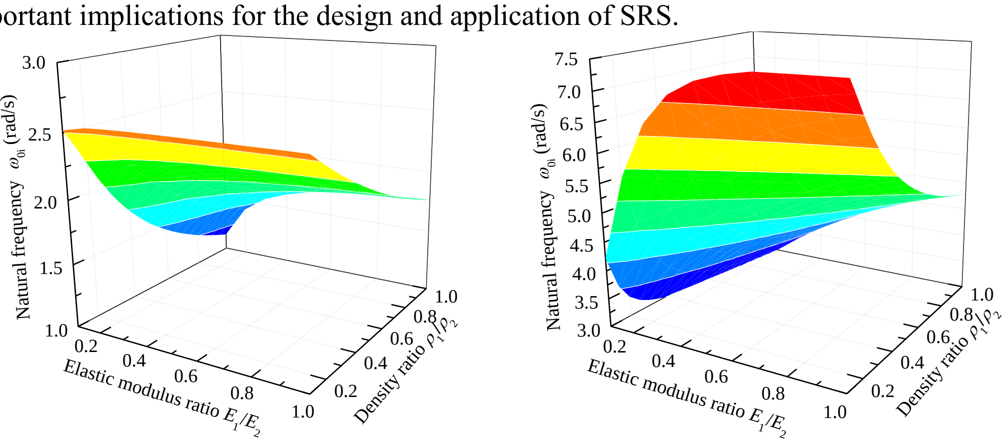


(a) First natural frequency (b) Second natural frequency

Figure 2. The natural frequency of SRS with length and diameter

When the elasticity modulus of SRS E_1 =2.1×10^{11} Pa, the density of SRS ρ_1=7850kg/m^3, the length of SRS L_1=3000m, L_2=2000m, the diameter of SRS D_1=0.022m, D_2=0.019m. The combination ratios of elasticity modulus and density are changed and three dimensional curves of natural frequency are obtained, in Fig.3.

In Fig.3, with elasticity modulus ratio decreasing and density ratio increasing, the natural frequency of SRS is increasing. When the elasticity modulus ratio is smaller areas, the density ratio has a significant impact on natural frequency. When the density ratio is bigger areas, the elasticity modulus ratio has a significant impact on natural frequency. Therefore, the above three-dimensional curves have important implications for the design and application of SRS.



(a) First natural frequency (b) Second natural frequency

Figure 3. The natural frequency of SRS with elasticity modulus and density

5. Conclusions

Considering the influence of SRA on longitudinal vibration of SRS, a simulation model of MSRWA is built. A computational model of natural frequency is corrected against the deficiency of current model. With the free vibration of SRS, a computational model of natural frequency is corrected against the deficiency of current model. With the forced vibration of MSRWA, a mode superposition method is used to find the numerical solution. The affecting factors of natural frequency are analyzed, such as the length, diameter, density and elastic modulus of SRS. Then three dimensional curves of natural frequency are obtained. The above conclusions are very important to the design and application of SRA in practical applications.

6. References

[1] Liu X.F, Qi Y.G. A Modern Approach to the Selection of Sucker Rod Pumping Systems in CBM Wells. Journal of Petroleum Science and Engineering, Vol.76, pp. 100–108, 2011.
[2] Xing Mingming, Dong Shimin. A new simulation model of beam pumping system applied in energy saving and resource consumption reduction. SPE Production & Operations, Vol. 30, pp.130-140, 2015.
[3] Shardakov I.N, Wasserman I.N. Numerical modelling of longitudinal vibrations of a sucker rod string, Journal of Sound and Vibration, Vol. 329, pp. 317-327, 2010.
[4] Gibbs S.G. Predicting the Behavior of Sucker-Rod Pumping System. Journal of Petroleum Technology, Vol. 15 , pp.769-778, 1963.
[5] Gibbs S.G., Neely A.B. Computer Diagnosis of Down-Hole Conditions in Sucker Rod Pumping Wells. SPE Journal Paper, 1165-PA, 1966.
[6] Lu, X, Vandiver, J. K. Damping and Natural Frequency Estimation Using the Least Pth Optimization Technique, Offshore Technology Conference, Houston, Texas, OTC-4283-MS, 1982.
[7] Dong S, Ma D, Huang X. The resonance characteristic and resonance condition of sucker rod string. China Petroleum Machinery, Vol. 29, pp. 22-24. 2001.
[8] Peng Y, Yan W, Wang L. Analysis of Twisting Vibration Eigenfrequencies of Rotary Sucker-rod String. Journal of Xi'an Petroleum Institute, Vol. 18, pp. 57-59, 2003.
[9] Zhu B. The Analysis and Calculation of the Inherent Frequency of Longitudinal Vibration of Rod String. Oil Field Equipment, Vol. 36, pp. 11-14, 2007.
[10] Chen J, Qi Y, Liu X, Liu C. Study on 3-D Vibrations of Sucker Rod String Based on the Elastic Body Vibration Theory. Oil Field Equipment, Vol. 38, pp. 15-19, 2009.
[11] Fernando C, María J. E. Longitudinal vibration of a damped rod. Part I: Complex natural frequencies and mode shapes. International Journal of Mechanical Sciences, Vol.48, pp. 969-975, 2006.
[12] Xing M, Dong S, Tong Z, Tian R, Chen H. Dynamic simulation and efficiency analysis of beam pumping system. Journal of Central South University, Vol.22, pp. 3367-3375, 2015,
[13] Takacs G. and Belhaj H. Latest Technological Advances in Rod Pumping Allow Achieving Efficiencies Higher than with ESP System. Journal of Canadian Petroleum Technology, Vol. 50, pp. 53-58, 2011.

Acknowledgments

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